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Dynamical Approach to the Nonleptonic Decays

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An assumption about the structure of single-particle matrix elements of the weak nonleptonic Hamiltonian is suggested and motivated. Using this assumption together with the algebra of currents, we calculate the S- and P-wave decays of hyperons, and the K decays, and find good agreement with experiment.

I. INTRODUCTION

THE current algebra as applied to the current-current form of the weak Hamiltonian combined with the $SU(3)$ symmetry has led to numerically satisfactory results for the S-wave nonleptonic decays and to very bad results for the P-wave nonleptonic decays.[1] In this work we shall introduce a simple dynamical assumption about the matrix elements of the weak Hamiltonian which will serve as the substitute for the $SU(3)$ assumptions usually used.

Since in the $SU(3)$ limit both the S- and P-wave amplitudes vanish in this new scheme, it is profoundly different from the usual theory. Consequently, we shall show that it is possible to predict the observed P-wave amplitudes if some of the as yet unmeasured axial-vector coupling constants for the semileptonic processes depart significantly from the values predicted by the $SU(3)$ symmetry. This departure is in accord with several recent theoretical predictions. In addition, our scheme predicts the S-wave decays very well in terms of one parameter which turns out to be simply related to the universal weak-coupling constant.

II. REVIEW OF USUAL TECHNIQUES

Before we proceed to the new method it will be convenient to outline the conventional formulation[1] so as to emphasize the points of departure. We introduce the nonleptonic Hamiltonian density which is assumed to have $\Delta I=\frac{1}{2}$ and $\Delta S=1$ selection rules. The decay $N'\rightarrow N\pi$, where N and N' are baryons, is then described by the matrix element

$$\langle N(p)\pi(q)|-i\int d^4x\, H_W(x)|N'(p')\rangle = -i(2\pi)^4\delta^4(p'-p-q)(2q_0V)^{-1/2}M(q). \quad (1)$$

For purposes of extrapolations that will be performed later, it is convenient to display explicitly the pion momentum in the above definition of $M(q)$. Contracting the pion, Eq. (1) becomes

$$M_j{}^i(q)=(q^2+m_\pi{}^2)\int d^4x\, e^{-iq\cdot x} \times\langle N(p)|T(\phi_j{}^i(x)H_W(0))|N'(p')\rangle. \quad (2)$$

Introducing the hypothesis of partially conserved axial-vector current (PCAC) in the form

$$\partial_\mu A_{\mu j}{}^i(x)=(c_\pi/\sqrt{2})\phi_j{}^i(x)\,,\quad c_\pi=-2M_Nm_\pi{}^2g_A/G_{NN\pi}\,,$$

Eq. (2) may be written as

$$M_j{}^i(q)=\frac{\sqrt{2}}{c_\pi}(q^2+m_\pi{}^2)\left\{iq_\mu\int d^4x\, e^{-iq\cdot x} \times\langle N(p)|T(A_{\mu j}{}^i(x)H_W(0))|N'(p')\rangle -\int d^3x\, e^{-i\mathbf{q}\cdot\mathbf{x}} \times\langle N(p)|[A_{0j}{}^i(\mathbf{x},0),H_W(0)]|N'(p')\rangle\right\}. \quad (3)$$

Assuming commutation relations of the form $[A_0(\mathbf{x},0), H_W(0)]\alpha\delta^3(\mathbf{x})$, the second term of Eq. (3) may be evaluated regardless of the value of q_μ. The evaluation of the first term is, however, in general, not so elementary and this difficulty forces the usual expansion of $M_j{}^i(q)$ about $q=0$. In this case, it is easily seen that contributions for $q=0$ occur only when intermediate states are degenerate in the mass with the initial or the final state. In actual computations, we shall avoid calculating any such contribution by introducing a fictitious mass difference where necessary and take the limit of mass degeneracy at the end of the calculation. Consequently we find

$$M_j{}^i(0)=\frac{\sqrt{2}}{c_\pi}m_\pi{}^2\langle N(p)|\left[\int d^3x\, A_{0j}{}^i(\mathbf{x},0) \times H_W(0)\right]|N'(p')\rangle. \quad (4)$$

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[1] M. Suzuki, Phys. Rev. Letters **15**, 986 (1965); M. Suzuki, Phys. Rev. **144**, 1154 (1966); H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); **15**, 997 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* **16**, 380 (1966); L. S. Brown and C. M. Sommerfield, *ibid.* **16**, 751 (1966); P. Babu, Phys. Rev. **148**, 1440 (1966); Fayyazuddin and Riazuddin, Trieste Reports, 1966 (unpublished); Y. T. Chiu and J. Schechter, Phys. Rev. Letters **16**, 1022 (1966); S. N. Biswas, A. Kumar, and R. P. Saxena, *ibid.* **17**, 268 (1966).

