Orientational Dependence of the Flux-Flow Resistivity and Critical Current Density in Type-II Superconducting Foils*

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Flux-flow measurements on type-II superconducting foils show that the Lorentz force is fully adequate to describe the angular dependence of the flux-flow resistivity. The orientational dependence of the critical-current density is not simply obtained, however, by assuming that flow begins when the Lorentz force achieves a critical value. Instead, the results imply an anisotropy in the pinning force which we interpret in terms of the natural anisotropy of the surface in a foil sample. The role of the surface in flux pinning is therefore emphasized again.

I. INTRODUCTION

THE problem of energy dissipation associated with flux motion in type-II superconductors has been investigated under several experimental situations.[1–3] One such situation is obtained when a transport current is made to flow in a superconductor in a perpendicular magnetic field.[4] Under these circumstances, a Lorentz force acts to drive flux across the sample, against pinning forces, and against a retarding viscous force.

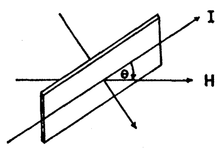

FIG. 1. Sample geometry. The applied field H makes an angle θ with the sample surface and with the transport current I.

We restrict our discussion to foil samples and use the arrangement shown in Fig. 1. Thus the field H makes an angle θ to the current I (current density J) and to the surface. The field penetration is in the form of quantized vortices, of flux quantum ϕ_0. The Lorentz force per unit length of vortex is

$$F_L=(J\phi_0/c)\sin\theta. \tag{1}$$

If pinning forces F_P are present, these inhibit flux motion until the Lorentz force becomes sufficiently large to overcome them. We assume that F_P can be written in terms of a critical pinning current density J_P, such that

$$F_P=J_P\phi_0/c. \tag{2}$$

We would expect that J_P would be dependent on field, but for isotropic sources of pinning, independent of orientation.

If the Lorentz force is large enough to overcome the pinning force, the fluxoids move across the sample with a velocity V_L, and energy dissipation is observed. We describe the motion in terms of a viscous force ηV_L, where η is a viscosity coefficient. The equation of motion[5] is simply

$$F_L-F_P=\eta V_L. \tag{3}$$

If there are N fluxoids per unit area in the sample, then at sufficiently high fields, $H\sim B=N\phi_0$.

The dissipation arises because the flux motion creates electric fields in the vortex cores, and these cause normal currents to flow within the cores.[6,7] A macroscopic electric field E is established along the sample, and the Joule heat EJ developed may be equated to the power supplied by the Lorentz force NF_LV_L. Combining this with Eqs. (1)–(3), we obtain for E,

$$E=(\phi_0H/\eta c^2)(J\sin^2\theta-J_P\sin\theta). \tag{4}$$

Kim[4,5] and others[8] have reported on the current-voltage curves in the perpendicular field orientation ($\theta=90°$). At sufficiently high currents, they find linear current-voltage characteristics. If we define a differential flow resistivity $\rho_F=dE/dJ$, then ρ_F is linear in H, in agreement with Eq. (4).

We have measured the orientational dependence of ρ_F and of the critical-current densities required to initiate flux flow. The present measurements indicate that the Lorentz force is fully adequate to describe ρ_F. However, the orientational dependence of the pinning force confirms our previous conclusions that pinning in cold-worked foils is principally associated with the surface.

Swartz and Hart[9] have also measured the orientational dependence of critical currents in samples similar to our own, but with H varied about a different set of axes. They have explained their results partially in terms of critical surface currents. We find certain differences between our results and theirs, and offer arguments opposed to the idea of critical surface currents below H_{c2}.

