Evaporation of 3 to 8 Neutrons in Reactions between ^{12}C and Various Uranium Nuclides*

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Cross sections for the production of ^{242}Cf, ^{243}Cf, ^{244}Cf, ^{245}Cf, and ^{246}Cf in reactions between ^{12}C and ^{233}U, ^{234}U, ^{235}U, ^{236}U, and ^{238}U have been measured in the energy range 60–110 MeV for ^{12}C. A good fit is obtained to the peaks of the cross-section curves. The fit involved (1) calculation of the compound nucleus cross section by the use of the parabolic approximation to the real part of the optical model, (2) modification of Jackson's formula for P_x to include fission and angular-momentum effects, and (3) use of the Γ_n/Γ_f formula due to Fujimoto and Yamaguchi. The analysis suggests that the value of $\langle\Gamma_n/\Gamma_f\rangle_{av}$ is independent of the energy of ^{12}C. The formula by Fujimoto and Yamaguchi reproduces the experimental $\langle\Gamma_n/\Gamma_f\rangle_{av}$ values with a standard deviation of 16%.

I. INTRODUCTION

HEAVY-ION reactions, that are characterized by the formation of a compound nucleus followed by neutron emission, constitute a powerful method for producing and identifying neutron-deficient nuclides. The excitation functions exhibit sharp peaks, and their positions depend upon the number x of neutrons emitted and can therefore be used for mass assignments.

In a region where fission and charged-particle emission can be ignored, the cross section σ_x as a function of energy fits well the formula[1]

$$\sigma_x=\sigma_{CN}P_x, \tag{1}$$

where σ_{CN} is the cross section for the formation of the compound nucleus, and P_x is the probability for the emission of exactly x neutrons and is calculated according to the Jackson formula[2] modified to include angular-momentum effects.[1]

In the heavy-element region, the cross sections are strongly influenced by fission competition. Equation (1) must then be modified to include this effect. Fission may take place at each step in the cascade and the cross section can then be written as[3,4]

$$\sigma_x=\sigma_{CN}P_x\prod_{i=1}^{x}[\Gamma_n/(\Gamma_n+\Gamma_f)]_i, \tag{2}$$

where Γ_n and Γ_f are level widths for neutron emission and fission, respectively. Again, other modes of decay have been ignored during the cascade. The last term in Eq. (2) represents the fraction of nuclei that survives fission through the cascade of x neutrons.

The present work was undertaken in order to investigate in some detail the validity of Eq. (2). Special emphasis is placed on the study of the ratio Γ_n/Γ_f and its variation with various nuclear quantities.

Similar studies have been undertaken in the heavy-element region with p, d, α,[4,5] and heavier ions[3,6–8] as projectiles. For most of these cases, the Jackson formula has been successful in reproducing the experimental data and the ratio Γ_n/Γ_f has been found to be independent of the energy of the ion. In a recent work with ^{18}O, ^{19}F, and ^{22}Ne incident on ^{238}U, Donets *et al.* concluded that Γ_n/Γ_f increased with increasing ion energy.[7]

We chose ^{233}U, ^{234}U, ^{235}U, ^{236}U, and ^{238}U as target nuclei which were bombarded with ^{12}C of energy up to 110 MeV to produce known californium nuclides with mass number from 242–246. (The nuclides ^{242}Cf and ^{243}Cf were discovered during these investigations and their decay properties have been reported elsewhere.[9,10]) This gives us the possibility of studying reactions with a wide range in x (3–8), excitation energy (30–80 MeV), and mass number of cascading nuclei (243–250).

The systems $^{238}U(^{12}C,4n)$, $^{238}U(^{12}C,5n)$, and $^{238}U(^{12}C,6n)$ had previously been measured,[3,6] but were included in our experiments to minimize relative errors.

II. EXPERIMENTAL

The targets were made by molecular plating, from an isopropyl-alcohol solution, uranyl nitrate onto 5-mg/cm² Be foils to a thickness of about 0.5 mg/cm². The amount of uranium on the target was determined by pulse-height analysis.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
† On leave of absence from the Institute of Nuclear Research, Prague.
[1] T. Sikkeland, Arkiv Fysik **36**, 539 (1967).
[2] J. D. Jackson, Can. J. Phys. **34**, 767 (1956).
[3] T. Sikkeland, S. G. Thompson, and A. Ghiorso, Phys. Rev. **112**, 543 (1958).
[4] R. Vandenbosch, T. D. Thomas, S. Vandenbosch, R. A. Glass, and G. T. Seaborg, Phys. Rev. **111**, 1358 (1958).
[5] For a review of these studies, see R. Vandenbosch and J. R. Huizenga, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Vol. **15**, p. 688.
[6] V. V. Volkov *et al.*, Zh. Eksperim. i Teor. Fiz. **36**, 762 (1959) [English transl.: Soviet Phys.—JETP **9**, 536 (1959)].
[7] E. D. Donets, V. A. Schegolev, and V. A. Ermakov, Yadern. Fiz. **2**, 1015 (1965) [English transl.: Soviet J. Nucl. Phys. **2**, 723 (1965)].
[8] V. L. Mihkeev, V. I. Ilyuschenko, and M. B. Miller, Joint Institute for Nuclear Research, Dubna USSR, 1966, Report No. P-2694 (unpublished).
[9] T. Sikkeland and A. Ghiorso, Phys. Letters **24B**, 331 (1967).
[10] T. Sikkeland, A. Ghiorso, J. Maly, and M. J. Nurmia, Phys. Letters **24B**, 333 (1967).

