Current-Algebra Sum Rules for Arbitrary Spin and Mass*

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Fubini–Dashen–Gell-Mann sum rules are derived for targets of arbitrary spin and mass, and implications concerning j-plane analyticity are pointed out.

I. INTRODUCTION AND SUMMARY OF RESULTS

In recent years the development of high-energy physics has stressed the importance of complications due to spin. In particular, the presence of particles with spin imposes definite constraints on the analytic structure in the complex angular momentum plane. For spinless targets it has been shown by Bronzan, Gerstein, Lee, and Low[1] and by Singh[2] that the Fubini–Dashen–Gell-Mann[3] sum rules imply the presence of fixed poles in the complex angular momentum plane for certain t-channel helicity amplitudes. (In the t channel two isovector currents create a particle-antiparticle pair.) In this paper we deal with the question of what j-plane analyticity emerges when current algebra is applied to the scattering of currents on particles with arbitrary spin.

In Sec. II, we derive Fubini–Dashen–Gell-Mann sum rules by making use of mixed amplitudes, which carry tensor indices for the currents and helicity indices for the particles. The use of these "hybrid" amplitudes allows us to extend a method developed by Fubini[4] for the spinless case to the arbitrary spin case.

Our final results are expressed in terms of parity-conserving helicity amplitudes.[5] These amplitudes are well suited for investigating j-plane analyticity.

If we denote the helicity difference of the currents and the particles by μ and λ, respectively, and the product of the intrinsic parities of the currents by η, the following results for $\mu=2$ are obtained: (1) The amplitudes with parity $(+\eta)$ and $|\lambda|\leq 1$ (nonsense-sense at $J=1$) have a right signature fixed pole at $J=1$ whose residue can be expressed in terms of form factors. (2) All t-channel parity-conserving amplitudes with parity $(-\eta)$ and all t-channel parity-conserving helicity amplitudes (both parities) with $|\lambda|\geq 2$ (nonsense-nonsense at $J=1$) superconvergence and hence there is no correct signature fixed pole at $J=1$.

The special case of equal-mass, spin-$\frac{1}{2}$ targets is contained in a paper by Gerstein.[6] Our sum rules agree with his for this case. We remark, however, that our results do not agree with those derived by Bander,[7] for arbitrary spin and mass.

Note added in proof. After the submission of this paper an erratum of M. Bander appeared [Phys. Rev. (to be published)]. The final sum rules given in this erratum agree with those derived here.

II. DERIVATION OF SUM RULES AND DISCUSSION

We consider the creation of a zero-baryon-number system composed of two particles by means of two isospin-one vector currents. (See Fig. 1.) We denote the masses, momenta, spins, and helicities of the particles

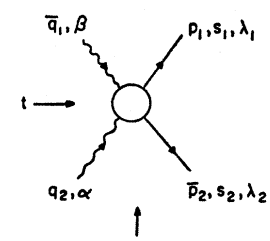

FIG. 1. The diagram considered in the derivation of sum rules.

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by m_1,m_2; $p_1,\bar{p}_2$; s_1,s_2; and λ_1,λ_2, respectively. The momenta and isospin indices of the vector currents shall be denoted by $\bar{q}_1,q_2$; α,β, respectively.

We first collect some kinematic relations:

$$P_\mu=(p_1-\bar{p}_2)_\mu;\quad y=q_1\cdot q_2;\quad \bar{q}_1^2=\mu_1^2;\quad q_2^2=\mu_2^2;$$
$$t=(p_1+\bar{p}_2)^2=(\bar{q}_1+q_2)^2;\quad s=(q_2-\bar{p}_2)^2;\quad u=(q_2-p_1)^2;$$
$$\nu=P\cdot(q_2-\bar{q}_1)=(s-u);\quad -\bar{q}_1\cdot P=\tfrac{1}{2}\nu-\tfrac{1}{2}(m_1^2-m_2^2);$$
$$q_2\cdot P=\tfrac{1}{2}\nu+\tfrac{1}{2}(m_1^2-m_2^2).$$

The scattering angle in the t-channel c.m. system is given by

$$\cos\theta_t=\left\{\nu-\frac{(m_1^2-m_2^2)(\mu_1^2-\mu_2^2)}{t}\right\}\bigg/\{4|\mathbf{p}_t||\mathbf{q}_t|\},\quad (1)$$

where $|\mathbf{p}_t|$ and $|\mathbf{q}_t|$ are the c.m. momenta of the particles and currents, respectively;

$$|\mathbf{p}_t|=[t^2-2t(m_1^2+m_2^2)+(m_1^2-m_2^2)^2]^{1/2}/2t^{1/2},\quad (2a)$$
$$|\mathbf{q}_t|=[t^2-2t(\mu_1^2+\mu_2^2)+(\mu_1^2-\mu_2^2)^2]^{1/2}/2t^{1/2}.\quad (2b)$$

