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Superheating of the Meissner State and the Giant Vortex State of a Cylinder of Finite Extent*

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The superheating field and the size-dependent critical field of the Meissner state for a cylinder of radius R, with Ginzburg-Landau (GL) parameters κ between 0.3 and 2.4 and size parameters R/λ between 2.5 and 20, have been calculated from the GL theory. For very large values of R/λ the superheating field of the cylinder approaches that of the semi-infinite half-space. Similiar studies of the giant vortex state show that the superheating fields are smaller than for the Meissner state. Under certain conditions, as the applied magnetic field is increased, the solutions to the GL equations may cease to exist for the Meissner and giant vortex state for a constant value of the fluxoid quantum number before the Gibbs free energy of the superconducting state reaches that of the normal state.

I. INTRODUCTION

By magnetic superheating of a superconductor, one usually means that as the magnetic field is increased, the total energy of the specimen is increased beyond that of the normal state while the specimen is still superconducting, so that the nucleation of the normal state is delayed. Early[1] experiments and theories on type-I superconductors met with little success in demonstrating this effect reliably, and the first serious theoretical treatment of superheating was given by Ginzburg,[2] who has solved the Ginzburg-Landau (GL) equations[3] for a semi-infinite half-space. Others[4] have followed the same approach or similar ones. Essentially, it was found that flux can be delayed from entering the Meissner state beyond H_c for a type-I superconductor, beyond H_{c1} for a type-II superconductor, that the maximum superheating field $H_{sh}=H_c$ for $\kappa\rightarrow\infty$, and that for $\kappa\rightarrow 0$ the value of $H_{sh}\rightarrow\infty$. For example, for $\kappa=1$ the value of $H_{sh}/H_c=1.27$ and for $\kappa=0.3$ it is 1.81, which means that large superheating effects should have a good chance of being observable. A recent calculation[5] of the superheating field of a cylinder with $\kappa=0.5$ and $R/\lambda=14.4$ (R=radius of cylinder; λ=low field penetration depth) is in agreement with that of the semi-infinite half-space.[2] However, Galaĭko[6] has calculated that the superheated Meissner state of a bulk specimen is stable up to approximately 0.8 H_c when $\kappa\gg 1$, and Takács[7] finds that flux can be delayed from entering the Meissner state of a semi-infinite half-space with $\kappa\gg 1$ up to approximately 0.5 H_c. It was shown[8] experimentally that flux can easily be delayed from entering type-II specimens for applied magnetic fields $H_0>H_{c1}$, and that for materials with κ values of about 3.6 the maximum experimentally observed field for flux delay beyond H_{c1} is in the close neighborhood of H_c. Other experimenters,[9] who have worked with small type-I spheres, with bulk type-II superconductors with κ values of about unity, and with thin type-I cylinders seem to get fair agreement with the above theory.[2]

We have already calculated the maximum superheating fields for a slab[10,11] of finite thickness $2L$ as a function of κ and L/λ. This calculation[10] is based on a method employed in computing the giant vortex state[12,13] and is different from that of Marcus,[14] who has also computed the superheating field of a slab of finite thickness, though over a very limited range of parameters κ and L/λ.

Here we have calculated, for a very long cylinder of radius R, the size-dependent critical field H_s of the Meissner state, the maximum superheating field H_{sh} of

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[1] For a review up to 1957 see T. E. Faber and A. B. Pippard, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1957), p. 159.

[2] V. L. Ginzburg, Zh. Eksperim. i Teor. Fiz. **34**, 113 (1958) [English transl.: Soviet Phys.—JETP **7**, 78 (1958)].

[3] V. L. Ginzburg and L. D. Landau, Zh. Eksperim. i Teor. Fiz. **20**, 1064 (1950).

[4] C. P. Bean and J. P. Livingston, Phys. Rev. Letters **12**, 14 (1964). P. G. de Gennes, Solid State Commun. **3**, 127 (1965); P. G. de Gennes, in *Proceedings of the International Symposium on Quantum Fluids, Brighton, England, 1965*, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), p. 26; J. Matricon and D. Saint-James, Phys. Letters **24A**, 241 (1967); L. Kramer, *ibid.* **24A**, 571 (1967); H. J. Fink and R. D. Kessinger, *ibid.* **25A**, 241 (1967).

