Effects of Nonparabolicity on Non-Ohmic Transport in InSb and InAs

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(Received 2 October 1967)

Hot-electron distribution functions, occupation densities, and drift velocities are calculated for InAs and InSb at room temperature, on the assumption that there are polar-optical-phonon interactions only. Exact energy-momentum dispersion and wave functions are taken into account. The distribution function and occupation density are compared with calculations based on an electron-temperature model in parabolic bands. These calculations give essentially the same result for the average energy prior to collective breakdown, but a rather different shape for the distribution function. The drift velocity, as calculated here, remains a linear function of the field up to fields well above avalanche breakdown. This behavior seems to be confirmed by experiment.

I. INTRODUCTION

IN the last decade the behavior of hot electrons in narrow band-gap semiconductors, like that of some of the III–V and II–VI compounds, has become of great interest in connection with various plasma effects, and related avalanche processes. Underlying these phenomena is the electron-lattice interaction via polar optical phonons, which frequently is the dominant intrinsic scattering mechanism. Furthermore, these materials are notorious for their nonparabolic energy bands. Some attempts have been made within the last two years to solve the appropriate Boltzmann equation,[1–4] and thus obtain the carrier distribution function at high electric field. However, no serious effort has been made at comparing theory with experiments throughout the whole range of electric fields for which data exist. This is difficult in many of the experiments reported because of the occurrence of several competing phenomena, such as impact ionization and the resulting phenomena of pinching and/or mixed scattering.[5,6] Effects due to external magnetic fields represent further complications (for example, the Suhl effect). A well-known property of electron–optical-phonon interaction in parabolic energy bands is hot-electron runaway,[7–9] a situation which is prevented in nonparabolic bands.[1,2] A more detailed study of this interaction for hot electrons thus appears worthwhile. The purpose of the present paper is to do this from two different points of view.

The first approach consists in carrying out a model calculation for InAs of the distribution function, occupation density, and average energy, for the case of nonparabolic bands. These results are compared with calculations based on the assumptions of parabolic bands and the existence of an electron temperature. The latter assumptions appear to form the basis of a successful model for a range of electric fields prior to collective breakdown, or avalanching. For this model to be theoretically meaningful, one requires an electron concentration n in excess of some critical concentration.[8,9] As recently pointed out by Conwell,[10] this critical concentration can be rather large in the III–V compounds. Similar considerations apply to nonparabolic bands. Thus, in InSb and InAs, the two materials to be studied here, $n \gg 2\times10^{16}$ cm^{-3} and $n \gg 8\times10^{16}$ cm^{-3}, respectively, for electron energies of the order of three optical-phonon energies, and room temperature. These concentrations are much higher than those in the samples on which experiments are usually done.

The second objective is to calculate the drift velocity for hot electrons in nonparabolic bands for InAs and InSb on the assumption that there is interaction with polar optical modes only, and for a range of electric fields well above the region of experimentally observed breakdown. An attempt will be made at comparison with experiment at room temperature. The experimental data at higher temperature ($T \gtrsim 200$°K) exhibit linear current-voltage characteristics up to the breakdown field[11] and beyond.[12,13] One may object that the competing phenomena, as mentioned earlier, make this comparison with experiment invalid. However, since lattice scattering is a very fast process ($\lesssim 10^{-12}$ sec), current measurements with very short voltage pulses ought to avoid most competing phenomena. This appears in fact to have been achieved in Ref. 13, where currents were measured for voltages of 2 to 3×10^{-10}-sec

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[13] A. C. Prior, J. Electron. Control **4**, 165 (1958).

duration. Some impact ionization may nevertheless occur, as indicated by a recent experimental estimate of the impact ionization cross section in InSb by McGroddy and Nathan[14] (the ratio of impact to total scattering cross sections is about one-twentieth). These scattering events would remove very energetic electrons from the tail of the distribution without, however, affecting the mobility too much, at least for not too high an electric field. With little impact ionization, one need not consider pinching[5,6] (which for suitably high drift velocities takes of the order 10^{-7} sec to occur), and electron-hole scattering.[12]

There remain two phenomena which complicate the solution of the Boltzmann equation: streaming (as contrasted to diffusion), and the inelastic nature of the electron-phonon scattering. Both can be avoided if one restricts oneself to higher lattice temperature[15] (and/or high electric field). It will be shown that, at room temperature, streaming is unlikely in InSb and InAs.

The tacit assumption was also made that there is no scattering to (possibly) higher valleys, although this may not be justified.[16,17]

In the next section the calculations are presented, and in Sec. III the results and discussion of an application to InSb (290°K) and InAs (300°K) are given.

