Study of the Ion–Vortex-Ring Transition*

G. W. RAYFIELD†
Physics Department, University of Oregon, Eugene, Oregon
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The velocity of bare ions and vortex rings in the region of the ion–vortex-ring transition as a function of applied electric field was experimentally studied and is described in detail. The velocity at which a bare ion produces quantized vortex rings has been determined. In the case of negative ions under pressure, this critical velocity is related to the radius of the negative-ion bubble. It was also found that creation of vortex rings by bare ions requires a considerably larger electric field than that necessary to support a singly charged vortex ring. An effect associated with the radioactive source (for producing ions) was observed which allowed vortex rings to be produced in the source region with comparatively weak electric fields. Vortex rings having a radius as small as 37 Å were studied, and no deviation from the classical hydrodynamic equation was observed. The data indicate that the vortex core size may be pressure- and temperature-sensitive.

INTRODUCTION

IN earlier work it has been shown that ions in liquid He^4 at temperatures well below the λ point can produce quantized vortex rings.[1] A velocity spectrometer which worked well at temperatures near 0.28°K was used to measure the velocity of the vortex rings. Energy loss by the vortex ring due to quasiparticle scattering was found to be negligible in this temperature regime. This allowed the energy of the vortex ring to be determined by measuring the potential rise experienced by the charged vortex ring moving between two grids at different potentials. By studying the velocity of a vortex ring as a function of its energy, the quantization of circulation in the superfluid was verified. In addition, the radius of the vortex line core was inferred by comparision with the classical hydrodynamic equation for a vortex ring and was found to be about 1 Å. The smallest vortex rings studied in this earlier work had a radius of about 500 Å. At higher temperatures, vortex rings were shown to lose a considerable amount of energy because of quasiparticle scattering. Near 0.7°K, for example, this energy loss was found to exceed 10 eV/cm. Energy-loss measurements across a one-centimeter drift space allowed the vortex-line–quasiparticle scattering cross section to be measured. The roton–vortex-line scattering cross section was found to be about 9 Å.

Several interesting effects have been observed by other authors which are apparently related to the creation of vorticity by ions in the superfluid.[2,3] The reader is referred to the original papers for a complete discussion.

The present work is concerned with a detailed study of the critical region where ions produce vortex rings. Results of preliminary studies of the critical region have already been presented.[4] This work was concerned primarily with the change in negative-ion velocity necessary for vortex-ring creation when hydrostatic pressure is applied to the liquid. Qualitatively, the rise in this critical velocity with increasing hydrostatic pressure was found to be consistent with the bubble model for the negative ion. Above about 14 atm the critical velocity decreased with increasing pressure, indicating roton creation by negative ions. This hypothesis was in agreement with studies of the penetrating ability of negative-ion complexes into field-free regions. Recently the pressure dependence of the bubble radius has been determined by negative-ion trapping on vortex lines.[5] Good quantitative agreement with these data has now been obtained. Small positively charged vortex rings having a radius as small as 37 Å have also been studied in the present work. The hydrodynamic vortex-ring equations seem to be valid for these small rings. Measurements of the vortex-line–roton scattering cross section as a function of pressure have been made. An increase of core radius with temperature and pressure is suggested by the data.[6]

EXPERIMENTAL APPARATUS AND MEASUREMENTS

The external apparatus is shown schematically in Fig. 1. A He^3 refrigerator is used to get to temperatures below 1.1°K. The temperature of the He^3 refrigerator is controlled by an electronic temperature regulator.[7] Particularly good temperature regulation is required if measurements of α, the vortex-ring attenuation coefficient for quasiparticle scattering, are to be made, since α varies exponentially with the reciprocal of temperature. The liquid He^4 is admitted to the chamber where the grids are located through a needle valve connected to the bath. All the liquid He^4 used in the experiment chamber is passed through a Millipore filter (10-mμ

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† Alfred P. Sloan Research Fellow.

[1] G. W. Rayfield and F. Reif, Phys. Rev. **136**, A1194 (1964).
[2] L. Bruschi, B. Maraviglia, and P. Mazzoldi, Phys. Rev. **143**, 84 (1966).
[3] G. Careri, S. Consolo, P. Hazzoldi, and M. Santini, Phys. Rev. Letters **5**, 392 (1965).
[4] G. W. Rayfield, Phys. Rev. Letters **16**, 934 (1966).
[5] B. E. Springett, Phys. Rev. **155**, 139 (1967). See also B. E. Springett and R. J. Donnelly, Phys. Rev. Letters **17**, 364 (1967).
[6] It is interesting and suggestive to note that Hall has estimated the vortex-line core size to be about 6.8 Å near 1°K. This estimate is based on experiments which study the velocity of vortex waves in liquid He^4 near 1°K and is suggestive of a temperature-dependent core radius [see H. E. Hall, Advan. Phys. **9**, 89 (1960)].
[7] C. Blake and C. E. Chase, Rev. Sci. Instr. **34**, 984 (1963).

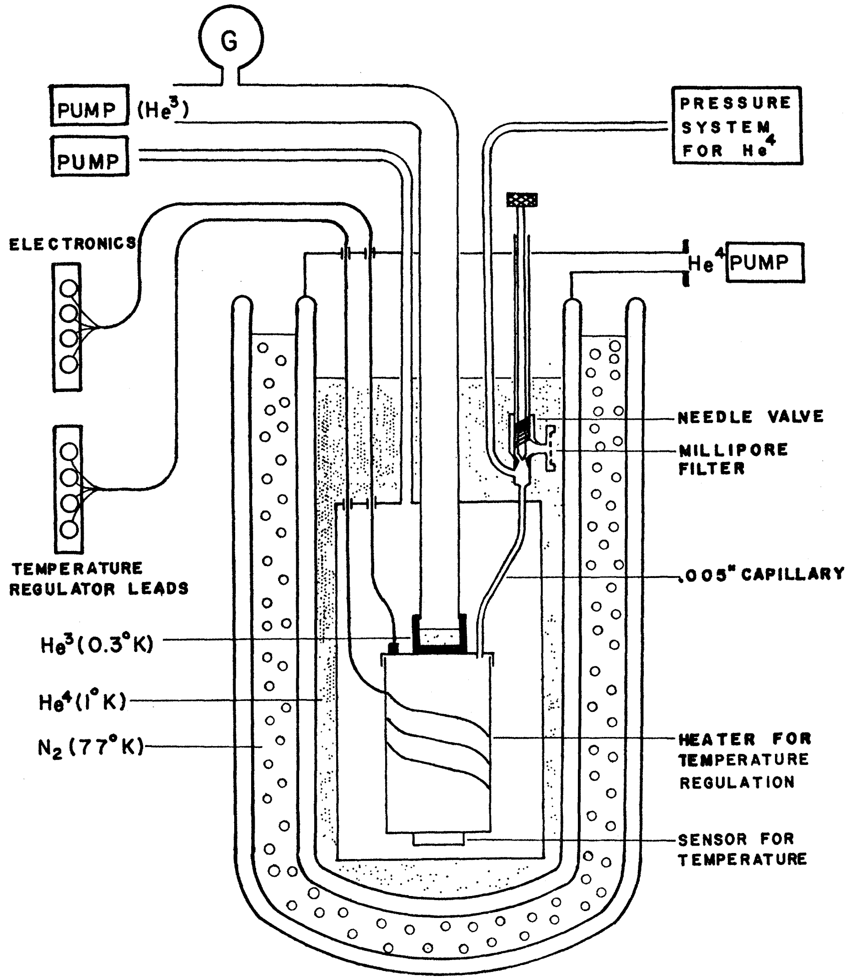

FIG. 1. External apparatus for velocity spectrometer.

mean pore diameter). No difficulties were experienced which could be traced to impurities in the liquid.

