Reconciliation of Experimental Lamb Shifts*

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We develop a detailed analysis of the metastable quenching problem encountered in our recent level-crossing measurements in the $n=2$ state of atomic hydrogen. The analysis allows calculation of a previously unaccounted systematic correction to the value of the Lamb shift $\mathcal{S}$ derived from these measurements. This correction, together with refinements in the calculation of $\mathcal{S}$ from a measured crossing, decreases our value to $\mathcal{S}=1057.86\pm0.10$ MHz. Within experimental error, this value agrees with the original result as measured by Lamb and collaborators. In addition, the analysis allows the calculation of a new upper limit to the parity impurity amplitude in the $2S$ state of atomic hydrogen, namely $|a_0|^2\leq 8\times10^{-9}$, at zero magnetic field.

I. INTRODUCTION

RECENTLY, we reported a remeasurement of the Lamb shift $\mathcal{S}$ in the $n=2$ state of atomic hydrogen by a level-crossing (LC) method.[1] The LC value of $\mathcal{S}$ exceeds the most recent theoretical value[2] by about 5 ± 2 parts in 10^4. Also, it exceeds the original Lamb value[3] by about 3 ± 2 parts in 10^4. The former discrepancy relates to questions on the validity of quantum electrodynamics.[4] The latter relates indirectly to questions regarding the value of the fine structure constant.[5]

These discrepancies amount to only a few tenths of one percent of the linewidths of the metastable quenching resonances measured in both the Lamb and LC experiments. Clearly, in order to achieve such high resolution, a rigorous analysis of the specific metastable quenching problem encountered in each experiment is needed. Lamb has developed the basic theory, and derived the necessary detailed analysis of specific quenching effects observed in his experiments.[6] We have outlined the application of the Lamb theory to the LC experiments in our previous work.[7] However, we derived only an approximate analysis of the observed metastable quenching, as various estimates of possible systematic effects were small, indicating that substantial detail was unnecessary.

In this paper, we extend our previous analysis of specific quenching effects observed in the LC experiments to include a detailed treatment of "accidental" quenching of the metastable beam by stray electrostatic fields, and by motional electric fields generated when the beam passes through a magnetic field. Initial motivation for this work was provided by an attempt to calculate the effects of a possible parity impurity in the metastable state. However, as the calculation revealed a new systematic correction to the LC value of $\mathcal{S}$, it became clear that a detailed analysis of the LC quenching problem was necessary. We present that analysis here, not only to calculate the new correction, which reconciles the LC value of $\mathcal{S}$ with the original Lamb value, but also to provide a firm basis for a rigorous analysis of present and continuing experiments employing techniques developed during the LC work. Results derived here provide a calculational method for treating all systematic effects thought to be important in the LC experiments and related work, and will be used in future publications in this series.

II. EXPERIMENTAL METHOD

A. General Remarks

The LC experiments have been described previously.[1,7] We produce a beam of hydrogen atoms in the metastable $2^2S_{1/2}$ state, and look for a resonant decrease of metastable intensity as the result of inducing $2S\rightarrow 2P$ transitions by application of a static electric field. The observed metastable quenching, plotted against magnetic field, shows a characteristic broad resonance at the $\beta(2^2S_{1/2}, m_J=-1)-e(2^2P_{1/2}, m_J=+1)$ crossing points near 574G. We denote the observable crossings in hydrogen by H(538) and H(605), which correspond, respectively, to crossings between hfs (hyperfine structure) levels $\beta(m_F=0)-e(m_F=+1)$ at 538G, and $\beta(m_F=-1)-e(m_F=0)$ at 605G. Location of a crossing point to 1 part in 10^4, together with an extrapolation to zero field along the Zeeman lines,[8] determines $\mathcal{S}$ to the desired 0.1 MHz precision (i.e., 1 part

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[5] E. R. Cohen, Nuovo Cimento Suppl. IV, 839 (1966).

[6] See Appendix II of HI, and HIII, Ref. 3.

[7] R. T. Robiscoe, Phys. Rev. **138**, A22 (1965).

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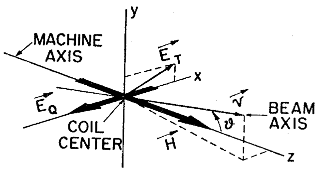

FIG. 1. Diagram of beam trajectory through transition region. The xy plane is the midplane of the Helmholtz coil. The Zeeman magnetic field H is applied along the z axis, which is lined up with the machine (geometric) axis. If the beam axis is "tilted" at angle θ to the z axis, then a motional electric field E_T is generated.

in 10^4). This requires that the resonance line be resolved to about 1 part in 10^3 of its FWHM (full width at half maximum).

The LC experiments can be described generally as Lamb experiments at zero frequency. However, there are several important differences in the methods employed which substantially change the detailed quenching analysis. First, by a state selection process, we prepare the metastable beam so that only one of the hfs states of level β is present in the transition region. This reduces the $\beta \rightarrow e$ quenching resonance from a superposition of hfs transitions to a single hfs transition, so that we can observe the H(538) and H(605) crossings separately. Second, rather than applying the Zeeman field transverse to the beam, we apply a longitudinal magnetic field by aiming the beam down the axis of a Helmholtz coil. In principle, this reduces accidental quenching of the metastables by motional electric fields. Finally, we work near a crossing point, where the metastable quenching is extraordinarily sensitive to any electric field. This amplifies accidental quenching by motional fields and by small stray electrostatic fields in the transition region.