[5] R. Doll and P. Graf, Z. Physik **197**, 172 (1966).

[6] V. P. Galaĭko, Zh. Eksperim. i Teor. Fiz. **50**, 717 (1966) [English transl.: Soviet Phys.—JETP **23**, 475 (1966)].

[7] S. Takács, Phys. Status Solidi **21**, 707 (1967).

[8] A. S. Joseph and W. J. Tomasch, Phys. Rev. Letters **12**, 14 (1964); R. W. DeBlois and W. DeSorbo, *ibid.* **12**, 499 (1964).

[9] J. Feder, S. R. Kiser, and F. Rothwarf, Phys. Rev. Letters **17**, 87 (1966); F. W. Smith and M. Cardona, Phys. Letters **24A**, 247 (1967); J. C. Renard and Y. A. Rocher, *ibid.* **24A**, 509 (1967); R. Doll and P. Graff, Phys. Rev. Letters **19**, 897 (1967).

[10] H. J. Fink and A. G. Presson, Phys. Letters, **25A**, 378 (1967).

[11] The H_{sh}/H_c curves in Ref. 10 for $L/\lambda>5$ are incorrect due to a computational error. For $L/\lambda>5$ these curves coincide approximately with the $L/\lambda=5$ curve and thus agree with Ref. 2 for the semi-infinite half-space. The conclusion drawn in Ref. 10 for $R/\lambda\rightarrow\infty$ is thus wrong.

[12] H. J. Fink and A. G. Presson, Phys. Rev. **151**, 219 (1966).

[13] We have now subdivided the length R (or L) into 19 equal or variable intervals instead of 10 equal intervals as in Ref. 12.

[14] P. M. Marcus, Rev. Mod. Phys. **36**, 294 (1964).

the Meissner state[15], and the superheating field of the giant vortex state[12,15] for κ values between 0.3 and 2.4 and size parameters R/λ between 2.5 and 20.

II. THEORY AND RESULTS

We follow here the same notation and definitions as in Ref. 12 except for the normalized energy, which we choose to be larger by a factor of 4. The GL equations and the normalized energy for a very long cylinder in an axial magnetic field are

$$\frac{1}{\chi^2}r^{-1}\frac{d}{dr}\left\{r\frac{dF}{dr}\right\} = \frac{1}{8}\left(\frac{\chi}{\kappa}\right)^2\left[h_0r+r^{-1}\left\{\phi-\frac{2\sqrt{2}\kappa}{\chi^2}b\right\}\right]^2F-(1-F^2)F \quad (1)$$

$$-(8\pi/c)(\chi/\kappa)j_n=(d/dr)\{r^{-1}(d\phi/dr)\} = \left(\frac{\chi}{\kappa}\right)^2\left[h_0r+r^{-1}\left\{\phi-\frac{2\sqrt{2}\kappa}{\chi^2}b\right\}\right]F^2, \quad (2)$$

$$\Delta g=\frac{G_{SH}-G_{NH}}{H_c^2V/8\pi} = 2\int_0^1\{(h-h_0)^2-F^4\}rdr. \quad (3)$$

The definitions of the symbols are: the order parameter $\Psi(r,\theta)=F(r)e^{ib\theta}$, where $F\equiv F(r)$ is assumed to be real function and b is the fluxoid quantum number (integer); $r=\rho/R$, where ρ is the distance from the symmetry axis of the cylinder and R is the radius of the cylinder; θ is the angle around and in the plane perpendicular to the symmetry axis; $\kappa=\lambda/\xi$, where λ is the low field penetration[3] depth and ξ is the coherence length; $\chi=R/\xi$; $\chi/\kappa=R/\lambda$; $h\equiv h(r)=H(r)/H_c=h_0+(1/2r)(d\phi/dr)$, where H is the internal magnetic field parallel to the z direction and H_c is the thermodynamic critical field; $h_0=H_0/H_c$, where H_0 is the applied magnetic field. $\phi\equiv\phi(r)$ is defined through the vector potential $\mathbf{A}\equiv(0;\ A_\theta;\ 0)$, $2A_\theta=RH_c[h_0r+\phi/r]$; $j_n=\lambda j(r)/H_c$, where the current density $j(r)$ flows in the $\boldsymbol{\theta}$ direction; V is the volume of the specimen; and $G_{SH}-G_{NH}$ is the difference between the Gibbs free energies of the superconductor in a magnetic field and that of the normal state (assumed nonmagnetic) in a magnetic field.

