$\bar{p}p$ Annihilations at Rest into $\pi^+\pi^-\pi^0$†

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The reaction $\bar{p}p \to \pi^+\pi^-\pi^0$ was studied for antiprotons stopped in the Columbia-Brookhaven National Laboratory 30-in. bubble chamber. For this reaction, 1193 events were selected after applying stringent criteria to reduce the contamination of the sample. Analysis of the distribution of events on the Dalitz plot shows a $\rho\pi$ contribution in addition to the dominant 3π contribution from the ${}^1S_0$ $\bar{p}p$ state. The annihilation from the ${}^3S_1$ $\bar{p}p$ state has contributions from both a $\rho\pi$ and 3π final states.

INTRODUCTION

THE annihilation of stopped antiprotons in hydrogen has been the subject of considerable investigation, both experimental[1–3] and theoretical.[4] The annihilation at rest has been shown experimentally to take place principally from an S state of the $\bar{p}p$ system.[5]

We report here results obtained in an analysis of the annihilation $\bar{p}+p \to \pi^+\pi^-\pi^0$. The data were obtained by analyzing pictures taken of the Columbia-BNL 30-in. liquid-hydrogen bubble chamber exposed to a stopping $\bar{p}$ beam. A fraction of these pictures have been analyzed previously.[3] The results presented here are based on measurements of events made at the University of Chicago. These events did not form part of the earlier sample.

The object of this experiment was to produce a sample of events of the kind $\bar{p}p \to \pi^+\pi^-\pi^0$ larger than and containing less background than earlier experiments. The event selection was similar to that performed earlier[3] with the following modifications:

(1) The χ^2 limit on the kinematic reconstruction (Grind fit) of an event was 4 in this experiment (previously it was 6).

(2) All measured two-prong events were fitted to the hypothesis $\bar{p}+p \to \pi^+\pi^-\eta$. Events which fitted this hypothesis were excluded from the sample of $\pi^+\pi^-\pi^0$ events. This criterion certainly rejects some $\pi^+\pi^-\pi^0$ events but not in such a way as to bias the distribution of events on the Dalitz plot. The advantage is that events with poorly determined momenta are eliminated from the analysis. Since poorly determined events from other reactions are more likely to fit the incorrect hypothesis of $\bar{p}+p \to \pi^+ + \pi^- + \pi^0$ than well-measured events, this criterion reduces the percentage of other reactions plotted on the Dalitz plot.

(3) A histogram of events versus the cosine of the angle θ between the incident $\bar{p}$ and the sum of the measured π^+ and π^- momenta from events that fit the reaction $\bar{p}p \to \pi^+ + \pi^- + \pi^0$ showed a deviation from isotropy with an excess of events with $\cos\theta > 0.99$. (The excess is presumably due to the reaction $\bar{p}p \to \pi^+\pi^-$ in flight.) To eliminate this background, no events with $\cos\theta > 0.99$ were accepted.

The experimental results are shown in Fig. 1 in terms of normalized Dalitz plot variable $X = (T_+ - T_-)/\sqrt{3}Q$ and $Y = T_0/Q$, where T_+, T_-, T_0 are the kinetic energies of the pions and $Q = 2m_p - m_{\pi^0} - 2m_{\pi^+}$. Projections of the data onto the E_{π^+}, E_{π^-}, and E_{π^0} axes are shown in Fig. 2. Charge conjugation invariance in the annihilation requires that the energy distribution of the π^+ and the π^- be the same. Applying a χ^2 test to the data, we find an 84% probability that the distributions are the same. The same χ^2 test when applied to the π^0 energy spectrum and the average of the π^+ and π^- spectra gives only a 0.2% probability of the distributions being the same. The π^0 energy spectrum is seen