As a consequence of these differences in method, the detailed analysis of the LC experiments is quite different from that of the Lamb experiments. The LC analysis is simplified by resolution of the hfs, since we deal with a single $2S \rightarrow 2P$ transition rather than a superposition. On the other hand, the LC analysis is complicated by the extreme sensitivity to accidental quenching effects near a crossing point. We must deal very carefully with the effects of all electric fields present in the transition region.

B. Fields in the Transition Region

The transition region is at the center of the Helmholtz coil, where the magnetic field H is uniform. Here we apply a transverse, well-localized electrostatic quenching field E_Q. Figure 1 shows the relative orientation of E_Q and H. During a run, H is swept through a particular $\beta - e$ crossing point, while a preset constant value of E_Q is switched on, then off, to determine its relative quenching effect on the β beam. For our beam, an applied quenching field $E_Q = \frac{1}{2}$ V/cm provides about 30% $\beta \rightarrow e$ quenching at resonance.

As indicated in Fig. 1, E_Q may not be the only field which quenches the beam. If the beam is accidentally

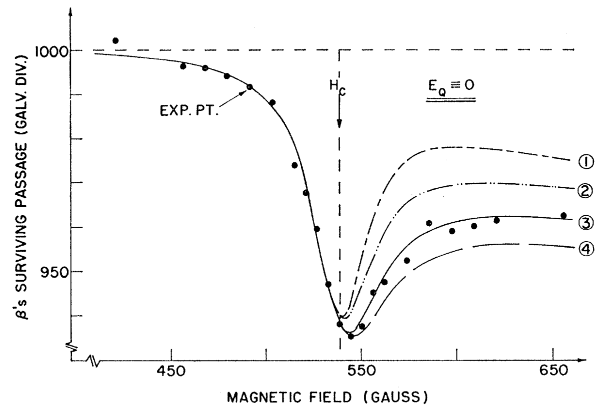

FIG. 2. Beam-notch curve for H(538) crossing. The total β beam is normalized to 1000 divisions at Zeeman magnetic field $H=0$. Accidental beam quenching, exhibiting a quasiresonance at the crossing point field H_C, is due to stray and motional electric fields. The experimental points are compared with curves calculated from Eq. (17). Curve parameters used are given in Table II.

tilted at a small angle θ to the coil axis, then a motional electric field E_T is generated in the xy plane, of magnitude $E_T = (v/c)H \sin\theta$, where v is the beam velocity. Despite careful lineup, an accidental beam tilt of as much as $\theta = 1°$ is possible for the machine geometry. Note that the component E_{Tx} of $\mathbf{E}_T$ which adds to $\mathbf{E}_Q$ may be positive or negative, depending on the beam orientation. Additional beam quenching arises from whatever stray fields E_S may be present (from charged insulating layers on electrode surfaces, contact potential differences, etc.). Values of E_T and E_S of magnitude only 0.1 V/cm give measurable effects in the LC experiments, namely order of 1% $\beta \rightarrow e$ quenching at resonance.

We can check for the presence of accidental quenching fields such as E_T and E_S by monitoring the β beam intensity versus magnetic field when E_Q is set to zero. Figure 2 shows the result of such an experiment during the H(538) work. Similar results, which we call "beam-notch" curves, were obtained during the H(605) work.[9] While the 6% loss of β intensity at the crossing point field H_C is undoubtedly due to accidental fields of net magnitude approximately 0.2 V/cm, a similar curve would result if there were a small parity impurity in the $2S$ state. In any case, the beam notch constitutes a varying background of β's available for quenching by the applied field E_Q. Appropriate normalization (namely, measuring the *fractional* beam quenching by E_Q) cancels most asymmetry effects in the observed resonance line shape due to this varying background. However, as the x-axis components of $\mathbf{E}_T$ and $\mathbf{E}_S$ effectively change the magnitude of $\mathbf{E}_Q$, we expect residual effects in the line shape due to the presence of these accidental fields.

C. Effects of Accidental Quenching

Nonzero components of $\mathbf{E}_T$ and $\mathbf{E}_S$ along $\mathbf{E}_Q$ (x axis in Fig. 1) can affect the line shape directly, as follows.

[9] See Fig. 5 of Ref. 7.