II. CALCULATIONS

It is the purpose here to calculate the equilibrium distribution function for nonparabolic, but spherical, energy bands. To that end, the same approximations as in I will be used: energy transport by diffusion, and elastic scattering. The same notation will also be adopted. Thus

$$k^2=(2m_n/\hbar^2)\gamma(E) \tag{1}$$

expresses the relation between the Bloch wave vector $\mathbf{k}$ and the energy E through the arbitrary function $\gamma(E)$; m_n is the effective mass at the band edge. The momentum effective mass m^* and density of states $N(E)$ are then given by

$$m^*=m_n\frac{d\gamma}{dE},\quad N(E)=\frac{(2m_n)^{3/2}}{2\hbar^3}\gamma^{1/2}\frac{d\gamma}{dE}. \tag{2}$$

The first task now is to obtain the scattering probability which is needed in the collision integral of the Boltzmann equation. Its general form is

$$W_{\mathbf{k},\mathbf{k}'}\begin{pmatrix}a\\e\end{pmatrix}=\frac{2\pi}{\hbar}B(\mathbf{k},\mathbf{k}')\begin{Bmatrix}\mathfrak{N}\delta[E(\mathbf{k}')-E(\mathbf{k})-k\Theta]\\(\mathfrak{N}+1)\delta[E(\mathbf{k}')-E(\mathbf{k})+k\Theta]\end{Bmatrix}, \tag{3}$$

a and e referring to absorption and emission, respectively, $\mathfrak{N}$ to the phonon occupation number, and Θ to the optical-phonon Debye temperature. Writing B as $B(k,k',y)$, where y equals the cosine of the scattering angle, one has for arbitrary energy bands and corresponding wave functions (compare Ehrenreich, Refs. 18 and 19)

$$B(k,k',y)=\frac{2\pi e^2k\Theta}{V|\mathbf{k}-\mathbf{k}'|^2}\left(\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_s}\right)\mathcal{G}(k,k',y), \tag{4}$$

where ϵ_∞ and ϵ_s are the high-frequency and static dielectric constants, respectively, V is the volume, and

$$\mathcal{G}(k,k',y)=\tfrac{1}{2}\sum_{\mu,\mu'}\left|\int_0\phi_{\mu'k'}{}^*(\mathbf{r})\phi_{\mu k}(\mathbf{r})d^3r\right|^2. \tag{5}$$

The functions $\phi_{\mu k}$ are the cell periodic parts of Bloch functions, and μ, μ' are spin labels (of two types). The integration is over a unit cell. For the purpose here, Kane's results of the band calculations for InSb, with the neglect of higher bands, are used.[20] Thus the $\phi_{\mu k}$'s for the conduction band are given in Ref. 20, Eq. (14). The energy-momentum relation in the form of Eq. (1) is obtained from Ref. 20, Eq. (10), neglecting terms of order $\hbar^2k^2/2m$ (m=free-electron mass), and is

$$\gamma=\frac{E(E+G)(E+G+\Delta)(G+\frac{2}{3}\Delta)}{G(G+\Delta)(E+G+\frac{2}{3}\Delta)}, \tag{6}$$

where G is the energy gap and Δ is the spin-orbit splitting of the valence bands.

The evaluation of Eq. (5) is straightforward, the result being

$$\begin{aligned}\mathcal{G}(k,k',y)=&\,a_k{}^2a_{k'}{}^2+2a_ka_{k'}(b_kb_{k'}+c_kc_{k'})y\\&+(b_kb_{k'}+c_kc_{k'})^2y^2+\{\tfrac{1}{4}b_k{}^2b_{k'}{}^2+\tfrac{1}{2}(b_kc_{k'}+c_kb_{k'})^2\\&-(1/\sqrt{2})b_kb_{k'}(b_kc_{k'}+c_kb_{k'})\}(1-y^2),\end{aligned} \tag{7}$$

where a_k, b_k, c_k are wave-function normalization coefficients given in Ref. 20, Eq. (15). It should be emphasized that Eq. (7) was calculated by taking the initial direction of $\mathbf{k}$ along a principal axis of the crystal. Following Ehrenreich,[19] it was then assumed that Eq. (7) is valid for $\mathbf{k}$ along all other initial directions. This assumption is equivalent to neglecting the anisotropy in the scattering introduced by the p-function

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[15] By restricting oneself to higher lattice temperature, one cannot justifiably attempt to analyze Glicksman's [M. Glicksman, in *Seventh International Conference on the Physics of Semiconductors* (Dunod, Paris, 1965), p. 137] interesting data on InSb at 77°K, for example. An attempt was made here, nevertheless, in the region of high electric field, but prior to pinching. The results were negative: Not only could one not reproduce the form of the drift-velocity dependence on the electric field, but also much too low a mobility was obtained, even for extreme values of m_n and F_0 (see later). This clearly indicates that the diffusion approximation used here does not apply at lower temperature in InSb.