At this stage, it is now necessary to make a very natural assumption about the structure of the Hamiltonian. It is *postulated* that its structure is such that

$$\int d^3x\,[A_0(\mathbf{x},0)_j{}^i,H_W(0)] = \int d^3x\,[V_0(\mathbf{x},0)_j{}^i,H_W(0)], \quad (5)$$

where $V_{0j}{}^i$ is the time component of the vector current. This assumption essentially states that the parity-violating part of the weak Hamiltonian $H_W{}^{\text{p.v.}}$ has the same weighting as the parity-conserving part $H_W{}^{\text{p.c.}}$ and, so, may be reexpressed as the two equations

$$\left[\int d^3x\,A_{0j}{}^i(\mathbf{x},0),H_W{}^{\text{p.v.}}(0)\right] = \left[\int d^3x\,V_{0j}{}^i(\mathbf{x},0),H_W{}^{\text{p.v.}}\right] \quad (6)$$

and

$$\left[\int d^3x\,A_{0j}{}^i(\mathbf{x},0),H_W{}^{\text{p.c.}}(0)\right] \times\left[\int d^3x\,V_{0j}{}^i(\mathbf{x},0),H_W{}^{\text{p.v.}}\right]. \quad (7)$$

It is understood that i and j are restricted to the values 1 and 2, so that $\int d^3x\,A_{0j}{}^i(x)$ is always a pion-type axial charge and that $\int d^3x\,V_{0j}{}^i(x)$ are the time-independent generators of isotopic spin transformations. These conditions are, of course, met by the usual current-current form, as well as its special form with octet dominance,

$$H_W=(G'/\sqrt{2})d_{6ij}(A_\mu{}^i+V_\mu{}^i)(A_\mu{}^j+V_\mu{}^j), \quad (8)$$

and also by the scalar density Hamiltonian[2]

$$H_W=\alpha(\bar\psi\gamma_5\lambda_7\psi+\bar\psi\lambda_6\psi). \quad (9)$$

In Eqs. (8) and (9) we have used the well-known $SU(3)$ notation of Gell-Mann, instead of the tensor notation. Substituting Eq. (5) into Eq. (4) and using the fact that $\langle N|\int d^3x\,V_{0j}{}^i$ is a combination of states in the same isotopic multiplet as the N, it follows that the problem of evaluating $M_j{}^i(0)$ is reduced to evaluating matrix elements of the form $\langle N^{\text{I}}|H_W(0)|N^{\text{II}}\rangle$, where N^{I} and N^{II} are members of the baryon octet. Now for $H_W(0)$ of the current-current form (8), this matrix element is, in general, extremely complex in structure and is not directly evaluated in terms of simple parameters. However, if $SU(3)$ is assumed, and $H_W(0)$ transforms like the sixth component of an octet, then using rather symbolic notation, it follows that[1]

$$\langle N^{\text{I}}|H_W{}^{\text{p.c.}}(0)|N^{\text{II}}\rangle=Xd_{6\,\text{I}\,\text{II}}+Yf_{6\,\text{I}\,\text{II}}$$

and

$$\langle N^{\text{I}}|H_W{}^{\text{p.v.}}(0)|N^{\text{II}}\rangle=0. \quad (10)$$

The first expression shows that $M_j{}^{i\,\text{p.c.}}(0)$ can be expressed in terms of two parameters, coupling strength Y and the $F/D=Y/X$ ratios of the weak scalar spurion. Thus, to the extent that $M_j{}^{i\,\text{p.c.}}(q)$ for a physical pion is described by the value for $q=0$, the S-wave nonleptonic decays of the baryons are described by two parameters. With proper adjustment of these parameters it is possible to get a good fit with the experiment. The second expression arises because of charge-conjugation invariance combined with the $SU(3)$ symmetry. It shows that $M_j{}^{i\,\text{p.c.}}(0)$ cannot be a good estimate of $M_j{}^{i\,\text{p.c.}}(q)$ with q corresponding to a physical pion for, if it were, the P-wave decays would not occur. In order to get around this difficulty, Brown and Sommerfield[1] have observed that the Born-approximation contribution to Eq. (2) does not extrapolate slowly to $M_p{}^{\text{p.c.}}(0)$. In fact, while $M_B{}^{\text{p.c.}}(q)\propto[M(N')+M(N'')]^{-1}$, one has $M_B{}^{\text{p.c.}}(0)\propto[M(N')-M(N'')]^{-1}$. Consequently, it was suggested that the correct expansion for the nonleptonic decay should be written as

