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Experimental Measurement of the Equivalence of Active and Passive Gravitational Mass*

L. B. KREUZER†

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

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A new type of gravitational experiment is reported, and a simple theoretical treatment in terms of Bondi's concepts of active, passive, and inertial mass is used to compare the results with other experiments. A homogeneous Teflon cylinder, which is completely immersed in a mixture of dibromomethane and trichloroethylene prepared to have about the same density as Teflon, was slowly moved back and forth in this liquid. The resultant time-varying gravitational field due to the density difference between the solid Teflon and the displaced liquid was detected by a Cavendish-type torsion balance placed outside of the liquid container. Buoyant forces on the Teflon were measured. The time-varying gravitational field detected by the balance was extrapolated to the condition of neutral buoyancy. The fractional density difference between the Teflon and the liquid required to produce this field was found to be $\Delta\rho/\rho=(1.2\pm4.4)\times10^{-5}$. This upper bound of approximately 5×10^{-5} is compared to 10^{-3}, the best value that can be deduced from other experiments.

INTRODUCTION

THIS paper is divided into three parts. The first describes a gravitational experiment, and the second presents a simple theoretical discussion, which is used to compare this experiment with previously published experiments. The third part presents a summary and conclusion. The work is based on the author's Ph.D. thesis.[1]

I. EXPERIMENTAL

A. Apparatus

A Cavendish-type torsion balance with a beam, suspended by a 2-mil-diam and 20-cm-long tungsten wire, consisting of two pure aluminum cylinders 1.18 in. long and 1.57 in. in diameter, separated by about 10 in., was constructed. The fiber torsion constant was measured to be approximately 1 dyn cm/rad. Pairs of electrodes, with a bias voltage across them, were mounted close to each side of the torsion beam near its ends. They produced an electrostatic torque which was linear in the voltage applied between the electrodes and the torsion beam. A servo used this electrostatic torque to balance other torques on the balance and to prevent beam rotation. This was accomplished by connecting the output of the photoelectric optical lever, which detected beam rotation through appropriate amplifiers and filters, to the electrodes. When the servo was operating, the dynamical properties of the balance were determined, for the most part, by the electrical properties of the servo system rather than by the mechanical properties of the balance suspension. External torques were measured by the size of the electrostatic torque needed to balance their effect on the beam.

The torsion balance was enclosed in a sealed aluminum container which contained air at atmospheric pressure. Operating the torsion balance in a high vacuum could have eliminated unwanted torques produced by thermally generated convection currents in the air surrounding the beam, but it seemed that this advantage would be completely offset by the deleterious effect of removing the air damping of the vibrational modes of the balance. These modes are driven by ground vibrations and, although the torsional motion of the beam is rather insensitive to vibrations, it is driven by nonlinear coupling to other vibrating modes. The air in the balance container reduces the

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† Present address: Bell Telephone Laboratories, Murray Hill, N. J.

[1] L. Kreuzer, Ph.D. thesis, Department of Physics, Princeton University, 1966 (unpublished).

coupling of ground vibrations to the torsional motion of the beam by damping these vibrating modes. Temperature gradients which cause convection currents were insulated against by surrounding the beam with three cylindrical, air-spaced aluminum containers.

A tank completely full of a mixture of trichloroethylene and dibromomethane, in which a Teflon[2] cylinder was immersed, was placed next to the balance. The cylinder of Teflon, which had a diameter of 9 in., a length of 9 in., and weighed about 20 kg was attached by fine nylon strings to a motor drive. This was arranged so that the motor could move the cylinder between the ends of the tank. Vertical motion of the cylinder due to buoyant forces was normally restricted by tension in these strings. The size and direction of these bouyant forces was measured by reducing this tension by a fixed amount and noting the direction and extent of the cylinder's vertical motion. The tank which was insulated by a 1-in. thickness of fiber glass and aluminum foil building insulation had a number of thermistors, which were used to measure the temperature of the tank and its contents, fastened in thermal contact to its walls. A uniform winding of electrical resistance wire around the tank could provide heat for temperature control of the tank and its contents.