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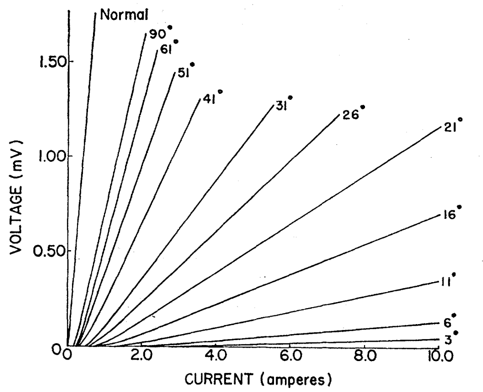

FIG. 2. Current-voltage characteristics at $H=1820$ Oe at different values of θ for well-annealed $Pb_{60}Tl_{40}$ foil 0.0072 mils thick. $H_{c2}=2850$ Oe for this sample. At sufficiently high currents, the characteristics are linear, and the extrapolation procedure used for defining the critical current seems justified.

II. EXPERIMENTAL PROCEDURE

Our measurements have been made on Pb–Bi and Pb–Tl alloys. Ingots are prepared by melting the high-purity constituents in glass under high vacuum. The ingots are first pressed into flat plates about 100 mils thick. Foils are then obtained by cold rolling or by further compression between glass plates.

The rolled foils undergo several passes through the rolling mill, with the orientation being changed in each pass so that no single direction of cold working should dominate. Therefore, our assumption that the volume-pinning sites are isotropic, on which Eq. (2) is based, seems justified. To ensure that this condition was met and to obtain variable critical-current densities, compressed foils were also prepared by pressing the alloy between glass microscope slides. Such a procedure produces much smoother surfaces and also much lower critical-current densities at $\theta=90°$.[10]

The resulting foils were cut into strips approximately 1.5 in.$\times$0.140 in. and with thicknesses ranging from $\sim$0.0025 to 0.010 in. The Pb–Bi samples (3–15% Bi) were annealed under vacuum for one day at 110°C, a temperature which relieved little of the cold work. The Pb–Tl samples (40% Tl) were vacuum-annealed for one day at 330°C and had very low critical currents.[9] These samples actually developed a shinier surface during the anneal and were highly reflecting until the vacuum was broken. During the mounting procedure, they began to tarnish, but were still quite shiny when inserted in the Dewar.

The sample rig was a standard four-probe device with knife-edge potential contacts 1.6 cm apart. Although these contacts may have reintroduced some cold work into the sample, the current-voltage characteristics were quite linear, and we believe that the use of pressure contacts created little sample inhomogeneity.

Samples were oriented by placing them in a magnetic field H such that $H_{c2}<H<H_{c3}$, and varying θ until a resistance minimum was obtained with fixed current. Although this procedure was sensitive to variations of about 0.1° at fixed field and current, the actual position of the minimum depended to some extent on the level of current and field. We do not understand the reason for this, but we note that these variations could be as large as 0.3°, and we take this as our uncertainty in determining $\theta=0$. For consistency, we define $\theta=0$ as the position of the resistance minimum just above H_{c2}.

Having determined this position, H_{c2} was measured in our earliest samples by observing the onset of the resistance transition at various current levels and choosing as H_{c2} the field where the onset of the transitions converge at higher currents.[11] In the majority of our samples, we also found H_{c2} by measuring the critical currents at decreasing fields and choosing as H_{c2} the discontinuity in this curve as the currents change from surface currents to bulk currents.[12] This was a much more sensitive procedure and reproducibly yielded values of H_{c2} to within a few oersted.

Current-voltage characteristics were obtained at 4.2°K for various fixed fields as a function of the angle θ. Continuous characteristics were plotted on a Moseley Model 7000-A X-Y recorder after first amplifying the sample voltage using a Keithley No. 148 Nanovoltmeter. Measurements were carried out at sufficiently high current densities to determine the linear portion of the characteristic. This is then extrapolated to $V=0$; the intersection with the current axis defines the critical current to initiate flux flow. A set of characteristics is reproduced in Fig. 2 for a well-annealed Pb–Tl sample 0.0072 mils thick at a field of 1820 Oe ($H_{c2}=2850$ Oe). The characteristics are clearly linear at sufficiently high currents.