Beams of 124-MeV ^{12}C from the Hilac were, after magnetic deflection through 30°, degraded to the desired energy by the use of weighed Be foils. The range-energy curve of ^{12}C in Be, as measured by Walton, was used to estimate the energy.[11] The degraded energy spectrum was also measured by the use of a diffuse-junction Si detector and was very nearly Gaussian in shape. The full width at half-maximum (FWHM) increased almost linearly with decreasing energy from 0.7 MeV at 110 MeV to about 2 MeV at 60 MeV. The most probable energy is believed to be accurate to within 2 MeV.

The collimator in front of the target had a diameter of 0.6 cm. The average beam current was about 1.5×10^{-6} A. At these intensities the degrader foils had to be in contact with a water-cooled copper surface.

The yield of the various α-emitting californium isotopes was determined by the use of an α grid chamber in conjunction with a 200-channel pulse-height analyzer. The decay of the various α groups was generally followed through several half-lives.

As energy calibration standards, the 5.80 and 7.68 MeV α group[5] from ^{244}Cm and ^{214}Po, respectively, were used.

Two methods were used to measure the cross sections. In one, the relative cross sections were determined as a function of ion energy by the use of the recoil technique as described in Ref. 9. The recoil atoms produced in the reaction were slowed down in helium at a pressure of about 700 Torr contained inside a cylindrical chamber of 2.5 cm diam and 4.4 cm length. A Faraday cup for beam-intensity measurement was located at the end of that chamber. In the middle of the chamber wall and vertical to the beam axis was a 0.2-mm orifice through which the helium gas with the recoils flowed into a larger chamber that was kept at a pressure of about 1 Torr. The recoils were collected on a platinum disk placed in front of the orifice at a distance of about 2 mm. After bombardment, the foil was flamed to remove β and α activities of volatile elements produced from the Be foils, and Pb and Bi impurities. The time between end of bombardment and start of analysis was about 1 min.

The over-all yield of this recoil technique was determined by measuring the absolute cross section at the peak of the reaction $^{238}U(^{12}C,4n)^{246}Cf$. In this experiment, the ^{238}U target was facing the beam such that the recoil products were caught in the target itself or in its backing. The actinides were separated from beryllium by the use of a NaOH precipitation with Fe^{3+} as carrier and from uranium and iron by the use of an ion-exchange column, and were finally electroplated from a NH_4Cl solution onto a Pt disk and then α pulse-height analyzed. ^{244}Cm tracer was added in the dissolving step to check the over-all chemical yield.

We found the cross section for this system to be 59 ± 6 μb, which is to be compared to the values 28 [3] and 62 μb,[6] as determined by other experimenters. The yield of the recoil technique was 10% and was reproduced with a standard deviation of 25%.

The possibility that the yield of this method varied with bombarding energy was not checked directly. The geometry of the chamber and the pressure of He were such that all recoils should have been stopped in the gas and not on the walls. With ^{238}U as target we find the ratios σ_4/σ_6 and σ_4/σ_5 to have the values 4.1 and 0.60, respectively, which are, within errors, in agreement with the values 5.4 (Ref. 3) and 0.7 (Ref. 6) obtained in earlier experiments, indicating no systematic change in yield.

In the analysis, we assumed the following values for the α energy, half-life, and α branching for californium isotopes: ^{242}Cf, 7.39 MeV, 3.4 min, 100% [9]; ^{243}Cf, 7.05 MeV, 10 min, 10% [10]; ^{244}Cf, 7.21 MeV, 20 min, 100% [10]; ^{245}Cf, 7.14 MeV, 45 min, 66% [12]; ^{246}Cf, 6.75 MeV, 36 h, 100%.[12]