The proximity of the tank to the balance, as shown in Fig. 1, caused the center of mass of the cylinder, which was periodically moved from one end of the tank to the other, first to be near one end of the torsion beam and then to be near the other. Gravitational attraction between the balance beam and the cylinder then produces a periodic torque on the beam. Since the liquid which is displaced by the immersed cylinder also causes a torque on the beam, the net torque on the beam is proportional to the difference in density between the Teflon and the liquid.

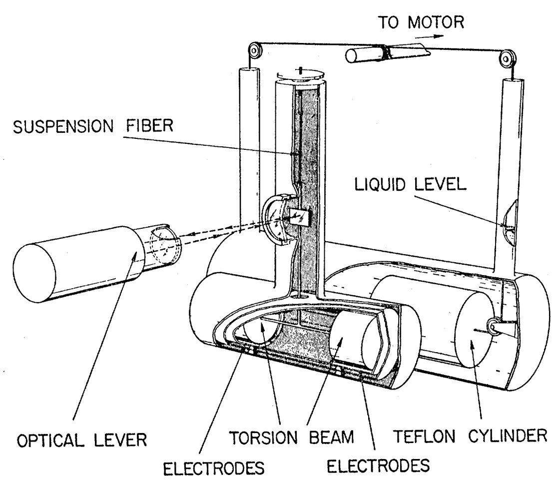

FIG. 1. Cut-away view of tank, Teflon cylinder, and torsion balance.

[2] Trade name for plastic manufactured by E. I. DuPont de Nemours and Company, Inc.

This torque was measured by a synchronous detector which correlated the torque signal from the balance with the position of the Teflon cylinder in the tank. Averaging the output of the synchronous detector over a long period of time allowed the detection of very weak signals.

B. Method

The liquid—a mixture of dibromomethane, which has a density greater than Teflon, and trichloroethylene, which has a density less than Teflon—in which the Teflon cylinder was immersed was prepared to have a density very close to that of Teflon. During the experiment the tank which contained this liquid and the torsion balance were placed in a thermally insulated basement room. The tank temperature changed from time to time, owing to changes in room temperature. The different rates of thermal expansion of Teflon

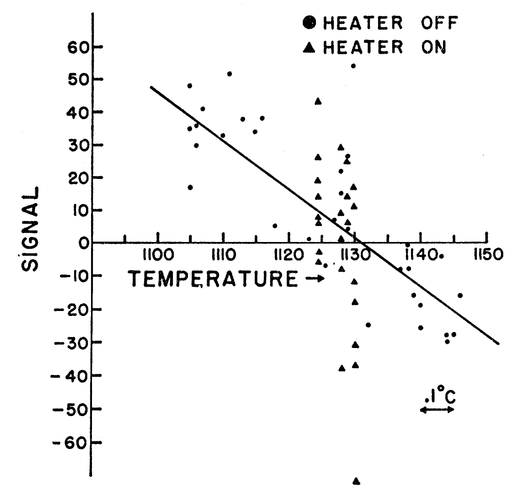

FIG. 2. Torsion balance signal averaged over 3-h periods as a function of liquid temperature.

and the liquid caused the density of the Teflon to be, at different times, both higher and lower than the density of the liquid.

The experimental procedure consisted of measuring synchronous detector output and density difference between the Teflon cylinder and liquid as a function of liquid temperature. The Teflon cylinder was moved back and forth between the ends of the tank with a period of 400 sec. Liquid temperature was recorded continuously on a chart recorder. The density difference between the liquid and the solid was measured twice a day.

Temperature changes of the liquid in which the Teflon cylinder was immersed were measured by recording resistance changes of a thermistor in contact with the liquid. These resistance changes were converted into temperature changes by using a conversion coefficient supplied by the manufacturer. In order to calibrate the experimental results, it was necessary to know the coefficient of thermal expansion of the liquid and that of the Teflon cylinder. The coefficient of expansion of the liquid was measured and that of Teflon was found in the literature.[3]

C. Results

The signal is recorded as a function of liquid temperature in Fig. 2. Each point represents an average of the output of the synchronous detector over a 3-h period. Since it was found that the temperature never changed by more than 0.02°C in any 3-h period, a single temperature has been assigned to each 3-h average. Some of the data were collected with the heater off and the tank in thermal equilibrium with the room, and some were collected with the tank heater on and the tank temperature a few tenths of a degree above room temperature. Although there is some indication of systematic differences between these two groups of data, they have both been retained. The straight line drawn in Fig. 2 is a least-squares fit to the data.