$$M(q)\simeq\{\lim_{q\to0}[M(q)-M_B(q)]+M_B(q)\}. \quad (11)$$

Proceeding in this manner and using the value of the weak spurion as calculated to yield the correct value of the S-wave decays, one still finds that the predicted P-wave decay amplitudes are roughly half as large as the experimental ones.[3]

III. NEW DYNAMICAL ASSUMPTION

It is possible that the technique outlined above with slight modifications can in fact correctly produce the P-wave rates. For example, if the Born amplitudes extrapolate badly there is no obvious reason forbidding other perturbative-type amplitudes to extrapolate badly also and perhaps, in principle, it would be possible to handle these in the same way as the Born terms are handled in Eq. (11). On the other hand, it could be that the extrapolation $q\to0$ is so bad that no simple finite number of perturbation corrections can properly relate $M(q)$ to $M(0)$. Some more insight to the problem may be gained by noting that $M_B{}^{\text{p.c.}}(q)$ as given by Eq. (10) contains only baryon poles, while work done previous to the development of current-algebra techniques suggested that there should be contributions from the K-meson poles as well.[4] Some attempts have been made recently to include by hand, in the above calculation, the contributions of a Born diagram and in this manner it has been possible to get

[2] M. K. Gaillard, Phys. Letters **20**, 533 (1966); Riazuddin and K. T. Mahanthappa, Phys. Rev. **147**, 972 (1966); C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966); R. Gatto, L. Maiani, and G. Preparta, Nuovo Cimento **41A**, 622 (1966).

[3] L. S. Brown and C. M. Sommerfield, Ref. 1.

[4] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961).

somewhat closer agreement with the experiment.[5] However, we wish to extract the content of this diagram in quite a different way, and in so doing will depart considerably from the normal structure of the theory.

All of the discussion up to Eqs. (10) and (11) seems to be very basic and we will accept it as valid. As mentioned, the $SU(3)$ assumptions leading to Eqs. (10) and (11) were made because of the basic difficulty of evaluating the matrix element of the product of two currents or, for that matter, of a scalar density. We have no general prescription to overcome this difficulty so we shall, instead, replace the $SU(3)$ assumption by an alternative dynamical assumption. We assume that the weak spurion has the following matrix elements between single-baryon states:

$$\langle N|H_W^{\text{p.v.}}(x)|N'\rangle=\lambda_P\partial_\mu\langle N|A_{\mu7}(x)|N'\rangle \qquad (12)$$

and

$$\langle N|H_W^{\text{p.c.}}(x)|N'\rangle=\lambda_S\partial_\mu\langle N|V_{\mu7}(x)|N'\rangle, \qquad (13)$$

where, of course, $V_{\mu3}{}^2(x)$ is the vector current and $A_{\mu3}{}^2(x)$ is the axial-strangeness-changing neutral current density. We emphasize that these equalities are *assumed only for the single-particle matrix elements* and not for the operators. For the time being, we will treat λ_S and λ_P as independent parameters.

At this point a few comments are in order about Eqs. (12) and (13) and their relation to the earlier formulations of this paper. From the above equations it might be conjectured that we could define

$$H_W=\lambda\partial_\mu A_7{}^\mu+\lambda\partial_\mu V_7{}^\mu. \qquad (14)$$

Indeed, in the initial formulations of this theory[6] it was suggested that (14) be considered seriously as a phenomenological interacting mediating the nonleptonic decays. Although all numerical calculation of a preceding paper[6] are in correspondence with calculations in this presentation, we will not consider that H_W as given by (14) is actually the nonleptonic Hamiltonian because of basic theoretical difficulties. These difficulties are considerably subtler than one might think and, consequently, we only discuss them briefly at this point. It is usually stated that adding a divergence to a Lagrangian is equivalent to a unitary transformation and, consequently, that no change in the physical predictions of the theory can occur. This is a gross oversimplification since, under certain circumstances, when either there is a spontaneously broken symmetry or the operator whose divergence is added contains a part linear in a field operator which has canonical commutation relations, the transformation induced is nonunitary and may change the physical predictions of the theory. Thus a divergence interaction is not of necessity trivial. Quite aside from this argument, it can be seen directly that if a decay $N'\to N+\pi$ is to be described by H_W as given by (14), then we must have for the first-order contribution