Since no chemical separation was performed, we considered possible interference from other nuclides with similar decay properties. We found that the following two series[12] in some cases hampered the analysis;

$$^{228}\mathrm{U}_{6.67\ \mathrm{MeV}}{}^{9.3\ \mathrm{min}}\!\longrightarrow{}^{224}\mathrm{Th}_{7.13\ \mathrm{MeV}}{}^{1\ \mathrm{sec}}\!\longrightarrow{}^{220}\mathrm{Ra}_{7.43\ \mathrm{MeV}}{}^{0.03\ \mathrm{sec}}$$

$$\longrightarrow{}^{216}\mathrm{Rn}_{8.01\ \mathrm{MeV}}{}^{10^{-4}\ \mathrm{sec}}\!\longrightarrow{}^{212}\mathrm{Po}_{8.78\ \mathrm{MeV}}{}^{3\times10^{-7}\ \mathrm{sec}}$$

$$\longrightarrow{}^{224}\mathrm{Ra}_{5.68\ \mathrm{MeV}}{}^{3.64\ \mathrm{d}}\!\longrightarrow{}^{220}\mathrm{Rn}_{6.28\ \mathrm{MeV}}{}^{51.5\ \mathrm{sec}}$$

$$\longrightarrow{}^{216}\mathrm{Po}_{6.78\ \mathrm{MeV}}{}^{0.16\ \mathrm{sec}}.$$

With ^{228}U present, the α groups in this series could interfere with ^{242}Cf, ^{243}Cf, ^{244}Cf, and ^{245}Cf activities, and with ^{224}Ra present the 6.78-MeV α group could interfere with ^{246}Cf activity. The presence of ^{228}U was spotted by the 8.01- and 8.78-MeV groups. The excitation functions for the production of ^{228}U were not determined. We observed this series with all targets used. The threshold for its production increases with increasing A of the target from about 70-MeV ^{12}C with ^{233}U to about 110-MeV with ^{238}U. The interference from the ^{228}U series was serious only at the tails of the functions for ^{242}Cf and to some extent for ^{243}Cf, ^{244}Cf, and ^{245}Cf. For the latter three a more difficult problem was the separation of their α groups at 7.05, 7.14, and 7.22 MeV in the cases when one of them was dominating. In such cases questionable data were eliminated.

The interference from the ^{224}Ra series in the analysis of ^{246}Cf was never serious over the main part of the peak. The possibility that at the highest energies, i.e., at the tail of the curve, we have a contribution from ^{216}Po is not ruled out.

[11] J. R. Walton, Lawrence Radiation Laboratory (private communication).

[12] D. Strominger, J. M. Hollander, and G. T. Seaborg, Rev. Mod. Phys. **30**, 585 (1958).

TABLE I. Results of the analysis of experimental maximum cross sections obtained in $U(^{12}C,xn)Cf$ reactions. Symbols not defined in the text are A_t, the mass number of target nucleus; A_p, the mass number of the product nucleus. The calculated values for $(\sigma_{CN}P_x)_{max}$ and $\langle\Gamma_n/\Gamma_f\rangle_{av}$ were obtained by the use of the formulas by Jackson (modified) and Fujimoto and Yamaguchi, respectively. The values for $E_{i,max}$ and $\sigma_{x,max}$ were taken from the curves in Figs. 1–4.

A_t	x	A_p	$E_{i,max}$ (MeV)	$\sigma_{x,max}$ (μb)	ΔE	$(\sigma_{CN}P_x)_{max}$ (mb)	A_{av}	$\langle\Gamma_n/\Gamma_f\rangle_{av}$ (exp)	$\langle\Gamma_n/\Gamma_f\rangle_{av}$ (calc)
238	4	246	67.5	62	2.0	26	248.5	0.28 ±0.03	0.30
238	5	245	73.5	100	1.5	260	248	0.26 ±0.01	0.25
238	6	244	83.5	15	0	400	247.5	0.22 ±0.01	0.23
238	7	243	~95	3.0	0.5	550	247	0.21 ±0.01	0.20
238	8	242	~115	0.29	1.0	520	246.5	0.20 ±0.01	0.17
236	3	245	67.5	2.5	0	1.5	247	0.13 ±0.03	0.19
236	4	244	70	22	0	48	246.5	0.17 ±0.02	0.17
236	5	243	77.5	9.8	0	300	246	0.15 ±0.01	0.14
236	6	242	88	2.1	~1.0	420	245.5	0.15 ±0.01	0.13
235	3	244	67.5	1.5	0	0.75	246	0.14 ±0.04	0.15
235	4	243	70.5	8.8	0	80	245.5	0.11 ±0.02	0.13
235	5	242	77.5	5.0	−0.5	270	245	0.13 ±0.01	0.12
234	4	242	72	4.0	1.0	90	244.5	0.089±0.009	0.098
233	3	242	67	0.37	−1.0	2.0	244	0.060±0.016	0.087

III. EXPERIMENTAL RESULTS

The experimental cross sections are plotted versus the bombarding energy E_i in Figs. 1–4. Typical errors are indicated by error bars and include (a) statistical errors in the counting, (b) standard deviation of 25% in recoil collection efficiency, and (c) uncertainty in target thickness. The maximum cross sections for σ_x and the corresponding energies for E_i are given in Table I.

The effects of energy spread of the beam on the width of the excitation functions were not taken into account. Such a correction might make some of the peaks as much as 2 MeV narrower.