The grid arrangement of the velocity spectrometer is shown in Fig. 2. A P_0^{210} radioactive source S with a gold window was used to generate the ions. The source strength was less than 10 μCi. A Cary vibrating-reed electrometer was used to measure the current arriving at the collector C. The distance from the source to grid G_1 is 3 mm; this is followed by a 5-mm drift space, a 1-mm gate, a 10-mm velocity measuring space, another 1-mm gate, and a 3-mm separation from grid G_5 to collector. A square-wave voltage is applied simultaneously to G_3 and G_4 as shown. Bias voltages are applied to all grids in such a manner that when the square-wave voltage is positive the electric field between the grids is as shown in (a) of Fig. 3. When the square-wave voltage is negative the fields are as shown in (b) of Fig. 3. The ion-extracting field is $\mathcal{E}_s$, the drift field is $\mathcal{E}$, and the field used to stop the charge complexes in the gates is $\mathcal{E}_1$. $\mathcal{E}_1$ may be made quite large compared to $\mathcal{E}$; thus both bare ions and charged vortex rings may be gated.

The operation of the spectrometer is simple. Assume the velocity of the ion complex has reached a steady-state value by virtue of moving through the drift space G_1-G_2. This condition exists when the energy lost per centimeter due to quasiparticle scattering is equal to the energy gain per centimeter due to the electric field. A unique relation then exists between the steady-state drift velocity v and electric field $\mathcal{E}$. By monitoring the current arriving at the collector C as a function of the frequency of the square-wave oscillator, the velocity of the ion complex is measured in the region from G_2 to G_5. A maximum in collector current occurs whenever the ion velocity v and oscillator frequency f are such that $f=nv/2L$, where n is an odd integer and L is the length of the measuring space plus gates ($L=1.2$ cm). The gates are assumed to stop the ion complex as soon as the field is reversed in the gating region. Rough checks on vortex-ring penetration into the gates when characteristic retarding fields are applied indicate this is a reasonable assumption. See Appendix A.

The external electronics required in the experiment are shown in Fig. 2. The frequency meter converts the

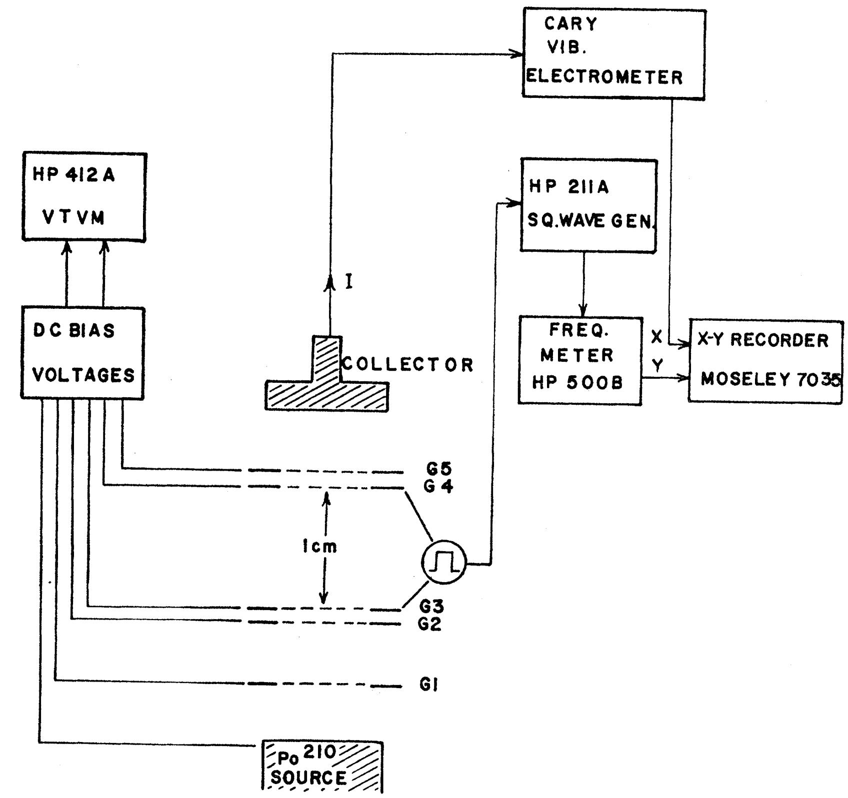

FIG. 2. Grid arrangement for velocity spectrometer and associated external electronics. The operation of the spectrometer is explained in the text.

frequency of the square-wave generator to a dc voltage which feeds the x axis of the x-y recorder. The vertical (y) axis is driven by the dc output voltage of the electrometer, which is proportional to the current collected on the collector. The dc bias is provided by a set of dry cells. A typical x-y recording trace which is used to determine the drift velocity of the ion in a given electric field is shown in (a) of Fig. 4. The collector current on the vertical axis is shown plotted against the frequency of the square-wave generator on the horizontal axis.

CRITICAL VELOCITY FOR VORTEX-RING CREATION BY POSITIVE AND NEGATIVE IONS

Some of the material presented here is an extension of previous work.[4] In the present measurements the electric field in the ion-extraction region (S to G_1) was kept small to insure against formation of vortex rings by ions in this region. Particular care in keeping this field small was necessary, because during the course of these experiments it was observed that the source was capable of producing vortex rings even though the electric fields involved were not large enough to accelerate the ion to the critical velocity necessary for vortex-ring production.[8] It will be convenient in presenting the data to refer to the ion without coupled vorticity as a "bare ion" and the ion-vortex-ring complex as simply a "vortex ring." For convenience in plotting data the drift velocity measured across the apparatus is plotted in Figs. 5 and 6 in terms of the reduced electric field $\mathcal{E}'=\mu(0)\times\frac{2}{3}\times10^{-3}\times T^{-1/2}\times\mathcal{E}$, where $\mu(0)$ is the zero-field mobility taken from Reif and Meyer,[9] T is the absolute temperature, and $\mathcal{E}$ is the applied electric field. This method of plotting yields a sort of temperature-independent "universal curve" and is consistent in approach with Reif and Meyer's earlier work. It is of interest that although the maximum ion

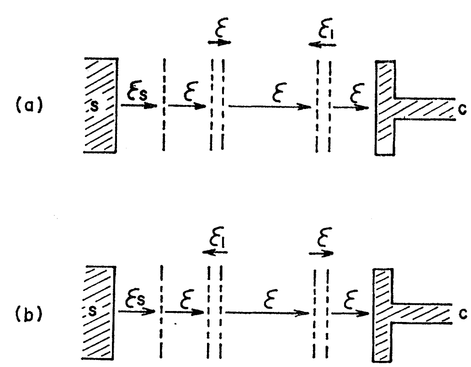

FIG. 3. Field configuration between grids when square wave positive (a) and when square wave is negative (b).

[8] The origin of this effect is not clear. Possibly the region very near the source is a tangle of vortex lines and this may initiate or nucleate vortex-ring formation by the bare ions.

[9] F. Reif and L. Meyer, Phys. Rev. **119**, 1164 (1960); and L. Meyer and F. Reif, Phys. Rev. Letters **5**, 1 (1960); Phys. Rev. **123**, 727 (1961).