$$\langle N'|\int d^4x\, H_W(x)|N\pi\rangle=(2\pi)^4\delta^4(p'-p-q)\times(p'-p-q)_\mu\langle N'|A_{\mu7}(0)+V_{\mu7}(0)|N\pi\rangle,$$

which vanishes unless $\langle N'|A_{\mu7}(0)+V_{\mu7}(0)|N\rangle$ is singular for $N'=N+\pi$. It may be confirmed that, with the known particles given these physically observed masses, no simple approximation can produce the necessary pole in the above matrix element. Consequently, we may reject (14) as a serious possibility for the Hamiltonian on the basis of physical, if not theoretical, arguments.

It should be emphasized at this point that these considerations for the above matrix element in no way imply that the matrix elements between baryon states of the actual H_W, whatever it may be, cannot behave as in Eqs. (12) and (13).

It is of interest to consider if there is, in the presumably realistic case of a current-current interaction as given by Eq. (8), any sort of mechanism which at least suggests the validity of Eqs. (12) and (13).

Note that using (13) combined with (4) and (5), we find that

$$M_j{}^i(0)=-\frac{\lambda_S\sqrt{2}}{c_\pi}m_\pi{}^2\langle N|\left[\int d^3x\, V_{0j}{}^i(x),V_{\mu7}(0)\right]|N'\rangle\times(p-p')_\mu. \qquad (15)$$

If for S waves $M_j{}^i(0)\simeq M_j{}^i(q)$, then the above when inserted into (1) becomes

$$\langle N\pi|-i\int d^4x\, H_W(x)|N'\rangle=(2\pi)^4\delta^4(p'-p-q)\frac{\lambda_S\sqrt{2}m_\pi{}^2}{c_\pi(2q_0v)^{1/2}}q_\mu\times\langle N|\left[\int d^3x\, V_{0j}{}^i(x),V_{\mu7}(0)\right]|N'\rangle. \qquad (16)$$

This structure looks very similar to the structure of the matrix element for baryon decay after using pion factorization. To this end we consider the analytically continued matrix element $\langle\pi|-iH_W(0)|\bar{N}N'\rangle$. Using H_W as given by (8) and inserting a complete set of intermediate states and using the hypothesis of partially conserved axial-vector current (PCAC), it easily follows on the particle mass shells that, for the S-wave amplitude,

$$\langle\pi|-iH_W(0)|\bar{N}N'\rangle=\frac{G'}{\sqrt{2}}\Bigg[\frac{c_\pi}{\sqrt{2}m_\pi{}^2}\frac{i(p'-p)_\mu}{(2q_0v)^{1/2}}\times\langle 0|\left[\int_{x_0=0}d^3x\, V_{0j}{}^i(x),V_{\mu7}(0)\right]|N\bar{N}\rangle+\sum_{S\text{-wave part}}\langle\pi|V_{\mu i}+A_{\mu i}|\alpha\rangle\langle\alpha|V_{\mu j}+A_{\mu j}|\bar{N}N'\rangle\Bigg]. \qquad (17)$$

[5] S. A. Bludman (unpublished).

[6] G. S. Guralnik, V. S. Mathur, and L. K. Pandit, University of Rochester Report Nos. UR-875-148, 1966 and UR-875-169, 1966 (unpublished). In this context, see also K. Nishijima and J. L. Swank, Phys. Rev. **146**, 1161 (1966).

If it is now conjectured that in the soft-pion limit every term but the first of the above expression becomes negligible, it is then possible to deduce (12) and (13) if the identification

$$\lambda_S=(G'/\sqrt{2})(c_\pi/\sqrt{2}m_\pi^2)^2 \tag{18}$$

is made.