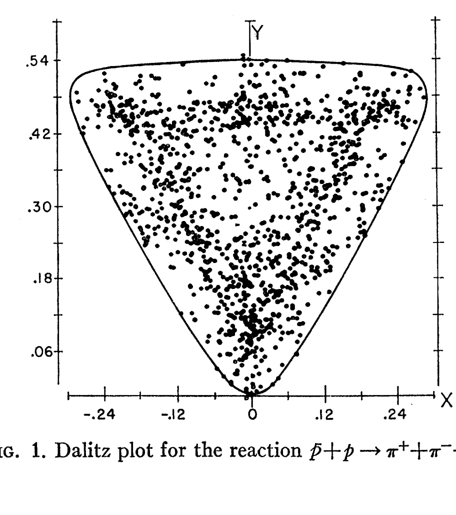

FIG. 1. Dalitz plot for the reaction $\bar{p}+p \to \pi^+ + \pi^- + \pi^0$.

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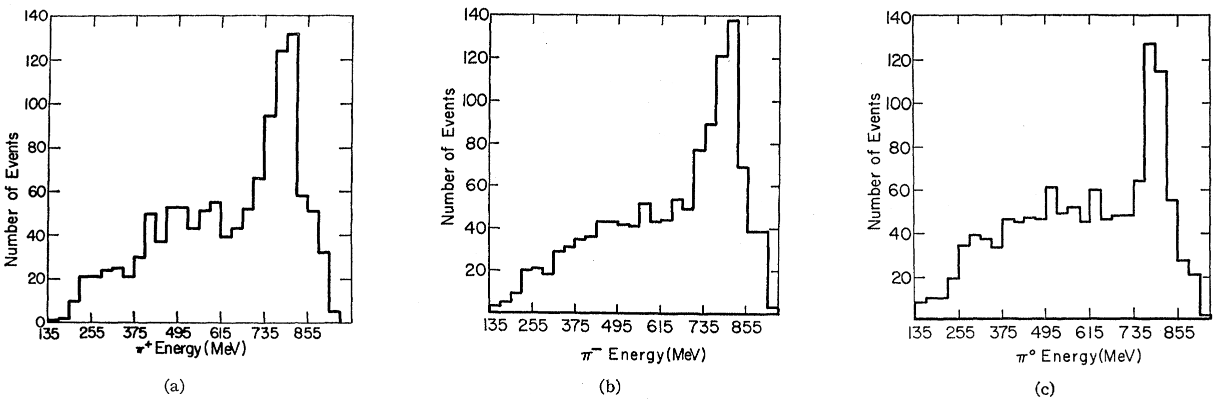

FIG. 2. (a) π^+ energy distribution, (b) π^- energy distribution, and (c) π^0 energy distribution.

to be significantly different from the π^+ or π^- energy spectra.

It is known that the annihilation of stopped antiprotons in hydrogen takes place from the S state in the $\bar{p}p$ system.[5] Since protons are fermions this means that the parity of the initial $\bar{p}p$ system and hence of the final three-pion system is negative. The angular momentum J of the system is just the spin s of the $\bar{p}p$ state; hence $J=0$ for the singlet state, and $J=1$ for the triplet state. The charge conjugation C is $(-1)^{L+s}$ and is therefore $+1$ when $J=0$ and -1 when $J=1$. The $\bar{p}p$ state is a coherent superposition of states with definite isotopic spin. Each of these component states has G parity equal to $C\times(-1)^T$. Since the final state of three pions has $G=-1$ it can come only from the ${}^1S_0$, $T=1$ and ${}^3S_1$, $T=0$ components of the $\bar{p}p$ system.

In the analysis, we attempt to find the amplitude for the annihilation of $\bar{p}+p$ into three pions from the experimental distribution of energies on the Dalitz plot. The total amplitude is the incoherent sum of the annihilation amplitudes, A_1 from the ${}^1S_0$ and A_3 from the ${}^3S_1$ $\bar{p}p$ states.

ANALYSIS

The amplitudes A_1 and A_3 are functions of the momenta and charge coordinates of the pions and the spin coordinates of the $\bar{p}$ and the p. Since no measurement is made of these spin coordinates, the observed amplitudes are functions only of the pion variables. The spatial properties and isotopic spin values of these functions are the same as for the initial $\bar{p}p$ state, i.e., the function of the pion variables appropriate to annihilation from the ${}^1S_0$ $\bar{p}p$ state must be a pseudoscalar with $T=1$ while the function appropriate to the annihilation from the ${}^3S_1$ $\bar{p}p$ state is a vector function with $T=0$.