IV. DISCUSSION

We shall make the assumption that Γ_n/Γ_f is independent of the bombarding energy. According to

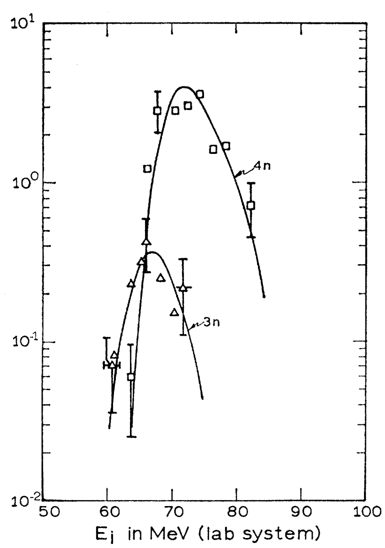

FIG. 1. Experimental cross sections σ_x plotted versus ^{12}C energy E_i for the systems $^{233}U(^{12}C,3n)\,^{242}Cf(\triangle)$ and $^{234}U(^{12}C,4n)\,^{242}Cf(\square)$. The curves represent the function $\sigma_{CN}P_x$ normalized at the peak to the experimental points. The energy scales for the curves are displaced ΔE MeV relative to that of the figure. Values for ΔE are given in Table I.

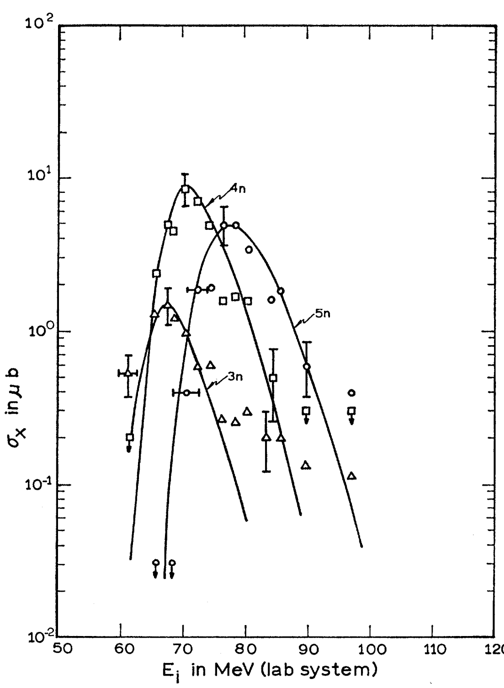

FIG. 2. Experimental cross sections σ_x plotted versus ^{12}C energy E_i for the $^{235}U(^{12}C,xn)\,^{247-x}Cf$ reactions. The symbols and corresponding values of x for the experimental points are $\triangle$, $3n$; $\square$, $4n$; and $\bigcirc$, 5. The curves represent the function $\sigma_{CN}P_x$ normalized at the peak to the experimental points. The energy scales for the curves are displaced ΔE MeV relative to that of the figure. Values for ΔE are given in Table I.

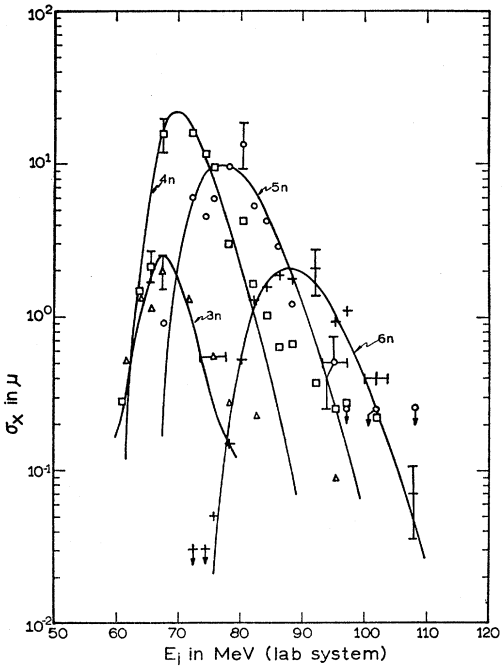

FIG. 3. Experimental cross sections σ_x plotted versus ^{12}C energy E_i for $^{236}U(^{12}C,xn)\,^{247-x}Cf$ reactions. The symbols and corresponding values of x for the experimental points are △, $3n$; □, $4n$; ○, $5n$; and +, $6n$. The curves represent the function $\sigma_{CN}P_x$ normalized at the peak to the experimental points. The energy scales for the curves are displaced ΔE MeV relative to that of the figure. Values for ΔE are given in Table I.