The T-matrix element for the process under consideration is given by

$$T_{\mu\nu;\lambda_1\lambda_2}{}^{\alpha\beta}=i(4p_{10}p_{20})^{1/2}\int d^4x\exp(-i\bar{q}_1\cdot x)\theta(x_0)\times\langle p_1\lambda_1\bar{p}_2\lambda_2|[j_\mu{}^\alpha(x),j_\nu{}^\beta(0)]|0\rangle.\quad (3)$$

We next expand this hybrid amplitude, which has part helicity, and part tensor indices, in terms of the three polar vectors P_μ, $q_{1\mu}$, and $q_{2\mu}$, and an axial vector K_μ defined by the relation

$$K_\mu=\varepsilon_{\mu\nu\rho\sigma}P^\nu q_1{}^\rho q_2{}^\sigma/[|\mathbf{q}_t|(\sin\theta_t)(t^{1/2})].$$

The coefficients will be functions of the helicities of the particles, the masses, and the variables t and ν. To wit:

$$\begin{aligned}T^{\alpha\beta}{}_{\mu\nu;\lambda_1\lambda_2}=&A_{\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu P_\nu+B_{1\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu q_{1\nu}+B_{2\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu q_{2\nu}+B_{3\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}P_\nu+B_{4\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}P_\nu\\&+C_{1\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}q_{2\nu}+C_{2\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}q_{1\nu}+C_{3\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}q_{1\nu}+C_{4\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}q_{2\nu}+C_{5\lambda_1\lambda_2}{}^{\alpha\beta}g_{\mu\nu}+E_{1\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu K_\nu+E_{2\lambda_1\lambda_2}{}^{\alpha\beta}K_\mu P_\nu\\&+R_{1\lambda_1\lambda_2}{}^{\alpha\beta}K_\mu q_{1\nu}+R_{2\lambda_1\lambda_2}{}^{\alpha\beta}K_\mu q_{2\nu}+R_{3\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}K_\nu+R_{4\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}K_\nu.\end{aligned}\quad (4)$$

In contrast to the spinless case, the additional axial vector K_μ must occur in the expansion; this arises because of the helicity-dependent factors appearing in our expansion. We further remark, in contrast to the spinless case, that the coefficients are not yet free of kinematic singularities.

In analogy to the procedure developed by Fubini[4] for spinless targets we also define three expressions $U_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}$, $\tilde{U}_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}$, and $W_{\lambda_1\lambda_2}{}^{\alpha\beta}$. They are constructed like $T_{\mu\nu;\lambda_1\lambda_2}{}^{\alpha\beta}$ [Eq. (3)], where one or both currents are replaced by their divergences:

$$\begin{aligned}U_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}&=-(4p_{10}p_{20})^{1/2}\int d^4x\exp(-i\bar{q}_1\cdot x)\theta(x_0)\langle p_1\lambda_1\bar{p}_2\lambda_2|[\mathfrak{D}^\alpha(x),j_\mu{}^\beta(0)]|0\rangle\\&=H_{\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu+G_{1\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}+G_{2\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}+I_{\lambda_1\lambda_2}{}^{\alpha\beta}K_\mu,\end{aligned}\quad (5)$$

$$\begin{aligned}\tilde{U}_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}&=-(4p_{10}p_{20})^{1/2}\int d^4x\exp(-i\bar{q}_1\cdot x)\theta(x_0)\langle p_1\lambda_1\bar{p}_2\lambda_2|[j_\mu{}^\alpha(x),\mathfrak{D}^\beta(0)]|0\rangle\\&=\tilde{H}_{\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu+\tilde{G}_{1\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}+\tilde{G}_{2\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}+\tilde{I}_{\lambda_1\lambda_2}{}^{\alpha\beta}K_\mu,\end{aligned}\quad (6)$$

$$W_{\lambda_1\lambda_2}{}^{\alpha\beta}=-i(4p_{10}p_{20})^{1/2}\int d^4x\exp-(i\bar{q}_1\cdot x)\theta(x_0)\langle p_1\lambda_1\bar{p}_2\lambda_2|[\mathfrak{D}^\alpha(x),\mathfrak{D}^\beta(0)]|0\rangle,\quad (7)$$

with

$$\mathfrak{D}^\alpha(x)=\partial_\mu j^{\mu,\alpha}(x).$$

Finally we introduce the equal-time commutators:

$$\begin{aligned}\phi_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}&=-(4p_{10}p_{20})^{1/2}\int d^4x\exp(-i\bar{q}_1\cdot x)\langle p_1\lambda_1\bar{p}_2\lambda_2|[j_0{}^\alpha(x),j_\mu{}^\beta(0)]|0\rangle\delta(x_0)\\&=F_{\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu+S_{1\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}+S_{2\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}+S_{3\lambda_1\lambda_2}{}^{\alpha\beta}K_\mu,\end{aligned}\quad (8)$$

$$\begin{aligned}\tilde{\phi}_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}&=-(4p_{10}p_{20})^{1/2}\int d^4x\exp(-i\bar{q}_1\cdot x)\langle p_1\lambda_1\bar{p}_2\lambda_2|[j_\mu{}^\alpha(x),j_0{}^\beta(0)]|0\rangle\delta(x_0)\\&=\tilde{F}_{\lambda_1\lambda_2}{}^{\alpha\beta}P_\mu+\tilde{S}_{1\lambda_1\lambda_2}{}^{\alpha\beta}q_{1\mu}+\tilde{S}_{2\lambda_1\lambda_2}{}^{\alpha\beta}q_{2\mu}+\tilde{S}_{3\lambda_1\lambda_2}{}^{\alpha\beta}K_\mu,\end{aligned}\quad (9)$$

$$\psi_{\lambda_1\lambda_2}{}^{\alpha\beta}=-i(4p_{10}p_{20})^{1/2}\int d^4x\,\exp(-i\bar{q}_1\cdot x)\langle p_1\lambda_1\bar{p}_2\lambda_2|[\mathfrak{D}^\alpha(0),j_0{}^\beta(x)]|0\rangle\delta(x_0)\,, \tag{10}$$

$$\tilde{\psi}_{\lambda_1\lambda_2}{}^{\alpha\beta}=-i(4p_{10}p_{20})^{1/2}\int d^4x\,\exp(-i\bar{q}_1\cdot x)\langle p_1\lambda_1\bar{p}_2\lambda_2|[j_0{}^\alpha(x),\mathfrak{D}^\beta(0)]|0\rangle\delta(x_0)\,. \tag{11}$$

The vector commutators shall be assumed to satisfy the usual current-algebra commutation relations. To wit:

$$\delta(x_0-y_0)[j_0{}^\alpha(x),j_\nu{}^\beta(y)]=i\epsilon_{\alpha\beta\gamma}j_\nu{}^\gamma(x)\delta^3(\mathbf{x}-\mathbf{y})+\text{Schwinger terms.} \tag{12}$$

On the basis of the fundamental relation (12) we can derive the Fubini–Dashen–Gell-Mann sum rules if we make the following two additional assumptions: (I) All amplitudes $A_{\lambda_1\lambda_2}{}^{\alpha\beta}$, $B_{1\lambda_1\lambda_2}{}^{\alpha\beta}$, $\cdots$, $\tilde{\psi}_{\lambda_1\lambda_2}{}^{\alpha\beta}$ which are defined by Eqs. (3)–(11) are bounded by $c\nu^b$, where b is less than one. (II) No Schwinger terms contribute to $F_{\lambda_1\lambda_2}{}^{\alpha\beta}$ [the coefficient of P_μ in the equal-time commutator defined by Eq. (8)].

We now proceed in four steps. First, we obtain several identities which give relations among the amplitudes $A_{\lambda_1\lambda_2}{}^{\alpha\beta}$, $B_{1\lambda_1\lambda_2}{}^{\alpha\beta}$, $\cdots$ etc.; next the amplitudes $A_{\lambda_1\lambda_2}{}^{\alpha\beta}$ and $(E_{1\lambda_1\lambda_2}{}^{\alpha\beta}+E_{2\lambda_1\lambda_2}{}^{\alpha\beta})$, which have simple connections with t-channel helicity amplitudes are singled out; then we take the limit $\nu\to\infty$; and finally the sum rules emerge after removing kinematical singularities and writing dispersion relations in ν.