The construction of these functions is facilitated if we consider the final three-pion state as composed of a dipion system and the third pion (Fig. 3). Taking this point of view, we let l represent the orbital angular momentum of the dipion and L the relative orbital angular momentum of the dipion with respect to the third pion. Angular momentum conservation implies: $\mathbf{l}+\mathbf{L}=\mathbf{J}$, where $J=0$ for ${}^1S_0$ annihilation and $J=1$ for ${}^3S_1$ annihilation. Parity conservation requires $(-1)^{l+L}=1$. Together the two conditions require $l=L$ for both ${}^1S_0$ and ${}^3S_1$ annihilation. It is therefore natural to expand the amplitudes A_1 and A_3 in a power series in L, i.e.,

$$A_i=\sum_{L=L_{\min}} C_i^L A_i^L\,,$$

where $i=1$ or 3, $L_{\min}=0$ for $i=1$, and $L_{\min}=1$ for $i=3$.

The isotopic wave function for three pions with $T=0$ is totally antisymmetric in the isotopic spin coordinates of the three pions. Bose symmetry then requires that under the interchange of the momenta of the pions the amplitude A_3 be antisymmetric. For $T=1$, the three-pion isotopic spin-wave function either is symmetric under interchange of the pions or is of mixed symmetry under interchange of the pion momenta.

We have analyzed our data in terms of the following functions for the amplitudes:

$$A_1{}^0=e^{i\phi},\quad \phi=\alpha+\beta p_{\pi^0}\,,$$
$$A_1{}^1=(\mathbf{P}_{+0}\cdot\mathbf{p}_- e^{i\delta_{+0}}\sin\delta_{+0}-\mathbf{P}_{0-}\cdot\mathbf{p}_+ e^{i\delta_{-0}}\sin\delta_{-0})\,,$$

where[6]

$$\mathbf{P}_{ij}=(\mathbf{p}_i-\mathbf{p}_j)+\mathbf{p}_k(E_i-E_j)/(E_i+E_j+m_{ij})\,,$$
$$\tan\delta_{ij}=\Gamma(m_{ij})/2(m_{ij}-m_\rho)\,,$$
$$\Gamma(m_{ij})=\Gamma_\rho k^3(m_{ij})/[1+k^2(m_{ij})]\,,$$

and

$$k(m_{ij})=[(\tfrac{1}{4}m_{ij}{}^2-m_\pi{}^2)^{1/2}]/m_\rho\,.$$
$$A_3{}^1(\rho\pi)=\mathbf{p}_+\times\mathbf{p}_-[e^{i\delta_{+0}}\sin\delta_{+0}+e^{i\delta_{-0}}\sin\delta_{-0}+e^{i\delta_{+-}}\sin\delta_{+-}]\,,$$
$$A_3{}^1(3\pi)=\mathbf{p}_+\times\mathbf{p}_-[e^{i\delta_{+0}}\cos\delta_{+0}+e^{i\delta_{-0}}\cos\delta_{-0}+e^{i\delta_{+-}}\cos\delta_{+-}]\,,$$
$$A_3{}^1=A_3{}^1(\rho\pi)+C_3A_3{}^1(3\pi)\,,$$

[6] B. Barsella and E. Fabri, Phys. Rev. **126**, 1561 (1961).

where $\mathbf{p}_+$, $\mathbf{p}_-$, and $\mathbf{p}_0$ are the momenta of the π^+, π^-, and π^0 in the $\bar{p}p$ rest system.