The induced $\beta \to e$ transition rate is proportional to $|\mathbf{E}|^2$, where $\mathbf{E}=\sum(\mathbf{E}_k)_\perp$ is the net electric field perpendicular to the magnetic field (xy plane of Fig. 1).[10] The observed $\beta \to e$ quenching line shape $F(H)$ is proportional to the rate difference when E_Q is switched on, then off.[11] Thus

$$F(H) \propto \{1+2[(E_{Tx}+E_{Sx})/E_Q]\}E_Q^2\mathcal{L}(H), \quad (1)$$

where $\mathcal{L}(H)$ is a Lorentzian resonance function, centered on the crossing point field H_C. As we sweep through a resonance, the cross term in E_{Sx} remains constant, and so does not asymmetrize the line. However, E_{Tx} does change. Depending on whether it increases or decreases the effective E_Q (beam tilted up or down in Fig. 1), either the right or left wing of the line is raised. This shifts the apparent line center, and results in an erroneously high or low value for $\mathcal{S}$. Estimates show that the shift can be as large as ± 0.1 MHz in $\mathcal{S}$, for a beam tilt angle $\theta = 1°$.

Although we were unaware of this possible shift during the H(605) work, we found out in time to eliminate it during the H(538) work.[12] We did this by switching the E_Q polarity at a rate fast compared to the beam signal readout interval, but slow enough to avoid line broadening. The cross terms in Eq. (1) are thereby time averaged to zero. The line is symmetrically broadened by at most the equivalent of a 0.2 MHz shift in $\mathcal{S}$; this amounts to only 0.2% of the line FWHM. A switching rate of 30 cps is suitable, and was used during the H(538) work to eliminate the direct motional field asymmetry.[13]

We have now discovered that an indirect motional field effect may be an important source of line asymmetry. This occurs as follows. A motional field scales as vH, so that it preferentially quenches high-velocity metastables.[14] This quenching becomes more pronounced as the magnetic field increases. As we scan a resonance, the metastables contributing to the observed E_Q quenching on the right wing of the line spend more time, on the average, than those contributing to the left wing. Since the fraction of metastables quenched is proportional to transit time, the right wing of the line is raised relative to the left. The apparent line center shifts upward, which results in an erroneously high value for $\mathcal{S}$. The shift is dependent on the size of such motional fields as E_T. We now believe that these fields were large enough in the LC experiments to systematically shift all $\mathcal{S}$ values upward by at least 0.1 MHz.

Motional field distortion of the beam velocity distribution resulting in such line asymmetries has been discussed by Lamb.[15] The question of whether such effects are important in the LC experiments depends on our knowledge of what fraction of the beam notch in Fig. 2 is due to motional fields, as distinguished from stray fields. The stray fields do not cause line asymmetry, since—although they preferentially quench low-velocity metastables—they distort the beam velocity distribution *symmetrically* with respect to the crossing point. On the other hand, whatever motional fields are present cause the line asymmetry discussed above, precisely because they distort the velocity distribution *asymmetrically* about the crossing point. During our earlier work, we mistakenly estimated that most of the beam notch was caused by stray fields. Consequently, we made no correction to the observed line centers for motional field distortion of the velocity distribution. Now, by careful analysis of the beam notch, we have learned that motional fields are the main cause. After determining the size of these fields from that analysis, we can calculate the new correction to $\mathcal{S}$ resulting from the previously unaccounted indirect motional field line asymmetry.

III. QUENCHING ANALYSIS

Here, by an analysis of possible metastable beam quenching due to accidental fields, we devise a fit to the beam notch curve (Fig. 2) which allows a determination of the motional fields present in the transition region. Before proceeding to Sec. IV, where we calculate the resultant shift in $\mathcal{S}$ due to the indirect motional field line asymmetry, we apply the analysis to the problem of determining an upper limit to the parity impurity in the metastable state.

A. Quench Integral

With state selection, we observe beam quenching via a *single* $2S \to 2P$ hfs transition. Let such a beam, at velocity v, pass through a magnetic field where the $2S \to 2P$ transition frequency is ν_{SP}. With ℓ the pathlength along the beam (z axis in Fig. 1), let the beam detector and source be at $\ell = \pm L$. Suppose there is a net electrostatic field E along the beam which couples the $2S$ and $2P$ levels via a Stark matrix element. A straightforward application of the Bethe-Lamb theory[6] of $2S$ quenching gives the fraction of metastables reaching the detector as

$$b = \exp\left\{-\frac{1}{v\tau}\int_{-L}^{+L} C(E/E_0)^2 d\ell / [1+(4\pi\tau\nu_{SP})^2]\right\}. \quad (2)$$

Here, τ is the $2P$ natural lifetime.[16] The scale electric

[10] This is the geometry for $\Delta m_J = -1$ transitions. A possible component of $\mathbf{E}_S$ parallel to the magnetic field drives $\Delta m_J = 0$ transitions. These have a negligible effect on the $\beta \to e$ quenching line shape, as E_S is small, and the $\Delta m_J = 0$ transitions are far off resonance.

[11] See Appendix I of Ref. 7.

[12] The author is indebted to Professor W. E. Lamb, Jr., for pointing out the possibility of this Lamb-shift shift.

[13] We measured neither the size nor the sign of this effect during the H(605) work (Ref. 7). Certain indirect evidence indicates the shift was bounded by $\Delta\mathcal{S} = \pm 0.05$ MHz. However, the H(605) value of $\mathcal{S}$ is clearly less reliable than the H(538) value.