This present definition of critical-current density is not one which has previously been employed. We believe it has a distinct advantage in that it depends on a well-behaved current-voltage characteristic. Previously em-

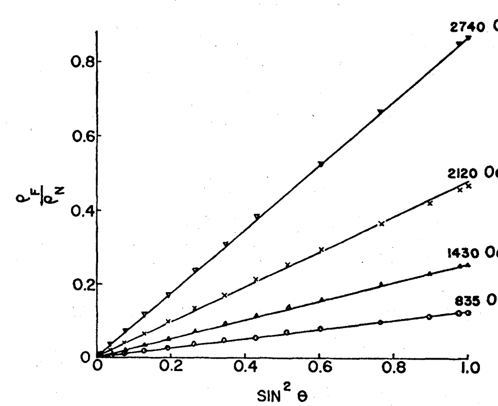

FIG. 3. Angular dependence of the flux-flow resistivity. The normalized flow resistivity ρ_F/ρ_N is plotted versus $\sin^2\theta$ at various fields for the same sample as in Fig. 2. Data demonstrate that flow resistivity depends only on the Lorentz force and on the viscous resistance to the flux motion.

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ployed criteria, such as the appearance of a minimum voltage across a sample, involve a certain ambiguity. Since critical-current densities can vary over several decades, depending on magnetic field, the question of the role of heat generation in the restoration of resistance is always vital. Many observers report samples burning out when the critical current is exceeded. The present method circumvents the question of heating effects, since a linear characteristic is not obtained when heat generation is important. We see this in our samples as we continue to increase the transport current to high values. Also, Jones, Rhoderick, and Rose-Innes[13] have shown that the nonlinear low-voltage portion of the characteristic is a function of sample inhomogeneity. It is therefore not clear that data obtained from this portion of the curve should have any direct relationship to bulk properties.

III. EXPERIMENTAL RESULTS

A. Flux-Flow Resistivity

The orientational dependence of the flux-flow resistivity ρ_F can be obtained from Eq. (4):

$$\rho_F = dE/dJ = (\phi_0 H/\eta c^2)\sin^2\theta. \qquad (5)$$

Thus, for a fixed field, the simple Lorentz force description predicts a $\sin^2\theta$ dependence for the flow resistivity.

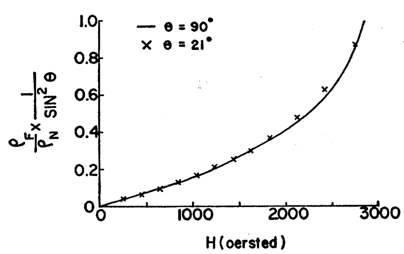

FIG. 4. Field dependence of the flux-flow resistance. The normalized flow resistivity ρ_F/ρ_N is plotted as a function of field at $\theta=90°$ (solid curve). The crosses show this ratio obtained at $\theta=21°$ divided by $\sin^2\theta$. Again the data demonstrate that the flux-flow resistivity is determined only by the Lorentz force and the viscosity coefficient.

We have verified this relationship for a number of samples and various field values, as demonstrated in Fig. 3, for the same Pb–Tl sample shown in Fig. 2. Similar data are obtained with all other samples even though the various samples possess vastly different pinning forces and critical currents. The flux-flow resistivity is therefore independent of the pinning forces and depends only on the Lorentz force and the viscous resistance to the motion.[14]

We emphasize these results by showing in Fig. 4 the flux-flow resistivity of this sample obtained as a function of field at $\theta=90°$ (solid curve) and the flux-flow resistivity obtained at $\theta=21°$ (crosses) normalized by $\sin^2\theta$. The coincidence of the data is apparent. We note that these results contradict those of Swartz and Hart,[9] who obtain data on the flux-flow resistivity when the field is rotated about the longitudinal axis of the sample so that the Lorentz force remains constant ($J \perp H$). They found that the flux-flow resistivity decreases as the field becomes more nearly parallel to the surface. We do not understand the reasons for these seemingly contradictory results.