The magnetization per unit volume $4\pi M$ is defined[12] by

$$4\pi M=H_c\phi(1). \quad (4)$$

In Ref. 12 we have disregarded, as stated, any overshoot and undershoot of Δg near $\Delta g\approx 0$; thus we have neglected superheating effects. The example which was discussed in detail[12] ($\chi=3$ and $\kappa=1$) had only a very small amount of overshoot. However, we have found that this is not correct in general for arbitrary parameters χ and κ; in what follows we shall deal with this point in detail.

We have solved Eqs. (1) and (2) by the same method as in Ref. 12 with improved accuracy in the vicinity of $\Delta g\approx 0$ with the boundary conditions: $dF/dr=0$ and $d\phi/dr=0$ at $r=1$ for all b values, and with $F=0$ and $\phi=0$ at $r=0$ for $b\neq 0$, and with $dF/dr=0$ and $\phi=0$ at $r=0$ for $b=0$.

The functions $F(r)$ and $\phi(r)$ and their derivatives were calculated and from these Δg [Eq. (3)] was calculated. Figures 1–3 show the results for Δg as a function of h_0 for various values of the fluxoid quantum number b for $R/\lambda=5$ for values of κ equal to 0.707, 1.4, and 1.7, respectively. For the Meissner state, $b=0$. In large magnetic fields ($H_{c2}\lesssim H_0<H_{c3}$) we have the exact equivalent description of the surface sheath on a

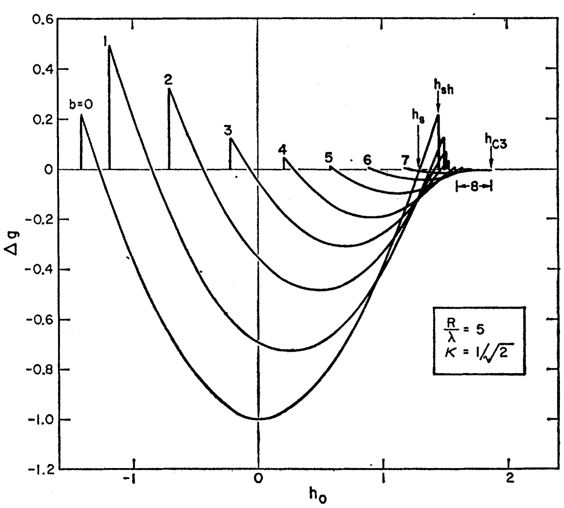

FIG. 1. Shown is the difference between the normalized magnetic Gibbs free energy of the superconducting state and the normal state Δg [Eq. (3)] as a function of $h_0=H_0/H_c$ (H_0 is the applied field) for $R/\lambda=5$ and $\kappa=0.707$ for all possible fluxoid quantum numbers b. For $b=8$ only the magnetic field interval is indicated because Δg is too small to be significantly different from zero. Note the regions where Δg collapses to zero for constant b. Note also that there are no undershoots of Δg near $\Delta g\approx 0$. Near h_{c3} there are no overshoots in Δg.