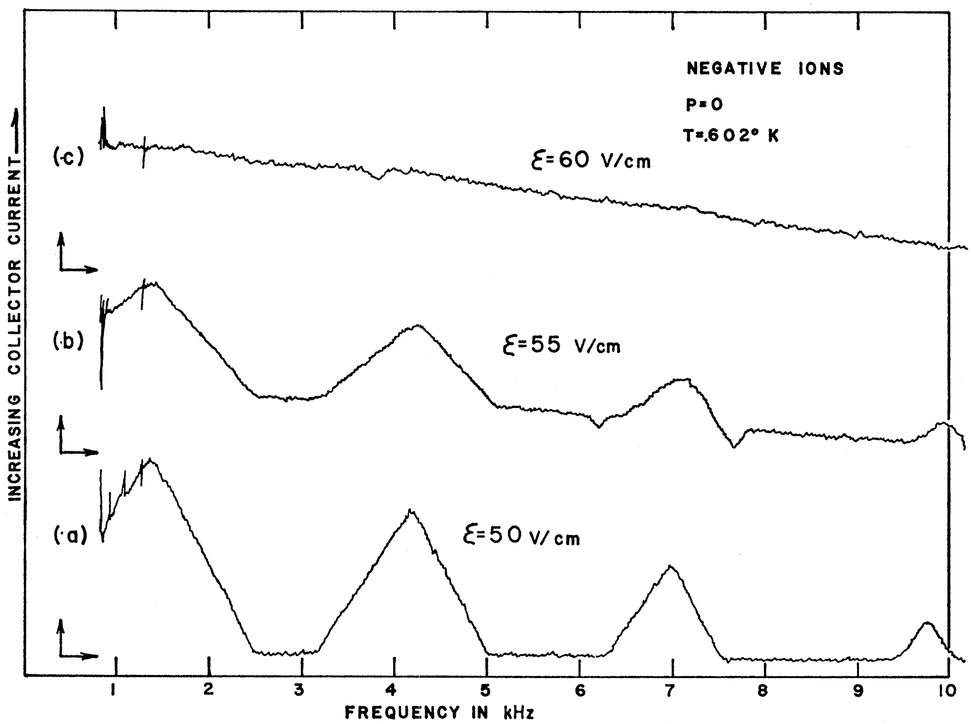

FIG. 4. Recorder traces used in determining the drift velocity of negative ions in a given electric field.

velocities measured are consistent with Reif and Meyer's results, the corresponding reduced electric field is different. The reason for this discrepancy is not clear. Negatively charged vortex rings have not yet been studied in detail, although indications are that positive and negative vortex rings do not behave very differently (this was also found in the earlier work).

The bare-ion–vortex-ring transition for negative ions is sudden and complete when the drift velocity reaches about 34 m/sec.[10] This rapid transition is made most evident by examining an actual x-y recorder trace in Fig. 4 as the critical velocity is approached. In (a), the collector current returns to zero between resonance peaks, indicating that no vortex rings are present.[11] In (b), a background current appears which indicates that vortex rings are being formed. In (c), the bare-ion spectrum has completely disappeared, indicating that essentially all the ions have formed vortex rings. The ion current in the apparatus for this field then has all the characteristics of vortex rings.

Since the bare-ion–vortex-ring transition is well defined, the critical velocity v_c for vortex-ring creation by negative ions can be measured with accuracy. Previous measurements[4] of this value were in error because of the source effect for vortex-ring production. Curves used to obtain v_c as a function of pressure are shown in Fig. 7. Note that the curve at 15 atm of pressure does not show a cutoff.[12] This is characteristic of negative ions above about 14 atm where roton creation limits the negative-ion velocity. Evidence for roton creation was presented earlier[4] by observing the field-penetrating ability of negative ions as a function of pressure. A curve of v_c as a function of pressure is shown in Fig. 8. The initial increase of v_c with increasing pressure is explained on the basis of a bubble model for the negative ion[13] and quantized circulation for vorticity.

By referring to Fig. 5, it is clear that the vortex-ring creation depends only on the velocity, not the temperature or electric field. This fact suggests the assumption now used in relating the constraint of quantized circulation to the critical velocity v_c. The superfluid velocity field about the ion is assumed to be curl-free or to

[10] Earlier (see Ref. 4 above) it was reported that the bare-ion spectrum and vortex-ring spectrum coexisted over some range in applied drift field $\mathcal{E}$. This effect was due to formation of vortex rings near the source and not in the apparatus by the bare ion. If a ring is formed near the source, a comparatively weak field is sufficient to support it through the apparatus.

[11] If vortex rings were present they would move slower than the ions, and a background current of slow moving charge carrier would be observed which would prevent the collector currents from returning to zero when the oscillator frequency is off resonance. When the vortex rings are produced, a low-frequency resonance is observed. Also, at the value of electric field required for the bare ion to reach critical velocity the vortex rings are quite large and able to penetrate the gates unless much larger retarding fields are used in the gate regions.

[12] The ion velocity for roton creation is best obtained at lower temperatures as was done in the earlier work (see Ref. 4).

[13] C. G. Kuper, in *Proceedings of the International School of Physics, "Enrico Fermi," Course XXI*, edited by G. Careri (Academic Press Inc., New York, 1963), p. 414. See also J. Jortner, N. R. Kestner, S. A. Rice, and M. Cohen, J. Chem. Phys. 43, 2614 (1965).

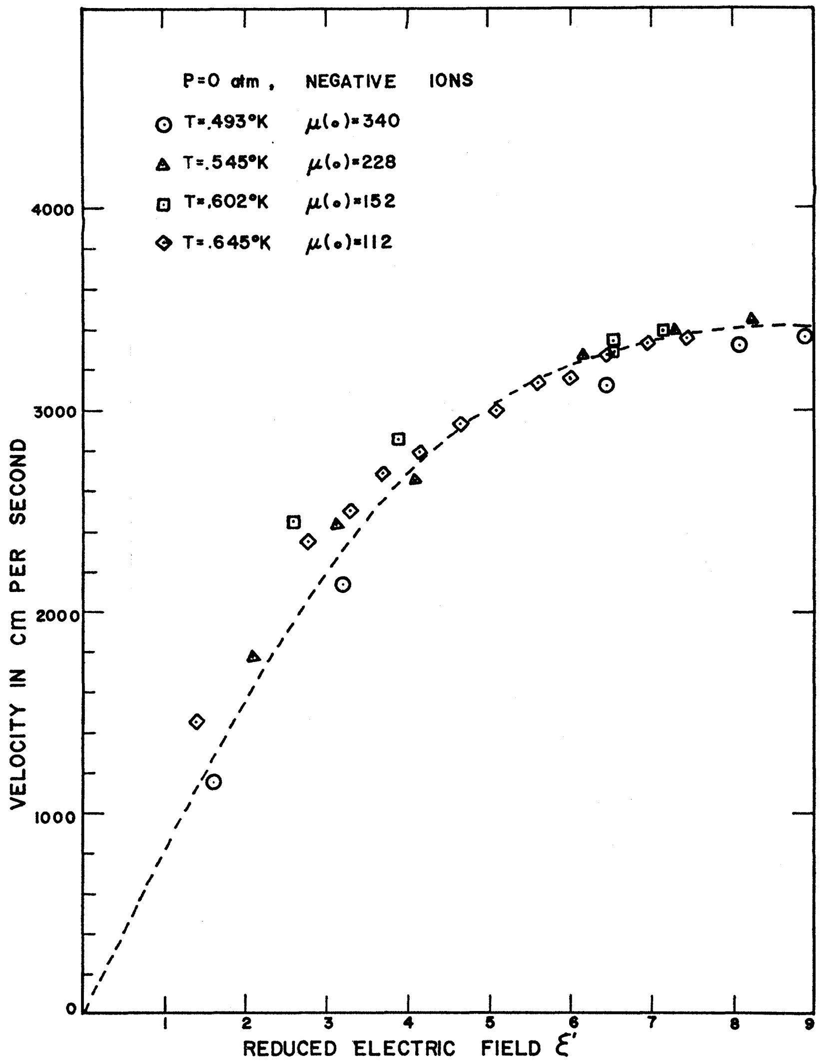

FIG. 5. Drift velocity of negative ions as a function of the reduced electric field ℰ' for various temperatures.