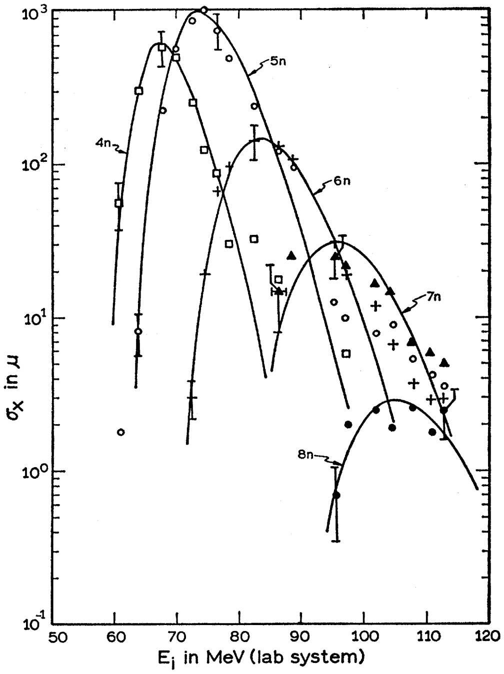

FIG. 4. Experimental cross sections σ_x plotted versus ^{12}C energy E_i for $^{238}U(^{12}C,xn)\,^{250-x}Cf$ reactions. The symbols and corresponding values for x for the experimental points are □, $4n$; ○, $5n$; +, $6n$; ▲, $7n$; and ●, $8n$. The curves represent the function $\sigma_{CN}P_x$ normalized at the peak to the experimental points. The energy scales for the curves are displaced $\triangle E$ MeV relative to that of the figure. Values for ΔE are given in Table I.

Eq. (2) this implies that the shape of the cross-section curve is determined by the product $P_x\sigma_{CN}$ only. We shall therefore separate the analysis into two parts. In Sec. IV A, we attempt to fit the shapes of $\sigma_{CN}P_x$ to those of the experimental curves. In Sec. IV B, experimental values for Γ_n/Γ_f are derived from Eq. (2) by the use of calculated $\sigma_{CN}P_x$ values and experimental σ_x values. Finally, calculated Γ_n/Γ_f values will be fitted to the experimental values.

A. Shape of the Excitation Function

Attempts were made to fit the shapes of the experimental curves by the use of the original Jackson formula that does not include angular-momentum terms.[2]

It turned out that the main part of a particular function could be fairly well reproduced with a value for T that was independent of the ion energy. However, T had to be increased as we increased x. Similar effects have been observed by Tarantin.[13] Typically, a temperature of about 1.2 MeV was required for a $4n$ reaction, whereas a value of 1.5 MeV had to be used for a $6n$ reaction. The main part of the peak of the former is at a lower bombarding energy than that of the latter. We believed that it was inconsistent not to use, at the same value of E^*, the same temperature for various xn reactions.

[13] N. I. Tarantin, Zh. Eksperim. i Teor. Fiz. **38**, 250 (1960) [English transl.: Soviet Phys.—JETP **11**, 181 (1960)].

Modified to include angular-momentum effects[1] the expression for $\sigma_{CN}P_x$ is

$$\sigma_{CN}P_x = \sum_{l=0}^{l_{CN}} \sigma_l P_{x,l}. \tag{3}$$

A brief outline of the definitions and calculations of the terms in Eq. (3) follows in parts (a)–(c).

(a) σ_l is the cross section for the lth partial wave of the incident ion. Using the optical model of the nucleus this cross section is given by[14]

$$\sigma_l = \pi\lambda\!\!\!^-{}^2(2l+1)T_l, \tag{4}$$

where $\lambda\!\!\!^-$ is the de Broglie wavelength of the projectile and T_l is the transmission coefficient of the wave. In the estimation of T_l, we use a parabolic approximation[15] to the real part of the effective optical-model potential with the following values for its parameters: $V_0=-70$ MeV, $r_0=1.24$ F, and $d=0.48$ F.

These values for the optical-model parameters were obtained in Ref. 16 by fitting the sum $\sum_{l=0}^{\infty}\sigma_l$, defined as the total interaction cross section, to the measured total fission cross sections for the system $^{238}U(^{12}C,f)$ from the barrier up to 124 MeV.

[14] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).
[15] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).
[16] V. E. Viola, Jr., and T. Sikkeland, Phys. Rev. **128**, 767 (1962).

(b) l_{CN} is a cutoff value above which only surface reactions take place and is adjusted such that the value of the ratio $\sum_{l=0}^{l_{CN}}/\sum_{l=0}^{\infty}\sigma_l$ is 0.8. This value is empirical and is based on results from fragment-fragment-angular-correlation measurements for the system ^{238}U+124 MeV ^{12}C.[17] It is then assumed that the value is independent of ion energy.[18]

(c) The last term in Eq. (3) is the probability for boiling out exactly x neutrons from a compound nucleus of angular momentum l and is given by[1]

$$P_{x,l}=I(\Delta_x 2x-3)-I(\Delta_{x+1}, 2x-1),$$

where $I(Z,n)$ is the incomplete γ function and

$$\Delta_x=(E^*-\sum_{i=1}^{x} B_i-E_R)/T,$$

$$\Delta_{x+1}=(E^*-\sum_{i=1}^{x} B_i-E_f-E_R{}^f)/T.$$

Here E^* is the excitation energy of the compound nucleus as estimated from the ion energy and masses involved; B_i is the binding energy of the ith neutron in the cascade; E_f is the fission barrier of the product nucleus, where $E_f<B_i$; E_R and $E_R{}^f$ are some average values of the rotational energies of the cascading nuclei at the equilibrium and saddle configurations, respectively; T is the nuclear temperature and it is assumed that the temperature for fission is equal to that for neutron evaporation.