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admixture in the wave functions. The present calculations are therefore strictly applicable only to materials where these anisotropies are small, like many of the III–V and II–VI compounds.

The next steps in the derivation of the distribution function consist in substituting Eqs. (4) and (7) into the collision integral and the relaxation time, replacing k and k' everywhere by $\gamma(E)$ and $\gamma(E')$, respectively, and performing relevant expansions and integrations (compare I). Some of the details of the calculation are given in the Appendix. The result is the following isotropic distribution function:

$$f_0(x)=\Omega(\lambda)\exp\left[-\left\{\alpha\int_0^x\frac{(\gamma')^4\mathcal{G}_0\mathcal{G}_1dz}{\frac{2}{3}(\alpha\lambda\gamma)^2+(\gamma')^4\mathcal{G}_0\mathcal{G}_1}\right\}\right], \quad (8)$$

where the energy has been expressed in units of G, $x=E/G$. The other parameters are

$$\gamma'=d\gamma/dE, \quad \alpha=G/k\Theta(\mathfrak{N}+\tfrac{1}{2}), \quad \lambda=F/F_0,$$

F being the electric field;

$$F_0=\frac{ek\Theta m_n}{k^2}\left(\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_s}\right);$$

$\Omega(\lambda)$ is a normalization constant. The functions $\mathcal{G}_0$ and $\mathcal{G}_1$ are

$$\mathcal{G}_0(x)=a_k^4+\tfrac{1}{3}(1-a_k^2)^2+\tfrac{2}{3}(\sqrt{2}b_kc_k-\tfrac{1}{2}b_k^2)^2,$$
$$\mathcal{G}_1(x)=\ln(4\gamma G/k\Theta\gamma')-1+a_k^4+(\sqrt{2}b_kc_k-\tfrac{1}{2}b_k^2)^2.$$

Here the a_k, b_k, c_k, γ, and γ' have been properly expressed in terms of the dimensionless energy parameter x. $\mathcal{G}_1$ and $\mathcal{G}_0$ arise from Eq. (7), occurring in the collision integral and the relaxation time, respectively. The latter is given by

$$1/\tau(x)=eF_0(2\mathfrak{N}+1)\gamma'(2m_nG\gamma)^{-1/2}\mathcal{G}_0(x). \quad (9)$$

At this point it is convenient to study the asymptotic behavior, for large energy, of the various quantities just discussed. This asymptotic region is reached when $x\gg1$ and $x\gg\Delta/G$. Note, however, that in practice this energy can be quite large, large enough to make invalid the description of the band structure employed here. One can readily verify the following:

$$\gamma\to x^2G\left(1+\frac{2}{3}\frac{\Delta}{G}\right)\Big/1+\frac{\Delta}{G}, \quad (10a)$$

$$a_k^2\to\tfrac{1}{2}, \quad b_k\to0, \quad c_k^2\to\tfrac{1}{2}, \quad (10b)$$

$$\mathcal{G}_1\to\ln(2Gx/k\Theta)-\tfrac{3}{4}, \quad (10c)$$

$$\mathcal{G}_0\to\tfrac{1}{3} \quad (\mathcal{G}_0=1 \text{ at } x=0), \quad (10d)$$

$$f_0\to\exp\left[-\left\{\frac{\alpha x}{1+\alpha^2\lambda^2/8[\ln(2G\bar{x}/k\Theta)-\frac{3}{4}]}\right\}\right], \quad (10e)$$

where $\bar{x}$ is some average energy.

From these equations one concludes the following: The largest nonparabolicity achieved, for the $E(k)$ dependence assumed here, is at best a linear dependence of E on k. The nonparabolicity is thus underestimated at higher energies; the $\mathbf{k}\cdot\mathbf{p}$ method which includes interactions with higher bands[20] would be more realistic in that region. Equations (10b)–(10d) show the importance of using the correct wave functions, i.e., not just plane waves. Thus, for example, the scattering rate is decreased by a factor of 3. This can affect the distribution function markedly because of its exponential form. The admixture of p functions to the total wave functions therefore counteracts the large effects of the energy-momentum nonparabolicity. Finally, note that the distribution function becomes a Maxwellian function at high energy. Electron runaway is therefore prevented by optical-phonon scattering alone. However, at larger electric fields, $f_0(E)$ can have a rather extended tail. A more nonparabolic $E(k)$ would, of course, reduce these tails. It is therefore clear that the prevention of electron runaway depends critically on the asymptotic behavior of E on k. Thus, if, asymptotically,

$$E\to k^\mu, \quad \mu\leq1$$

runaway is prevented. This is evidently the case here, but just barely, since $\mu=1$.