We have not been able to substantiate our conjecture. In the special case of $SU(3)$ symmetry with $M(N)=M(N')$, it cannot be valid for $H_W{}^{\text{p.v.}}$ since by C invariance $\langle 0|H_W{}^{\text{p.v.}}|\bar{N}N'\rangle=0$ which is not consistent with Eq. (12). However, when $SU(3)$ is not valid, the case of concern in this paper, C invariance places no restriction on the amplitude and it is conceivable that the conjecture is valid and that (13) does really derive from a current-current form of the nonleptonic decay interaction. If, then, the identification (18) is also valid, we still find that a comparison with the experimental numbers shows that $G'=G$, the universal Fermi coupling constant. This is remarkable since, if the Cabibbo angle were associated only with the strong currents, one would expect that $G'=G\cos\theta\sin\theta$.[7] A less ambitious program, but one which makes more sense in terms of $SU(3)$ limits, results from considering H_W to be a scalar density as given by (9). In this case it is postulated that the dynamics is such that the solution of the field equations would demonstrate the validity of relations (12) and (13). It follows at once that with this representation of $H_W(x)$ it is not necessary for $\langle 0|H_W{}^{\text{p.v.}}|N'\bar{N}\rangle$ to vanish in the $SU(3)$ limit, so that C invariance does not cause any difficulty with the identification of relations (12) and (13) with the Hamiltonian as given by (9). Having dropped the requirement that the $SU(3)$ matrix element of the pseudoscalar spurion vanishes by C invariance, one might think that the dynamical scheme which we have proposed is unnecessary and that the $SU(3)$ invariance can be used to calculate the matrix elements. This is not the case because of an elementary but remarkable theorem of Coleman and Glashow.[8] If the total Hamiltonian is given by

$$H=H_{SU(3)}+H_{\text{m.s.}}+H_{\text{p.c.}}+H_{\text{p.v.}}\,,$$

with

$$H_{\text{m.s.}}=\delta M\bar{\psi}\lambda_8\psi$$

and H_W given by (9), it is easily seen that under the transformation $\psi\to\Psi'=e^{-(i\alpha/\delta m)\lambda_7}\psi$, to first order in α,

$$H'=H_{SU(3)}+H_{\text{m.s.}}+H_{\text{p.v.}}\,.$$

Consequently, if the medium-strong $SU(3)$ symmetry breaking is of the mass-splitting type as given above, a simple canonical transformation demonstrates that there are no nonleptonic P-wave decays. We conclude that the medium-strong interaction must be of a more complicated nature than the simple mass-splitting type indicated above if a scalar density weak Hamiltonian is retained. The departures from $SU(3)$ are very important. This effect will be reflected in the formulas we derive for the P-wave decays, which will vanish when the coupling constants[9] involved take on their $SU(3)$ values. We note at this point that the S-wave amplitudes also vanish in the $SU(3)$ limit corresponding to their relation to the divergence of a conserved vector current. Thus the theory proposed here, unlike others suggested to date, is meaningful only for $SU(3)$ broken.

IV. TABULATION OF THE AMPLITUDES

Without further conjecture as to the dynamical orgins of Eqs. (12) and (13), it is possible to take them as given and to calculate the S- and the P-wave decay amplitudes. As has been previously mentioned, one should be particularly careful in extrapolating the Born part of the amplitudes to vanishing pion four-momentum. In order to account for any difficulties that might arise in this process, we shall determine $M(q)$ through Eq. (11). Using Eqs. (12) and (13), it is easily determined that the off-shell P-wave Born amplitudes are of the order $\delta M/2M$ and, hence, negligible, so that

$$M_p(q)\simeq M_p(0)+M_p{}^{\text{Born}}(q)\,. \tag{19}$$

For the S waves, on the other hand, it is found that the Born terms on and off the mass shell are of the same order of magnitude, so that

$$M^S(q)=M^S(0)\,. \tag{20}$$

We define in the standard manner the A (S-wave) and the B (P-wave) amplitudes by the following expression:

$$M(q)=-i(M_NM_{N'}/p_0p_0'V^2)^{1/2}\bar{u}(p)\times(A+B\gamma_5)u(p')\,. \tag{21}$$

Using the standard definitions for the vector and the axial-vector coupling constants, the result for the various hyperon decays are tabulated below:

$$A(\Lambda\to p\pi^-)=-\sqrt{2}(m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_S(M_\Lambda-M_N)g_V(\Lambda\to p)\,, \tag{22a}$$

$$A(\Sigma^+\to n\pi^+)=0\,, \tag{22b}$$

$$A(\Sigma^+\to p\pi^0)=(m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_S(M_\Sigma-M_N)g_V(\Sigma^+\to p)\,, \tag{22c}$$

$$A(\Sigma^-\to n\pi^-)=-(\sqrt{2}m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_S(M_\Sigma-M_N)g_V(\Sigma^-\to n)\,, \tag{22d}$$

$$A(\Xi^-\to \Lambda\pi^-)=-(\sqrt{2}m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_S(M_\Xi-M_\Lambda)g_V(\Xi^-\to \Lambda)\,. \tag{22e}$$

[7] This result is in contradiction of the trend indicated by saturation calculations. See, e.g., E. Ferrari, V. S. Mathur, and L. K. Pandit, Phys. Letters, **21**, 560 (1966); Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1966). It is in accord with the observation independently made by Sakurai on the basis of a vector-particle-dominance model; see Ref. 10.