It can be easily seen, by using partial integration that the following relations hold:

$$\begin{aligned}
q_1{}^\mu T_{\mu\nu;\lambda_1\lambda_2}{}^{\alpha\beta}&=U_{\nu;\lambda_1\lambda_2}{}^{\alpha\beta}+\phi_{\nu;\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
q_2{}^\nu T_{\mu\nu;\lambda_1\lambda_2}{}^{\alpha\beta}&=\tilde{U}_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{\phi}_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
q_1{}^\mu \tilde{U}_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}&=W_{\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{\psi}_{\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
q_2{}^\nu U_{\nu;\lambda_1\lambda_2}{}^{\alpha\beta}&=W_{\lambda_1\lambda_2}{}^{\alpha\beta}+\psi_{\lambda_1\lambda_2}{}^{\alpha\beta}\,.
\end{aligned} \tag{13}$$

By comparing the coefficients of the vector P_μ, $q_{1\mu}$, $q_{2\mu}$, and K_μ in Eqs. (13) we obtain the following identities:

$$\begin{aligned}
A_{\lambda_1\lambda_2}{}^{\alpha\beta}q_1\cdot P+B_{3\lambda_1\lambda_2}{}^{\alpha\beta}\mu_1{}^2+B_{4\lambda_1\lambda_2}{}^{\alpha\beta}y&=H_{\lambda_1\lambda_2}{}^{\alpha\beta}+F_{\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
B_{1\lambda_1\lambda_2}{}^{\alpha\beta}q_1\cdot P+C_{1\lambda_1\lambda_2}{}^{\alpha\beta}\mu_1{}^2+C_{3\lambda_1\lambda_2}{}^{\alpha\beta}y+C_{5\lambda_1\lambda_2}{}^{\alpha\beta}&=G_{1\lambda_1\lambda_2}{}^{\alpha\beta}+S_{1\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
B_{2\lambda_1\lambda_2}{}^{\alpha\beta}q_1\cdot P+C_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_1{}^2+C_{4\lambda_1\lambda_2}{}^{\alpha\beta}y&=G_{2\lambda_1\lambda_2}{}^{\alpha\beta}+S_{2\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
E_{1\lambda_1\lambda_2}{}^{\alpha\beta}q_1\cdot P+R_{3\lambda_1\lambda_2}{}^{\alpha\beta}\mu_1{}^2+R_{4\lambda_1\lambda_2}{}^{\alpha\beta}y&=I_{\lambda_1\lambda_2}{}^{\alpha\beta}+S_{3\lambda_1\lambda_2}{}^{\alpha\beta}\,,
\end{aligned} \tag{14}$$

$$\begin{aligned}
A_{\lambda_1\lambda_2}{}^{\alpha\beta}P\cdot q_2+B_{1\lambda_1\lambda_2}{}^{\alpha\beta}y+B_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2&=\tilde{H}_{\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{F}_{\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
B_{3\lambda_1\lambda_2}{}^{\alpha\beta}P\cdot q_2+C_{1\lambda_1\lambda_2}{}^{\alpha\beta}y+C_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2&=\tilde{G}_{1\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{S}_{1\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
B_{4\lambda_1\lambda_2}{}^{\alpha\beta}P\cdot q_2+C_{3\lambda_1\lambda_2}{}^{\alpha\beta}y+C_{4\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2+C_{5\lambda_1\lambda_2}{}^{\alpha\beta}&=\tilde{G}_{2\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{S}_{2\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
E_{2\lambda_1\lambda_2}{}^{\alpha\beta}P\cdot q_2+R_{1\lambda_1\lambda_2}{}^{\alpha\beta}y+R_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2&=\tilde{I}_{\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{S}_{3\lambda_1\lambda_2}{}^{\alpha\beta}\,,
\end{aligned} \tag{15}$$

$$\begin{aligned}
H_{\lambda_1\lambda_2}{}^{\alpha\beta}P\cdot q_2+G_{1\lambda_1\lambda_2}{}^{\alpha\beta}y+G_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2&=W_{\lambda_1\lambda_2}{}^{\alpha\beta}+\psi_{\lambda_1\lambda_2}{}^{\alpha\beta}\,,\\
\tilde{H}_{\lambda_1\lambda_2}{}^{\alpha\beta}q_1\cdot P+\tilde{G}_{1\lambda_1\lambda_2}{}^{\alpha\beta}\mu_1{}^2+\tilde{G}_{2\lambda_1\lambda_2}{}^{\alpha\beta}y&=W_{\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{\psi}_{\lambda_1\lambda_2}{}^{\alpha\beta}\,.
\end{aligned} \tag{16}$$