These remarks therefore justify the normalization constant $\Omega(\lambda)$. It is

$$\Omega(\lambda)=\frac{2\pi^2\hbar^3n_0}{(2m_nG)^{3/2}}K(\lambda), \quad (11)$$

where n_0 is the carrier density and

$$K(\lambda)=\int_0^\infty\gamma^{1/2}\gamma'\exp\left[-\left\{\alpha\int_0^x\frac{(\gamma')^4\mathcal{G}_1\mathcal{G}_0dz}{\frac{2}{3}(\alpha\lambda\gamma)^2+(\gamma')^4\mathcal{G}_0\mathcal{G}_1}\right\}\right]dx. \quad (12)$$

Quantities like the carrier occupation density, drift velocity, and average energy can readily be obtained by use of Eqs. (8), (9), (11), and (12). Note that all integrations above were started at $x=0$. This represents no serious error in the calculation of most quantities, since in the approximation of high temperature and/or high fields used here, the energy region near $x=0$ is little occupied. The only difficulty encountered is in the integration of the function in the exponential of f_0: It contains a singularity (near $x\cong0.02$, depending slightly on $G/k\Theta$), due to the log term in $\mathcal{G}_1$, which has the shape of the derivative of a δ function. This double spike, which is very narrow in extent, was avoided in the numerical integration; the integrand was interpolated monotonically to 1 as x goes to zero.

III. APPLICATIONS TO InSb AND InAs

The preceding theory will now be applied to InSb (290°K) and InAs (300°K). First, a comparison will be made with the electron-temperature model for parabolic

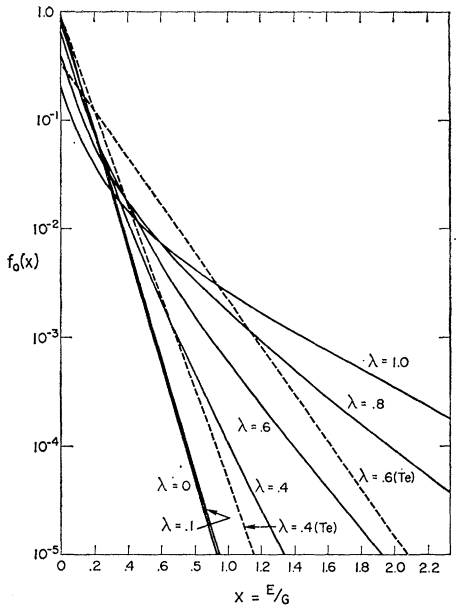

FIG. 1. Distribution functions for various $\lambda(=F/F_0)$ as a function of $x(=E/G)$ for InAs at 300°K.

bands. For that purpose, InAs at 300°K was chosen,[21] with the following parameters[22,23]: $G=0.35$ eV, $\Delta=0.43$ eV, $m_n=0.022m$, $\Theta=334$°K, $\epsilon=14.3$, $\epsilon_\infty=11.5$. These numbers give $\Omega/K=2.03\times10^{-19}n_0$, $F_0\cong1980$ V/cm, $K(\lambda=0)=38$. In Fig. 1, several normalized distribution functions [in units of $\Omega(\lambda=0)$] are given for different λ's. Note the following points:

(a) The curve at zero field (labeled $\lambda=0$) corresponds to a Maxwellian at $T=330$°K, instead of $T=300$°K, as one should have. There is therefore not much error, even at zero electric field.

(b) One requires a substantial increase in field to change the distribution function appreciably from its form at zero field. The case $\lambda=0.1$ is hardly different from $\lambda=0$, even though it corresponds to a field of 200 V/cm.

(c) For larger λ ($\lambda=0.6$, 0.8, 1.0), f_0 tends towards a superposition of two Maxwellians, corresponding to the fact that the asymptotic limit of $E\propto k$ is being approached. This indicates that an effective electron-temperature model for nonparabolic bands would not describe the shape of f_0 very well.

(d) Curves marked $\lambda=0.4(T_e)$ and $\lambda=0.6(T_e)$ are the normalized distributions for parabolic bands (electron-temperature model). It seems that the electron-temperature model differs little from the present theory at small λ, whereas at high λ there is a sizable reduction

[21] The conclusions reached here for InAs apply equally well to InSb. They are not too valid for wider gap materials, because these are more parabolic over the energy range considered here.