The calculations of $\sigma_{CN}P_x$ were performed on a CDC 6600 computer. Values for the nuclear masses and B_i were taken from the tables by Foreman and Seaborg.[19] Their values are in excellent agreement with the known decay data in this region. Values for the fission barrier are taken from Viola and Wilkins,[20] who obtained their values from an analysis of spontaneous fission half-life.

The nuclear temperature was used as an adjustable parameter.

The values for the rotational energies depend on the angular-momentum distributions and the moments of inertia of the nuclei in the neutron cascade.

The l distributions depend mainly on the variation of Γ_n/Γ_f with l since the average angular momentum carried off by a neutron is negligible, and γ emission presumably does not compete favorably with neutron emission and fission when the excitation energy is larger than B_i and E_f.

We shall make the extreme assumption that Γ_n/Γ_f is independent of l. At each step in the cascade the l distribution of the nuclei is then equal to that of the compound nucleus. In a simple model, Γ_n/Γ_f is predicted to be proportional to $\exp(E_R{}^f-E_R)/T$ (Ref. 21) hence, independent on l when $E_R{}^f=E_R$. The latter energy is estimated from the expression $(\hbar^2/2\mathfrak{J})l(l+1)$, where $\mathfrak{J}$ is the effective moment of inertia.[21] We shall use $\mathfrak{J}^0/\mathfrak{J}$ as an adjustable parameter assumed to be independent of E^* and l. Here $\mathfrak{J}^0$ is the rigid-body moment of inertia of a spherical nucleus of constant density and is given by $\mathfrak{J}^0=(2/5)Mr_0{}^2A^{2/3}$, where M and A are the nucleidic mass and mass number, respectively, and r_0 is the radius parameter for which we used the value 1.22×10^{-13} cm.

Best over-all fit was obtained with $T=1.20$ MeV and $\mathfrak{J}^0/\mathfrak{J}=1.25$, with an uncertainty of 0.05 MeV and 0.25, respectively.

The calculated curves for $\sigma_{CN}P_x$ are compared to the experimental σ_x values in Figs. 1–4. For each curve, the peak value for $\sigma_{CN}P_x$ is normalized to that for σ_x. The energy scales of the calculated curves have been displaced a certain amount ΔE relative to those of the experimental ones. The values for ΔE are listed in Table I. They were never larger than 2 MeV, which is within the experimental uncertainties.

As is seen from the figures, when data are available, the experimental curves exhibit a tail that is not reproduced by the calculated ones. The effect is small, i.e., the cross section at the tail is of the order of 1% of that at the peak. However, the discrepancy is regarded as significant.

Similar tails were observed for the reactions ^{238}U(^{12}C,4n)^{246}Cf, where the yield was determined after chemical separation.[2,6] It is believed that the tails cannot fully be explained by the presence of low-energy carbon ions in the beam. The discrepancy is due to a breakdown of either the Jackson formula or the assumption that Γ_n/Γ_f is independent of E_i.

B. Experimental Γ_n/Γ_f Systematics

1. Experimental Γ_n/Γ_f Values

We define a mean value of Γ_n/Γ_f as[4]

$$\langle\Gamma_n/\Gamma_f\rangle_{av}=\bar{G}/(1-\bar{G}). \qquad (5)$$

Here $\bar{G}$ is a mean value of $\Gamma_n/(\Gamma_n+\Gamma_f)$ defined as

$$\bar{G}=[\prod_{i=1}^{x}\Gamma_n/(\Gamma_n+\Gamma_f)]_i{}^{1/x}, \qquad (6)$$

which, according to Eq. (2), is given by

$$\bar{G}=[\sigma_x/(\sigma_{CN}P_x)]^{1/x}. \qquad (7)$$

Values for $\langle\Gamma_n/\Gamma_f\rangle_{av}$, estimated at the peak of σ_x and $\sigma_{CN}P_x$, are listed in Table I together with the quantity

[17] T. Sikkeland and V. E. Viola, Jr., in *Proceedings of the Third Conference on Reactions between Complex Nuclei, Asilomar, 1963*, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963).
[18] T. Sikkeland, Phys. Rev. **135**, B669 (1964).
[19] B. M. Foreman, Jr., and G. T. Seaborg, J. Inorg. Nucl. Chem. **7**, 305 (1958).
[20] V. E. Viola, Jr., and B. D. Wilkins, Nucl. Phys. **82**, 65 (1966).
[21] J. R. Huizenga and R. Vandenbosch, in *Nuclear Reactions*, edited by P. M. Endt and P. B. Smith (North-Holland Publishing Co., Amsterdam, 1962).

