constraint conditions would lead to similar equations relating several of the Hearn-Leader amplitudes.

The low-energy theorem gives the value $B_6(s=m^2, t=0)=2\mu^2$. In the forward direction, however, the Born term of B_6 vanishes (for all s) and the low-energy theorem emerges from the continuum of B_6, which is the statement of the Drell-Hearn sum rule. However, the roles of Born term and continuum interchange if we write separate dispersion relation for $\bar{A}_+$ and $\bar{A}_-$. The contribution of $\bar{A}_+$ vanishes in the forward direction. The continuum of $\bar{A}_-$ also vanishes in the low-energy limit, because of odd crossing, and the low-energy theorem for B_6 emerges solely from the Born term of $\bar{A}_-$. The Drell-Hearn sum rule emerges from the dispersion relation for $\bar{A}_-$ as a superconvergence condition, showing that it is the decreasing high-energy behavior which relates the Born term and continuum of this amplitude.

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Model of Positive-Parity Baryon Excited States*

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The occurrence in nature of very high-spin baryons leads to the suggestion that they can be described as composites of high-spin baryons and mesons with low orbital angular momentum. As a consequence, there appears to be no limit beyond which the baryon spectrum cannot extend. We implement these ideas with a dynamical bootstrap model of positive-parity baryons employing $SU(2)\times O(3)$ symmetry. We prove the consistency of this symmetry with our dynamics, and predict a spectrum in accord with the latest phase-shift analysis of the pion-nucleon system. We indicate how our model is described in a Regge theory of angular momentum trajectories, and we conclude with a comment on the negative-parity baryon spectrum.

1. INTRODUCTION

THERE is now direct experimental evidence exhibiting baryon resonance structure above 3 GeV center-of-mass energy,[1] and even indirect evidence that resonance structure at 10 GeV may produce observable polarization phenomena in scattering reactions.[2] This leads one to suspect that the baryon spectrum may continue indefinitely to states of arbitrarily high spin. Such behavior may place fairly restrictive demands on the possible form a theory of hadrons can have, and accordingly has been the subject of several interesting papers.[3] However, it appears that before any firm conclusions can be reached, one must have a clear picture of the excitation mechanism, i.e., of the physics of the excited states.

The most interesting early work on this subject was done by Carruthers, who used bootstrap arguments to study excited states of the nucleon.[4] He used an "L-excitation" mechanism whereby an excited state is generated by increasing the orbital angular momentum of the ground-state constituents. For instance, if the $N^*(1236)\ \frac{3}{2}^+$ resonance is primarily coupled to a P-wave πN composite then its first excited state, the $\frac{7}{2}\,N^{*\prime}(1924)$, is primarily coupled to an F-wave πN composite. The most appealing aspect of this approach is its consistency with the dynamics of long-range forces, in that the very forces which bind the πN system to form a P-wave nucleon are also attractive in the F wave.[4] It is very likely that this mechansim will play an important role in any realistic model of the lowest excited states. However, there are both experimental and theoretical reasons for believing that this approach cannot account for the over-all or global structure of the baryon spectrum, in particular the striking linear relation between spin and mass squared. On an empirical level, we may calculate coupling constants $g(B'B\pi)$ from the observed decay widths of the high-spin baryons B' into low-spin baryons B. We then find that as the difference in spin between B' and B becomes large, the coupling implied by the reduced decay width becomes extremely small, perhaps exponentially so. For instance, the F-wave decays of $N'(1688)$ and $N^{*\prime}(1924)$ into πN have coupling constants $\simeq 1/100$ those of $NN\pi$ and $N^*N\pi$, respectively (when expressed as pseudovector coupling). On a theoretical level, the "L-excitation" mechanism cannot explain the long chains of baryon excited states implied by experiment since in such a scheme the singularities

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† Present address.
1 A. Citron *et al.*, Phys. Rev. **144**, 1101 (1966); S. W. Kormanyos *et al.*, Phys. Rev. Letters **16**, 709 (1966).
2 B. R. Desai *et al.*, Phys. Rev. Letters **18**, 565 (1967).
3 N. N. Khuri, Phys. Rev. Letters **18**, 1094 (1967); C. E. Jones and V. L. Teplitz, *ibid.* **19**, 135 (1967).
4 P. A. Carruthers, *Lectures in Theoretical Physics* (University of Colorado Press, Boulder, Colo., 1965), Vol. VIIb.

used as forces to bootstrap the high-spin, high-mass states no longer lie "nearby" them, i.e., there is no real justification for the dynamics used.