[22] C. Hilsum, in *Semiconductors and Semimetals*, edited by R. K. W. Willards and A. C. Beer (Academic Press Inc., New York, 1966), Vol. I, p. 3.

[23] O. Madelung, *Physics of III–V Compounds* (John Wiley & Sons, Inc., New York, 1964).

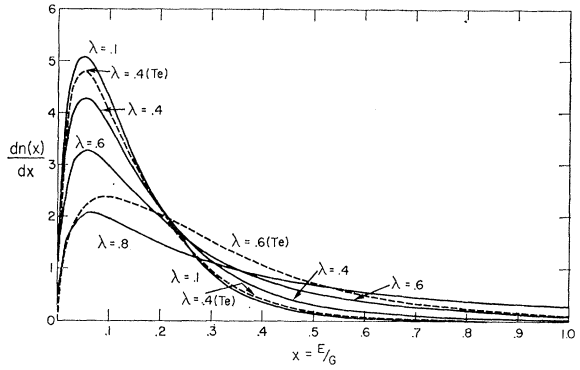

FIG. 2. Carrier occupation density for different λ as a function of x for InAs at 300°K.

of the distribution function, in the case of nonparabolic bands, at intermediate energies. However, extensive tails appear as expected from Eq. (10c).

Figure 2 presents the number-occupation density (in units of $\frac{1}{2}n_0$) as a function of $x=E/G$, for different λ. Comments (a) and (b) apply here too. In addition, note the electron-temperature-model curve labeled "$\lambda=0.6\times(T_e)$." At higher energies, the difference between it and the nonparabolic case for $\lambda=0.6$ is not as great as for the corresponding distribution functions shown in Fig. 5. The reason is that the larger density of states for nonparabolic bands offsets this difference somewhat.

In Fig. 3, curve 1 is the average energy, $\bar{x}(=\langle E\rangle/G)$, as obtained here, whereas curves 2 and 3 are the corresponding plots in the case of parabolic bands at temperatures of 300 and 330°K. The latter temperature was chosen since it represents the effective temperature for which the present calculations have been done. It is apparent that there is very little difference between the three curves up to about $\lambda=0.6$. Thereafter, collective breakdown occurs for the electron-temperature

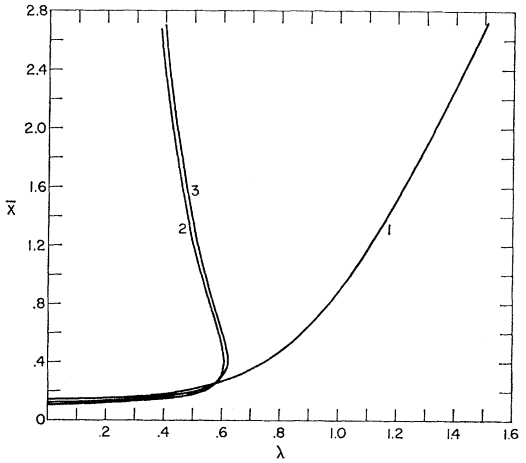

FIG. 3. Average energy as a function of λ for InAs: (1) present calculation at 300°K; (2) electron temperature, parabolic band model at $T=300$°K; (3) same as in (2) but with $T=330$°K.

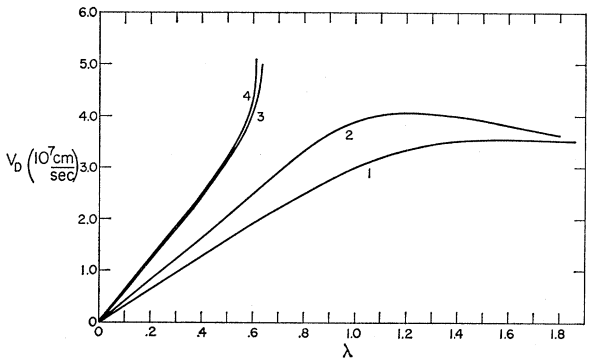

FIG. 4. Drift velocity as a function of λ: (1) InSb at 290°K, present calculation; (2) InAs at 300°K, present calculation; (3) InSb at 290°K, electron temperature, parabolic band; (4) InAs at 300°K, electron temperature, parabolic band.

model, whereas in the present work $\bar{x}$ is always monotonically increasing with λ.