satisfy the quantum condition of circulation. Just below v_c we have $\oint \mathbf{v}\cdot d\mathbf{l}=0$,[14] while just above $\oint \mathbf{v}_s\cdot d\mathbf{l}=h/m$. Assume the line integral can be written in terms of average quantities $\oint \mathbf{v}_s\cdot d\mathbf{l}=\langle v_s\rangle\times\langle\oint dl\rangle \propto v_c\times R$, where R is the radius of the charged bubble representing the negative ion. We have then $v_c\times R=$const or $v_c\propto 1/R$. From the pressure dependence of v_c below 15 atm we can thus obtain the pressure dependence of the bubble radius. Figure 9 shows bubble radius R as a function of pressure plotted on curves obtained earlier by Springett.[5] These curves were obtained by observing the trapping of negative ions on an array of vortex lines in the rotating superfluid. The constant of proportionality between v_c and R was obtained by taking $R=16$ Å at the vapor pressure. Recent measurements by Northby and Sanders[15] indicate the bubble radius corresponding to the vapor pressure should be $R_{\text{ion}}=21$ Å. This value was obtained by observing the photoejection of electrons from bubble states. Recent work by Pratt and Zimmerman on negative-ion binding to vortex lines in the rotating superfluid also indicates a bubble radius of about 20 Å.[16] Unfortunately, from the observation of a critical velocity for the negative ion one cannot obtain absolute values for the bubble radius, only the pressure dependence. Hopefully, a theory may be developed in the future which will allow this computation to be made.

A universal curve of velocity versus reduced electric field for positive ions is shown in Fig. 6. Both the bare-ion spectrum and vortex-ring spectrum are shown. Again, particularly in the case of positive ions, the source was found to play a dominant role in producing vortex rings in weak electric fields. Typically, when the reduced electric field through the apparatus was greater than about 7.5, a vortex-ring spectrum could be ob-

[14] The line integral shown is to be taken only in the superfluid.
[15] J. A. Northby and T. M. Sanders, Phys. Rev. Letters 18, 1184 (1967).
[16] (Private communication.)

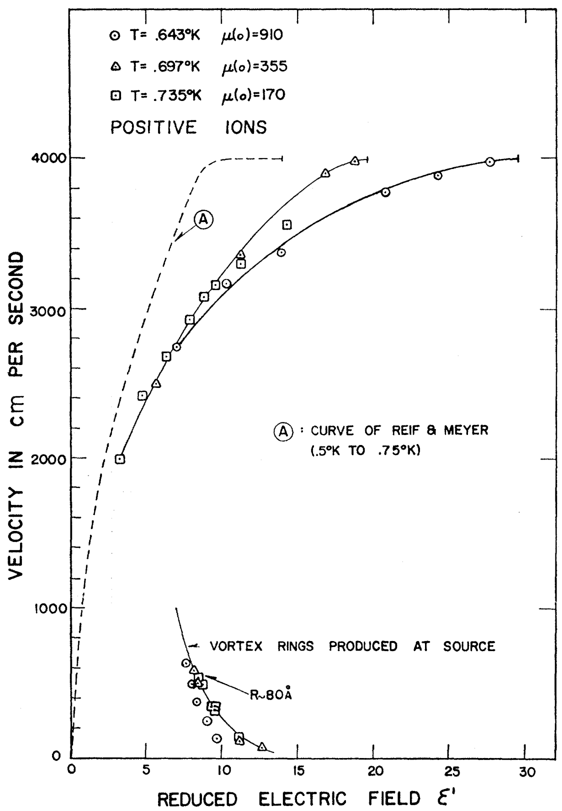

FIG. 6. Drift velocity of positive ions (both bare ions and vortex rings) as a function of the reduced electric field for various temperatures.

served. However, when the electric field in the source region was reduced so $\mathcal{E}'$ was less than 7.5, a much larger electric field through the remainder of the apparatus was needed in order to produce vortex rings: typically, $\mathcal{E}'>19$.[17] The disagreement between the present velocity curves and those made earlier by Reif and Meyer could be due to the source effect.

The curve of ion velocity versus electric field indicates that the bare positive ion must move at an average velocity of 40 m/sec in order to produce a vortex ring. No pressure or temperature variation of this critical velocity has been observed, which, using the same argument put forward in the negative-ion case, indicates the positive-ion radius is not pressure sensitive.

In liquid He^4 under the vapor pressure, the critical velocity for negative ions is 34 m/sec. Using the relation $v_c \propto 1/R$, one finds the ratio of positive- to negative-ion radii, $R_+/R_-=34/40=0.85$. This gives a positive-ion radius of from 13.6–17.9 Å, depending on the value used for the negative-ion radius. Estimates[18,19,13] of the positive-ion radius indicate that it is about 6 Å. Possibly the scaling relation $R_+/R_-=v_c^-/v_c^+$ does not work because the boundary condition is very different for a bubble (negative ion) and a solid ball (positive ion). Exactly how the boundary condition should be taken into account in computing the radius of the ion from its critical velocity is not clear at this point.

Another interesting feature of Fig. 5 and Fig. 6 is the leveling off of the velocity as the critical velocity is approached by the negative and positive ion. Since the flattening occurs simultaneously with the onset of vortex-ring creation,[20] it seems the two must be linked in some manner. It follows, therefore, that a theoretical treatment of vortex-ring creation by ions in the superfluid should also explain the flattening of the velocity curve near the critical velocity. Unfortunately, the ions are in a regime where one can not consider them as either very massive or very light compared to the momentum transfer by rotons. The mass of both the positive and negative ions is about 100 He^4 masses.[21,13] The momentum of a roton is $P_0=2\times10^{-19}$ g cm/sec.

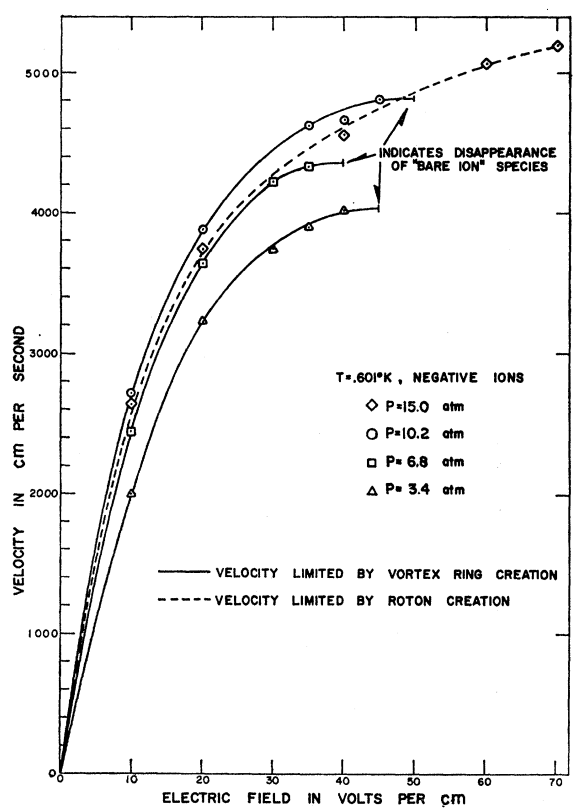

FIG. 7. Drift velocity of negative ions versus drift field ℰ at 0.601°K in liquid He^4 under various hydrostatic pressures.

[17] At low temperatures ($T<0.4$°K), the field in the source region could even be reduced to zero and vortex rings were still observed moving across the apparatus after having apparently been produced in the source region.

[18] K. R. Atkins, in *Proceedings at the International School of Physics, "Enrico Fermi," Course XXI* (Academic Press Inc., New York, 1963), p. 407.

[19] R. J. Donnelly, *Experimental Superfluidity* (The University of Chicago Press, Chicago, 1967), p. 195.