We feel that the high-spin excited states must be described by configurations of virtual particles with *low* values of the orbital angular momentum.[5] Postponing detailed discussions of models until Secs. 2 and 3, we note here two properties which descriptions of these high-spin baryons should contain: (i) The predicted states should be consistent with the same type of dynamics which governs the ground states, i.e., we assume that the most strongly attractive of the long-range forces in two-body composites determine the quantum numbers of the populated states. (ii) Since the gap in spin between baryon excited states is apparently $\Delta J=2$, we expect that exchange forces are at least as important as the direct potential given by meson exchange processes. Perhaps the most significant consequence of this paper is that there appears to be no limit to the spin and mass of the excited states we generate, for at any given energy only the nearest singularities are being used in the dynamics. This point has been stressed by the author in a preliminary report of this work.[6]

We now summarize the contents. In Sec. 2 we study a model of the N-$N^*(1236)$ excited states which is a very simple extension of the Chew reciprocal bootstrap[7] of the N-N^* system. Section 3 is devoted to a study of a model containing $SU(2)\times O(3)$ symmetry. In Sec. 4, we study properties of the $SU(2)\times O(3)$ model in greater detail and in Sec. 5 discuss a consequence of this model for the negative-parity baryons. A discussion of our results concludes the paper in Sec. 6.

2. EXTENSION OF THE N-N^* RECIPROCAL BOOTSTRAP

One of the earliest successes of bootstrap theory was Chew's calculation with the nucelon and the 3-3 resonance (hereafter referred to as N and N^*, respectively) in which the $NN\pi$ coupling constant and $N^*N\pi$ width were shown to satisfy the consistency requirements of a simple S-matrix model.[7] Later, Lin and Cutkosky used the Bethe-Salpeter equation to generate bootstrap equations in a study of the N, N^*, π system and found reasonable agreement with experiment in their estimate of the $NN\pi$ and $N^*N\pi$ coupling constants and the N^*-N mass difference.[8] Our first model of baryon excited states involves a very natural extension of the above ideas. For simplicity we restrict our calculations to systems having zero strangeness. (All of the following results remain qualitatively the same upon the inclusion of strange particles.) Suppose that N and N^* each have just a single lowest excited state, the $\frac{5}{2}^+$ $N'(1688)$ and $\frac{7}{2}^+$ $N^{*\prime}(1924)$ with isospin $\frac{1}{2}$ and $\frac{3}{2}$, respectively, which form a reciprocal bootstrap analogous to the ground-state particles. That is, N' and $N^{*\prime}$ are primarily P-wave bound states in the $N'\pi$, $N^{*\prime}\pi$ channels, and, with the assumption of static kinematics, interact by means of N' and $N^{*\prime}$ exchange forces. Consider the following set of bootstrap equations implied by the Bethe-Salpeter equation.[8] As shown in Fig. 1, they are of two types,

vertex:

$$g_{ab}=\sum_{e,f} g_{af}g_{eb}g_{ef}D_{ab}{}^{ef} \tag{1a}$$

and normalization:

$$1=\sum_{b,e,f} g_{af}g_{eb}g_{ef}g_{ab}W_{bf}{}^{ae}, \tag{1b}$$

where

$$D_{ab}{}^{ef}=\frac{C_{ab}{}^{ef}}{12\pi^2}\int_1^\Lambda \frac{(\omega^2-1)^{3/2}d\omega}{(\omega+M_f-M_a)(\omega+M_e-M_b)} \tag{2a}$$

and

$$W_{bf}{}^{ae}=K_a{}^b\frac{\partial D_{ab}{}^{ef}}{\partial M_a}. \tag{2b}$$

In the above, M_i is the mass of baryon i, the meson mass is unity, Λ is a P-wave cutoff, $K_a{}^b$ is a two-particle Green's function, and $C_{ab}{}^{ef}$ is a factor proportional to crossing matrix elements. The equations (1a)–(2b) relate coupling constants and baryon mass differences, yielding for example 235 MeV for the N^*-N mass difference in the N, N^*, π model.[8] In our N', $N^{*\prime}$, π model, there are three coupling constants $f(N'N'\pi)$, $f(N^{*\prime}N^{*\prime}\pi)$, $f(N^{*\prime}N'\pi)$, and one mass difference $N^{*\prime}$-N'. By expanding the equations (1a)–(2b) to first order in the baryon mass difference, we can express the bootstrap equations in terms of the dimensionless variables