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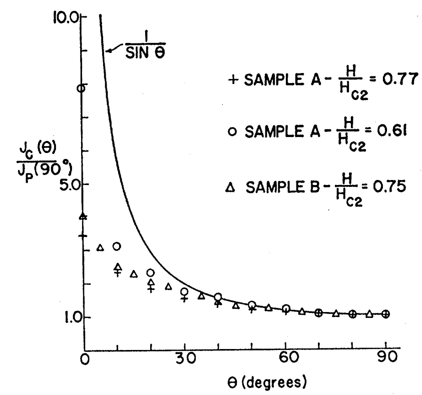

FIG. 5. Angular dependence of the critical pinning current density for a $Pb_{90.7}Bi_{9.3}$ rolled foil. The normalized critical current density $J_c(\theta)/J_P$ is shown as a function of θ. The solid curve is $1/\sin\theta$. Sample B was heavily etched and had half the critical current density of sample A.

B. Critical-Current Density

We can also obtain the orientational dependence of the critical-current density from Eq. (4). Using our definition of J_c as the extrapolation of the flow characteristic to $E=0$, we obtain the value of J_c at the

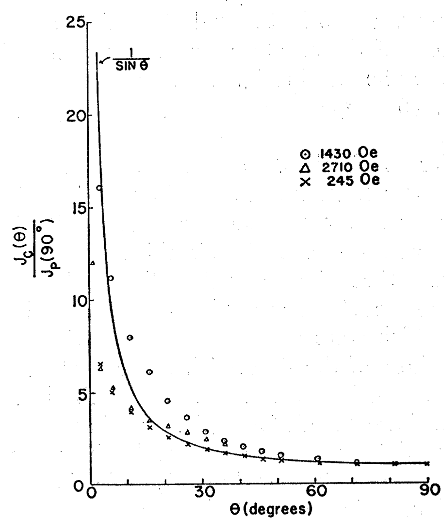

FIG. 6. Angular dependence of the critical pinning current density for $Pb_{60}Tl_{40}$ sample on which data were presented previously. The solid curve is $1/\sin\theta$. The data show the smallest deviation from this curve at small angles for $H/H_{c2}\sim0.5$. Note that with this sample $J_c(\theta)/J_P$ is greater than $1/\sin\theta$ at intermediate angles. Pb–Bi samples which were pressed rather than rolled, and which therefore had smoother surfaces, showed intermediate behavior in this angular range.

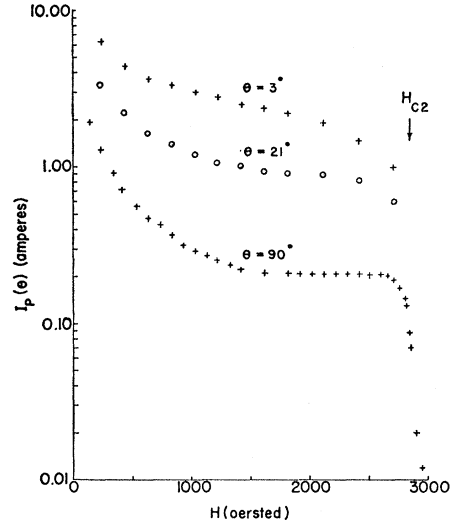

FIG. 7. Magnetic-field dependence of critical current for same $Pb_{60}Tl_{40}$ foil. Data are presented for $\theta=3°$, 21°, and 90°.

angle θ, $J_c(\theta)$;

$$J_c(\theta)/J_P=1/\sin\theta, \tag{6}$$

where J_P is the value of J_c at $\theta=90°$.

Data representative of our measured results are shown in Figs. 5 and 6. In Fig. 5, we show data on a rolled 9.3%-Bi sample before and after etching. In the latter case, J_P is 50% of what it was before etching. Clearly the results are not described by Eq. (6). Instead, for angles less than about 60° the critical-current density is less than that predicted by assuming that flow begins when the Lorentz force achieves a certain critical value. Qualitatively, for samples prepared in this manner, the orientational dependence is nearly independent of the absolute magnitude of the pinning forces, as can be seen in Fig. 5. Note that the deviations from Eq. (6) appear to be less as the applied field decreases.

