Table I. The differences in Z/A and E/A between fluorine and bromine indicate that they are interesting materials to compare. The results of this experiment, given above, indicate that the difference between the ratios of active to passive mass for bromine and fluorine is less than 5×10^{-5}.

III. SUMMARY AND CONCLUSIONS

This experimental technique of comparing the field produced by a homogeneous solid with the field of the fluid which it displaces has made it possible to measure an upper bound for $\Delta m/m$ which is smaller than any value which may be deduced from previous experiments. Although it is difficult to evaluate the possible sources of error in previous experiments and to deduce an upper bound for $\Delta m/m$, the scatter in values for G between various experiments makes it unreasonable to set this upper bound smaller than 10^{-3}. The present experimental result of 5×10^{-5} for an upper bound between fluorine and bromine is both a significant numerical improvement and also a more reliable estimate because it results from a direct measurement of the effect. Improvement by one and possibly two orders of magnitude should be possible by careful application of currently known experimental techniques. To improve the accuracy beyond that point would be very difficult and might require a completely different type of experiment. The present experimental technique would be severely limited by problems of measuring and controlling the temperature of the liquid and solid, by gravitational gradients caused by inhomogeneities in the solid, by noise generated in the balance by thermal effects and ground noise, and by the difficulty of measuring such small density differences.

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Validity of Special Relativity at Small Distances and the Velocity Dependence of the Charged-Pion Lifetime

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The velocity dependence of the lifetime of the pion is investigated under the assumption that the Hamiltonian contains a spatial form factor which in the lab frame (the frame at rest with respect to the neighboring macroscopic bodies) vanishes for distances larger than some length α. In this model, there is a violation of the principles of special relativity at small distances. In particular, space-time is anisotropic at distances smaller than α. The lifetime of the pion is calculated to second order in α, and it is shown that there will be about 1% deviation from the usual formula $\tau(v)=(1-v^2/c^2)^{-1/2}\tau(0)$, (which holds if special relativity is valid at arbitrarily small distances) if, e.g., the pion energy $E_\pi=10^4$ MeV and $\alpha\sim5\times10^{-16}$ cm. The measurement of the velocity dependence of the pion lifetime at high energies could thus serve as a possible check on the validity of special relativity at small distances. The deviation is of the same order of magnitude as that previously obtained for the muon decay.

IN a recent article by one of us,[1] the velocity dependence of the muon mean lifetime was investigated under the assumption that the principles of special relativity are violated for small distances. More specifically, it was assumed that the weak-interaction Hamiltonian contained a noncausal form factor which permitted interaction between simultaneous space-time events in the lab frame provided their spatial distance is less than some fundamental distance α. In this type of violation of relativity, the lab frame constitutes a preferred frame.[2] It was then shown in Ref. 1 that there will be a detectable deviation from the usual formula

$$\tau(v)=(1-v^2/c^2)^{-1/2}\tau_0 \qquad (1)$$

if, e.g., one assumes 1% accuracy on the measurement of the lifetime $\tau(v)$ and muon energy $E\sim10^4$ MeV provided α is not less 7×10^{-16} cm. An accurate high-energy measurement of the time-dilation formula for the decay of an unstable particle could thus serve as a possible check on the validity of special relativity in high-energy physics.

Subsequently, it was pointed out to the authors[3] that it might be easier to measure the velocity dependence

[1] L. B. Rédei, Phys. Rev. **162**, 1299 (1967).
[2] D. I. Blokhintsev, Phys. Letters **12**, 272 (1964); L. B. Rédei, Phys. Rev. **145**, 999 (1966).

[3] T. Alväger (private communication).

of the charged pion at high energies than that of the muon because of the relatively long lifetime of the muon. The purpose of this short note is to sketch the derivation and state the results of the calculations for the π^+ decay under the assumption that there is a breaking of causality at distances smaller than α.

We consider the decay

$$\pi^+ \to \mu^+ + \nu. \tag{2}$$

It is assumed that the decay goes through the virtual process[4]

$$\pi^+ \to \bar{N} + N \to \mu^+ + \nu.$$

The virtual decay of π^+ into $\bar{N}+N$ proceeds through strong interaction and we assume that there is no violation of relativity in the corresponding matrix element. This assumption agrees with the usual hypothesis that the stronger interaction as a rule has higher symmetry then the weaker one.