[20] Figure 7 seems to indicate that the flattening of the velocity curve is not so pronounced in the case of roton creation by the negative ions.

[21] A. Dahm and T. M. Sanders, Phys. Rev. Letters **17**, 126 (1966).

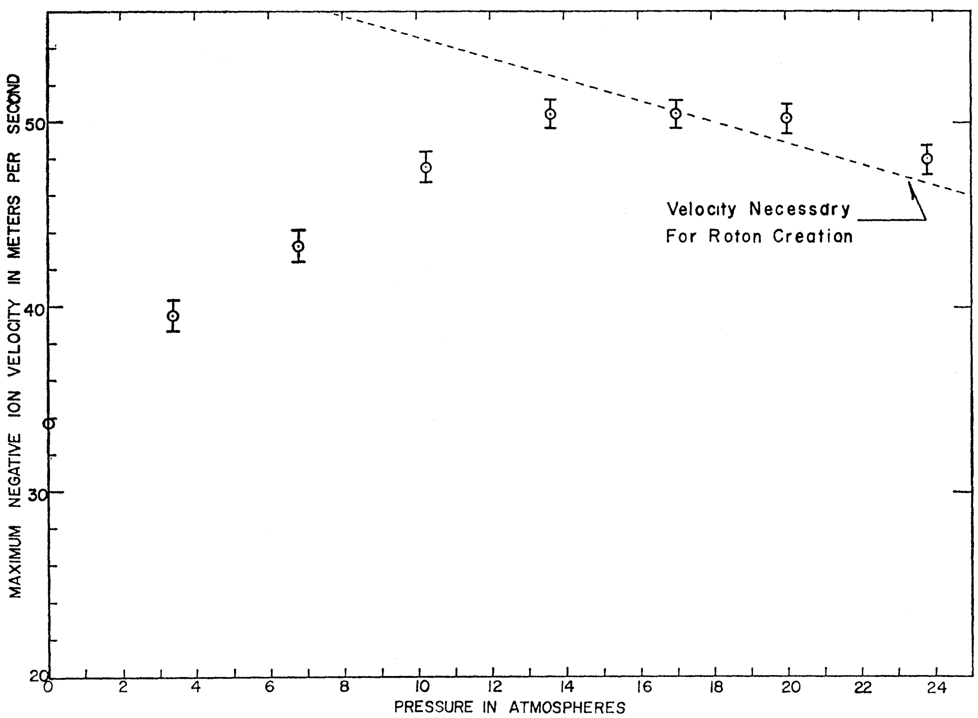

FIG. 8. Critical velocity v_c versus pressure for negative ions in liquid He^4.

The maximum possible momentum transferred to the ion is about $2P_0$, corresponding to a change in ion velocity Δv of $2P_0/m^*=6$ m/sec.[22] This corresponds to a 15% change in velocity of the ion near the critical velocity. Perhaps the best approximation is to assume the mass of the ion to be very large, and set the rate of momentum loss by the ion due to quasiparticle collision equal to the force acting on the ion due to the electric field. The assumption of a large mass for the ion allows one to neglect the fluctuation in its velocity due to quasiparticle collisions. The average velocity of the ion is then independent of its effective mass and depends only on the cross section for ion-quasiparticle scattering. The flattening of the velocity curve near the critical velocity would then indicate that the cross section for ion-quasiparticle scattering has gone up, possibly be-

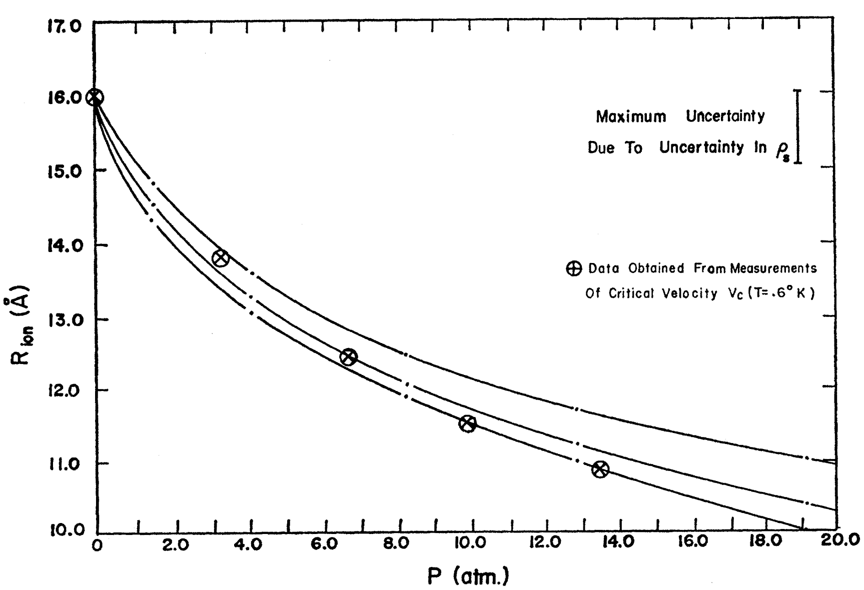

FIG. 9. Pressure dependence of negative-ion bubble radius derived from data of Fig. 8 up to 13.6 atms.

[22] The thermal velocity of the ion would also be approximately 6 m/sec.

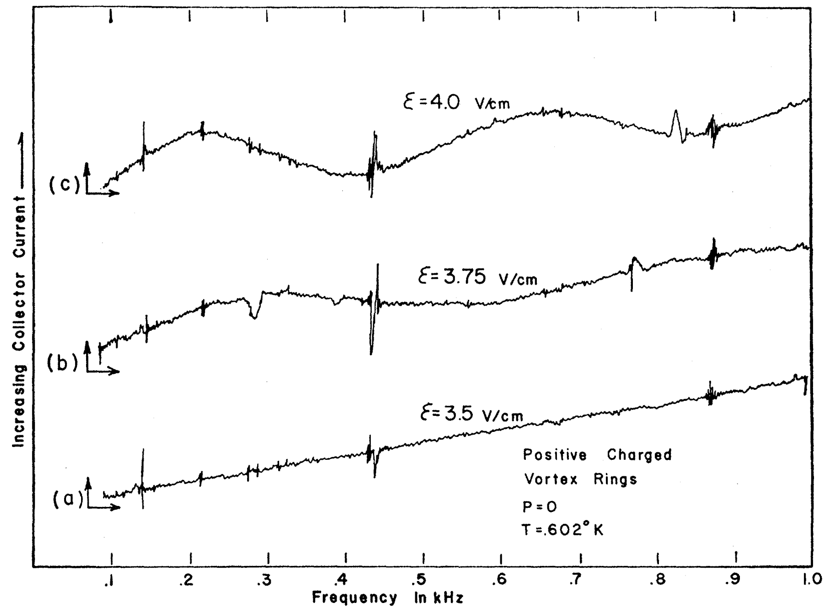

FIG. 10. Recorder traces used in determining the drift velocity of positive charged vortex rings in a given electric field.

cause of an increase in size of the velocity field about the ion associated with the motion of the ion through the superfluid. The nature of the transition from back flow to an ion–vortex-ring complex is not clear, but it seems reasonable to assume the two are connected in some way. Arguments relating to persistence of velocity effects by ions in liquid helium have previously been presented by Meyer and Reif.[9]

VORTEX-RING MEASUREMENTS

In this paper, we have shown that a gated velocity spectrometer may be used to study experimentally the ion–vortex-ring transition. It has also been shown that the same spectrometer may be used in studying the behavior of small vortex rings. The results of these measurements will now be described in detail.

