Investigation of the Pion Trajectory in a Regge-Pole Model

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Reactions of π, K, $\bar{p}$, and p resulting in $\Delta(1238)$ production are examined. It is shown that whenever an exchange with $P=(-)^{J+1}$ is allowed by the known selection rules, this exchange dominates the production mechanism at small momentum transfers. This phenomenon is described in terms of a Regge-pole model for pion exchange. Arguments are given to show that although the pion trajectory is lower than the ρ and A_2 trajectories, π exchange can dominate at small $|t|$ values. The pion trajectory is determined from the experimental data, and fits for the differential cross sections at various energies are given for $|t|\leq 0.25$ (GeV/c)^2.

I. INTRODUCTION

CONSIDERABLE progress has been achieved recently in the study of the theoretical complications connected with the kinematics of unequal-mass scattering and Regge poles.[1] Nevertheless, there still remain some practical problems when one considers the parametrization of inelastic processes and has to treat several trajectories and residue functions simultaneously. Regge-pole analysis of resonance production has been confined, therefore, to simple reactions in which both the number of exchanged trajectories and independent helicity amplitudes are rather small.[2] A puzzling question in this context is the role played by the pion trajectory. The fixed pole one-pion-exchange (OPE) model in its various modifications has scored a remarkable success in the interpretation and correlation of various high-energy reactions resulting in resonance production.[3] The important role played by the OPE mechanism is intuitively understood to result from the small pion mass and its strong couplings. This advantage seems to be lost in a Regge-pole model, where the pion trajectory (if existing at all) should be well below other trajectories such as α_ρ and α_{A_2}, and can be neglected altogether in the asymptotic limit.

In the present investigation we shall confine ourselves to various reactions on a proton target resulting in $\Delta(1238)$ production. In these reactions we should consider the π, ρ, and A_2 trajectories from the known unitary multiplets, and possibly a member of the 1^+ nonet. The reactions discussed are

$$\pi^+p\rightarrow\rho^0\Delta^{++}, \tag{1}$$

$$K^+p\rightarrow K^{*0}\Delta^{++}, \tag{2}$$

$$\pi^+p\rightarrow f^0\Delta^{++}, \tag{3}$$

$$p\bar{p}\rightarrow\Delta^{++}\bar{\Delta}^{--}, \tag{4}$$

$$pp\rightarrow p\Delta^{+}, \tag{5a}$$

$$pp\rightarrow n\Delta^{++}. \tag{5b}$$

The purpose of this paper is to point out that at moderate incoming energies the OPE mechanism cannot be neglected in a Regge-pole model. In Sec. II we discuss the possibility of isolating the pion exchange contributions to reactions (1)–(5). We show that these contributions dominate the production mechanism at small momentum transfers. Our analysis is consistent with the assumption that the pion lies on a Regge trajectory, which is calculated from the experimental data. Section III is devoted to a presentation of a Regge-pole parametrization suitable for our analysis. Fits to the experimental cross-section distributions are presented in Sec. IV. Section V contains a discussion and possible interpretations of our results.

II. DATA ANALYSIS

In the following we shall examine the available experimental decay correlations of reactions (1)–(5) in order to extract information on the corresponding production mechanisms. We shall make use of the fact, noted first by Gottfried and Jackson,[4] that parity restrictions on vertex couplings can be extended to Regge trajectories with the same spin-parity relation [e.g., $P=(-)^J$ or $P=(-)^{J+1}$].

Let us first consider reaction (1) and denote its 12 helicity amplitudes in the t channel by

$$\begin{aligned}
&F_1=F_{\frac12\frac32,01}{}^t, && F_5=F_{\frac12\frac32,00}{}^t, && F_9=F_{\frac12\frac32,0-1}{}^t,\\
&F_2=F_{\frac12\frac12,01}{}^t, && F_6=F_{\frac12\frac12,00}{}^t, && F_{10}={}_{\frac12\frac12,0-1}{}^t,\\
&F_3=F_{\frac12-\frac12,01}{}^t, && F_7=F_{\frac12-\frac12,00}{}^t, && F_{11}=F_{\frac12-\frac12,0-1}{}^t,\\
&F_4=F_{\frac12-\frac32,01}{}^t, && F_8=F_{\frac12-\frac32,00}{}^t, && F_{12}=F_{\frac12-\frac32,0-1}{}^t.
\end{aligned} \tag{6}$$

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* Research sponsored by Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. AFEOAR 66/39, through the European Office of Aerospace Research.

[1] D. Z. Freedman and J. M. Wang, Phys. Rev. Letters **17**, 569 (1966); **18**, 863 (1967); Phys. Rev. **153**, 1596 (1967).