The density difference between the Teflon cylinder and the liquid is plotted as a function of liquid temperature in Fig. 3. The method of measuring density differences, which has been described above, was linear only for small density differences, and for this reason Fig. 3 only contains data taken near neutral bouyancy.

A least-squares fit, of a straight line to the data represented in Fig. 3, gives the temperature of neutral buoyancy to be 1128.7±0.6 Ω. The temperature is given in units of thermistor resistance. The error of ±0.6 Ω is the deviation of the mean of the experimental points from the least-squares fit. The least-squares fit to the signal indicates a signal at this temperature of 3.2±3.4. The units are arbitrary and the error is due to the deviation of the mean about the least-squares fit. This signal may be expressed in terms of an equivalent mass Δm which would have produced this signal if it had been placed in air at the position of the center of the Teflon cylinder and moved back and forth at the same frequency as the Teflon cylinder was moved. Knowledge of the thermal coefficients of volume expansion of the liquid and the Teflon, combined with the slope of the least-squares fit to the data of Fig. 2, provides sufficient information to express the signal at neutral bouyancy in terms of an equivalent mass. If m is the mass of the cylinder, then

$$\Delta m/m=(4.2\pm4.4)\times10^{-5}.$$

The above expression for $\Delta m/m$ contains systematic errors produced by the finite mass of the string used to move the Teflon cylinder. A simple calculation[4] showed that the gravitational attraction between the string

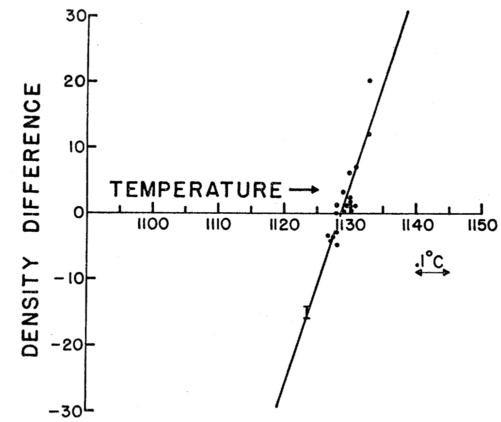

FIG. 3. Density difference between the Teflon cylinder and the liquid as a function of liquid temperature.

and the torsion beam produces an error of

$$\Delta m/m=1.3\times10^{-5}.$$

The difference in density between the nylon string and the Teflon cylinder causes an error in density measurements, which is reflected by an error in the measured temperature of neutral buoyancy. The resultant error introduced into the measured value for $\Delta m/m$ is

$$\Delta m/m=1.7\times10^{-5}.$$

The corrected value of $\Delta m/m$ is then

$$\Delta m/m=(1.2\pm4.4)\times10^{-5}.$$

These experimental results show no detected signal at neutral buoyancy and also set an upper bound to the size of any undetected signal.

A detailed study of the noise sources and possible systematic errors that limited the accuracy of this experiment was not made. It is, however, still possible to make some statements which may be useful in interpreting the results of this experiment and in designing similar types of measurements. The torsion balance was placed on a concrete pier, which went through a hole in the basement floor and penetrated some distance into the ground. Although this did provide some isolation, building vibrations due to machinery and other activities were still detectable on the pier. It seems reasonable to believe that by operating at a location isolated from man-made ground vibrations, a reduction in ground noise by as much as one order of magnitude might be achieved.

Noise was introduced into the measurements by the electrical noise in the photoelectric optical lever and by servo amplifiers. The state of the art of optical lever and electronic amplifier design is such that it would have been possible to use more complex components to achieve lower noise levels.

Mechanical coupling between the tank with the moving Teflon cylinder and the torsion balance could

[3] *Handbook of Chemistry and Physics*, edited by Charles D. Hodgman (Chemical Rubber Publishing Co., Cleveland, Ohio, 1963), 44th ed., p. 1557.