The results of the last three figures then suggest the following conclusion: In the region prior to collective breakdown, the electron-temperature model in parabolic bands works fairly well, because the assumed hot-carrier containment by electron-electron collisions is effectively replaced by electron-phonon collisions alone in nonparabolic bands.[24]

The other quantity of interest here is the drift velocity as a function of field. This has been calculated for both InSb (290°K) and InAs (300°K). For InSb the following parameters were used[22,23]: $G=0.18$ eV; $\Delta=0.81$ eV; $m_n=0.014m$, $\Theta=264$°K; $\epsilon_s=18.7$, $\epsilon_\infty=15.7$; $\Omega/K=1.08\times10^{-18}n_0$; $F_0\cong600$ V/cm; $K(\lambda=0)=12.8$. The results are shown in Fig. 4. Curve 1 is for InSb and curve 2 for InAs. Curves 3 and 4 are the corresponding ones for the parabolic-band case. Note the difference, both in magnitude and in curvatures, in the two sets of curves. It is remarkable that the drift velocity remains approximately linear with electric field (i.e., the mobility is independent of field) up to $\lambda\cong0.9$ ($F\cong540$ V/cm) for InSb and $\lambda\cong0.9$ ($F\cong1800$ V/cm) for InAs. These values of λ correspond to average energies $\langle E\rangle=0.66$ G for InAs (Fig. 3) and $\langle E\rangle=0.63$ G for InSb. The mobilities are 20 000 cm²/V sec and 53 000 cm²/V sec for InAs and InSb, respectively. They agree fairly well with mobilities reported in other calculations[22,23,25] (for the same F_0) and measurements. Experimental evidence in support of the constant mobilities with field is indicated by the papers of Steele and Tosima,[12] and of Prior.[13] Steele and Tosima's data suggest a constant mobility in InAs at 300°K up to $F\cong1000$ V/cm ($\lambda\cong0.5$), which extends above the avalanche threshold field ($F=750$ V/cm) and above which electron-hole scattering occurs. The ohmic mobility measured by these authors, however, is only 13 000 cm²/V sec, which is low and suggests complications due to impurity banding effects.[12] Prior reports a constant mobility in InSb at 290°K up to 800 V/cm ($\lambda\cong1.3$) and the low-field mobility of one of his samples (42 000 cm²/V sec) agrees fairly well with the ohmic mobility used here. His data are probably not reliable at higher fields (nor is the present calculation; see below), because impact ionization appears to set in, as stated in the Introduction.

Physically, the constant mobility can be understood in terms of the nonparabolic band. It is a result of canceling effects of increasing mass and decreasing scattering rate (although not as much as for parabolic bands) with energy. The correct wave functions play an important role in the scattering rate and the distribution function.

It is noteworthy that beyond 540 V/cm in InSb and 1800 V/cm in InAs the drift velocity starts deviating from linearity.[26] In terms of the physical picture, this occurs at the point at which the increasing mass becomes the dominant influence. Beyond $\lambda=1.2$ in InAs and $\lambda=1.6$ in InSb, the drift velocity actually decreases with field, because of the increasingly heavy mass. However, this result is not completely trustworthy, since at $\lambda=1.2$, $\langle E\rangle\cong1.5$ G for InAs, and at $\lambda=1.6$, $\langle E\rangle\cong1.8$ G for InSb. Many carriers are therefore at energies much greater than $\langle E\rangle$. At these energies, the γ used here is most likely becoming invalid. The effect of higher bands must be considered in the $\mathbf{k}\cdot\mathbf{p}$ method. Furthermore, other energy-loss mechanisms, such as acoustic modes, intervalley scattering, or impact ionization, may become effective. Most likely, however, the inclusion of higher bands and/or other energy-loss mechanisms[27] would give rise to a negative differential drift velocity at higher fields. This would manifest itself in an instability in the current-voltage characteristic. It could perhaps be observed by the application of short voltage pulses, of the order of 10^{-9} sec or less. Note that this instability is similar in nature, but not quite the same as the one proposed in I. That instability would arise because of a double-valued relation between $\langle E\rangle$ and F over a certain range of F (an unstable region energetically), which does not occur in InSb or InAs (Fig. 3). It is expected that this instability might occur in some of the wider band-gap materials like GaAs, InP, CdTe, etc.

It is unlikely that, at the temperature for which the present calculations have been done, any streaming occurs; most electrons are already at an energy greater than $k\Theta$ at zero field, a situation which should be enhanced at higher fields by the nature of the polar-phonon interaction. If one plots the ratio of the drift to root-mean-square velocities (assuming $m^*\approx0.03m$

[24] It should be remembered that the shape of f_0 is not very important for a quantity like the average energy. It is important, however, for phenomena like impact ionization, and the transfer of electrons to upper valleys in the Gunn effect (see Ref. 25).

[25] E. M. Conwell and M. O. Vassell, Phys. Letters **25A**, 302 (1967).

[26] A decrease in mobility has also been reported at much lower fields in a calculation by Dykman and Tomchuk (Ref. 1) for InSb, using $\gamma=E+E^2/G$, and plane waves for the wave functions.