The Dalitz-plot densities resulting from the presence of symmetric functions A_1 or the antisymmetric functions in A_3 in the annihilation will not result in any difference in the energy distributions between charged and neutral pions. The observed difference in the energy distributions requires some contribution from the states of mixed symmetry in A_1. The amplitude A_1^0 was constructed as a symmetric state in agreement with what was done in earlier analyses[1–3] and as is suggested from τ decay. The amplitude A_1^1 must belong to a state of mixed symmetry if C is equal to $+1$. Since the dipion system has $l=1$ and $T=1$ it is natural to assume that the dipion system forms a ρ meson. Since the three-pion system has $T=1$ only charged ρ mesons can be produced. The relative phase of A_1^0 and A_1^1 is not known *a priori*. There is no reason to expect that it will remain constant over the Dalitz plot since it certainly involves final-state $\pi\pi$ interactions. A convenient though by no means unique choice of this relative phase is $\phi=\alpha+\beta p_0$. With this choice of phase, A_1^0 satisfies all our conditions for the amplitude.

All the amplitudes could, in principle, be multiplied by arbitrary symmetric scalar functions of the momenta. The functions A_1^L could also be multiplied by scalar functions of P_0 and $|\mathbf{P}_+-\mathbf{P}_-|$. Except for the relative phase of A_1^0 and A_1^1 these functions are all taken to be constants.

The variation of the ρ width, Γ_ρ, with M_ρ is that suggested in Blatt and Weisskopf.[7] It corresponds to an elastic p-wave resonance without Coulomb corrections. The radius of interaction which appears in the expression for the width was chosen to be $\hbar/M_\rho c$ since the ρ is thought to dominate the p-wave $\pi\pi$ interaction.

The form of the background contribution in A_3^1 is suggested by noticing that time reversal and unitarity require that when a $\pi\pi$ combination has the mass of the ρ meson it must form a ρ. Since the ρ is a completely elastic resonance, this saturates the two-particle unitarity and requires that the background vanish when two of the pions have a mass equal to m_ρ.[8]

In fitting the experimental distributions the mass and width of the ρ may be considered either as free parameters or as constrained by previous measurement. The apparent position of the ρ peak $\sim$730 MeV/c^2 is very far from any currently accepted value of the ρ mass. The shift in position must be due to the background term proportional to C_3. Since there is no previous knowledge about C_3 it is unlikely that a meaningful value of m_ρ could be found since it will be strongly correlated to the value found for C_3.

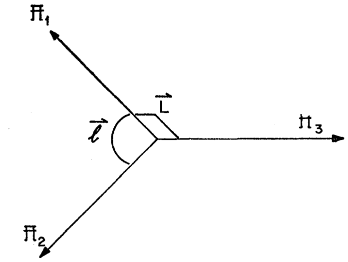

FIG. 3. Angular-momentum description of the three-pion system.

The difficulty in the second approach is that the analyses of the ρ do not give consistent values for its mass and width. The analysis of Rosenfeld *et al.*[9] finds that both charged and neutral ρ mesons have a mass $m_\rho=760$ MeV/c^2 and a width $\Gamma_\rho=134$ MeV/c^2. Roos,[10] using a different analysis procedure, finds for the charged ρ a mass $m_\rho=774$ MeV/c^2 and for the neutral ρ a mass $m_\rho=780$ MeV/c^2. In both cases he finds $\Gamma_\rho=128$ MeV/c^2. We have tried to find fits with the charged ρ mass different from the neutral ρ mass. All the fits found have very similar masses for the charged and neutral ρ mesons. This experiment cannot support Roos's evidence for different values for the mass of the charged and neutral ρ.

Since we do not know how to choose between the values of Rosenfeld *et al.*,[9] Roos,[10] or Auslander *et al.*[11] (from a colliding beam experiment), we have searched for fits with the ρ mass and width fixed at the values given in these different analyses.