[2] D. P. Roy, Nuovo Cimento **40**, 513 (1965); H. Coprasse and H. Stremnitzer, *ibid.* **44**, 1245 (1966); R. L. Thews, Phys. Rev. **155**, 1624 (1967).

[3] See, e.g., J. D. Jackson, in Proceedings of the Stony Brook Conference on High-Energy Two-Body Reactions, 1966 (to be published).

[4] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

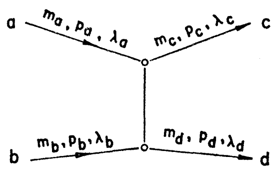

FIG. 1. A peripheral diagram for the reaction $a+b \to c+d$. p, m, and λ denote 4-momentum, mass, and helicity, respectively.

The diagonal spin density matrix elements for the Δ decay are defined as

$$\begin{aligned}\rho_{33}&=\tilde{\rho}_{33}/2(\tilde{\rho}_{33}+\tilde{\rho}_{11}),\\ \rho_{11}&=\tilde{\rho}_{11}/2(\tilde{\rho}_{33}+\tilde{\rho}_{11}),\end{aligned} \tag{7}$$

where

$$\begin{aligned}\tilde{\rho}_{33}&=|F_1|^2+|F_5|^2+|F_9|^2+|F_4|^2+|F_8|^2+|F_{12}|^2,\\ \tilde{\rho}_{11}&=|F_2|^2+|F_6|^2+|F_{10}|^2+|F_3|^2+|F_7|^2+|F_{11}|^2.\end{aligned} \tag{8}$$

Similarly the diagonal spin density matrix elements corresponding to the ρ decay are

$$\begin{aligned}\rho_{00}&=\tilde{\rho}_{00}/(2\tilde{\rho}_{11}+\tilde{\rho}_{00}),\\ \rho_{11}&=\tilde{\rho}_{11}/(2\tilde{\rho}_{11}+\tilde{\rho}_{00}),\end{aligned} \tag{9}$$

where

$$\begin{aligned}\tilde{\rho}_{00}&=2(|F_5|^2+|F_6|^2+|F_7|^2+|F_8|^2),\\ \tilde{\rho}_{11}&=|F_1|^2+|F_2|^2+|F_3|^2+|F_4|^2+|F_9|^2\\ &\quad+|F_{10}|^2+|F_{11}|^2+|F_{12}|^2.\end{aligned} \tag{10}$$

Experimental data on reaction (1) are available at 3.65, 4.0, and 8.0 GeV/c.[5–7] A characteristic feature is that in the forward direction, $|t|\leq 0.25$ (GeV/c)^2, the measured elements are $\rho_{00}(\rho)>0.8$; $\rho_{33}(\Delta)<0.05$. Referring to Eqs. (6)–(10), we deduce that

$$(|F_6|^2+|F_7|^2)/\sum_{i=1}^{12}|F_i|^2>0.7. \tag{11}$$

We note furthermore that the ρ-exchange contribution to reaction (1) is excluded because of the G-parity selection rule, and that the A_2 trajectory cannot couple to a final ρ in a zero-helicity state. We conclude that the production mechanism responsible for reaction (1) at small momentum transfers is dominated by a π exchange, or possibly an exchange of an A_1 (assuming that the A_1 belongs to a 1^+ nonet).

Exactly the same arguments may be advanced in connection with reaction (2), where the measured spin density matrix elements at 3.0, 3.5, and 5.0 GeV/c [8] in the forward direction show the same behavior as observed in reaction (1). Although ρ exchange is allowed for this reaction, we again note that both the ρ and A_2 trajectories cannot couple to the final K^* in a zero-helicity state. We obtain for reaction (2) the same conclusion we had for reaction (1), viz., that the production mechanism at small $|t|$ values is dominated by $P=(-)^{J+1}$ exchanges. A similar analysis of the data published on reaction (3) at 8 GeV/c [9] yields

$$(|F_{\frac{1}{2}\frac{1}{2},00}{}^t|^2+|F_{\frac{1}{2}-\frac{1}{2},00}{}^t|^2)/\sum_{i=1}^{16}|F_i|^2>0.75, \tag{12}$$

where the amplitudes $F_{\frac{1}{2}\frac{1}{2},00}{}^t$ and $F_{\frac{1}{2}-\frac{1}{2},00}{}^t$ are nonzero for $P=(-)^{J+1}$ exchanges only.

The importance of a contribution by an axial-vector meson Regge trajectory has been conjectured by Barmawi.[10] However, at least a partial success in the understanding of reactions (1)–(3) can be obtained using the absorptive OPE model.[3] The problems involved in such an interpretation are mostly in the normalization of the calculated cross sections rather than in the energy dependence or decay correlations. The normalization problem seems to arise from the unitarity violation of the OPE amplitudes written in the pole approximation. Such a deficiency is naturally removed when the OPE amplitudes are Reggeized. We shall conjecture, thus, that axial-vector exchanges may be neglected and that reactions (1)–(3) are dominated by OPE at small momentum transfers. Our aim is to examine the experimental data to see if they support the assumption that the pion lies on a Regge trajectory.