[14] After thermal dissociation and electron bombardment excitation, the metastable beam follows a Maxwell v^3 distribution, with a beam most probable velocity $\alpha = 6.8 \times 10^5$ cm/sec.

[15] See Sec. 64 of HIII and Sec. 93 of HV, Ref. 3.

[16] We ignore the $2S$ natural decay rate, since it is nonresonant and negligibly small compared to the $2P$ channel.

field E_0 measures the $2S-2P$ Stark coupling strength; $E_0=22.5$ V/cm for $\beta\rightarrow e$ transitions. C accounts for the magnetic-field variation of the Stark matrix element calculated in a (J,m_J) representation. We call the exponent in Eq. (2) the "quench integral." In general, it must be evaluated numerically, as the integrand is a complicated function of ℓ, due to the variation of C, E, and ν_{SP} with beam pathlength and magnetic field.

Here, as we are mainly interested in effects due to varying electric fields, we make several convenient approximations to Eq. (2). Near the $\beta-e$ crossing point, the magnetic field $H\simeq 574$G is small compared to the fine structure decoupling field (7.8kG in hydrogen). Consequently, C varies slowly with H; we evaluate C at the crossing point field H_C, and take it outside the integral. As well, the variation of ν_{SP} with H is nearly linear, so we set

$$4\pi\tau\nu_{SP}=\kappa_0[1-rh(\xi)]. \quad (3)$$

$\xi=\ell/R$ is a dimensionless pathlength parameter, with R the Helmholtz coil radius. $h(\xi)$ is the *fractional* coil field on the beam axis. $r=H/H_C$ is a dimensionless magnetic-field variable, with H the coil field at center. The constant $\kappa_0\simeq 4\pi\tau\mathcal{S}$ measures the $2S-2P$ separation at zero magnetic field. By use of Eq. (3), the quench integral now contains the resonant function

$$g(\xi,r)=\{1+\kappa_0^2[1-rh(\xi)]^2\}^{-1}. \quad (4)$$

Further, as the quenching is well localized at coil center (over a region of size $\ll L$), we extend the integration to ∞. Finally, since the fields are symmetric about coil center, we need only integrate from 0 to ∞. Thus Eq. (2) becomes

$$b(r,u)=\exp\left\{-(2RC/u\alpha\tau)\int_0^\infty (E/E_0)^2g(\xi,r)d\xi\right\}. \quad (5)$$

$u=v/\alpha$ is a dimensionless velocity parameter, with α the beam most probable velocity.[14] We anticipate averaging b over the beam velocity distribution, which (to sufficient approximation) is

$$M(u)du=(4/\sqrt{\pi})u^2e^{-u^2}du. \quad (6)$$

B. Beam Notch

Evaluation of the Integral

The quench integral which determines the detailed shape of the beam notch is

$$\mathcal{G}(r,u)=(2RC/u\alpha\tau)\int_0^\infty [E(\xi,r,u)/E_0]^2g(\xi,r)d\xi, \quad (7)$$

with E the residual electric field present when the applied quenching field E_Q is set to zero. This E may depend on beam orientation, as well as pathlength ξ, magnetic field r, and velocity u. The beam tilt field E_T is an example; in convenient notation

$$E_T/E_0=uQ_Mrh(\xi)\sin\theta, \quad (8)$$

where θ is the tilt angle, and $Q_M=(\alpha/c)H_C/E_0$ is a constant. Another type of motional field, E_t, acts on those metastables which travel off axis and explore transverse components of the coil field. With η the ratio of off-axis distance to coil radius R, the magnitude of E_t is conveniently written as

$$E_t/E_0=uQ_Mrh_t(\xi,\eta). \quad (9)$$

Here h_t is the ratio of transverse field to coil central field; h_t is an odd function of ξ, and is zero on the axis ($\eta=0$). The vector $\mathbf{E}_t$ lies in the xy plane of Fig. 1. Finally, we parametrize whatever stray fields are present by

$$\mathbf{E}_S/E_0=(Q_{Sx},Q_{Sy})e_S(\xi), \quad (10)$$

where Q_{Sx} and Q_{Sy} are the component stray field quench levels at coil center, and $e_S(\xi)$ is the *fractional* stray field along the beam axis. We assume $e_S(\xi)$ is an even function of ξ, and is about as well localized as the applied quenching field.[17]

Combining the fields $\mathbf{E}_T$, $\mathbf{E}_t$, and $\mathbf{E}_S$, we find we can write the beam-notch integral as a sum of terms

$$\mathcal{G}(r,u)=u\rho(r)+u^{-1}\psi_S(r)+(\sqrt{\pi})kX(r), \quad (11)$$

where the constant $k=4RC/(\sqrt{\pi})\alpha\tau$. The terms are as follows:

(1) X is a complicated cross term which depends on the azimuthal angle φ of a metastable trajectory in the beam, as

$$X(r)=Q_Mr(Q_{Sx}\sin\varphi-Q_{Sy}\cos\varphi)\sin\theta \times\int_0^\infty h(\xi)e_S(\xi)g(\xi,r)d\xi. \quad (12)$$

If we assume a uniform intensity distribution over φ, X averages to zero.