[15] We have found, as in Refs. 14 and 5, another solution in addition to that of the usual Meissner state which corresponds to that of a superconducting core and a normal surface of the cylinder and the slab. This solution is marginally stable when $H_{c2}<H_s<H_{sh}$, it exists between H_{c2} and H_{sh} and it is stable when $H_{c2}\gg H_{sh}$. In the latter case the lower field limit of this state is smaller than H_{sh} and in the vicinity of this lower field limit the solution is marginally stable. When H_{c2} is in the vicinity of H_s and H_{sh} we could not find a stable or marginally stable solution of this state. In the present work we have disregarded this state as not being characteristic of the Meissner state. Similarly, a superconducting state has been found whose order parameter is zero at the center and at the surface of the cylinder but finite near the surface of the cylinder. In the present work we have disregarded this state as not being characteristic of the giant vortex state.

macroscopic cylinder. It is found that $\Delta g(h_0)$ is not symmetric except for $b=0$. For the latter case, h_{sh} can be larger than h_s [which is defined as that field for which $\Delta g(b=0)=0$ (see Fig. 1)], or equal to the size-dependent critical field h_s (this is approximately satisfied in Fig. 2), or smaller than the extrapolated[16] value of h_s (shown in Fig. 3). The values $h_s(b=0)$ and $h_{\mathrm{sh}}(b=0)$ depend strongly on κ and R/λ, and the results are summarized in Fig. 4. They are similar to those of a slab of finite thickness[10,11] though there are numerical differences. For $R/\lambda=2.5$, a second-order phase transition of Δg at h_{sh} was observed (shaded area), which, for smaller values of R/λ, becomes more distinct. It appears that h_s and h_{sh} coincide when $R/\lambda \lesssim 1$. When $R/\lambda \gg 1$, the value of $h_s \to 1$, and the values of H_{sh}/H_c approach that of the semi-infinite half-space, which happens to be already satisfied when $R/\lambda \approx 10$ is reached.

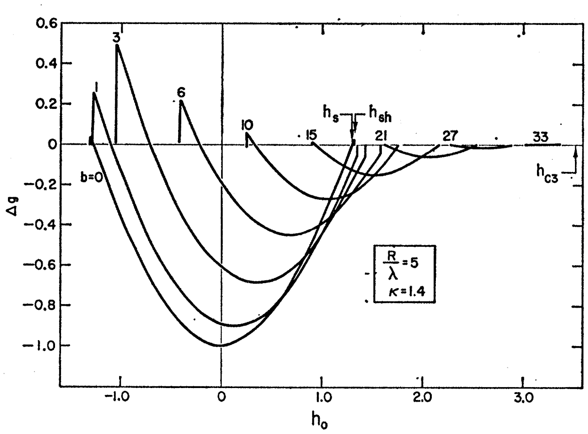

FIG. 2. This figure is similar to Fig. 1 except that $\kappa=1.4$. For the sake of clarity, not all possible quantum states are shown. Note that for $b=0$, the overshoot in Δg is small, and for larger b values (but not for b values near h_{c3}) Δg undershoots.

From Figs. 1–3 it can be seen when $b>0$ overshoots and undershoots of Δg exist in the vicinity of $\Delta g \approx 0$. For large magnetic fields (near H_{c3}) the overshoots and undershoots virtually disappear, because at H_{c3} the value of $G_{SH}=G_{NH}$. In general, it was found that the overshoot in the Gibbs free energy is largest for $b=0$ and becomes smaller for $b>0$, which means that if superheating exists in the sheath and giant vortex state, it should be considerably smaller than in the Meissner state. Therefore, the amount of superheating should be correspondingly smaller above H_{c2}. The critical current of the surface sheath[17] above H_{c2} is defined as the maximum lossless current in the sheath when the metastable superheated states are disregarded. Experi-

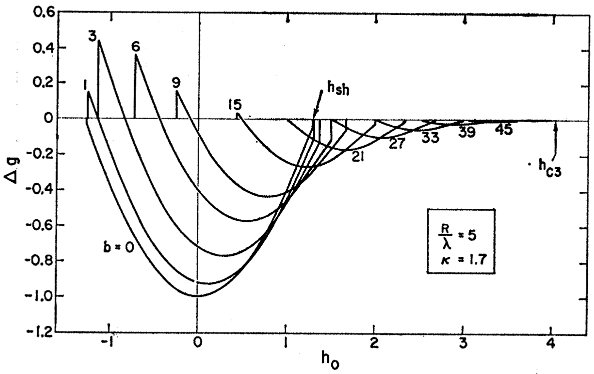

FIG. 3. This figure is similar to Fig. 1 except that $\kappa=1.7$. For the sake of clarity, not all possible quantum states b are shown. Note that for the smaller values of b including $b=0$ (but not for b values near h_{c3}) Δg undershoots.