In order to obtain data at lower fields and small angles, it was necessary to obtain samples with still lower critical currents, because sample heating became a problem with the rolled foils in this region. It was for this reason that the pressed well-annealed foils of Pb–Tl were used. These samples in general had critical currents which were at least one order of magnitude less than those of the Pb–Bi foils. Even with these reduced currents, it was not possible to obtain reliable data at the lowest fields for $\theta\lesssim 3°$. Limiting ourselves to $\theta\geq 3°$, our data at the lowest fields shown are still probably defined only to within 5%, taking into account the ambiguity in extrapolating to $V=0$ to define $J_c(\theta)$. At larger angles and fields, $J_c(\theta)$ is, of course, defined much more precisely.

Typical data for these latter foils are shown in Fig. 6 for the same Pb–Tl specimen shown previously. Again, at small angles, the critical currents are smaller than that predicted by Eq. (6). These results differ from those on the rolled foils in that at intermediate angles $J_c(\theta)/J_P$ is slightly *larger* than $1/\sin\theta$. The data presented in Fig. 6 show results for $H/H_{c2}=0.95$, 0.50, and 0.09. At small angles, the deviations from the $1/\sin\theta$ curve are smallest at $H/H_{c2}\sim 0.5$. The fluxoid pinning strength therefore has a field dependence which is a function of angle.

We demonstate this latter fact in Fig. 7, where data are presented for the same sample showing the field dependence of the critical current at $\theta=3°$, 21°, and 90°. The different field dependence for the pinning at different angles is also demonstrated in Fig. 8, where $J_c(\theta)/J_P$ is plotted as a function of field at different values of θ. The peaking at $H/H_{c2}\sim 0.5$ and small angles seems to be a general feature of all samples, independent of the method of preparation.

IV. DISCUSSION

Kim and co-workers[15,16] have previously analyzed critical currents in tube-magnetization measurements in terms of a critical-state concept. Thus flux-flow is observed when the driving force exceeds some critical parameter α_c. In terms of Kim's data, critical-current densities are obtained from the relationship[11]

$$J_c=\alpha_c/(H+B_0). \tag{7}$$

The Lorentz force is thus modified through the constant B_0, the origin of which remains obscure.

The relationship has been extended to include the case where H is at the angle θ to J. Cody *et al.*[17] and Swartz and Hart[9] thus find

$$J_c=\alpha_c/(H\sin\theta+B_0). \tag{8}$$

Our data on the angular dependence of the flux-flow

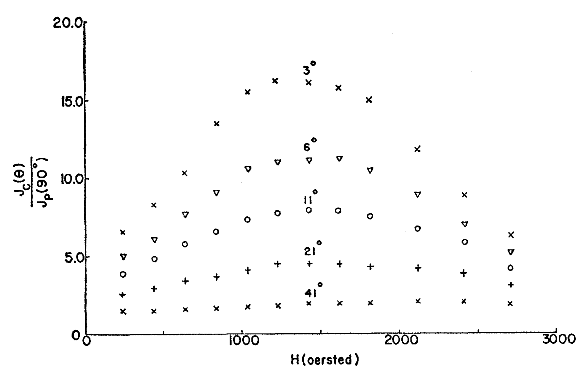

FIG. 8. Magnetic-field dependence of the ratio $J_c(\theta)/J_P$ for same $Pb_{60}Tl_{40}$ foil at various angles. At small angles, this ratio has a maximum at $H/H_{c2}\sim 0.5$.

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resistivity indicate that the Lorentz force description is perfectly valid in type-II foils and that the modification required by Eq. (8) is therefore unjustified. On the other hand, the angular dependence of the critical-current is not obtained simply by equating the Lorentz force to some critical value. The critical current increases more slowly at small angles than such an approach requires, as is suggested by Eq. (8).