(2) ψ_S measures the quenching due to stray fields. As these fields are well localized [that is, $e_S(\xi)$ is appreciable only near the coil center, where the magnetic field $h(0)=1$], we evaluate the resonance function $g(\xi,r)$ at $\xi=0$. Then ψ_S is symmetric about the coil center, as

$$\psi_S(r)=(\sqrt{\pi}/2)\mathfrak{f}_Sg(0,r),$$

where

$$\mathfrak{f}_S=kQ_S^2\int_0^\infty e_S^2(\xi)d\xi,\quad g(0,r)=[1+\kappa_0^2(1-r)^2]^{-1}. \quad (13)$$

$\mathfrak{f}_S$ is the fractional stray field quenching loss at the crossing point. $g(0,r)$ exhibits a Lorentzian resonance, centered at the crossing point, $r=1$.

[17] This is reasonable, as the quench electrodes which supply E_Q, and are expected to be the source of E_S, are precisely the surfaces nearest the metastable beam during transit.

(3) ρ measures the motional field quenching, in terms of integrals over the Helmholtz coil magnetic field, as

$$\rho(r)=(\sqrt{\pi}/2)[\mathfrak{f}_T\gamma_T(r)+\mathfrak{f}_t\gamma_t(r)],$$

where

$$\gamma(r)=r^2\mathfrak{M}(r)/\mathfrak{M}(1),$$

with

$$\mathfrak{M}_T(r)=\int_0^\infty h^2(\xi)g(\xi,r)d\xi,$$
$$\mathfrak{M}_t(r)=\int_0^\infty [h_t(\xi,\eta)/\eta]^2 g(\xi,r)d\xi. \quad (14)$$

To first order, $\mathfrak{M}_t$ is independent of the off-axis distance η. We have evaluated the $\gamma(r)$ numerically, using field functions for a perfect coil. Finally, the coefficients are

$$\mathfrak{f}_T=kQ_M^2\mathfrak{M}_T(1)\sin^2\theta, \quad \mathfrak{f}_t=kQ_M^2\mathfrak{M}_t(1)\eta^2. \quad (15)$$

They are the fractional motional field quenching losses at the crossing point, and can be calculated directly from apparatus parameters.

Fit to the Beam-Notch Data

Having calculated the beam-notch integral $\mathfrak{g}$, we can now fit the beam notch (Fig. 2). The fractional beam *loss* due to accidental quenching is

$$1-\exp[-\mathfrak{g}(r,u)]\simeq\mathfrak{g}(r,u). \quad (16)$$

The approximation is reasonable since most of the beam survives transit. We average over the beam velocity distribution of Eq. (6), and over the azimuthal angle φ. The cross term X drops out, and we get

$$\langle\mathfrak{F}(r)\rangle_{u,\varphi}\simeq\mathfrak{f}_T\gamma_T(r)+\mathfrak{f}_t\gamma_t(r)+\mathfrak{f}_Sg(0,r), \quad (17)$$

as the over-all fractional loss. The coefficients $\mathfrak{f}_T$, $\mathfrak{f}_t$, and $\mathfrak{f}_S$ are the fractional quenching losses due to the accidental fields E_T (beam tilt), E_t (motional transverse), and E_S (stray electrostatic), respectively. For the H(538) crossing we calculate $\mathfrak{f}_T=4\%$ for a tilt angle $\theta=1°$, and $\mathfrak{f}_t=\frac{1}{2}\%$ (averaged over beam cross section) for a beam of maximum radius $\eta=0.07$.[18] If stray fields are of order 0.1 V/cm, then $\mathfrak{f}_S\sim1\%$.

In Fig. 2, we fit H(538) beam-notch data by curves calculated from Eq. (17). Although the coefficients $\mathfrak{f}$ can be estimated, they are considered here to be best fit parameters. Table I gives the $\mathfrak{f}$ values used. Curve 3 provides a satisfactory fit, with

$$\mathfrak{f}_T=4.0\pm0.5\%, \quad \mathfrak{f}_t=0.5\%, \quad \mathfrak{f}_S=1.7\mp0.5\%. \quad (18)$$

The quoted errors span $\mathfrak{f}$ values obtained from a number of H(538) beam notches. Variation of the fit with $\mathfrak{f}_t$ is not shown. Doubling the calculated value $\mathfrak{f}_t=0.5\%$ produces a curve which falls off much too quickly with increasing magnetic field. We believe the calculated value is good to $\pm0.1\%$. Although it is not possible to decisively resolve the contributions from E_T and E_S quenching, we believe that within the above error limits, we have determined reliable values for the quenching fractions in ρ of Eq. (14), and for the stray field fraction $\mathfrak{f}_S$. This conclusion is strengthened by the fact that the estimated $\mathfrak{f}$ values agree quite well with those determined by the curve fitting.

[18] Relevant apparatus dimensions are: $R=4.95$ cm, Helmholtz coil mean winding radius; $d=6.8$ mm, beam diameter in the transition region. The maximum radius parameter used here is $\eta=d/2R$.

TABLE I. Beam-notch parameters.