[8] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964); B. W. Lee, *ibid.* **140**, B152 (1965).

[9] J. Schwinger, Phys. Rev. Letters **13**, 355 (1964); **13**, 500 (1964).

$$B(\Lambda\to p\pi^-)=-(\sqrt{2}m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_P(M_\Lambda+M_N)[-g_A(\Lambda\to p)+(2M_N/(M_\Lambda+M_N))g_V(\Lambda\to n)g_A(n\to p) -((M_\Lambda+M_\Sigma)/(M_\Lambda+M_N))g_A(\Lambda\to\Sigma^+)g_V(\Sigma^+\to p)], \quad (23a)$$

$$B(\Sigma^+\to n\pi^+)=-(\sqrt{2}m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_P(M_\Sigma+M_N)[(2M_N/(M_\Sigma+M_N))g_V(\Sigma^+\to p)g_A(p\to n) -((M_\Sigma+M_\Lambda)/(M_\Sigma+M_N))g_A(\Sigma^+\to\Lambda)g_V(\Lambda\to n)], \quad (23b)$$

$$B(\Sigma^+\to p\pi^0)=(-m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_P(M_\Sigma+M_N)[g_A(\Sigma^+\to p)+(2M_N/(M_\Sigma+M_N))g_V(\Sigma^+\to p)g_A(p\to p) -(2M_\Sigma/(M_\Sigma+M_N))g_A(\Sigma^+\to\Sigma^+)g_V(\Sigma^+\to p)], \quad (23c)$$

$$B(\Sigma^-\to n\pi^-)=-(\sqrt{2}m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_P(M_\Sigma+M_N)[-g_A(\Sigma^-\to n)-(2M_\Sigma/(M_\Sigma+M_N))g_A(\Sigma^-\to\Sigma^0)g_V(\Sigma^0\to n) -(M_\Sigma+M_\Lambda/(M_\Sigma+M_N))g_A(\Sigma^-\to\Lambda)g_V(\Lambda\to n)], \quad (23d)$$

$$B(\Xi^-\to\Lambda\pi^-)=-(\sqrt{2}m_\pi^2/c_\pi)\tfrac{1}{2}\lambda_P(M_\Xi+M_\Lambda)[-g_A(\Xi^-\to\Lambda)+(M_\Sigma+M_\Lambda/(M_\Sigma+M_\Lambda))g_V(\Xi^-\to\Sigma^-)g_A(\Sigma^-\to\Lambda) -(2M_\Xi/(M_\Xi+M_\Lambda))g_A(\Xi^-\to\Xi^0)g_V(\Xi^0\to\Lambda)]. \quad (23e)$$

Note that, as mentioned previously, the $SU(3)$ limit of both the S- and the P-wave amplitudes vanish.

By using pion and kaon PCAC, the P-wave amplitudes may be recast into a very interesting form. Equation (23a), for example, becomes

$$B(\Lambda\to p\pi^-)=-\tfrac{1}{2}\lambda_p[-(f_K/f_\pi)G_{\Lambda NK} +\sqrt{2}g_V(\Lambda\to N)G_{NN\pi}-g_V(\Sigma^+\to p)G_{\Lambda\Sigma\pi}]. \quad (24)$$

Here f_π and f_K are the π and the K decay constants, known to be nearly equal, and the G's are the strong-coupling constants with the off-shell strong-vertex form factors equated to unity. The first term on the right-hand side of (24) represents the purely current-algebraic contribution and can clearly be interpreted as the contribution from the K pole diagram mentioned previously. This is, we emphasize, the contribution which identically vanishes in the usual treatment. The last two terms which, in our treatment, arise because of corrections to the extrapolated amplitude, represent the Σ and the N pole contributions. Consequently, we find the general structure of the P-wave amplitudes to be as suggested by the pole model of Feldman, Matthews, and Salam.[3] The S-wave amplitudes may be interpreted as contributions from the K^* pole diagrams,[10] which, in this case, make the only contribution from baryon poles. We emphasize that although all our results have a pole-model interpretation, they arise from the more systematic current-algebraic approach, and the coefficients of all pole terms are determined precisely by this approach.