A_{av} which represents the mass number of the intermediate fissioning nucleus halfway along the evaporation chain.

The errors for $\langle\Gamma_n/\Gamma_f\rangle_{av}$, given in Table I, include experimental errors in σ_x, uncertainties in σ_{CN} (arising from an error of 0.02 F in r_0 and d), and in P_x (due to uncertainties of 0.05 MeV in T and 0.25 in $\mathfrak{J}^0/\mathfrak{J}$). It is apparent from the table that $\langle\Gamma_n/\Gamma_f\rangle_{av}$ within errors is independent of E_i.

2. Semiempirical Formula for Γ_n/Γ_f

In the estimation of $P_{x,l}$ we made the assumption that Γ_n/Γ_f is independent of l. A sufficient condition for $\langle\Gamma_n/\Gamma_f\rangle_{av}$ to be independent of E_i will then be that Γ_n/Γ_f is also independent of the excitation energy. A formula that expresses such an independence of excitation energy and angular momentum is the following one, that was developed by Fujimoto and Yamaguchi,[22] and modified by Vandenbosch and Huizenga[5] to include odd-even effects:

$$\Gamma_n/\Gamma_f=(2T/K_0)A^{2/3}\exp(E_f'-B_n')/T. \quad (8)$$

Here T is the nuclear temperature,

$K_0\simeq 9.8$ MeV,

$E_f'=E_f+\alpha\Delta_f$; $\alpha=2$ for even-even fissioning nucleus,
$=1$ for even-odd fissioning nucleus,

$B_n'=B_n+\alpha\Delta_n$; $\alpha=2$ for even-even nucleus after emission of one neutron,
$=1$ for even-odd nucleus after emission of one neutron,

and Δ_f and Δ_n are the pairing energies at saddle and equilibrium, respectively, and are assumed to be constants. It is then assumed that the exponential-level density dependence on excitation energy is determined from the mass surface of the odd-odd nuclei, and that the temperature for fission is equal to that of neutron evaporation.[21]

In a cascade of x neutrons the geometric mean value for Γ_n/Γ_f can be written as:

$$\langle\Gamma_n/\Gamma_f\rangle_{av}=cA_{av}^{2/3}[\exp(\beta\Delta/x)] \times\exp(\textstyle\sum E_f-\sum B_n)/xT, \quad (9)$$

where

$$c=(2T/K_0)\exp[(1.5/T)(\Delta_f-\Delta_n)]=\text{const},$$

$$\beta=0,\quad n_{ee}=n_{e0}$$
$$=1,\quad n_{ee}>n_{e0}$$
$$=-1,\quad n_{ee}>n_{e0},$$

$$\Delta=(\Delta_f+\Delta_n)/2T=\text{const}.$$

(n_{ee} and n_{e0} are the numbers of even-even and even-odd nuclides in the cascade, respectively.) Values for $\langle\Gamma_n/\Gamma_f\rangle_{av}$ calculated according to this formula were now fitted to experimental ones by adjusting the constants c, Δ, and T. Taking values for B_n and E_f from Refs. 19 and 20, respectively, we obtained a best fit with $c=0.33$, $\Delta=1.5$, and $T=0.59$ MeV with which experimental values were reproduced with a standard deviation of 16%. Calculated and experimental $\langle\Gamma_n/\Gamma_f\rangle_{av}$ values are compared in Table I.

We shall in the following make a few comments about the values of the parameters used in Eq. (9).

It has been suggested from spontaneous fission systematics that Δ_f has the value 1.2 MeV [20] and Δ_n is about 0.7 MeV. Inserting these values and $T=0.59$ and $K_0=9.8$ MeV into the expression for c, we estimate its value to be 0.43 as compared to 0.33 found in the analysis. This good agreement is to be regarded as fortuitous. Considering the uncertainties in the values of the parameters in the expression for c, its estimated value must have an error of at least 50%.

The first exponential term in Eq. (13) represents the odd-even effect. The importance of this term is demonstrated by the fact that, for the cases where x is an odd number, the average deviations of calculated $\langle\Gamma_n/\Gamma_f\rangle_{av}$ values from experimental ones, with and without that term, were 16 and 32%, respectively,

From the values of 1.5 for Δ and 0.6 MeV for T we obtain the value 1.8 for the sum $\Delta_f+\Delta_n$, in agreement with the expected value of 1.9 MeV.

Our value for the parameter T is in excellent agreement with the value of 0.6 MeV obtained by Vandenbosch and Huizenga[5] in a similar analysis, using experimental Γ_n/Γ_f values from p-, d-, and α-induced reactions.