Referring to Fig. 2 and Fig. 3, vortex rings are generated at the source by the field between S and G_1. The drift region between G_1 and G_2 allows the vortex ring to reach a steady-state velocity for the given electric field $\mathcal{E}$. The velocity of the ring is then measured in the region between G_2 and G_5 as a function of applied field $\mathcal{E}$. Typical results of a velocity measurement as a function of applied field $\mathcal{E}$ is shown in Fig. 10. In (a) the electric field is too weak to support a vortex ring, while in (b) the vortex-ring spectrum is observed for a slightly increased value of electric field. Curve (c) shows the decrease of vortex-ring velocity for a slightly larger electric field. The field in the region from S to G_1 is kept constant during these measurements. A gradual increase of background current is due to ions moving through the apparatus which are not trapped in vortex rings. For the value of electric field shown in (b), the bare-ion drift velocity is 670 cm/sec corresponding to a radius of 69 Å. Only positively charged vortex rings will be discussed.

The velocity of a vortex ring of radius R and circulation κ is[23]:

$$v=(\kappa/4\pi R)[\ln(8r/a)-\tfrac{1}{2}]. \tag{1}$$

The core of the vortex ring is assumed to be hollow and of radius a. Previous measurements[1] of a at low temperatures $T\simeq 0.3$°K show $a=1$ Å. At elevated temperatures ($T>0.55$°K), the energy loss by the vortex ring due to quasiparticle scattering becomes appreciable. It has been shown[1] that the effect of this scattering is to introduce a frictional force $\mathfrak{F}$ acting on the vortex ring

$$\mathfrak{F}=\alpha[\ln(8r/a)-\tfrac{1}{2}],$$

where α is an attenuation constant, which is temperature-dependent, but does not depend on the radius (energy) of the vortex ring. For a ring moving through the helium at a constant velocity (steady state), it is required that the frictional force $\mathfrak{F}$ is balanced by the force due to the electric field:

$$e\mathcal{E}=\alpha[\ln(8r/a)-\tfrac{1}{2}], \tag{2}$$

where $\mathcal{E}$ is the applied electric field across the apparatus from G_1 to C, and α, the attenuation constant, is measured in electron volts per centimeter. The vortex ring is assumed to be singly charged. Equations (1) and (2) may be used to eliminate R and find a relation between

[23] The core of the vortex is assumed hollow [see W. M. Hicks, Phil. Trans. Roy. Soc. London **175A**, pp. 183 and 190 (1884)]. It is interesting to note that the additive constant on the logarithm term varies depending on the model used for the core and the method used in computing the vortex-ring velocity. (For example, defining the velocity by $v=dE/dP$, where E is the energy of the ring and P the momentum, gives a somewhat different value for the constant.) A change in this additive constant changes the value of a at $T=0$, but relative changes in a with temperature are not affected.

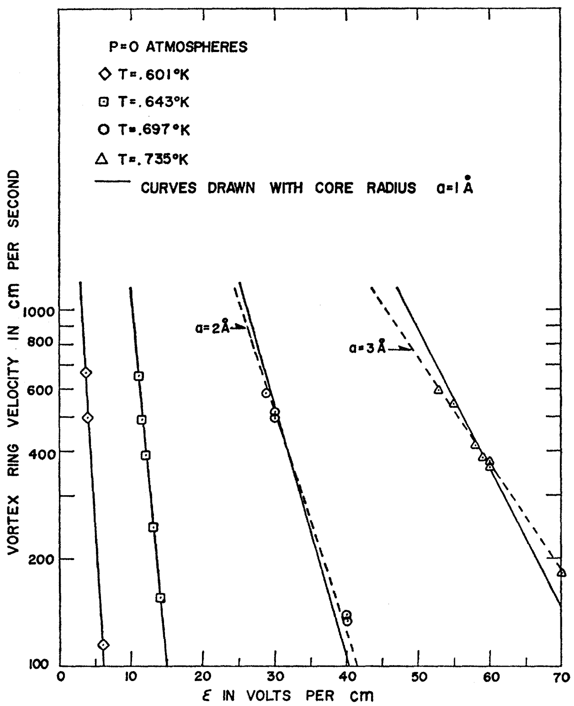

FIG. 11. Log of vortex-ring velocity versus electric field ℰ at various temperatures and normal vapor pressure.

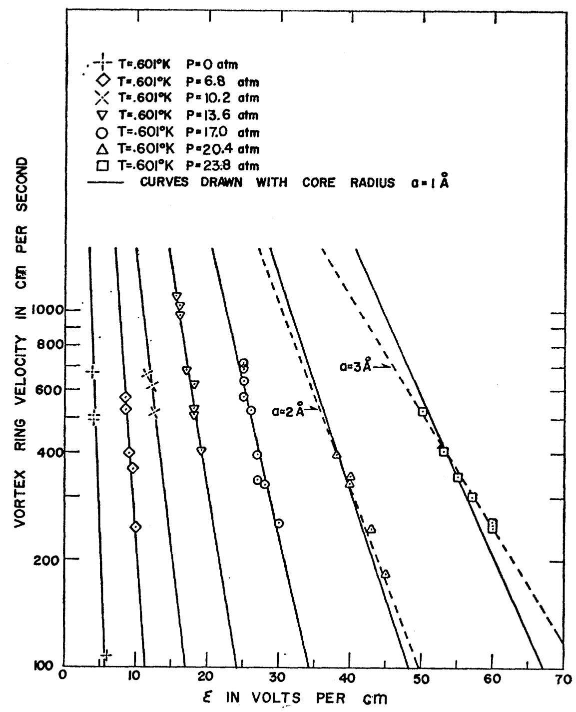

FIG. 12. Log of vortex-ring velocity versus electric field ℰ at various pressures for fixed temperature $T=0.601$°K.

$\mathcal{E}$ and v.

$$\ln v=-\frac{e\mathcal{E}}{\alpha}-\tfrac{1}{2}+\ln\left[\frac{8\kappa e\mathcal{E}}{4\pi a\alpha}\right].$$

For the range of parameters used in these experiments $\log v$ versus $\mathcal{E}/\alpha$ is essentially a straight line with the slope and intercept determined by α and a as shown in Fig. 11.

It is important to insure that equilibrium exists between the electric field force and frictional force acting on the vortex ring. Three tests were used: (1) It is required that the measured velocity and electric field fall on a straight line when $\log v$ is plotted versus $\mathcal{E}$. (2) In Appendix A the equation of motion for a vortex ring not in equilibrium with the electric field is examined. The distance traveled before equilibrium is reached is computed and compared with the length of the drift space. (3) Any change of source voltage should not have an effect on the velocity of the vortex rings. So far as could be determined all vortex rings corresponding to the data of Figs. 11 and 12 were in equilibrium with the applied electric field.

Figures 11 and 12 show the logarithm of the measured vortex-ring velocity plotted versus the applied electric field. The attenuation constant α and core radius a were adjusted for a best fit between theory and experiment. Surprisingly, the best fit is obtained if a is allowed to be pressure and temperature sensitive as shown in Figs. 11 and 12. Previous[1] measurements of α were obtained by actually measuring the amount of energy lost in traversing a field-free region. The present measurements give good agreement. Previous measurements of a near $T=0.3$°K were obtained by direct measurement of vortex-ring velocity in a field-free region with very little quasiparticle scattering present. The present data at low temperatures give the same value for core radius $a=1$ Å.