The fit was made simultaneously to the π^0 energy spectrum and the average of the π^+ and π^- energy

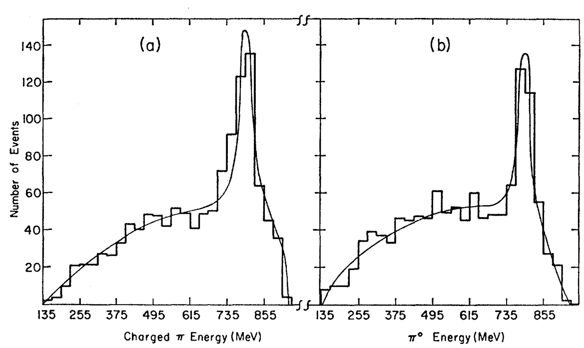

FIG. 4. (a) Charged-pion energy distribution with the curve representing the fit; (b) π^0 energy distribution with the curve representing the fit.

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[11] V. L. Auslander, G. I. Budker, Ju. N. Pestov, V. A. Sedorov, A. N. Skrimsky, and A. G. Kabakpasheve, Phys. Letters **25B**, 429 (1967).

spectrum with the following function:

$$|A|^2=a_1|A_1{}^0+C_1A_1{}^1|^2+a_3|A_3{}^1|^2\,,$$

with a_1, a_3, C_1, C_3, α, and β as free parameters to be determined.

The fits are:

$m_\rho=764$ MeV/c^2, $\Gamma_\rho=93$ MeV/c^2, $\chi^2=50$ (48 d.f.),
$a_1=0.52\pm0.02$, $a_3=0.48\pm0.02$,
$C_1=-0.28\pm0.05$, $C_3=0.33\pm0.01$,
$\alpha=0.28\pm0.03$ rad,
$\beta=(1.9\pm0.1)\times10^{-3}$ rad/(MeV/c).

$m_\rho=760$ MeV/c^2, $\Gamma_\rho=134$ MeV/c^2, $\chi^2=40$ (48 d.f.),
$a_1=0.41\pm0.02$, $a_3=0.59\pm0.02$,
$C_1=-0.50\pm0.03$, $C_3=0.44\pm0.01$,
$\alpha=0.16\pm0.02$ rad,
$\beta=(1.7\pm0.1)\times10^{-3}$ rad/(MeV/c)

$m_\rho=766$ MeV/c^2, $\Gamma_\rho=128$ MeV/c^2, $\chi^2=45$ (48 d.f.),
$a_1=0.46\pm0.02$, $a_3=0.54\pm0.02$,
$C_1=-0.38\pm0.04$, $C_3=0.45\pm0.01$,
$\alpha=0.13\pm0.09$ rad,
$\beta=(1.86\pm0.02)\times10^{-3}$ rad/(MeV/c).

(The normalization chosen is $|A|^2=|A_1{}^0|^2=|A_1{}^1|^2=|A_3{}^1|^2=1$.) The fit is shown in Fig. 4, with $m_\rho=764$ MeV/c^2 and $\Gamma_\rho=93$ MeV/c^2.[10]

CONCLUSIONS

The principal result of this experiment is that a fit has been obtained to the experimental distribution with the functions we have assumed for the amplitudes for all three values of m_ρ and Γ_ρ. The fit differs from earlier ones[1–3] in the large contribution from the three-pion (as distinguished from the $\rho\pi$) final state in the amplitude A_3. The effect of this term is to shift the peak in the $\pi\pi$ mass spectrum due to the ρ from the ρ mass to a lower value. The large value of the coefficient C_1, corresponding to $\rho\pi$ production from the ${}^1S_0$ state, is another new feature of our analysis. It is this term that produces the difference between the charged and neutral pion energy distributions.

The fit we have found is almost certainly not unique. We cannot rule out the possibility of a nonresonant 3π contribution to $A_1{}^1$. A possible σ or ϵ meson, while not necessary to obtain a fit to the data, also cannot be ruled out. The freedom available in constructing A_1 makes it seem reasonable that fits could be found which incorporate the σ or the ϵ. We have not constructed such an A_1 since we have found a solution for A_1 which fits the data and seems to have very desirable properties. In particular, for $A_1{}^0$ the three pions are in a symmetric state. In the decay of the η meson or in τ decay the symmetric three-pion state dominates. We might expect it to dominate also at these higher energies and a fit consistent with this assumption is found.

ACKNOWLEDGMENTS

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