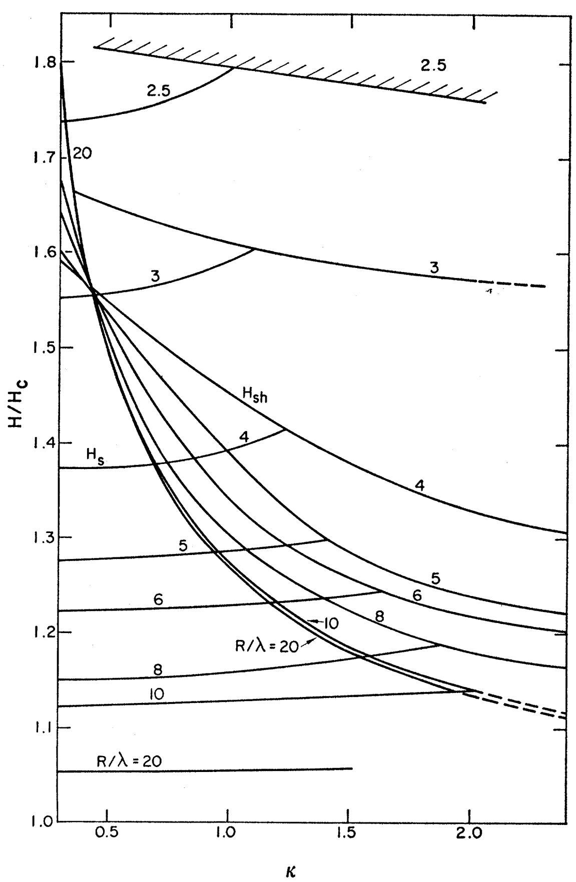

FIG. 4. Shown is the κ dependence of the critical field H_s at which the Gibbs free energy difference Δg [Eq. (3)] becomes zero, and also the field H_{sh} above which a solution of the Meissner state ceases to exist, as a function of the size parameter R/λ. For $R/\lambda=2.5$, a second-order phase transition of Δg at H_{sh} was observed, and the region of uncertainty of H_{sh} is indicated by the shaded area. Other uncertainties and extrapolations are indicated by broken lines.

[16] In Fig. 10 of Ref. 12 the extrapolated values of h_s were plotted. These should be modified by the results of Fig. 4.

[17] H. J. Fink and L. J. Barnes, Phys. Rev. Letters 15, 792 (1965).

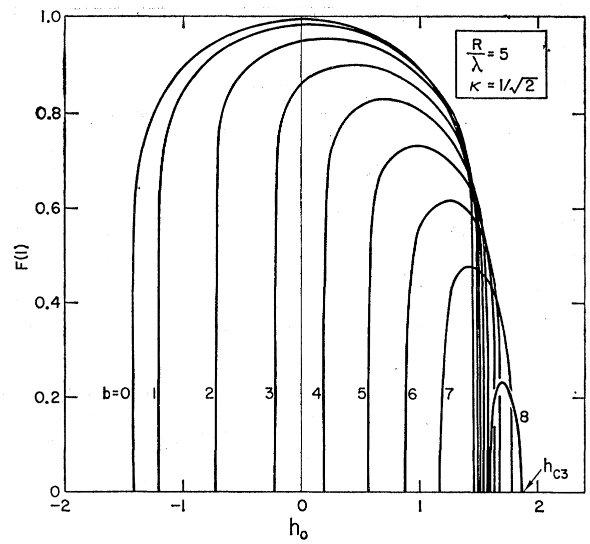

FIG. 5. Shown is the dependence of the absolute value of the order parameter at the surface of the cylinder as a function of the applied magnetic field $H_0(h_0=H_0/H_c)$ for $R/\lambda=5$ and $\kappa=0.707$ for all possible fluxoid quantum numbers (see Fig. 1).