Less stringent bounds on the number of dominating helicity amplitudes can be obtained from the measured decay correlations[11,12] corresponding to reactions (4) and (5). For reaction (4) one finds that

$$\sum_{\lambda_b\lambda_a}|F_{\lambda_b 1,\lambda_a,1}{}^t|^2/\sum_{i=1}^{32}|F_i|^2>0.6, \tag{13}$$

where λ_b means the λ of the antiparticle in the t channel. The notation is given in Fig. 1. For reaction (5) the statement made is

$$\sum_{\lambda_b\lambda_a\lambda_c}|F_{\lambda_b 1,\lambda_a\lambda_c}{}^t|^2/\sum_{i=1}^{16}|F_i|^2>0.65. \tag{14}$$

Moreover, in these reactions we do not have the separation between $P=(-)^J$ and $P=(-)^{J+1}$ exchanges. Nevertheless, analysis of reactions (4) and (5) in terms of the absorptive OPE model is quite successful with problems again arising because of an overestimation of the calculated cross sections. It is rather tempting to assume that the N-Δ vertex which is common in reactions (1)–(5) is dominated at small momentum transfers by a Reggeized π exchange.

[5] B. C. C. Shen, University of California Radiation Laboratory Report No. UCRL-16170, 1965 (unpublished).
[6] Aachen-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, B897 (1965).
[7] Aachen-Berlin-CERN Collaboration, Phys. Letters **19**, 608 (1965); *ibid.* **22**, 533 (1966).
[8] M. Ferro-Luzzi *et al.*, Nuovo Cimento **39**, 417 (1965); R. George *et al.*, *ibid.* **49**, 1 (1967).
[9] Aachen-Berlin-CERN Collaboration, Ref. 7.
[10] M. Barmawi, Phys. Rev. Letters **16**, 595 (1966).
[11] K. Böckmann *et al.*, Phys. Letters **15**, 356 (1965); V. Alles-Borelli *et al.*, Nuovo Cimento **48**, 360 (1967).
[12] G. Alexander *et al.*, Phys. Rev. **144**, 1122 (1965).

Consequences of such an assumption can be tested readily as cross sections for reaction (5) are available over a wide range of primary momenta.[13]

If we assume dominancy of a one-Regge-trajectory exchange, the differential cross sections

$$\frac{d\sigma}{dt}=\frac{f(t)}{sp^2}\left(\frac{s-u}{s_0}\right)^{2\alpha(t)} \tag{15}$$

are expected to yield some shrinkage of the forward peak. The availability of data at different energies enables us to extract a fit for the trajectory α. The experimental data for reaction (5) summarized along these arguments are presented in Figs. 2 and 3. The experimental points were fitted by the method of least squares to Eq. (15). Whenever the data were given without errors quoted we have assumed a statistical distribution. Figure 3 also contains the fitted α values obtained from reactions (1) and (2). For reaction (1) we have used the published data at 3.65 and 8 GeV/c.[5,7] For reaction (2) we took the α values obtained by the CERN-Brussels group.[8] These α values were obtained from an expression slightly different from Eq. (15), but the deviations are very small compared to the statistical errors.

The best fit obtained for the pion trajectory from data corresponding to reaction (5), assuming a straight line, is

$$\alpha(t)=-(1.75\pm0.45)(-t+m_\pi^2). \tag{16}$$

A slightly different slope of 1.50 ± 0.50 was obtained when points corresponding to reactions (1) and (2) were included. The slope calculated is bigger than $\alpha'(0)\simeq1$, which is the value obtained for other known trajectories. However, because of the present experimental limitations, we consider our value for the slope to be preliminary.

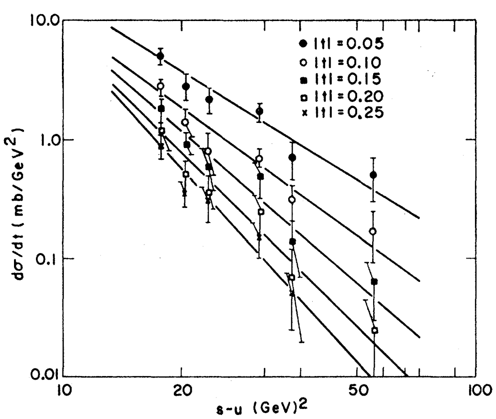

FIG. 2. Differential cross sections of reaction (5) at fixed momentum transfer as a function of $s-u$. The lines were obtained from Eqs. (15) and (16).