[4] Reference 1, Appendix 3.

have been a source of systematic error. In this experiment this coupling was reduced to a satisfactory level by using a smooth vibration-free drive mechanism and by mechanically isolating the balance and tank by placing the balance on a concrete pier, which penetrated through a hole in the floor on which the tank was supported. It would seem that mechanical isolation will not become a problem until other sources of noise are substantially reduced below the levels of this experiment.

Temperature measurements were made by measuring the resistance of glass bead thermistors with a dc-operated Wheatstone bridge. It is known that these thermistors are very stable and free from drifts, and that more sophisticated means of measuring resistance changes could have increased the precision of the temperature measurements by at least one order of magnitude.

Inhomogeneities in the density of the Teflon cylinder could cause gravitational gradients which would produce systematic errors. This was not a problem in this experiment, and if it became a problem because of the reduction of other sources of noise, the effect could be reduced by rotating the cylinder during the experiment to average out the gradients and by careful preparation of the Teflon to reduce inhomogeneities.

Systematic errors were introduced by the difference in density between the nylon strings used to move the Teflon cylinder and the liquid. This effect could be reduced by using Teflon instead of nylon strings, although the mechanical properties of Teflon are not as desirable.

The torsion balance is sensitive to gravitational gradients produced by objects in its vicinity. In order to reduce this source of noise, it is necessary to conduct the experiment at a reasonable distance from other laboratory activities. The remainder of this paper will be devoted to a theoretical discussion of the above result and to a comparison with other experiments.

II. THEORY

Bondi[5] distinguishes three kinds of mass." Inertial mass is the quantity that enters (and is defined by) Newton's second law; passive gravitational mass is the mass on which the gravitational field acts, that is, it is defined by $F=-m \operatorname{grad} V$; active mass is the mass that is the source of the gravitational field and is hence the mass that enters Poisson's equation and Gauss's law." Every object has three kinds of mass; or more exactly, there exist three numerically valued measures defined on the set of material objects. Each measure assigns a number to each body. In discussing this concept, it will be assumed that the accelerations are measured with all velocities, relative to the observer, small compared to the velocity of light; thus, relativistic effects are negligible.

Physicists assume today that these three properties of a body are measured by a single quantity. Identities between different types of mass are implied by the postulates of the mechanics of Newton and Einstein. Newton's third law, which states that the sum of the forces in a closed system is zero, implies that the ratio of active to passive mass for a body is a universal constant independent of the size or composition of the body. This may be interpreted to mean that active and passive gravitational mass are measures of the same property of matter and can be made to have identical numerical values by a proper choice of units. The principle of equivalence[6] in general relativity states that, neglecting the effects of gravitational gradients, a body experiences an acceleration in a gravitational field that is independent of its structure. This principle implies that for any body the ratio of passive to inertial mass is a constant independent of size and composition and that these masses are measures of the same property and may have identical numerical values by a proper choice of units.

In order to establish full equivalence between the three types of mass, it is necessary to establish equivalence between two sets of pairs. Roll, Kratkov, and Dicke[7] performed an experiment where the difference between the ratios of inertial to passive mass for gold and aluminum was measured. They concluded that this difference was less than 3×10^{-11}. This indicates that to a high degree of accuracy inertial and passive mass are measures of the same material property, and it will be assumed for the rest of this paper that they are identical. This remarkable degree of precision is in marked contrast to the rather poor experimental confirmation of the equivalence between passive (or inertial) mass and active mass.

The only precise experimental results, other than those reported in this paper, which are relevant to the problem of active gravitational mass come from the Cavendish-type determinations of the gravitational constant. In these experiments a torque is exerted on the torsion beam due to the gravitational attraction between the "small masses" on the torsion beam and the "large masses" which are placed near the balance. By carefully measuring the distance between the masses, the suspension-fiber torque constant, and the weight of the masses, it is possible to calculate the gravitational constant from the beam deflection. If the ratio of active to passive mass for the large masses is a function of their composition, then the value of the gravitational constant will depend on the composition of the large masses. Table I summarizes the results of a

[5] H. Bondi, Rev. Mod. Phys. **29**, 423 (1957).

[6] R. H. Dicke, *The Theoretical Significance of Experimental Relativity* (Gordon and Breach Science Publishers, Inc., New York, 1964).

[7] P. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N. Y.) **26**, 442 (1964).