V. CONCLUSION

A good fit has been obtained to the peaks of measured cross-section curves using formulas that are based on the assumptions that the temperature is independent of excitation energy, the temperature for fission is equal to that for neutron evaporation, and Γ_n/Γ_f is independent of angular momentum. Angular-momentum effects have to be introduced into Jackson's formula for P_x when used in the heavy-element region, as was the case in the rare-earth region.[1]

We shall make a few remarks about some of the quantitative results of our analysis.

The value of the nuclear temperature as used in the formula for $P_{x,l}$ is 1.20±0.05 MeV, which is significantly higher than that of 0.59±0.05 MeV found to fit the $\langle\Gamma_n/\Gamma_f\rangle_{av}$ data. It is interesting to note that the former is the average temperature of the nuclides that survive fission through the cascade, whereas the latter is the corresponding one of all nuclides except the product. In the framework of the level-density formula $\rho=\rho_0\exp(aE/A)^{1/2}$ this difference in the temperatures

[22] Y. Fujimoto and Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) 5, 76 (1950).

suggests that Γ_n/Γ_f increases with increasing excitation energy.[23] The same level-density formula does in fact predict such an energy dependence.[21]

The assumption that the angular-momentum distribution is the same at each step of the cascade is not necessarily valid. In our analysis the adjustment of the value for $\mathfrak{J}^0/\mathfrak{J}$ can compensate for any breakdown of this assumption. However, the value of 4.5 keV obtained for the quantity $\hbar^2/2\mathfrak{J}$ is not unreasonable. The value for the $\hbar^2/2\mathfrak{J}^\circ$ is 3.6 keV for $A=250$. The deformed nuclei in this region of the periodic table have $\hbar^2/2\mathfrak{J}$ values, as deduced from the rotational energies near ground state, of about 7 keV. If, as is predicted,[21] E_R^f is smaller than E_R, and thus Γ_n/Γ_f decreases with increasing l, the value for $\hbar^2/2\mathfrak{J}$ will be less than 4.5 keV.

It is apparent from these results that one cannot, on the basis of excitation functions, draw any detailed quantitative conclusions about the effect of angular momentum and excitation energy on the level widths for neutron emission and fission. However, the usefulness of the formulas for $P_{x,l}$ and Γ_n/Γ_f should be evident. They have few adjustable parameters, are relatively easy to use, and can be used in mass assignments and in the prediction of cross sections in nucleosynthesis.

[23] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).

As a final note, we shall make a few remarks regarding the conclusion drawn by Donets *et al.* that Γ_n/Γ_f increased with increasing excitation energy.[7] They used the unmodified Jackson formula and values for σ_{CN} taken from those calculated by Thomas, using the square-well model.[24] These values are too high at the barrier.[13] This error decreases with increasing E_i. As shown in calculations,[25] this will result in experimental $\langle\Gamma_n/\Gamma_f\rangle_{av}$ values that are too high for the lowest x, i.e., for the lowest excitation energies, and thus give the apparent effect that Γ_n/Γ_f increases with E^*. We believe that a variation of Γ_n/Γ_f with E^* has not yet been experimentally demonstrated in reactions between heavy elements and heavy ions.

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[24] T. D. Thomas, Phys. Rev. **116**, 703 (1959).

[25] T. Sikkeland, Lawrence Radiation Laboratory Report No. UCRL-16348, 1965 (unpublished).

Errata

Hypernuclear Spectroscopy for $A=6$*, K. ANANTHANARAYANAN [Phys. Rev. **163**, 985 (1967)]. On p. 988, line 9 of the right-hand column should read

. . . the 1^- ground state, they consider predominantly a $p_{3/2}$ α-n . . .

instead of

. . . the 2^- ground state, they consider only a $p_{1/2}$ α-n . . .

Three-Body Correlations in Reaction-Matrix Calculations, B. S. BHAKAR AND R. J. MCCARTHY [Phys. Rev. **164**, 1343 (1967)].

(1) The expression (23) should read

$$t(\mathbf{k},\mathbf{p},\gamma^2)=\sum_l g_l(\mathbf{k})g_l(\mathbf{p})(\mathbf{k}^2+\gamma^2)^{-1}P_l(\hat{k}\cdot\hat{p}). \quad (23)$$

(2) The Pauli operator defined in Appendix B should be replaced by

$$\begin{aligned} Q(\mathbf{p},K,\mathbf{k}_F)&=0 \quad \text{if} \quad (\mathbf{p}^2+\tfrac{1}{4}K^2)^{1/2}<\mathbf{k}_F \\ &=1 \quad \text{if} \quad |\mathbf{p}-\tfrac{1}{2}K|>\mathbf{k}_F \\ &=\frac{\mathbf{p}^2+\tfrac{1}{4}K^2-\mathbf{k}_F{}^2}{\mathbf{p}K} \quad \text{otherwise.} \end{aligned} \quad (B2)$$