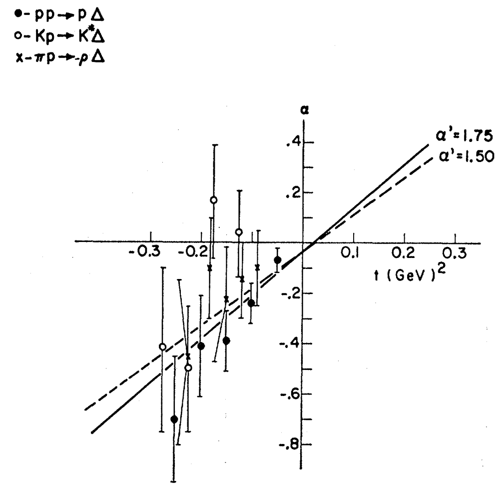

FIG. 3. Chew-Frautschi plot for the pion trajectory. The experimental points correspond to reactions (1), (2), and (5).

III. FORMALISM

Although Regge-pole models have been in vogue for the past five years, the problem of how to parametrize the trajectories and residues associated with specific poles is as yet unsettled.

Most fits to date[14] have assumed a linear dependence on t for the trajectories and have obtained reasonable agreement with experimental results both for the t dependence of the differential cross section and for polarization data.[15] Certain authors[16] have introduced a more complicated form for the trajectory, i.e., they assume a quadratic dependence on t. However, the slight improvement resulting does not seem to justify the additional parameters introduced.

The problem of what parametrization to assume for the residue is much more open, and no one trend seems to have asserted itself. Phillips and Rarita[16] have suggested an exponential form for the residues. A failing in this approach is its arbitrariness. Barmawi[10] took for the residue the expression obtained from the field-theoretical amplitudes. This assumes a smooth behavior from the pole to the physical region. Höller,[17] who has investigated π-N charge-exchange scattering, finds that

[13] G. G. Chadwick *et al.*, Phys. Rev. **128**, 1823 (1962); G. Cocconi *et al.*, Phys. Letters **15**, 134 (1964); C. M. Ankenbrandt *et al.*, Nuovo Cimento **35**, 1052 (1965); E. W. Anderson *et al.*, Phys. Rev. Letters **16**, 855 (1966); I. M. Blair *et al.*, *ibid.* **17**, 789 (1966).

[14] R. K. Logan, Phys. Rev. Letters **14**, 414 (1965); G. Höhler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966); M. Barmawi, Phys. Rev. **142**, 1088 (1966); see also Ref. 2.

[15] For elastic scatterings see, e.g., W. Rarita, R. J. Riddell, C. B. Chiu, and R. J. N. Phillips, University of California Radiation Laboratory Report No. UCRL-17523, 1967 (unpublished). For resonance production, see M. L. Paciello and A. Pugliese, Phys. Letters **24B**, 431 (1967); M. Krammer and U. Maor, Nuovo Cimento **50A**, 963 (1967); *ibid.* **25A**, 308 (1967).

[16] A. Pignotti, Phys. Rev. Letters **10**, 416 (1963); R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).

[17] G. Höhler, J. Baacke, and G. Eisenbeiss, Phys. Letters **22**, 203 (1966).