It has previously been shown,[1] using kinetic-theory arguments, that the attenuation constant α is simply related to the roton–vortex-line scattering cross section $\bar{\sigma}_{r0}$

$$\alpha_r=\frac{3\pi^2}{8}\frac{\kappa}{h^3}P_0{}^4\bar{\sigma}_{r0}(T)e^{-\Delta/kT}.$$

The value of α to be used in determining $\bar{\sigma}_{r0}$ depends on whether a is allowed to be temperature- and pressure-dependent. The curves for $\bar{\sigma}_{r0}$ shown in Figs. (13) and (14) are shown both with a variable and a fixed at $a=1$ Å. It is interesting that $\bar{\sigma}_{r0}$ seems to decrease with increasing temperature if a is held fixed but increases if a is allowed to vary. This increase of $\bar{\sigma}_{r0}$ with increasing a would be consistent with a hard-core scattering in addition to the usual $\mathbf{P}\cdot\mathbf{v}_s$ interaction for roton scattering. As the pressure is increased $\bar{\sigma}_{r0}$ seems to decrease if a is assumed constant, but remains approximately constant if a is allowed to vary. In all of these calculations the scattering due to phonons and He^3 impurities has been neglected. The assumption is reason-

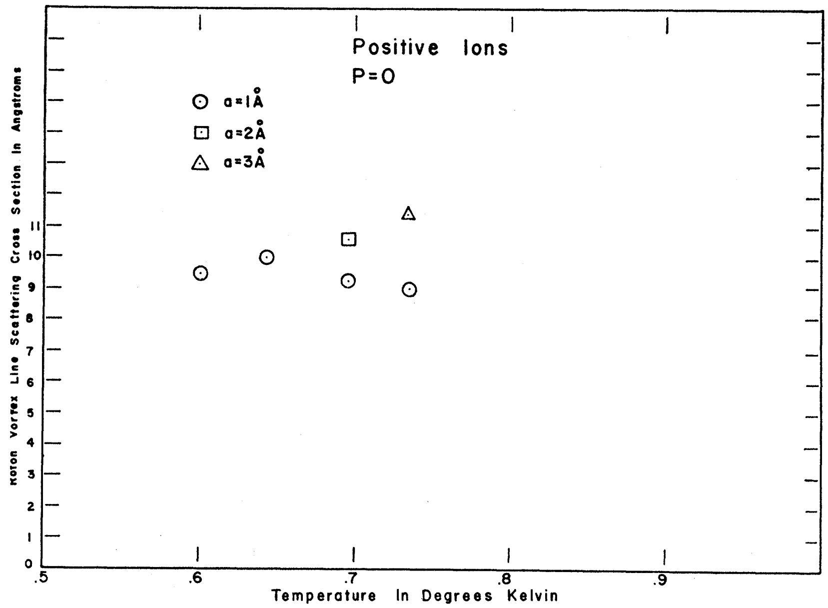

FIG. 13. Roton-vortex line scattering cross section at various temperatures.

able at this temperature.[1] The change of the roton parameters with pressure has been included.[24]

The smallest vortex ring which was observed in the present experiment had a radius of 37 Å.

It is interesting to note in Figs. 11 and 12 that the classical hydrodynamic equations (1) and (2) seem to hold quite well for rings of this size. Also the scattering due to the positive ion in the vortex core would appear to be negligible.

Huang and Olinto[25] have presented a phenomenological theory for vortex-ring creation by ions to explain data obtained by Careri[26] *et al.* In this theory, Huang and Olinto postulate that vortex-ring creation takes place at low-ion velocities, on the order of 5 m/sec. However, in order for singly charged vortex rings to be created, some sort of match is required between the vortex-ring velocity and ion velocity in a given electric field. At lower values than this electric field, the charge–vortex-ring complex is assumed to break up because the frictional force on the ion tends to make the ion move slower while the frictional force on the ring tends to make it move faster.

The basic postulate of Huang and Olinto is that the electric field necessary to produce a stable vortex ring is that field capable of supporting a charged vortex ring. This postulate is not supported by the present experiment. In Fig. 6 it is observed that singly charged vortex rings can exist when $\mathcal{E}'\approx 8$, while bare ions apparently produce vortex rings only when $\mathcal{E}'\approx 20$. Indeed, it is found that singly charged vortex rings are produced only when the positive-ion velocity is 40 m/sec. The corresponding singly charged vortex ring velocity is much lower, typically less than 1 cm/sec.[27] In the case of negative ions, Fig. 5, vortex rings exist for $\mathcal{E}'\approx 1$, while the reduced field necessary to produce singly charged vortex rings is $\mathcal{E}'\approx 8$. Another interesting feature is that an additional quasiparticle scattering by the positive ion attached to the core of a small vortex ring has not been observed, although this is basic to the theory of Huang and Olinto.[28]

Attempts to associate the vortex-ring creation with roton creation are also inconsistent with the data, since the critical velocity for roton creation drops as hydrostatic pressure is applied to the fluid, while the critical velocity for vortex-ring creation increases for negative ions and remains constant in the case of positive ions.

Perhaps a better approach to the problem would be to associate the vortex-ring creation process with a quantized circulation, as was done in this paper in the case of negative ions. Thus the parameter of central importance would be the ion velocity necessary so the

[24] D. G. Henshaw and A. D. B. Woods, in *Proceedings of the Seventh International Conference on Low-Temperature Physics, Toronto, 1960,* edited by G. M. Graham and A. Hollis Hallet (University of Toronto Press, Toronto, Canada, 1961), p. 64. $\Delta=8.65°$K, $P_0/h=1.91$ for p_0 atms and $\Delta=7.0$, $P_0/h=2.05$ for 25.3 atms.

[25] K. Huang and A. C. Olinto, Phys. Rev. **139**, A1441 (1965).

[26] G. Careri, S. Consolo, P. Mazzoldi, Phys. Rev. **136**, A303 (1964).

[27] As a typical example at $T=0.643°$K, the electric field required to produce vortex rings from the bare ion is 40 V/cm. The attenuation constant at this temperature is about 1.88 eV/cm. This gives $\mathcal{E}/\alpha\approx 20$. This value of $\mathcal{E}/\alpha$ should support a ring almost 1 cm in radius traveling considerably less than 1 cm/sec. However, a very long drift space would be required for this vortex ring to reach the steady-state value.

[28] It is interesting to compare the frictional force due to quasiparticle scattering from the ion with that due to scattering from the vortex ring. For constant velocity in a uniform electric field, we have $v=\mu\mathcal{E}$ for the ion, and $e\mathcal{E}=\alpha(\eta-\frac{1}{2})$ for the vortex ring. These may be used to estimate the two frictional forces at 0.643°K for a complex moving at 1000 cm/sec ($R=40$ Å), $\mathfrak{F}_{\text{v.r.}}=\alpha(\eta-\frac{1}{2})=1.88\times 5.32=10$ eV/cm while $\mathfrak{F}_{\text{ion}}=ev/\mu=1000/910=1.1$ eV/cm.

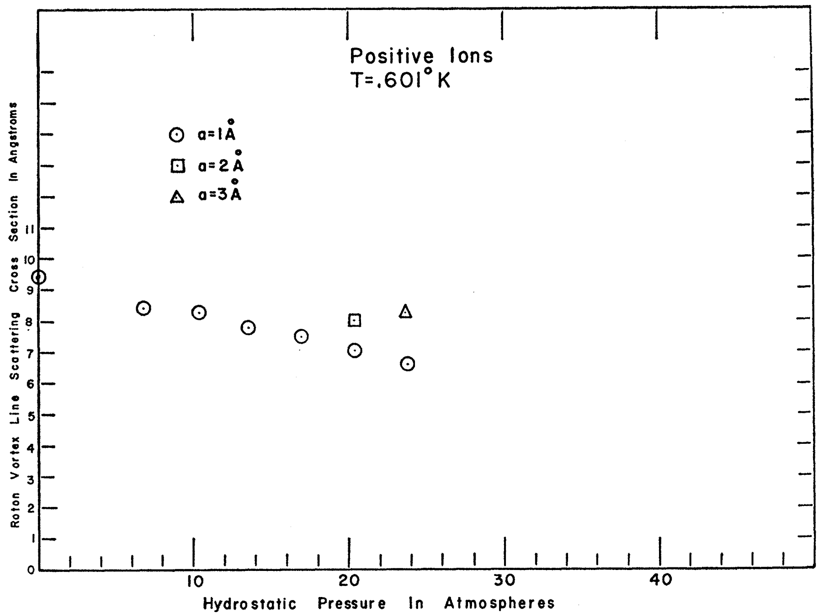

FIG. 14. Roton-vortex line scattering cross section at various pressures from $\alpha_r=(3\pi^2/8)(\kappa/h^3)p_0^4e^{-\Delta/kT}\bar{\sigma}_{r0}(T)$. The pressure-dependent roton parameters have been obtained from neutron scattering data (see Ref. 24).

flow field about the ion satisfies $\oint \mathbf{v}_s \cdot d\mathbf{l}=h/m$. The critical velocity in this case would be large and the mobility jumps which have been observed would have to be explained in some other way or fit into a theory of the flow field about the ion when it is below its critical velocity.