Equations (14)–(16) are used to express the amplitudes $A_{\lambda_1\lambda_2}{}^{\alpha\beta}$ and $E_{1\lambda_1\lambda_2}{}^{\alpha\beta}+E_{2\lambda_1\lambda_2}{}^{\alpha\beta}$ in terms of other amplitudes, viz.,

$$\begin{aligned}
A_{\lambda_1\lambda_2}{}^{\alpha\beta}=\frac{F_{\lambda_1\lambda_2}{}^{\alpha\beta}}{q_1\cdot P}+\frac{1}{(q_1\cdot P)(q_2\cdot P)}&[W_{\lambda_1\lambda_2}{}^{\alpha\beta}+\psi_{\lambda_1\lambda_2}{}^{\alpha\beta}-G_{1\lambda_1\lambda_2}{}^{\alpha\beta}y-G_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2\\
&-\mu_1{}^2(\tilde{G}_{1\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{S}_{1\lambda_1\lambda_2}{}^{\alpha\beta}-C_{1\lambda_1\lambda_2}{}^{\alpha\beta}y-C_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2)\\
&-y(\tilde{G}_{2\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{S}_{2\lambda_1\lambda_2}{}^{\alpha\beta}-C_{3\lambda_1\lambda_2}{}^{\alpha\beta}y-C_{4\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2)]\,,
\end{aligned} \tag{17}$$

$$\begin{aligned}
E_{1\lambda_1\lambda_2}{}^{\alpha\beta}+E_{2\lambda_1\lambda_2}{}^{\alpha\beta}=\frac{1}{(q_1\cdot P)(q_2\cdot P)}&[(\tilde{I}_{\lambda_1\lambda_2}{}^{\alpha\beta}+\tilde{S}_{3\lambda_1\lambda_2}{}^{\alpha\beta}-R_{1\lambda_1\lambda_2}{}^{\alpha\beta}y-R_{2\lambda_1\lambda_2}{}^{\alpha\beta}\mu_2{}^2)P\cdot q_1\\
&+(I_{\lambda_1\lambda_2}{}^{\alpha\beta}+S_{3\lambda_1\lambda_2}{}^{\alpha\beta}-R_{3\lambda_1\lambda_2}{}^{\alpha\beta}\mu_1{}^2-R_{4\lambda_1\lambda_2}{}^{\alpha\beta}y)P\cdot q_2]\,.
\end{aligned} \tag{18}$$

Using the key assumption (I) stated above, one obtains from Eqs. (17) and (18) in the limit as ν goes to infinity the following results:

$$\lim_{\nu\to\infty}\{\tfrac{1}{2}\nu A_{\lambda\lambda_2}{}^{\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)-F_{\lambda_1\lambda_2}{}^{\alpha\beta}(\nu,t)\}=0\,, \quad (19)$$

$$\lim_{\nu\to\infty}\{E_{1\lambda_1\lambda_2}{}^{\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)+E_{2\lambda_1\lambda_2}{}^{\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)\}=0\,. \quad (20)$$

These Eqs. (19) and (20) contain implicitly all sum rules which we can derive.

Next we express the amplitudes $A_{\lambda_1\lambda_2}{}^{\alpha\beta}$, $E_{1\lambda_1\lambda_2}{}^{\alpha\beta}+E_{2\lambda_1\lambda_2}{}^{\alpha\beta}$ and $F_{\lambda_1\lambda_2}{}^{\alpha\beta}$ in more familiar terms. We relate $A_{\lambda_1\lambda_2}{}^{\alpha\beta}$ and $E_{1\lambda_1\lambda_2}{}^{\alpha\beta}+E_{2\lambda_1\lambda_2}{}^{\alpha\beta}$ to t-channel helicity amplitudes. Because of the transversality and orthogonality of the current helicity vectors $\epsilon_\mu{}^{\pm1}$, the t-channel helicity amplitudes,

$$M_{\pm1,\pm1;\lambda_1\lambda_2}{}^{\alpha\beta}=\epsilon^{\pm1\mu}\epsilon^{\mp1\nu}T_{\mu\nu;\lambda_1\lambda_2}{}^{\alpha\beta}\,, \quad (21)$$

will only involve the amplitudes $A_{\lambda_1\lambda_2}{}^{\alpha\beta}$, $E_{1\lambda_1\lambda_2}{}^{\alpha\beta}$, and $E_{2\lambda_1\lambda_2}{}^{\alpha\beta}$. Taking suitable linear combinations one obtains

$$M_{+1,-1;\lambda_1\lambda_2}{}^{\alpha\beta}+M_{-1,+1;\lambda_1\lambda_2}{}^{\alpha\beta}=4|\mathbf{p}_t|^2\sin^2\theta_t A_{\lambda_1\lambda_2}{}^{\alpha\beta}\,, \quad (22)$$

$$M_{+1,-1;\lambda_1\lambda_2}{}^{\alpha\beta}-M_{-1,+1;\lambda_1\lambda_2}{}^{\alpha\beta}=-4i|\mathbf{p}_t|^2\sin\theta_t \times(E_{1\lambda_1\lambda_2}{}^{\alpha\beta}+E_{2\lambda_1\lambda_2}{}^{\alpha\beta})\,. \quad (23)$$