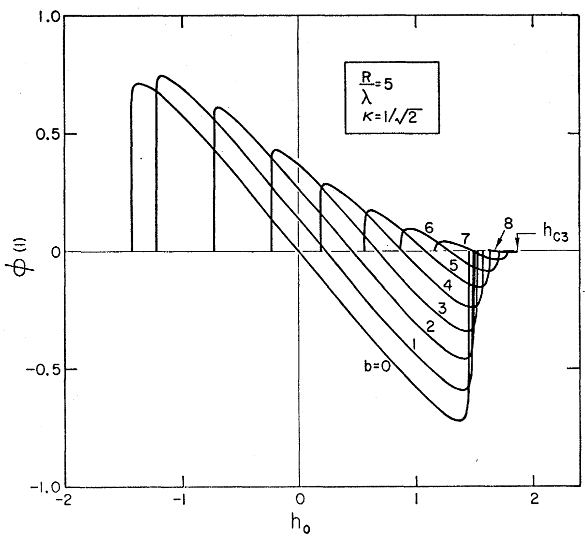

FIG. 6. Shown is the dependence of the normalized magnetization per unit volume $\phi(1)$ [Eq. (4)] as a function of the applied magnetic field $H_0(h_0=H_0/H_c)$ for $R/\lambda=5$ and $\kappa=0.707$ for all possible fluxoid quantum numbers (see Fig. 1).

ments[18,19] on bulk specimens do not seem to indicate any appreciable superheating effects in the sheath state.

The Δg-versus-h_0 curves for $|b|=\text{const}>0$ are unsymmetric, which is a consequence of the unsymmetric behavior of F and ϕ as a function of h_0. For $R/\lambda=5$ and $\kappa=0.707$, the order parameter at the surface of the cylinder $F(1)$; the ϕ function at the surface of the cylinder $\phi(1)$ are shown in Figs. 5 and 6, respectively. $\phi(1)$ is the normalized magnetization per unit volume of the cylinder [see Eq. (4)]. Results similar to Figs. 5 and 6 were obtained for other R/λ and κ values. As to irreversibility, flux locking, etc., the same conclusions apply here as in Ref. 12 for the giant vortex state.[20]

When b is a noninteger value, solutions for F and ϕ [Eqs. (1) and (2)] exist, and they are similar to those obtained for the integer b values. For an equilibrium solution such as we have here and in Ref. 12, a noninteger value of b is not permissible because the order parameter Ψ would be multivalued. However, during the transition process from one quantum state to another, b could temporarily be a noninteger value and similarly when fluctuations should occur.

III. CONCLUSIONS

We have calculated the size-dependent critical field and the superheating field for a long cylinder for which demagnetization effects can be ignored. We find as R/λ becomes very large, that the H_{sh}/H_c values of the cylinder approach that of the semi-infinite half-space. Similar superheating effects were found for the giant vortex states when the fluxoid quantum number is larger than zero and the specimen is of finite extent. Under certain circumstances, namely, for larger κ values, the Meissner state and giant vortex states for the smaller (and constant) fluxoid quantum numbers may cease to exist for Gibbs free energies $G_{SH}<G_{NH}$ when the specimens are comparable in thickness to λ. In large magnetic fields, there is very little or no overshoot and undershoot, and at H_{c3} the Gibbs free energy[21] $G_{NH}=G_{SH}$. Therefore, in a type-II bulk specimen the amount of superheating above H_{c2} is considerably smaller than that of the Meissner state.

[18] L. J. Barnes and H. J. Fink, Phys. Rev. **149**, 186 (1966).
[19] R. W. Rollins and J. Silcox, Phys. Rev. **155**, 404 (1967).
[20] In Ref. 12 it was stated incorrectly, due to computational errors, that no solutions for the giant vortex state exist for intermediate b values and $R/\xi \gtrsim 10$. Mathematical solutions for $R/\xi \gtrsim 10$ do exist for all b values.

[21] For details near H_{c3} which are beyond the accuracy of our calculations see: R. Doll and P. Graf, Z. Physik **204**, 205 (1967).