TABLE I. Summary of experimental determinations of the gravitational constant G.

Experimenter	Date	Small mass	Large mass	G^E (10^{-8} dyn cm^2/g^2)	Ref.
Boys	1889–94	Gold	Lead	6.6576±0.002	a
Braun	1887–96	Gilded brass	Brass	6.655	a
Braun	1887–96	Gilded brass	Iron filled with mercury	6.665	a
Poynting	1878–90			6.6984	a
Koning *et al.*	1884–97			6.685 ±0.011	a
Heyle	1930	Gold	Steel	6.678 ±0.003	b
Heyle	1930	Platinum	Steel	6.664 ±0.002	b
Heyle	1942	Glass	Steel	6.674 ±0.002	b
Heyle	1942	Platinum	Steel	6.6755±0.0008	c
Heyle	1942	Platinum	Steel	6.6685±0.0016	c

[a] A. Stanley Mackenzie, *The Laws of Gravitation* (American Book Co., New York, 1900).
[b] P. Heyle, J. Res. Natl. Bur. Std. **5**, 1243 (1930).
[c] P. Heyle, J. Res. Natl. Bur. Std. **29**, 1 (1942).

number of determinations of the gravitational constant. The errors that are listed are those quoted by the authors and include in most cases only a measure of the statistical spread of the data. This table shows that differences in the gravitational constant as large as 3 parts in 10^3, which depend on the composition of the large mass, could exist without contradiction to the measured values.

Both systematic and random errors contribute to the uncertainty in the results of the Cavendish-type experiments. Measurements of the balance deflection contain noise due to ground vibrations which shake the balance, temperature changes and gradients which cause balance deflections, and random errors produced in measuring the deflection. The size of these errors may be estimated from the statistical properties of the data. Systematic errors are also present and it is much more difficult to estimate their size from published data. They are predominantly due to errors in measuring the distance between and the size and shape of the large and small masses. An estimate of their size may be obtained by comparing the values of G obtained in different experiments. The 1942 experiments of Heyle[8] give two values for G which differ by 0.007, while the errors are claimed to be 0.0008 and 0.0016. This would seem to indicate, since the experiments were almost identical, that these error values are rather optimistic and that 0.007 is some measure of the systematic errors. The experiment reported in this paper was designed to overcome these systematic errors.

The quantity of interest, the difference between the ratios of active to passive mass for two dissimilar substances, is a small or zero difference between two large quantities. By measuring the difference directly, rather than the large quantities themselves and then subtracting them, the need for high precision was eliminated. It is clear from the description of this experiment that the signal, detected by the balance, is proportional to the difference in active mass between the solid Teflon cylinder and an equal volume of the liquid which fills the tank. The measured density difference between the liquid and the Teflon is proportional to the difference in passive mass, since the buoyant forces which are due to the gravitational pull of the earth on the liquid and Teflon are proportional to passive mass. At neutral buoyancy the passive mass densities of the solid and liquid are equal, and any signal detected by the balance must be due to a difference in active mass densities. As described above, the balance is calibrated by use of the coefficients of thermal expansion for Teflon and for the liquid, which do not have to be known accurately.

The data were expressed above in terms of a signal produced by a fraction $\Delta m/m$ of the mass of the cylinder. The gravitational attraction between the cylinder and balance is also proportional to G, the gravitational constant, and if the signal is expressed as a difference ΔG between the liquid and Teflon, then $\Delta G/G=\Delta m/m$. The data of Table I do not exclude the possibility that $\Delta m/m$ between lead and steel is as large as 10^{-3}. If such a difference did exist, it might be a function of nuclear structure, and for this reason the two materials compared in this experiment were selected to have nuclear structures as different as possible. This choice was restricted by the requirements that the solid and liquid have equal densities and that they do not react chemically. Teflon was selected because the solid is homogeneous and chemically inert. It is 76% fluorine by weight. The liquid, a mixture of trichloroethylene and dibromomethane, was 74% bromine by weight. Table II lists the ratio Z/A of protons to nucleons and E/A, the binding energy per nucleon for bromine, fluorine, and the materials used in the experiments of

TABLE II. Binding energy per nuclear (E/A) in MeV and the ratio of protons to nucleons (Z/A).[a]

Element	E/A	Z/A
Lead	7.9	0.39
Iron	8.8	0.46
Brass (copper)	8.7	0.45
Mercury	7.9	0.40
Fluorine	7.8	0.47
Bromine	8.7	0.44

[a] R. Leighton, *Principles of Modern Physics* (McGraw-Hill Book Co., New York, 1959), pp. 736–783.