The amplitude $F_{\lambda_1\lambda_2}{}^{\alpha\beta}$ is now expressed in terms of s-channel Breit-system form factors. This is done in two steps. First, we cross the amplitude $\phi_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}$ by replacing the outgoing antiparticle with momentum $\bar{p}_2$ by an incoming particle with momentum $p_2=-\bar{p}_2$. This yields the rulation

$$\phi_{\mu;\lambda_1\lambda_2}{}^{\alpha\beta}(\nu,t)=(-1)^{s_2-\lambda_2}\hat{\phi}_{\mu;\lambda_1,-\lambda_2}{}^{\alpha\beta}(\nu,t)\,, \quad (24)$$

where $\hat{\phi}_{\mu;\lambda_1,-\lambda_2}{}^{\alpha\beta}$ is the corresponding s-channel amplitude. Secondly, we expand $\hat{\phi}_{\mu;\lambda_1,-\lambda_2}{}^{\alpha\beta}$ in terms of an orthonormal basis $X_\mu{}^{(i)}$, $i=0,1\cdots3$, which has the following properties: in the s-channel Breit frame (specified by $\mathbf{P}=0$, $p_{1x}=p_{1y}=0$) the ith component of the vector $X_\mu{}^{(i)}$ is one and the other three components are zero. Thus the coefficients $\Gamma_{\lambda_1,-\lambda_2}{}^{i;\gamma}(t)$ in the expansion

$$\hat{\phi}_{\mu;\lambda_1,-\lambda_2}{}^{\alpha\beta}(\nu,t)=\sum_i \epsilon_{\alpha\beta\gamma}\Gamma_{\lambda_1,-\lambda_2}{}^{i;\gamma}X_\mu{}^{(i)} \quad (25)$$

are the usual Breit-system form factors.[8] We can disregard the Schwinger terms since by assumption (II) they do not contribute to $F_{\lambda_1\lambda_2}{}^{\alpha\beta}(\nu,t)$.

The basis vectors $X_\mu{}^{(i)}$ can be expressed in terms of the vectors P_μ, $q_{1\mu}$, $q_{2\mu}$, and K_μ, viz:

$$X_\mu{}^{(i)}=c^iP_\mu+d^iq_{1\mu}+e^iq_{2\mu}+f^iK_\mu\,. \quad (26)$$

[8] L. Durand III, P. C. DeCelles, and R. B. Marr, Phys. Rev. **126**, 1882 (1962).

The coefficients c^i can be easily computed to be

$$\begin{aligned} &c^0=(2m_1{}^2+2m_2{}^2-t)^{-1/2}\,,\\ &c^1=\cos\theta_t/(2(\sin\theta_t)|\mathbf{p}_t|)\,,\\ &c^2=0\,,\\ &c^3=(m_2{}^2-m_1{}^2)/[(2m_1{}^2+2m_2{}^2-t)^{1/2}2|\mathbf{p}_t|t^{1/2}]\,.\end{aligned} \quad (27)$$

Combining (8) and (24)–(27) we obtain

$$F_{\lambda_1\lambda_2}{}^{\alpha\beta}(\nu,t)=(-1)^{s_2-\lambda_2}\epsilon_{\alpha\beta\gamma}\left\{(2m_1{}^2+2m_2{}^2-t)^{-1/2}\times\left(\Gamma_{\lambda_1,-\lambda_2}{}^{0;\gamma}(t)+\frac{m_2{}^2-m_1{}^2}{2|\mathbf{p}_t|t^{1/2}}\Gamma_{\lambda_1,-\lambda_2}{}^{3;\gamma}(t)\right)+\frac{\cos\theta_t}{\sin\theta_t 2|\mathbf{p}_t|}\Gamma_{\lambda_1,-\lambda_2}{}^{1;\gamma}(t)\right\}\,. \quad (28)$$