[27] C. Hilsum and J. Welborn, J. Phys. Soc. Japan Suppl. **21**, 532 (1967).

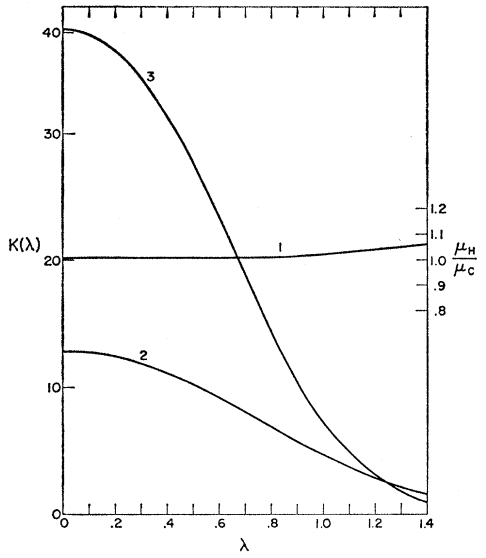

FIG. 5. (1) Ratio of Hall to drift mobilities at low magnetic fields as a function of λ (right scale); (2) $K(\lambda)$ as a function of λ for InSb at 290°K; (3) $K(\lambda)$ as function of λ for InAs at 300°K.

for InSb and $m^*\approx 0.05$ for InAs as average effective masses in the nonparabolic bands), one obtains maximum values of 0.25 and 0.31 at $\lambda=0.9$ and $\lambda=0.7$ for InSb and InAs, respectively. These values were obtained by using theoretical drift velocities, which are higher than the reported experimental ones.

Finally, the ratio of the Hall to drift mobility has been calculated numerically for vanishing magnetic field. The result is shown for InSb at 290°K in Fig. 5. The relative insensitivity to the electric field, the ratio remaining almost 1, is not very surprising. It is consistent with the constant mobility, a manifestation of a weakly dispersive scattering mechanism.

On the same figure (Fig. 5), the normalization constants K for InSb (290°K) and for InAs (300°K) are plotted. They are of interest because their existence comes about only because of no electron runaway.

ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to J. Pabrinkis for performing the programming and computing associated with this work.

APPENDIX

The derivation of Eq. (8) of the text will be presented here in greater detail. To this end, the same notation as in I is used. Equation (21) of I, giving the collision integral, will be the starting point. Using Eq. (2) for $N(E)$, and Eq. (4) for $B(k,k',y)$, one has for the collision integral

$$L_0f_0(E)=\frac{eF_0}{(2m_n)^{1/2}}\int_u^{V_+}\frac{1}{q}\{\gamma'(E+k\Theta)\mathcal{G}(q;E;E+k\Theta)\times[\mathfrak{N}f_0(E)-f_0(E+k\Theta)(\mathfrak{N}+1)]\}dq$$

$$+\frac{eF_0}{(2m_n)^{1/2}}\int_u^{V_-}\frac{1}{q}\{\gamma'(E-k\Theta)\mathcal{G}(q;E;E-k\Theta)\times[(\mathfrak{N}+1)f_0(E)-\mathfrak{N}f_0(E-k\Theta)]\}dq, \quad \text{(A1)}$$

where

$$u=\tfrac{1}{2}k\Theta\gamma'/\gamma^{1/2}, \quad V_+=(4\gamma)^{1/2}+u, \quad V_-=(4\gamma)^{1/2}-u.$$

$\mathcal{G}(q;E;E\pm k\Theta)$ has been obtained from Eq. (7) by relating k to $\gamma(E)$, k' to $\gamma(E\pm k\Theta)$, and y to q by the relation (compare I)

$$\gamma(E)+\gamma(E\pm k\Theta)-2[\gamma(E)\gamma(E\pm k\Theta)]^{1/2}y=q^2. \quad \text{(A2)}$$

The problem now is to expand Eq. (A1) in the ratio of $k\Theta/E$ to second order, and thus obtain a readily integrable differential equation of the second order for f_0. A complication arises, however, from the dependence of $\mathcal{G}$ on $E\pm k\Theta$. This may be overcome if it is realized that $\mathcal{G}$ can be written in the form

$$\mathcal{G}(q;E;E\pm k\Theta)=\sum_n P_n(E)Q_n(E\pm k\Theta)R_n(q), \quad \text{(A3)}$$

where the function P_n, Q_n, R_n can readily be obtained by comparison with Eqs. (7) and (A2). Their detailed form is not important here and therefore will not be given.