Before proceeding to the detailed numerical comparison of the above results with experiment, note that for the S-wave decays we have extra relations besides the usual $\Delta I=\frac{1}{2}$ sum rules. Using the $SU(3)$ values of the g_V's which are accurate to second order in the $SU(3)$ breaking, we find,[6] in excellent agreement with the experimental data,[11]

$$A(\Sigma_+^+)=0, \quad (25a)$$

$$A(\Lambda_-^0)/(M_\Lambda-M_N)=(\sqrt{\tfrac{3}{2}})A(\Sigma_-^-)/(M_\Sigma-M_N), \quad (25b)$$

$$2A(\Xi_-^-)/(M_\Xi-M_\Lambda)=A(\Lambda_-^0)/(M_\Lambda-M_N). \quad (25c)$$

The standard shorthand notation for the decay processes is employed in the above relations. Note that (25c) differs from the well-known Lee-Sugawara triangle[12] only through the mass-difference denominators. If we use the Gell-Mann–Okubo (GMO) mass formula, Eqs. (25b) and (25c) reduce to the exact Lee-Sugawara triangle

$$2A(\Xi_-^-)+A(\Lambda_-^0)=\sqrt{3}A(\Sigma_0^+). \quad (26)$$

It should be noted that the Lee-Sugawara triangle, which is conventionally derived in the $SU(3)$ symmetry limit, is valid here even when the symmetry-breaking effects are retained in the first order.

As an alternative procedure we may determine the parameters λ_S and λ_P by the experimental value of one amplitude. It is interesting that numerically the expression (18) is valid. Table I shows the values for the S-wave amplitudes constructed in this manner to be in very good agreement with experiment.[13] Note that while the conventional theory has two parameters corresponding to the F and D content of the weak spurion, here we have only one parameter.

For the case of the P-wave amplitudes, unfortunately, we do not have precise experimental information on all the g_A's required. We take the following values as given

TABLE I. The S-wave decay amplitudes.

S-wave amplitude $[10^5(\text{MeV sec})^{-1/2}]$	$A(\Lambda_-^0)$	$A(\Sigma_+^+)$	$A(\Sigma_0^+)$	$A(\Sigma_-^-)$	$A(\Xi_-^-)$
Theory	0.135	0	−0.112	0.158 (input)	−0.153
Expt[11]	0.132	0	−0.079[13]	0.158	−0.169

[10] K. Nishijima and L. J. Swank, Phys. Rev. **146**, 1161 (1966); J. J. Sakurai, *ibid.* **156**, 1508 (1967); B. W. Lee and A. R. Swift, *ibid.* **136**, B229 (1964).

[11] N. P. Samios, in Proceedings of the Argonne International Conference on Weak Interactions, Argonne National Laboratory Report No. ANL-7130, 1965 (unpublished); we have changed some of the signs in conformity with our phase convention. For experimental verification of Eq. (25a), see also D. Cline and J. Robinson, Wisconsin Report, 1966 (unpublished).

[12] H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964); B. W. Lee, Phys. Rev. Letters **12**, 83 (1964).

[13] The data quoted by N. Samios (Ref. 11) do not satisfy the the $I=\frac{1}{2}$ Σ triangle very well. More recent data, however, are in much better agreement with the triangle; see R. O. Bangerter *et al.*, Phys. Rev. Letters **17**, 495 (1966).

TABLE II. The *P*-wave decay amplitudes.

p-wave amplitude $[10^6(\text{MeV sec})^{1/2}]$	$B(\Lambda_-{}^0)$	$B(\Sigma_+{}^+)$	$B(\Sigma_0{}^+)$	$B(\Sigma_-{}^-)$	$B(\Xi_-{}^-)$
Theory	0.82	1.632 (input)	1.25	−0.14	0.697 (input)
Expt[11]	0.858	1.632	1.443	−0.127	0.697

from the experiments[14]:

$$g_A(n\to p)=1.18\,,\quad g_A(\Lambda\to p)=-0.88\pm0.06\,,$$

and

$$g_A(\Sigma^-\to n)=0.49\pm0.05\,.$$

In the absence of reliable experimental estimates we shall be guided by the work of Calucci *et al.*,[15] who have estimated the corrections to the $SU(3)$ symmetry values of the g_A's. They find, in particular, that the $g_A(\Sigma^+\to\Lambda)$ should be larger by about 30% than the $SU(3)$ symmetry value ($\simeq$0.64). For the $g_A(\Xi^-\to\Lambda)$, they do not find much difference from the symmetry value ($\simeq$0.16); we will therefore adopt the values $g_A(\Sigma^+\to\Lambda)\simeq1.0$ and $g_A(\Xi^-\to\Lambda)\simeq0.16$. Failing any information on the remaining g_A's, namely, $g_A(\Sigma^-\to\Sigma^0)$ and $g_A(\Xi^-\to\Xi^0)$, we have to treat these as parameters. We shall show in Sec. V that consistency of our theory with the experimental values of the nonleptonic meson decay requires that $|\lambda_S|=|\lambda_P|$. For $\lambda_S=\lambda_P$ and $g_A(\Sigma^-\to\Sigma^0)\simeq1.19$ and $g_A(\Xi^-\to\Xi^0)\simeq0.26$, we find reasonable agreement, as shown in Table II. If experiment should confirm the values of the g_A which we have used, we may conclude that we have satisfactorily predicted the *P*-wave amplitudes.

V. NONLEPTONIC DECAYS OF THE *K* MESONS

Equations (12) and (13) as they stand, of course, give us no information about matrix elements of H_W between members of the meson octet. If the decomposition (17) is made, it is even less probable here that, in the baryon case, the first term alone dominates since all particles involved in the decay are of more comparable mass. Nevertheless, we postulate that, for members of the meson octet, Eqs. (12) and (13) are valid with λ as determined from the nonleptonic decay of baryons. Using the commutation relations (5) alone, we may relate in the usual way[15] the processes $K_2{}^0\to\pi^+\pi^-\pi^0$ to $K_1{}^0\to\pi^+\pi^-$ to find

$$\left|\frac{B(K_2{}^0\to\pi^+\pi^-\pi^0)}{A(K_1{}^0\to\pi^+\pi^-)}\right|=\frac{m_\pi{}^2}{c_\pi}\left|\frac{\lambda_p}{\lambda_s}\right|\,,\qquad(27)$$

in excellent agreement with experiment for $|\lambda_p|=|\lambda_s|$ but, of course, not a test of our model *per se*. Next, we may easily calculate the K_1 decays[16] in terms of the K_{l3} form factor $F_+(0)$, and obtain

$$A(K_1{}^0\to\pi^+\pi^-)=(m_\pi{}^2/\sqrt{2}c_\pi)\lambda(M_K{}^2-m_\pi{}^2)\times[F_+(K^0\to\pi^-)-F_+(K^0\to\pi)]\,.\qquad(28)$$

This amplitude may be related to the *S*-wave hyperon decays if, in accord with the Ademollo-Gatto theorem,[17] we use the $SU(3)$ values of F_+ and $g_V(\Sigma^-\to n)$; we find[18]

$$\frac{A(K_1{}^0\to\pi^+\pi^-)}{A(\Sigma^-\to n\pi^-)}=-\frac{M_K{}^2-M_\pi{}^2}{M_\Sigma-M_N}\simeq-6.4m_\pi\,.\qquad(29)$$

The experimental absolute value of this ratio is 4.96 m_π, so that our results are off by almost 30%. We may interpret this as indicating that we should have chosen a slightly different value for the parameter λ for mesonic processes. Of course, it could be that Eqs. (12) and (13) need correction in this case.

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[14] We have calculated the g_A's from the rates quoted by N. Brene *et al.*, Phys. Rev. **149**, 1288 (1966). The errors on the rates for the $\Sigma\to\Lambda e\nu$ and the $\Xi\to\Lambda e\nu$ decays are very large; hence we have not used these rates to obtain $g_A(\Sigma\to\Lambda)$ and $g_A(\Xi\to\Lambda)$.

[15] G. Calucci, G. Denardo, and C. Rebbi, Torino Report, 1966 (unpublished); Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966); D. K. Elias and J. C. Taylor, Nuovo Cimento **44**, 518 (1966); S. K. Bose and S. N. Biswas, Phys. Rev. Letters **16**, 330 (1966); M. Suzuki (Ref. 1).

[16] The two-pion state must be space-symmetrized to preserve Bose statistics. See, e.g., M. Suzuki (Ref. 1).

[17] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964).

[18] This result is the same as obtained in their scheme by Riazuddin and Mahanthappa (Ref. 2).