From this it follows that the transition amplitude for the pion decay can be written as

$$\langle \mu^+,\nu | S | \pi^+ \rangle = \sum_{N,\bar{N}} \int d^4x \langle \mu^+,\nu | H_{\text{int}}(x) | N,\bar{N} \rangle \times \langle N,\bar{N} | X | \pi \rangle, \tag{3}$$

where $H_{\text{int}}(x)$ is the weak-interaction Hamilton density and $\langle N,\bar{N} | X | \pi \rangle$ is some unknown Lorentz-invariant amplitude for the virtual strong transition. Following Ref. 1 we assume that $H_{\text{int}}(x)$ is noncausal; more specifically, we put

$$\int H_{\text{int}}(x) d^4x = \tfrac{1}{4}\sqrt{2}\, g \int d^4x d^4y\, \bar{\Psi}_N(x)(1-\gamma_5)\gamma_\lambda(1+\gamma_5) \times \Psi_N(x) F(x-y) \bar{\Psi}_\nu(y)(1-\gamma_5)\gamma_\lambda(1+\gamma_5)\Psi_\mu(y), \tag{4}$$

where

$$F(x) = (\tfrac{4}{3}\pi\alpha^3)^{-1}\delta(nx)\Theta(\alpha^2 - x^2 + (nx)^2). \tag{5}$$

In this expression, α is some fundamental length below which causality is violated, Θ is a step function, and n a timelike unit vector characteristic of the frame [in the lab system $n = (\mathbf{0}, i)$]. In the lab system

$$F(x) = (\tfrac{4}{3}\pi\alpha^3)^{-1}\delta(x_0)\Theta(\alpha^2 - |\mathbf{x}|^2), \tag{6}$$

which shows that in this model causality is violated for $|\mathbf{x}|^2 < \alpha^2$. In the limit $\alpha \to 0$, $F(x) \to \delta(x)$ and the Hamiltonian defined by (4) reduces to the customary local Hamiltonian. For details see Ref. 1. Applying the usual technique for evaluating the S-matrix element, one obtains after some straightforward algebra

$$\langle \mu^+,\bar{\nu} | S | \pi^+ \rangle = (m/E_\pi)^{1/2}\delta(p_\mu + p_\nu - q_\pi) G(\mathbf{q}) C \times \bar{u}(\mathbf{p}_\nu)(1-\gamma_5) q_\mu \gamma_\mu v(\mathbf{p}_\mu)/(E_\mu E_\nu)^{1/2}, \tag{7}$$

where $\mathbf{q}$ is the pion momentum, $G(\mathbf{q})$ is the Fourier transform of $F(x)$, and C is some constant. Squaring the matrix element and summing over all decay states, one obtains the decay rate

$$dW/dt = (m/E_\pi)\tau_0^{-1} |G(\mathbf{q})|^2 \tag{8}$$

or the lifetime

$$\tau(v) = (1-v^2)^{-1/2}\tau_0[|G(q)|^2], \tag{9}$$

where the units used are such that $\hbar = c = 1$. The Fourier transform $G(q)$ of Eq. (6) is given by

$$G(\mathbf{q}) = 1 - \tfrac{1}{10}\alpha^2 |\mathbf{q}|^2 \tag{10}$$

to second order in α, which gives

$$\tau = (1-v^2)^{-1/2}\tau_0(1 + \tfrac{1}{5}\alpha^2 |\mathbf{q}|^2). \tag{11}$$

In the high-energy limit $v \sim c$ this becomes

$$\tau = (1-v^2/c^2)^{-1/2}\tau_0[1 + \tfrac{1}{5}(m_\pi c/\hbar)^2 (E_\pi/m_\pi c^2)^2 \alpha^2] \tag{12}$$

in cgs units. Substituting the numerical values for the constants in Eq. (12), we obtain finally

$$\tau = (1-v^2/c^2)^{-1/2}\tau_0[1 + 10^{25}(E_\pi/m_\pi c^2)^2 \alpha^2] \tag{13}$$

(α in cm).

Assuming, e.g., an accuracy of 1% on the measurement of the lifetime $\tau(v)$ and $E_\pi \sim 10^4$ MeV, Eq. (13) predicts an observable deviation from the usual formula

$$\tau = (1-v^2/c^2)^{-1/2}\tau_0$$

if $\alpha > 4.6 \times 10^{-16}$ cm.

[4] See e.g., G. Källén, *Elementary Particle Physics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1964), Chap. 15.