Curve No.[a]	% beam loss $\mathfrak{f}_T$	$\mathfrak{f}_t$	$\mathfrak{f}_S$
1	0.0	0.5	5.5
2	2.0	0.5	3.5
3	4.0	0.5	1.7
4	5.5	0.5	0.0

[a] These curves correspond to those plotted in Fig. 2.

C. 2S Parity Impurity Amplitude

The possibility of a parity impurity in the 2S state of atomic hydrogen has been discussed by several authors.[19] If the 2S state wave function contains some 2P state, so that it becomes $\Phi_S'=\Phi_S+a\Phi_P$, then the 2S decay rate is augmented by $|a|^2/\tau$, where τ is the 2P natural lifetime. The LC experiments are particularly sensitive to a possible parity mixing matrix element μ between 2S and 2P states, as the amplitude a is

$$a=\mu/[(E_S-E_P)+i(\hbar/2\tau)]. \quad (19)$$

The effect of μ (assumed to be independent of magnetic field) is "magnified" at a crossing point, where the $2S-2P$ energy difference E_S-E_P is zero. Predicted values of μ give an unobservably small effect in the LC experiment. We can set an upper limit on μ, however, by claiming that it induces a beam loss at the crossing point which is not larger than the stray field fraction $\mathfrak{f}_S$ determined above.

Define the dimensionless matrix element $\mu'=(2\tau/\hbar)\mu$. The expected fractional beam loss due to μ' is

$$\mathfrak{F}'(r)\simeq k|\mu'|^2\int_0^\infty g(\xi,r)d\xi, \quad (20)$$

where k is defined in Eq. (11). If we claim this "unaccountable" loss is not larger than $\mathfrak{f}_S$ at the crossing point, then

$$|\mu'|^2\leq\mathfrak{f}_S/k\int_0^\infty g(\xi,1)d\xi=1.87\times10^{-4}\mathfrak{f}_S. \quad (21)$$

We have evaluated this expression for the H(538) crossing; the value of the integral is 0.527. Using $\mathfrak{f}_S=1.7\%$ from Eq. (18), we get $|\mu'|^2\leq3\times10^{-6}$. The amplitude at zero magnetic field is then

$$|a_0|^2=|\mu'|^2/(1+\kappa_0^2)\leq8\times10^{-9}, \quad (22)$$

[19] E. E. Salpeter, Phys. Rev. **112**, 1642 (1958); R. A. Carhart, *ibid.* **132**, 2337 (1963); F. C. Michel, *ibid.* **138**, B408 (1965); P. G. H. Sandars, Phys. Letters **14**, 194 (1965).

where the "magnification" factor $\kappa_0=(2\tau/\hbar)(E_S-E_P)_0=19.6$ for the H(538) crossing. This value of a_0 constitutes a new upper limit to the parity impurity amplitude in the $2S$ state.[20] Sandars and Lipworth have established a much more stringent limit in the ground state of the cesium atom.[21]

IV. LINE SHAPE ANALYSIS

Here we treat the problem of calculating the line center shift in an observed metastable quenching resonance resulting from any one of a number of factors which slightly distort the line shape. Since even the largest line asymmetries encountered in the LC work generate shifts of order only 0.1% of the line FWHM, a perturbation approach proves satisfactory, and we can derive a general line center shift formula in closed form. This analysis is applied to a calculation of the previously unaccounted line shift resulting from the indirect motional field asymmetry described in Sec. II C. We then discuss the resultant new correction to the value of $\mathcal{S}$.

A. General Line Asymmetry

We ignore accidental quenching for the moment, and suppose that the applied field $E_Q(\xi)$ is the only important source of beam quenching. The fractional beam surviving transit [Eq. (5)] can be written as

$$b(r,u;\psi)=\exp[-(1/u)A(r)\psi(r)],$$

where

$$\psi(r)=\left[(2RC/\alpha\tau)Q^2\int_0^\infty e^2(\xi)d\xi\right]g(0,r). \quad (23)$$

The symmetric resonance function ψ, similar to ψ_S of Eq. (13), measures quenching by the applied field, which is known to have a well-localized distribution $e(\xi)$ along the beam axis. $Q=E_Q(0)/E_0$ is the applied field quench level at coil center. We introduce $A(r)$ as a general line asymmetry factor. An example of such a factor is the Stark matrix-element correction C in Eq. (2).

The fractional beam quenched when E_Q is switched on, then off, is

$$f(r,u;\psi)=b(r,u;0)-b(r,u;\psi). \quad (24)$$

The expected quenching resonance line shape is obtained by averaging f over the beam velocity distribution of Eq. (6). Using the beam signal of Eq. (23), we get the line shape function

$$\bar{F}(r,\psi)=G(r,\psi)/G(r,\infty)\simeq(2/\sqrt{\pi})A\psi-(A\psi)^2+\cdots,$$

where

$$G(r,\psi)=\int_0^\infty f(r,u;\psi)M(u)du. \quad (25)$$

As we normally work at low quench levels, the first term of this expansion represents the observed quenching resonance quite well. A better approximation is obtained by using the line shape function

$$F(r,\psi)=1-\exp[-(2/\sqrt{\pi})A(r)\psi(r)], \quad (26)$$

which differs from $\bar{F}$ by a small term second order in ψ. A typical resonance line fits F to within 5% of the line FWHM.