CONCLUSIONS

The ion-vortex ring transition has been studied experimentally and several characteristics regarding the transition have been noted. The critical velocity for vortex-ring production by negative ions has been demonstrated to depend on the radius of the negative ions until a pressure of about 15 atms is reached whereupon the ion velocity is limited by roton emission. The critical velocity for positive ions was found to be 40 m/sec and is essentially pressure-independent. The critical velocity for both negative and positive ions was found to be temperature-independent. Creation of vortex rings by bare ions requires considerably larger electric fields than those fields necessary to support a singly charged vortex ring. A source effect was observed which allowed vortex rings to be produced near the source much more easily than in the space between grids. Apparently, small singly charged positive vortex rings obey the classical hydrodynamic equations down to a radius of about 37 Å. No evidence for quasiparticle scattering by the ion trapped on the vortex core was observed. However, the vortex-ring data indicate that the size of the vortex core may be temperature- and pressure-dependent.

Future experiments should be performed to determine the effect of He^3 impurities on the ion–vortex-ring transition. The behavior of small negatively charged vortex rings would be interesting, since the quasiparticle scattering due to negative ions is much larger than that due to positive ions. Finally, and perhaps most important, the temperature and pressure dependence of the vortex core should be examined carefully. Work along these lines is now in progress.

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APPENDIX A

In order to calculate the penetration of a vortex ring into a gate when the field is reversed, one requires the equation of motion. Let the total force acting on the vortex ring in electron volts per cm be $e\mathcal{E}+\alpha(\eta-\frac{1}{2})$, where $\mathcal{E}$ is the retarding field, α is the attenuation coefficient, and $\eta=\ln(8R/a)$. The question is: Given a ring of radius R_0, how far can it penetrate this region? It will be assumed that the ring vanishes when $R=10$ Å; this assumption for the size is not critical. The Magnus effect[1] can be used to find the response of a vortex line element to a given force. Let $\mathbf{F}'$ be the force acting on the line per unit length

$$2\pi RF_z'=e\mathcal{E}+\alpha(\eta-\tfrac{1}{2}).$$

Then setting the magnus force opposite and equal to this force we have

$$F_z'=-G_z'=-\rho(\boldsymbol{\kappa}\times\mathbf{U})_z=-\rho\kappa\dot{R}$$

or substituting for F_z'

$$2\pi\rho\kappa R\dot{R}=(d/dt)(\pi\rho\kappa R^2)=-e\mathcal{E}-\alpha(\eta-\tfrac{1}{2}).$$

Let the vortex ring be moving in the axial z direction.

The instantaneous velocity of the ring is taken to be

$$\frac{dz}{dt}=v=\frac{\kappa}{4\pi R}(\eta-\tfrac{1}{2}),$$
$$\frac{dR}{dz}\frac{dz}{dt}=\frac{-e\mathcal{E}-\alpha(\eta-\frac{1}{2})}{2\pi R\rho\kappa}, \tag{A1}$$
$$\frac{dR}{dz}=\frac{1}{v}\left(\frac{-e\mathcal{E}-\alpha(\eta-\frac{1}{2})}{2\pi\rho\kappa R}\right)=\frac{-e\mathcal{E}-\alpha(\eta-\frac{1}{2})}{\frac{1}{2}\rho\kappa^2(\eta-\frac{1}{2})}.$$

This equation was integrated (on a computer) to find the vortex-ring radius as a function of axial position z. For the range of parameters used in the experiment, the penetration of vortex rings into the gates was negligible when the fields were reversed.

The procedure used in determining the distance at which the vortex-ring velocity reaches a steady-state value is similar except that the electric field is now tending to increase the energy of the vortex ring. Now $\mathfrak{F}=e\mathcal{E}-\alpha(\eta-\frac{1}{2})$, and R as a function of z may be computed as above. Since the velocity of the ring depends on its radius, the velocity as a function of axial position may now be determined and compared with the steady-state value. The steady-state value is determined by setting $e\mathcal{E}=\alpha(\eta-\frac{1}{2})$, solving for R, and then for the steady state velocity, $v=(\kappa/4\pi R)(\eta-\frac{1}{2})$. In general, the drift space was chosen large enough so the velocity of the vortex ring before entering the velocity-measuring region was within a few percent of the steady-state value.

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Many-Body Theory for Quantum Kinetic Equations*

BANDEL BEZZERIDES
Lawrence Radiation Laboratory, University of California, Livermore, California
AND
D. F. DUBOIS
Hughes Research Laboratories, Malibu, California
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A quantum kinetic equation is obtained which includes scattering events beyond the binary collision process of the Boltzmann equation. The time rate of change of the Wigner function due to collisions is given as the difference of two scattering rates analogous to the collisional side of the Boltzmann equation. These total rates are given in terms of partial rates which are identified with probability amplitudes for scattering events in the *medium*. Thus, the method is inherently free of the divergence difficulties of theories which calculate the total rates in terms of scattering events in vacuum. Special attention is devoted to the question in what sense does some form of Fermi's golden rule apply to the calculation of the partial rates for a given event. It is shown that certain modifications to this picture are necessary, but the changes may be understood from a simple physical point of view.

I. INTRODUCTION

MODERN research in the transport properties of a macroscopic system has been focused on the rigorous derivation of a class of Markovian equations used to describe the irreversible approach to thermal equilibrium. A well-known example of such an equation is the Boltzmann kinetic equation. These derivations should be regarded as complete only if a more refined description is characterized precisely by correction terms, and if the corrections are small in some sense. Attempts in these directions have met with limited success.

It is a simple matter to derive coupled sets of equations for the various distribution functions of the system. For definiteness, consider a moderately dense gas; here the first and second distribution functions F_1 and F_2, respectively, are usually adequate. Bogoliubov assumed that the gas relaxed toward equilibrium in three well-defined stages, and that after a time $t\gg t_{\text{collision}}$ the joint probability is "synchronized" to F_1 in the sense that F_2 becomes a functional of F_1 as far as time dependence is concerned.[1] Thus a kinetic equation for F_1 is obtained of the form

$$\partial_T F_1+LF_1=C(F_1), \tag{1.1}$$

where C describes the effects of correlations or collisions, and the operator L is usually regarded as describing the time rate of change of F_1 in the absence of collisions. Strictly speaking, this theory is incomplete since no direct estimate of the error in the assumption is possible. Further, certain of the correction terms which can be calculated are found to diverge.[2]

In an effort to circumvent the synchronization assumption, a cluster-expansion approach has been used

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[1] Essentially the same idea is employed in the Chapman-Enskog solution of the Boltzmann equation in which F_1 becomes "locked" to the temporal behavior of the local properties of the system.

[2] A. H. Kritz and G. Sandri, Phys. Today 19, 57 (1966).