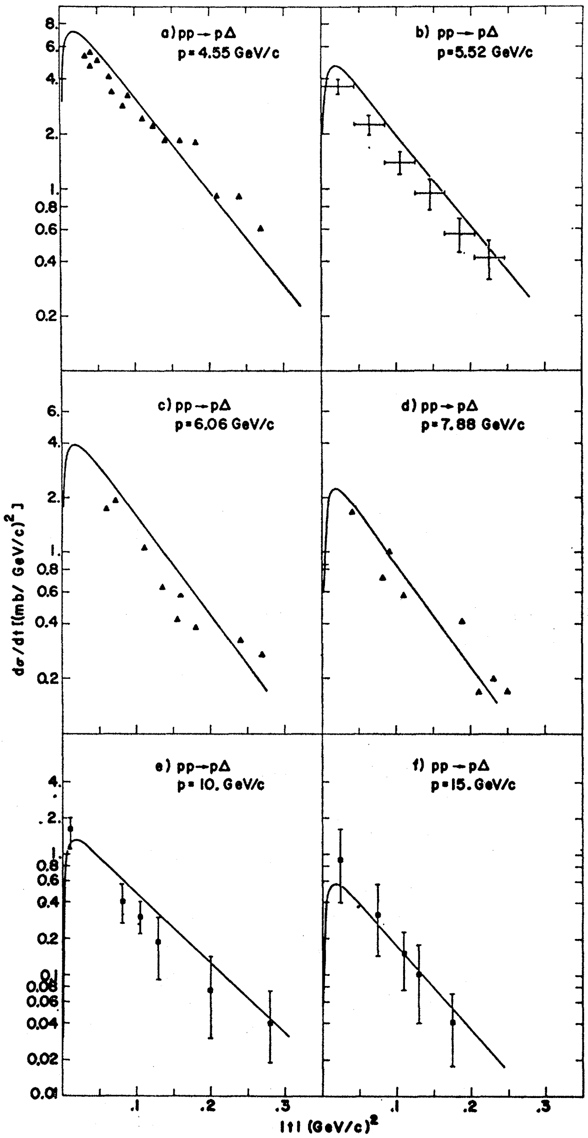

FIG. 4. Experimental and fitted differential cross sections for reaction (5).

he can obtain agreement with the experimental data if the field-theoretical expression for the residues have a strong t dependence. This contradicts Barmawi's assumption. From this point of view, there is some advantage in the method suggested by Wang,[18] to separate out the kinematic singularities, and then take the residue to be a constant. This constant does not necessarily have to be the same value as the residue at the pole. However, for the pion trajectory, since the pole at $t=m_\pi^2=0.02$ $(\text{GeV}/c)^2$ is so close to the physical region, we expect that the difference in the Barmawi and Wang approaches should be small.

Our t-channel amplitudes can be parametrized in the form

$$F_{\lambda_b\lambda_d,\lambda_a\lambda_c}{}^t=(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}\times K(t)\gamma(t)[(s-u)/s_0]^{\alpha-\lambda_m}, \quad (17)$$

where s, t, and u are the Mandelstam variables, s_0 is a scaling constant, and θ_t is the scattering angle in the t channel. $\lambda_m=\max(|\lambda|,|\mu|)$, where $\lambda=\lambda_a-\lambda_c$, $\mu=\lambda_b-\lambda_d$. $K(t)$ denotes the kinematic singularity factors and $\gamma(t)$ is the residue function. α denotes the pion trajectory.

We note that the dominant contribution to the differential cross section in the forward direction will come from those amplitudes for which $\lambda_m=0$. For $\lambda_m\neq 0$, $\gamma(t)$ is proportional to $\alpha(t)$ because of the nonsense transition involved.[19] Consequently, we will concern ourselves only with the amplitudes for which $\lambda=\mu=0$. These amplitudes were found in the analysis carried out in Sec. II to dominate the production mechanism in the forward direction. For these amplitudes we use the field-theoretical residues assuming a smooth continuation from the pole to the physical t region. A disadvantage in this approach is that we cannot calculate deviations of the decay density matrix elements from pure OPE predictions. Nevertheless, we have already observed that these deviations are quite small.

A few additional remarks should be added about the approximations leading to the Regge parametrization which we have adopted in Eq. (17):

(a) In obtaining Eq. (17) we have followed the Mandelstam extension[20] of the Regge formalism for negative α, keeping the leading term in the expansion.

(b) We assume the existence of daughter trajectories[21] to assure the proper s^α asymptotic behavior in the s-channel forward direction.

(c) By neglecting the amplitudes with $\lambda_m\neq 0$ we avoid the necessity of considering possible conspiracies at $\theta_s=0^0$.

(d) Conspiracy relations that occur at $t=0$ are of interest only for reaction (4), where the crossing matrix contains a $t^{-1/2}$ singularity. We have neglected such possible constraints since data are not available in the very narrow forward cone.

Effectively, our Reggeization of the scattering amplitude consists of replacing the pion propagator:

$$1/(t-m_\pi^2)\rightarrow P(t),$$

$$P(t)=[2\alpha(t)+1]\pi\alpha'(0)\times\frac{1+e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}\frac{\Gamma[\alpha(t)+\frac{1}{2}]}{2^{\alpha(t)}\pi^{1/2}\Gamma[\alpha(t)+1]}\left(\frac{s-u}{s_0}\right)^{\alpha(t)}. \quad (18)$$

The resulting expressions for the cross sections cor-

[18] L. L. Wang, Phys. Rev. **142**, 1187 (1966).

[19] M. Gell-Mann, M. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[20] S. Mandelstam, Ann. Phys. (N. Y.) **19**, 254 (1962).