Substitution of (A3) into (A1) puts the collision integral into a form suitable for expansion in a manner similar to that given by Stratton.[28] After the expansion is carried out for each term in $\mathcal{G}$, one has

$$L_0f_0(E)=\frac{1}{(2m_n\gamma)^{1/2}\gamma'}\sum_n\left(\frac{P_n}{Q_n}\right)\frac{d}{dE}\times\left[(\gamma'Q_n)^2G_{n1}\left\{f_0+\eta\frac{d}{dE}f_0\right\}\right], \quad \text{(A4)}$$

with

$$G_{n1}=eF_0k\Theta\int_u^{(4\gamma)^{1/2}}\frac{R_n(q)}{q}dq, \quad \eta=k\Theta(\mathfrak{N}+\tfrac{1}{2}). \quad \text{(A5)}$$

If $P_n=Q_n$ for all n, Eq. (A4) would simplify greatly. However, inspection of Eq. (7), together with Eq. (A2), reveals that this is not so: For eight terms, $P_n\neq Q_n$. One avoids this difficulty as follows. The drift term in the Boltzmann equation is

[28] R. Stratton, Proc. Roy. Soc. (London) A242, 355 (1957).

$$\frac{2}{3}\frac{eF}{\gamma'(2m_n\gamma)^{1/2}}\frac{d}{dE}\gamma f_1(E)$$
$$=-\frac{2}{3}\frac{e^2F^2}{m_n\gamma'(2m_n\gamma)^{1/2}}\frac{d}{dE}\left(\frac{\gamma}{\gamma'}(2m_n\gamma)^{1/2}\tau\frac{df_0}{dE}\right), \quad \text{(A6)}$$

where τ is the relaxation time. Equating (A6) to (A4) and integrating both sides, Eq. (A4), by parts, yields

$$\frac{2}{3}\frac{e^2F^2}{m_n}\frac{\gamma}{\gamma'}(2m_n\gamma)^{1/2}\tau\frac{df_0}{dE}=(\gamma')^2\left(f_0+\eta\frac{df_0}{dE}\right)\left(\sum_n P_nQ_nG_{n1}\right)$$
$$+\int^E(\gamma')^2\left(f_0+\eta\frac{df_0}{dE'}\right)\left(\sum_n\left[P_n\frac{dQ_n}{dE'}-Q_n\frac{dP_n}{dE'}\right]G_{n1}\right)dE'. \quad \text{(A7)}$$

One can now show that the sum over n in the integral term of Eq. (A7) equals zero. The sum over the eight terms for which $P_n\neq Q_n$ gives zero, and for the remainder of the terms, $P_n=Q_n$. One has furthermore

$$\sum P_nQ_nG_{n1}=eF_0k\Theta\int_u^{(4\gamma)^{1/2}}\frac{1}{q}\mathcal{G}(q;E;E)dq$$
$$=eF_0k\Theta\mathcal{G}_1, \quad \text{(A8)}$$

where $\mathcal{G}_1$ is given in the text.

The relaxation time is also readily obtained (compare I), and is given by Eq. (9) of the text; $\mathcal{G}_0$, which is also given there, is

$$\mathcal{G}_0=\frac{1}{2}\int_{-1}^{1}\mathcal{G}(y;E;E)dy.$$

Putting all these equations together, one readily integrates Eq. (A7) to obtain Eq. (8) of the text.

PHYSICAL REVIEW VOLUME 168, NUMBER 3 15 APRIL 1968

de Haas–van Alphen Effect and Fermi Surface of White Tin*

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We report here the results of an extensive investigation of the de Haas–van Alphen effect in white tin. These results agree qualitatively with the predictions of Weisz's pseudopotential model for the tin Fermi surface; for every case in which Weisz's model differs qualitatively from the single-orthogonalized-plane-wave model, our data agree with the former. Quantitative agreement, however, has not been obtained. The experimentally determined cross-sectional areas of the Fermi surface typically differ by about 30% from those calculated by Weisz. Area branches are also reported that are due to coupled orbits generated by magnetic breakdown of the spin-orbit-coupling-induced energy gap between the third and fourth bands near the XL and XP zone lines.

I. INTRODUCTION

THE electronic properties of metallic ("white") tin have been studied in considerable detail over the past few years. One of the primary reasons for this is the relatively long electron mean free path that can be obtained in pure tin crystals at temperatures of about 1°K. These experimental studies have fallen into two basic categories; those which have used the electronic properties relevant to the Fermi surface to study the Fermi surface itself,[1–18] and those which have studied the purely dynamical aspects of the electronic prop-

* Supported in part by the U. S. Army Research Office (Durham), the National Science Foundation, and the Advanced Research Projects Agency.
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