We measure line centers from the observed quenching resonance by locating the midpoint between magnetic fields at "working points" on the line. These are defined to be points at the same fractional quenching λF_M, where $F_M=\bar{F}(1,\psi)$ is the observed maximum quenching fraction, and $0<\lambda<1$. Normally we work at the $\lambda=\frac{3}{4}$ points on the line, which is the region of greatest slope. As the line is quite symmetric, the working points are almost equally disposed above and below the crossing point field H_C. However, the measured line center is shifted from H_C if any line asymmetry is present. Using the approximate line shape function of Eq. (26), we calculate the fractional shift

$$(\Delta H_C/H_C)_\lambda\simeq\tfrac{1}{8}(\delta H_0/H_C)^2[(dA/dr)/A]_{r=1}B(\lambda,F_M),$$

where

$$B(\lambda,F_M)=\ln(1-F_M)/\ln(1-\lambda F_M). \quad (27)$$

Here $\delta H_0\simeq 2H_C/\kappa_0$ is the line FWHM for the Lorentzian resonance function ψ. The approximation is good to within the 5% linewidth discrepancy between F and $\bar{F}$. The shift is positive for any asymmetry factor $A(r)$ for which $dA/dr>0$ at the crossing point, $r=1$.

B. Motional Field Asymmetry

If we include the accidental quenching fields as well as the applied field E_Q, the beam signal becomes

$$b(r,u;\psi)=\exp\left\{-\left[\frac{1}{u}\psi(r)+\mathcal{J}(r,u)+\text{cross terms}\right]\right\}. \quad (28)$$

Here $\mathcal{J}$ is the beam-notch integral of Eq. (11). The cross terms between E_Q and the accidental fields E_S and E_T are similar to X of Eq. (12).[22] In first order, they are averaged to zero either by the E_Q polarity switching scheme, or by an average over the beam azimuthal angle φ.

The velocity averaging function of Eq. (25) now becomes

$$G(r,\psi)=(4/\sqrt{\pi})e^{-(\sqrt{\pi})kX}\times\int_0^\infty e^{-(u\rho+\psi_S/u)}[1-e^{-\psi/u}]u^2e^{-u^2}du. \quad (29)$$

[20] Compare with $|a_0|^2\lesssim 7\times10^{-7}$, obtained by W. L. Fite *et al.*, Phys. Rev. **116**, 363 (1959).

[21] P. G. H. Sandars and E. Lipworth, Phys. Rev. Letters **13**, 718 (1964). See also J. P. Carrico, E. Lipworth, and P. G. H. Sandars in Proceedings of the Berkeley Conference on the Physics of Free Atoms, Berkeley, California, 1966 (unpublished); C. O. Thornberg and J. G. King, *ibid.* (unpublished).

[22] There is no cross term between E_Q and E_t, as the latter field is an odd function of ξ about the coil center.

While the cross term in X cancels in the ratio which gives the line shape function $\bar{F}$, the term in ρ and ψ_S effectively distorts the beam velocity distribution. To the extent that this distortion is asymmetric about the crossing point, the measured line center is shifted. We find the appropriate asymmetry factor by employing the new G function in the $\bar{F}$ expansion. To first order in the small quantities ψ, ρ and ψ_S, we get

$$\bar{F}(r,\psi)\simeq(2/\sqrt{\pi})A(r)\psi(r),$$

where

$$A(r)=1+(2/\sqrt{\pi})A_1\rho(r)-A_2\psi_S(r). \quad (30)$$

Here A_2 and $A_1=1-(\pi/4)$ are numerical constants. As the ψ_S derivative is zero at the crossing point, only the motional field term in ρ can shift the measured center.

Using Eq. (27), we calculate the motional field asymmetry shift

$$(\Delta H_C/H_C)_\lambda\simeq\tfrac{1}{8}(\delta H_0/H_C)^2B(\lambda,F_M)\times A_1[\mathfrak{f}_T(d\gamma_T/dr)+\mathfrak{f}_t(d\gamma_t/dr)]_{r=1}. \quad (31)$$

The γ derivatives are quite insensitive to the specific crossing point in question; they are $(d\gamma_T/dr)=7.16$ and $(d\gamma_t/dr)=13.7$ at $r=1$. Calculating for a typical H(538) resonance, with $\delta H_0=55G$, $\lambda=\frac{3}{4}$ and $F_M=0.30$, and using the $\mathfrak{f}$ values in Eq. (18), we find $(\Delta H_C/H_C)\simeq140\pm20$ ppm. The measured H(538) resonance center must be *decreased* by this amount to account for the new line asymmetry.