We believe that the resolution of this apparent contradiction can be found in the surface dependence of pinning. Previously,[10] we have shown that for identically prepared foils the critical-current density is proportional to the surface-to-volume ratio of the foil, approaching zero as this ratio becomes zero. Without a surface there would therefore be no pinning. Other effects emphasize surface pinning. Plating the surface with a nickel film, etching the surface, or otherwise providing a smoother surface reduces the critical-current density markedly.[9,10]

In obtaining Eq. (4), we assumed that pinning was isotropic. This was justified on the basis of isotropy in the volume defects because of the manner of sample preparation. The failure of Eq. (4) in describing the orientational dependence of the critical-current density may be interpreted in terms of the existence of an anisotropy in the pinning forces. For a foil sample, the surface provides a natural source of such anisotropy, and one which will be similar for various samples independent of the absolute magnitude of the pinning force. We therefore take the present results as further evidence of the role played by the surface in pinning.

Previously, we explained the importance of the surface in pinning in terms of the results of Pearl,[18] who found that the electromagnetic region of the vortex spreads at the surface. Thus at the surface the range of interaction is extended, and the fluxoid lattice is stiffened. If fluoxids are to move, they must do so collectively at the surface, and hence must wait until a driving force equal to the maximum pinning force is provided. Inside the foil, individual fluxoids may slip past weaker pinning sites at a lower driving force. Since for a given current density the Lorentz force is proportional to the thickness, the observed surface-to-volume ratio dependence of the critical-current density is obtained.

We suggest that the present results can also be understood qualitatively in terms of the influence of the spread in the field region of the vortex at the surface. As the field deviates from a perpendicular orientation to the surface, the surface plane cuts a larger cross section of the vortex, and this adds to the intrinsic spread of the electromagnetic region at the surface. The fluxoid lattice is therefore stiffened further, and the critical current increases above $1/\sin\theta$ because of the enhanced surface pinning (Fig. 6). Since the intrinsic spread described by Pearl only exists within a penetration depth of the surface, we can also explain why $J_c(\theta)/J_P$ falls below $1/\sin\theta$ at small angles. This is because now only a small fraction of the fluxoids will intersect a surface, and the entire fluxoid lattice will not be stiffened.

As an alternative to this explanation, Swartz and Hart[9] have previously suggested that the critical currents are surface currents, and that these surface currents increase as the magnetic field becomes aligned with the surface (in our case, as θ decreases). They were led to this conclusion by the observation[9] that Cu plating reduces the critical current at all orientations. We have concluded that plating reduces surface pinning and hence reduces the *bulk* critical current.

The results of these two mechanisms differ in one important respect. If transport currents flow principally in the surface, then the Lorentz force in the bulk is greatly reduced from what one would expect, assuming a uniform current distribution. This would be manifest in a reduced flux-flow resistivity, and, in particular, one which would decrease more rapidly with decreasing θ than the sine-square dependence of Eq. (5). We do not observe such an effect. We do note, however, that even though a number of our samples had critical-current densities as low as or lower than the annealed unplated samples of Swartz and Hart,[9] we never observed a "peak effect." There may, therefore, have been an intrinsic difference in the two sets of specimens.

We cannot offer even a qualitative explanation for the peaking of $J_c(\theta)/J_P$ at $H/H_{c2}\sim 0.5$. However, there is no existing theory which accounts for the field dependence of the critical current at $\theta=90°$. Attempting to understand this more complicated ratio is even more difficult.

Finally, we note that Deltour and Tinkham[19] have reported on the angular dependence of the critical field for thin type-I films carrying a constant current. When this experiment was performed with our type-II foils, we found a much more gradual dependence of the critical field on θ at small angles. This is not surprising, since the vortex structure and flow characteristics for films is determined only by the field component perpendicular to the surface.[20] We believe that this factor accounts for the differences between our results and the results of Swartz and Hart[9] for films. We take as confirmation of this the difference in their own data for the angular dependence of the critical current when H remains in the plane of the surface but varies with respect to the direction of the current. In their bulk foils, the angular dependence of Eq. (8) was obtained, but in their films no systematic θ dependence was observed. This is understandable, since in the latter case the critical currents are not limited by vortex motion.

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