As a final step we express the usual helicity amplitudes, as defined in Eq. (21), by parity-conserving helicity amplitudes $W_{+1,-1;\lambda_1\lambda_2}{}^{\pm}$.[5] Using the definition given in Ref. 5, one obtains for the expressions which enter Eqs. (22) and (23):

$$\begin{aligned} M_{+1,-1;\lambda_1\lambda_2}&\pm M_{-1,+1,\lambda_1\lambda_2}\\ &=\tfrac{1}{2}W_{+1,-1}{}^{+}{}_{;\lambda_1\lambda_2}\{(1+z)^{|2+\lambda|/2}(1-z)^{|2-\lambda|/2}\\ &\quad\pm(-1)^{\lambda+\lambda_m}(1+z)^{|2-\lambda|/2}(1-z)^{|2+\lambda|/2}\}\\ &\quad+\tfrac{1}{2}(-1)^{\lambda+\lambda_m}W_{+1,-1;\lambda_1\lambda_2}{}^{-}\{(1+z)^{|2+\lambda|/2}(1-z)^{|2-\lambda|/2}\\ &\quad\mp(-1)^{\lambda+\lambda_m}(1+z)^{|2-\lambda|/2}(1-z)^{|2+\lambda|/2}\}\,, \end{aligned} \quad (29)$$

where $z=\cos\theta_t$, $\lambda=\lambda_1-\lambda_2$, and $\lambda_m=\max(2,\lambda)$.

There is a natural division into the following three cases: $\lambda=0$, $\lambda=\pm1$, and $|\lambda|>1$, which we now discuss separately.

A. $\lambda=(\lambda_1-\lambda_2)=0$

Inserting Eqs. (29), (22), and (23) in Eqs. (19) and (20) yields the following results:

$$\lim_{\nu\to\infty}\{\tfrac{1}{2}\nu W_{+1,-1;\lambda_1\lambda_1}{}^{+;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2) (4|\mathbf{p}_t|^2)-F_{\lambda_1,\lambda_1}{}^{\alpha\beta}(\nu,t)\}=0\,, \quad (30)$$

$$\lim_{\nu\to\infty}\{W_{+1,-1;\lambda_1\lambda_1}{}^{-;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)\sin\theta_t/ (-4i|\mathbf{p}_t|^2)\}=0\,. \quad (31)$$

B. $\lambda=(\lambda_1-\lambda_2)=\pm1$

Here we obtain the equations

$$\lim_{\nu\to\infty}\left\{\frac{\nu}{8|\mathbf{p}_t|^2\sin\theta_t}[\pm W_{+1,-1;\lambda_1,\lambda_1\mp1}{}^{+;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)\cos\theta_t - W_{+1,-1;\lambda_1,\lambda_1\mp1}{}^{-;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)] -F_{\lambda_1,\lambda_1\mp1}{}^{\alpha\beta}(\nu,t)\right\}=0 \quad (32)$$

and

$$\lim_{\nu\to\infty}\left\{\frac{1}{4i|\mathbf{p}_t|^2}[W_{+1,-1;\lambda_1,\lambda_1\mp1}{}^{+;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2) \mp W_{+1,-1;\lambda_1,\lambda_1\mp1}{}^{-;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)\cos\theta_t]\right\}=0. \quad (33)$$

Combining Eqs. (32) and (33), we note that the term $W_{+1,-1;\lambda_1,\lambda_1\mp1}{}^{-}$ in Eq. (32) and the term $W_{+1,-1,\lambda_1,\lambda_1\mp1}{}^{+}$ in Eq. (33) can be neglected, and so we are led to the results

$$\lim_{\nu\to\infty}\left\{\frac{\pm\nu}{8|\mathbf{p}_t|^2}\frac{\cos\theta_t}{\sin\theta_t}W_{+1,-1;\lambda_1,\lambda_1\mp1}{}^{+;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2) - F_{\lambda_1,\lambda_1\mp1}{}^{\alpha\beta}(\nu,t)\right\}=0 \quad (34)$$

and

$$\lim_{\nu\to\infty}W_{+1,-1;\lambda_1\lambda_1\mp1}{}^{-;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)\cos\theta_t=0. \quad (35)$$

C. $|\lambda|=|\lambda_1-\lambda_2|>1$

By similar considerations to those in case (B), we obtain

$$\lim_{\nu\to\infty}\left\{\frac{\nu(\sin\theta_t)^{|\lambda|-2}}{8|\mathbf{p}_t|^2}W_{+1,-1;\lambda_1,\lambda_1-\lambda}{}^{+;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2) - F_{\lambda_1,-\lambda_1+\lambda}{}^{\alpha\beta}(\nu,t)\right\}=0 \quad (36)$$

and

$$\lim_{\nu\to\infty}\{W_{+1,-1;\lambda_1,\lambda_1-\lambda}{}^{-;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)(\sin\theta_t)^{|\lambda|-1}\}=0. \quad (37)$$