[8] P. Heyle, J. Res. Natl. Bur. Std. **29**, 1 (1942).

Table I. The differences in Z/A and E/A between fluorine and bromine indicate that they are interesting materials to compare. The results of this experiment, given above, indicate that the difference between the ratios of active to passive mass for bromine and fluorine is less than 5×10^{-5}.

III. SUMMARY AND CONCLUSIONS

This experimental technique of comparing the field produced by a homogeneous solid with the field of the fluid which it displaces has made it possible to measure an upper bound for $\Delta m/m$ which is smaller than any value which may be deduced from previous experiments. Although it is difficult to evaluate the possible sources of error in previous experiments and to deduce an upper bound for $\Delta m/m$, the scatter in values for G between various experiments makes it unreasonable to set this upper bound smaller than 10^{-3}. The present experimental result of 5×10^{-5} for an upper bound between fluorine and bromine is both a significant numerical improvement and also a more reliable estimate because it results from a direct measurement of the effect. Improvement by one and possibly two orders of magnitude should be possible by careful application of currently known experimental techniques. To improve the accuracy beyond that point would be very difficult and might require a completely different type of experiment. The present experimental technique would be severely limited by problems of measuring and controlling the temperature of the liquid and solid, by gravitational gradients caused by inhomogeneities in the solid, by noise generated in the balance by thermal effects and ground noise, and by the difficulty of measuring such small density differences.

ACKNOWLEDGMENTS

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Validity of Special Relativity at Small Distances and the Velocity Dependence of the Charged-Pion Lifetime

L-E. Lundberg and L. B. Rédei
Institute of Theoretical Physics, Umeå University, Umeå, Sweden
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The velocity dependence of the lifetime of the pion is investigated under the assumption that the Hamiltonian contains a spatial form factor which in the lab frame (the frame at rest with respect to the neighboring macroscopic bodies) vanishes for distances larger than some length α. In this model, there is a violation of the principles of special relativity at small distances. In particular, space-time is anisotropic at distances smaller than α. The lifetime of the pion is calculated to second order in α, and it is shown that there will be about 1% deviation from the usual formula $\tau(v)=(1-v^2/c^2)^{-1/2}\tau(0)$, (which holds if special relativity is valid at arbitrarily small distances) if, e.g., the pion energy $E_\pi=10^4$ MeV and $\alpha\sim5\times10^{-16}$ cm. The measurement of the velocity dependence of the pion lifetime at high energies could thus serve as a possible check on the validity of special relativity at small distances. The deviation is of the same order of magnitude as that previously obtained for the muon decay.

In a recent article by one of us,[1] the velocity dependence of the muon mean lifetime was investigated under the assumption that the principles of special relativity are violated for small distances. More specifically, it was assumed that the weak-interaction Hamiltonian contained a noncausal form factor which permitted interaction between simultaneous space-time events in the lab frame provided their spatial distance is less than some fundamental distance α. In this type of violation of relativity, the lab frame constitutes a preferred frame.[2] It was then shown in Ref. 1 that there will be a detectable deviation from the usual formula

$$\tau(v)=(1-v^2/c^2)^{-1/2}\tau_0 \quad (1)$$

if, e.g., one assumes 1% accuracy on the measurement of the lifetime $\tau(v)$ and muon energy $E\sim10^4$ MeV provided α is not less 7×10^{-16} cm. An accurate high-energy measurement of the time-dilation formula for the decay of an unstable particle could thus serve as a possible check on the validity of special relativity in high-energy physics.

Subsequently, it was pointed out to the authors[3] that it might be easier to measure the velocity dependence

[1] L. B. Rédei, Phys. Rev. **162**, 1299 (1967).
[2] D. I. Blokhintsev, Phys. Letters **12**, 272 (1964); L. B. Rédei, Phys. Rev. **145**, 999 (1966).
[3] T. Alväger (private communication).