[21] See D. Z. Freedman and J. M. Wang, Ref. 1.

responding to reactions (1)–(5) are

$$\frac{d\sigma_1}{dt}=\frac{2\pi}{3sq^2}\frac{g_{\pi\pi\rho}^2}{4\pi}\frac{g_{\pi p\Delta}^2}{4\pi m_\pi^2}P(t)a_c^2b_d^2[t-(m_\Delta+m_p)^2], \quad (19)$$

$$\frac{d\sigma_2}{dt}=\frac{2\pi}{3sq^2}\frac{g_{K\pi K^*}^2}{4\pi}\frac{g_{\pi p\Delta}^2}{4\pi m_\pi^2}P(t)a_c^2b_d^2[t-(m_\Delta+m_p)^2], \quad (20)$$

$$\frac{d\sigma_3}{dt}=\frac{16\pi}{9sq^2}\frac{g_{\pi\pi f}^2}{4\pi m_f^2}\frac{g_{\pi p\Delta}^2}{4\pi m_\pi^2}P(t)a_c^4b_d^2[t-(m_\Delta+m_p)^2], \quad (21)$$

$$\frac{d\sigma_4}{dt}=\frac{\pi}{9sq^2}\left(\frac{g_{\pi p\Delta}^2}{4\pi m_\pi^2}\right)^2P(t)b_d^4[t-(m_\Delta+m_p)^2]^2, \quad (22)$$

$$\frac{d\sigma_5}{dt}=\frac{\pi}{6sq^2}\frac{g_{\pi pp}^2}{4\pi}\frac{g_{\pi p\Delta}^2}{4\pi m_\pi^2}P(t)tb_d^2[t-(m_\Delta+m_p)^2], \quad (23)$$

where

$$q^2=(1/4s)[s-(m_a+m_b)^2][s-(m_a-m_b)^2],$$
$$a_c^2=(1/4m_c^2)[t-(m_c+m_a)^2][t-(m_c-m_a)^2],$$
$$b_d^2=(1/4m_d^2)[t-(m_d+m_b)^2][t-(m_d-m_b)^2].$$

The slope of the pion trajectory was deduced from experimental data as discussed in Sec. II. This leaves only one free parameter, s_0. In Sec. IV we adjust this parameter to give best fits to the experimental cross sections.

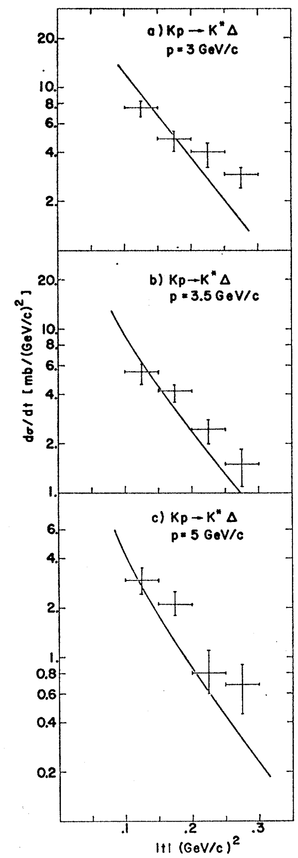

FIG. 6. Experimental and fitted differential cross sections for reaction (2).

IV. COMPARISON WITH EXPERIMENT

The best value obtained from reaction (5) for the pion trajectory yielding a slope of $\alpha'(t)=1.75$ was used throughout the fitting procedure. As we noted, the over-all fit to $\alpha'(t)$ was slightly lower. However, because pp data are available over a wider range of s than are available for reactions (1) and (2), we consider this fit more significant.

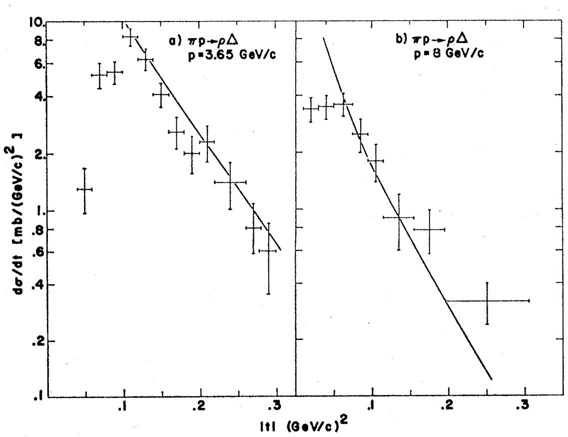

FIG. 5. Experimental and fitted differential cross sections for reaction (1).

Differential cross sections corresponding to reactions (1)–(5) at various incoming energies were fitted after adjusting the free parameter s_0 in Eqs. (19)–(23). The physical interpretation of this scaling factor is not very clear. It is associated with the external particles, and as such might be classified according to some symmetry scheme; however, previous fits[14–16] have made very different assumptions. In the present approach we have determined the best value of s_0 in each reaction from a χ^2 fit to the experimental data.

(a) $pp\to p\Delta^+$. Fits were made for differential cross sections at incoming proton momenta of 4.5–15.0 GeV/c.[12,13] The fits are presented in Fig. 4 together with the experimental distributions. Experimental errors are quoted only when given in the original papers. The coupling constants used in our calculations are $g_{p\pi^0p}^2/4\pi=14.5$, $g_{p\pi^0\Delta^+}^2/4\pi=0.25$. The best value obtained for the free parameter is $s_0=0.8$ GeV^2.