The results (30), (31), and (34)–(37) obtained above can be transformed into sum rules in the usual way, by dispersing the parity-conserving amplitudes which are free of singularities in ν. As the amplitudes $W_{+1,-1;\lambda_1\lambda_2}{}^{\pm}(\nu,t,\mu_1{}^2,\mu_2{}^2)$ vanish as ν goes to infinity, one can write an unsubtracted dispersion relation,

$$W_{+1,-1;\lambda_1\lambda_1}{}^{\pm;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2) = \frac{1}{\pi}\int_{-\infty}^{+\infty}d\nu'\frac{w_{+1-1;\lambda_1\lambda_2}{}^{\pm;\alpha\beta}(\nu',t,\mu_1{}^2,\mu_2{}^2)}{\nu'-\nu},$$

where $w_{+1,-1;\lambda_1\lambda_2}{}^{\pm;\alpha\beta}$ is the absorptive part of $W_{+1,-1;\lambda_1\lambda_2}{}^{\pm;\alpha\beta}$. Inserting the dispersion integral into Eqs. (30), (31), and (34)–(37) and expressing $F_{\lambda_1\lambda_2}{}^{\alpha\beta}$ by Eq. (28) yields the following sum rules and superconvergence relations:

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}d\nu\, w_{+1,-1;\lambda_1\lambda_2}{}^{+;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2) = (-1)^{s_2-\lambda_2}\varepsilon_{\alpha\beta\gamma}\left\{8|\mathbf{p}_t|^2(2m_1{}^2+2m_2{}^2-t)^{-1/2} \times\left[\Gamma_{\lambda_1,-\lambda_2}{}^{0;\gamma}(t)+\frac{m_2{}^2-m_1{}^2}{2|\mathbf{p}_t|t^{1/2}}\Gamma_{\lambda_1,-\lambda_2}{}^{3;\gamma}(t) +2\sqrt{2}|\mathbf{p}_t|[\Gamma_{\lambda_1,-\lambda_2}{}^{+;\gamma}(t)+\Gamma_{\lambda_1,-\lambda_2}{}^{-;\gamma}(t)]\right]\right\}, \quad (38)$$

where

$$\Gamma_{\lambda_1\lambda_2}{}^{\pm}(t)=\mp[\Gamma_{\lambda_1\lambda_2}{}^{1}(t)\pm i\Gamma_{\lambda_1,\lambda_2}{}^{2}(t)]/\sqrt{2}$$

and

$$\int_{-\infty}^{+\infty}w_{+1,-1;\lambda_1\lambda_2}{}^{-;\alpha\beta}(\nu,t,\mu_1{}^2,\mu_2{}^2)d\nu=0. \quad (39)$$

Using the properties of the Breit-system form factors[8]

$$\Gamma_{\lambda_1\lambda_2}{}^{0,3}(t)=\delta_{\lambda_1,-\lambda_2}\Gamma_{\lambda_1\lambda_2}{}^{0,3}(t)$$

and

$$\Gamma_{\lambda_1,\lambda_2}{}^{\pm}(t)=\delta_{\lambda_1,-\lambda_2\mp1}\Gamma_{\lambda_1\lambda_2}{}^{\pm},$$

it is apparent that $F_{\lambda_1\lambda_2}{}^{\alpha\beta}(\nu,t)\equiv0$ when $|\lambda_1-\lambda_2|>1$. Thus it follows that the amplitudes $w_{+1,-1;\lambda_1\lambda_2}{}^{\pm}$ with $|\lambda_1-\lambda_2|>1$ are superconvergent, viz.,

$$\int_{-\infty}^{+\infty}\nu^n w_{+1,-1;\lambda_1,\lambda_2}{}^{\pm}(\nu,t,\mu_1{}^2,\mu_2{}^2)d\nu=0, \qquad n=0,1,\cdots,|\lambda|-1. \quad (40)$$

The amplitudes $W_{+1,-1;\lambda_1\lambda_2}{}^{\pm}$ may be expanded in partial wave amplitudes which can be continued into the complex angular momentum plane as discussed in Ref. 5. In complete analogy with Bronzan, Gerstein, Lee, and Low[1] and Singh,[2] the nonvanishing right-hand side of Eq. (38) implies the presence of a fixed pole at $J=1$. This leads immediately to the conclusions stated in Sec. 1.

Till now we have considered only vector-vector commutation relations; however, we can easily generalize our results for the case of the full $SU(2)\times SU(2)$ algebra. If we consider the creation of a baryon-number-zero two-particle system by means of an axial-vector and a vector current, the following changes must occur. First, the positive and negative parity-conserving helicity amplitudes change their roles, because of the definition of these amplitudes. Secondly, the vector form factors become axial-vector form factors. No changes occur for the case where the two particles are created by two axial-vector currents.

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