The shift calculated in Eq. (31) is independent of crossing point field H_C. For, from Eqs. (15), both $\mathfrak{f}$ values scale as αH_C^2, in terms of parameters (other than geometry) which vary from crossing to crossing. The dependence on beam velocity α predicts the beam notch for deuterium should be roughly $(1/\sqrt{2})$ as deep as for hydrogen. We have observed just such a ratio. Using this in Eq. (31), where the factors δH_0 and B are about the same for all crossings, we find

$$\Delta H_C/H_C\propto\alpha[p\langle\sin^2\theta\rangle+q\langle\eta^2\rangle], \quad (32)$$

where p and q are numerical constants. The $\langle\ \rangle$ are averages over beam geometry. The shift should be roughly $(1/\sqrt{2})$ as large for a deuterium crossing as for a hydrogen crossing, due to the velocity dependence.

C. Lamb-Shift Shift

In the LC experiments, the derived value of the Lamb shift $\mathcal{S}$ is (very nearly) directly proportional to the measured crossing point field H_C. The immediate effect of the above motional field asymmetry is to decrease the H(538) value of $\mathcal{S}$ by 0.15 ± 0.01 MHz. Similarly, the H(605) value is decreased by 0.17 ± 0.05 MHz.[23]

TABLE II. Corrected Lamb-shift values.

Crossing point in hydrogen	H(538)	H(605)
Original $\mathcal{S}$ value,[a] MHz	1058.04±0.10	1058.07±0.10
MF asymmetry correction	−0.15±0.01	−0.17±0.05
Correction due to BP[b]	−0.05	−0.01
Final $\mathcal{S}$ value, MHz	1057.84±0.10	1057.89±0.15

[a] Taken from Eq. (2) of Ref. 1, and Eq. (20) of Ref. 7, respectively.
[b] Calculated from Tables IV and V of Brodsky and Parsons; see Ref. 8.

In Table II, we give values of $\mathcal{S}$ originally derived from the LC experiments.[1,7] These values resulted from a perturbation calculation by which we extrapolated from a measured H_C to the zero-field interval which determines $\mathcal{S}$. The H_C values used were corrected for all line asymmetry effects known at that time. The next entry in Table II is the new correction due to the indirect motional field asymmetry. Next, we include a correction to the perturbation calculation relating H_C to $\mathcal{S}$. We use the results of Brodsky and Parsons (BP),[8] which claim an intrinsic accuracy of 0.001 MHz in $\mathcal{S}$. The 0.05 MHz difference between our result and the BP result for H(538) is due to several errors in our calculation[24]; after correcting them, we agree with BP to within 0.02 MHz. The BP result has been confirmed to this accuracy in an independent calculation by Rebane.[25] The last entry in Table II is the final value of $\mathcal{S}$ from the LC experiments, corrected for all known line asymmetries.

The weighted average LC value of $\mathcal{S}$ in hydrogen is now

$$\mathcal{S}(\text{LC expt})=1057.86\pm0.10\ \text{MHz}. \quad (33)$$

The quoted accuracy is at least twice the standard deviation of the mean of the means of ten independent runs comprising over 200 line center measurements. We believe that all known systematic effects have now been calculated to an accuracy of better than ±0.03 MHz in $\mathcal{S}$. This value of $\mathcal{S}$ now agrees, within experimental error, with the value measured by Lamb and collaborators.[26] The average of the experimental values exceeds Soto's result[2] by about 3±2 parts in 10^4. If the value of the fine structure constant derived from the recent ac Josephson effect experiment[27] is used in the theory, this discrepancy is reduced to 0.25±0.15 MHz.

[23] The H(605) beam notch (Fig. 5 of Ref. 7) is fitted according to the theory of Sec. III above. The derived quenching amplitudes are: 5.5±0.5, 0.7, and 1.7∓0.5%, in the same order as Eq. (18). The calculated shift is $\Delta\mathcal{S}=-0.17\pm0.02$ MHz. The somewhat larger error quoted in Table II allows for the uncertainty as to the direct motional field asymmetry effect (Ref. 13).

[24] See Sec. IV of Ref. 7. We used an incorrect value of g_S/g_P (see Ref. 55). Additional higher-order hfs terms, with inclusion of the nuclear g_I value, account for the difference.

[25] T. Rebane (private communication).

[26] $\mathcal{S}$(Lamb expt) = 1057.77±0.10 MHz. See HV, Ref. 3.

[27] W. H. Parker, B. N. Taylor, and D. N. Langenberg, Phys. Rev. Letters **18**, 287 (1967).

V. SUMMARY

In the present paper, we have extended our previous approximate analysis of the level-crossing experiments in the $n=2$ level of atomic hydrogen, so as to establish a firm basis for the high resolution claimed in that work. We have developed a quantitative treatment of observed quenching of a beam of metastable atoms near a crossing point by accidental electric fields, which provides information regarding a possible parity impurity in the metastable state. More importantly, the detailed analysis shows that the beam velocity distribution may be substantially distorted near a crossing by the presence of small motional electric fields, previously estimated to have a negligible effect on beam quenching. This distortion results in a new systematic correction to the value of the Lamb shift $\mathcal{S}$ derived from the level-crossing measurements. The correction reconciles the level-crossing value of $\mathcal{S}$ with the value originally measured by Lamb and collaborators, and considerably reduces the apparent discrepancy between experiment and theory for $\mathcal{S}$.

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