(b) $\pi^+p\to\rho^0\Delta^{++}$. Experimental[5,7] and fitted cross sections at 3.65 and 8.0 GeV/c are given in Fig. 5. Data corresponding to this reaction at 4.0 GeV/c [6] were omitted because their normalization is not consistent with the 3.65-GeV/c experiment. A clarification of this ambiguity would shed more light on the validity of our approach. The coupling constants used are $g_{\pi^+\rho^0\pi^-}^2/4\pi=1.85$, $g_{p\pi^+\Delta^{++}}^2/4\pi=0.38$. The best value obtained for s_0 is 0.7 GeV^2.

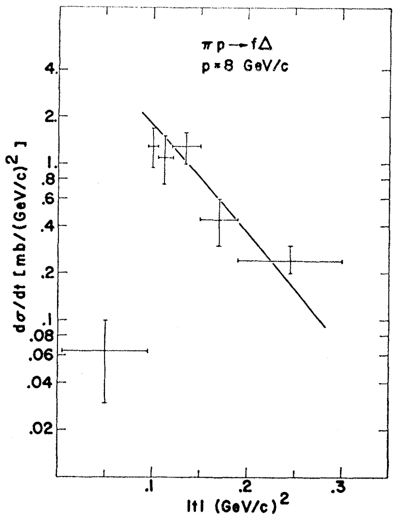

FIG. 7. Experimental and fitted differential cross sections for reaction (3).

(c) $K^+p \to K^{*0}\Delta^{++}$. Figure 6 contains the experimental[8] and fitted cross sections at 3.0, 3.5, and 5.0 GeV/c. The relevant coupling constant is $g_{K^+K^{*0}\pi^-}{}^2/4\pi = 1.5$. The fitting yields $s_0 = 0.7$ GeV^2.

(d) $\pi^+p \to f^0\Delta^{++}$. Data on this reaction are available at 8.0 GeV/c [9] and are presented together with our fit in Fig. 7. The coupling constant used is $g_{\pi^+f^0\pi^-}{}^2/4\pi = 2.4$. Obviously, the energy dependence of Eq. (21) could not be examined. The best fit was obtained for $s_0 = 1.0$ GeV^2.

(e) $p\bar{p} \to \Delta^{++}\bar{\Delta}^{--}$. Differential cross sections for this reaction were published for incoming momenta of 3.6 and 5.7 GeV/c.[11] Because different background subtractions were employed in these experiments it is difficult to compare the energy dependence of the reported cross sections; we have confined ourselves to the high-statistics experiment of Alles-Borelli *et al.*[11] at 5.7 GeV/c. The results are shown in Fig. 8, with $s_0 = 0.6$ GeV^2.

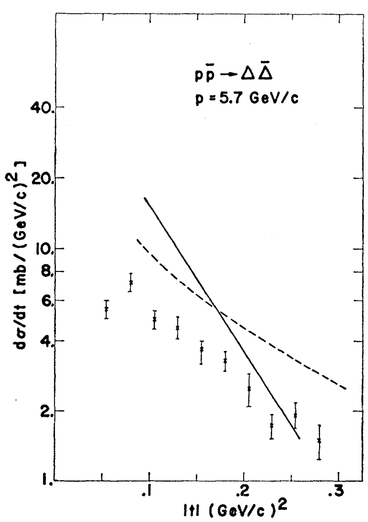

FIG. 8. Experimental and fitted differential cross sections for reaction (4). The dashed line corresponds to the experimental cross section corrected to include the full isobar mass distributions. (See Alles-Borelli *et al.*, Ref. 11.)

A few general remarks follow our analysis. Generally speaking, our fits are quite reasonable in the forward direction, $|t| < 0.25$ (GeV/c)^2, considering our crude approximations. The fits deteriorate badly for $|t| > 0.3$ (GeV/c)^2, which is to be expected. The least satisfactory fit was obtained for reaction (5) but, as we have noted, background approximations are most difficult in this case. Our model is expected to be unsatisfactory at very high energies. We therefore expect that reactions (1)–(3) will exhibit at these energies polarization properties which are very different from those seen in the presently available data. This observation provides us with a qualitative test for the validity of the present model.

V. DISCUSSION

Until fairly recently, physicists labored under the impression that the higher the Regge trajectory the greater is its contribution to the scattering amplitude. However, such over-all generalizations have now been shown to be inaccurate[22] and one has to examine each helicity amplitude separately and ascertain whether there is a compensation for the lowness of the trajectory.

We have considered processes for which pion exchange is permitted and for which the isospin of the exchanged trajectory is limited to $T \geq 1$. We find that we can obtain reasonable results by just considering the pion exchange and neglecting the contributions of the ρ and A_2 trajectories, similar results are obtained in the absorption model.[3] The relative importance of the pion trajectory at moderate incoming momenta can be explained by the fact that its pole is very close to the physical region, and that it has large couplings to other particles. In addition, for small momentum transfers it receives impetus from the corresponding kinematic singularities compared to those for amplitudes to which the ρ and A_2 trajectories contribute.

The approximation which we have used, neglecting all helicity amplitudes with $\lambda_m \neq 0$, does not provide us with any new insight to the decay correlations. Contrary to the procedure of calculating cross sections, the predicted spin density matrix elements depend on a delicate balance between the various t-channel helicity amplitudes.[3] We know[23] that no joint decay correlations can exist if a single Regge trajectory is exchanged. Consequently, it is obvious that any extension of the present model must include not only contributions of the pion trajectory to other helicity amplitudes, but

[22] S. Frautschi and L. Jones, Phys. Rev. **163**, 1820 (1967).
[23] A. B. Kaidalov, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 4, 484 1966 [English transl: Soviet Phys.—JETP Letters **4**, 325 (1966)].

also contributions from other trajectories as well. Such a calculation necessarily requires many more parameters.

The fit that we obtain for the π trajectory is steeper than the one usually associated with other Regge trajectories.[14–16] Our preliminary result does not seem to support the assumption of an exchange degeneracy between the π- and B-meson trajectories.[24] If we take seriously the theory of Regge recurrences, we would expect to find a 2^{--} resonance with mass in the range of 1.0 GeV. This is yet to be discovered, or is it connected with the double peak structure of the A_2 meson?[25]

[24] D. G. Sutherland, Nucl. Phys. **B2**, 157 (1967).
[25] G. Chikovani *et al.*, Phys. Letters **25B**, 44 (1967).

An unsatisfactory feature of our fit is the dependence on s_0, which sets the scale for the absolute differential cross sections. The ideas associating s_0 with the radius of interaction and taking $s_0^2=m_am_bm_cm_d$ do not seem to work in our case. Our data are more consistent with the association of s_0 with a universal constant of the order of m_p^2.

The main conclusion of our analysis is that the transition vertex of a target nucleon to a Δ isobar is dominated at moderate energies and small momentum transfers by an exchange of a $P=(-)^{J+1}$ object when this is allowed by the selection rules imposed by the other vertex. Our calculations support the assumption that this exchange is well described by the pion Regge trajectory.

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Interaction Radii of Strongly Interacting Particles

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Interaction radii for pairs of excited elementary particles are computed from differential production cross sections by a simple optical model. It is found that the excited states of baryons and mesons consistently have interaction radii smaller than the ground states, and that they expand slowly with energy, corresponding to a shrinkage of the diffraction region, up to center-of-mass energies of 15 GeV or so. It is probable that there is very little more expansion at higher energies.

USING a modification[1] of the coherent drop model[2] for a particle-particle reaction, we have estimated the effective interaction radii of various two-particle systems. Effective radii of interaction are moderately well known for particles in their ground states (e.g., pion with proton, kaon with proton, antiproton with proton, nucleon with nucleon); these have been estimated by application of optical models to differential elastic scattering cross sections. The modification[1] used here allows interaction radii for particles in excited states to be evaluated from differential production cross sections. The method has been extended to estimate these radii as functions of center-of-mass energy.

A constrained test[3] for self-consistency of the method has been made using experimental data for the reaction

$$\pi^+ + d \to p + p. \tag{1}$$

Here the experimental differential elastic scattering cross sections for pion on deuteron and for proton on proton were used to compute the differential production cross section for reaction (1), all at the same center-of-mass energy, and a reasonable fit to the experimental differential production data was obtained.

We now apply the method to the two-body reactions

$$\pi + p \to A + B \quad \text{and} \quad K + p \to C + D, \tag{2}$$

where the final product is a proton plus an excited meson, or where both final particles are excited, and also to the reaction

$$\bar{p} + p \to E + F, \tag{3}$$

where EF is the pair, excited baryon and antibaryon.

The model is applicable at high energies where the elastic and production reactions are diffractive, i.e., have a dependence on the squared four-momentum transfer t of the form

$$d\sigma/dt = S\exp(-\gamma t). \tag{4}$$

We write the elastic scattering cross section in the usual small-angle approximation

$$(d\sigma/dt)_{\text{elastic}} = \pi\left[\int_0^\infty a(b)J_0(bt^{1/2})bdb\right]^2 \tag{5}$$

from which an analytic evaluation of $a(b)$ follows, if (4)

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[3] L. Marshall Libby and S. Miyashita, Phys. Rev. **160**, 1447 (1967).