$$g_0=D_2{}^{1/2}f(N'N'\pi)\,,\quad g_1=D_2{}^{1/2}f(N^{*\prime}N'\pi)\,,\quad g_2=D_2{}^{1/2}f(N^{*\prime}N^{*\prime}\pi)\,,$$

and $\chi=(D_3/D_2)[M(N^{*\prime})-M(N')]$, where

$$D_n=\frac{1}{12\pi^2}\int_1^\Lambda \frac{k^3d\omega}{\omega^n} \tag{3}$$

and $k=(\omega^2-1)^{1/2}$ is the pion momentum. Thereupon, the bootstrap equations for the N', $N^{*\prime}$, π system take the form

$$g_0=-(31/105)g_0{}^3+(40/21)g_2g_1{}^2(1-2\chi)+(64/63)g_0g_1{}^2(1-\chi)\,,$$

$$g_1=[(4/21)g_0{}^2(1+\chi)+(5/7)g_0g_2+(1/63)g_1{}^2+(4/21)g_2{}^2(1-\chi)]g_1\,,$$

$$g_2=(649/945)g_2{}^3+(5/7)g_1{}^2g_0(1+2\chi)+(8/21)g_1{}^2g_2(1+\chi)\,,$$

[5] This approach has also been stressed by G. F. Chew (private communication); S. Y. Chu and C. I. Tan, University of California Radiation Laboratory Report No. UCRL 17511, 1967 (unpublished).
[6] E. Golowich, Phys. Rev. Letters 18, 633 (1967).
[7] G. F. Chew, Phys. Rev. Letters 9, 233 (1962).
[8] K. Y. Lin and R. E. Cutkosky, Phys. Rev. 140, B205 (1965).

and

$$-(31/105)g_0^4+g_1^2\{(64/63)g_0^2[1-(1+\tfrac{1}{2}\eta)\chi]+(32/63)g_0^2(1-\eta\chi)+(8/189)g_1^2[1-2(1+\tfrac{1}{2}\eta)\chi]$$
$$+(32/63)g_2^2[1-2(1+\tfrac{1}{2}\eta)\chi-\eta\chi]+(80/21)g_0g_2[1-(1+\tfrac{1}{2}\eta)\chi-\eta\chi]\}=(649/945)g_2^4+g_1^2\{(4/21)g_2^2(1+\eta\chi)$$
$$+(8/21)g_2^2[1+(1+\tfrac{1}{2}\eta)x]+(1/63)g_1^2[1+2(1+\tfrac{1}{2}\eta)x]+(10/7)g_0g_2[1+(1+\tfrac{1}{2}\eta)\chi+\eta\chi]$$
$$+(4/21)g_0^2[1+2(1+\tfrac{1}{2}\eta)\chi+\eta\chi]\}\,. \quad (4)$$

In the above, $\eta=D_2D_4/D_3^2$, and we choose Λ to have the same value as the cutoff in the N, N^*, π model, $\Lambda=5.4m_\pi$. (There, it serves to pin down the scale of the bootstrap model by giving the empirical numerical value of the $NN\pi$ coupling constant.) The only self-consistent solution to Eqs. (4) is $g_0=0.68$, $g_1=0.39$, $g_2=1.2$, and $\chi=-0.4$, the latter corresponding to a mass difference, $M(N^{*\prime})-M(N')=-250$ MeV. Hence this simple extension gives an $N^{*\prime}$ with mass substantially *less* than N', in obvious disagreement with experiment. The reason that the N, N^*, π mode does not extend simply to N', $N^{*\prime}$, π can be understood by studying several of the forces. For instance, the process contributing to $NN\pi$ coupling in which a nucleon is exchanged in elastic πN scattering has a positive crossing coefficient $(-1/3)\times(-1/3)=1/9$ leading to a weak but attractive force. The analogous process for the $N'N'\pi$ coupling, in which N' is exchanged in elastic $N'\pi$ scattering has a fairly large, negative crossing coefficient, $(-1/3)\times(31/35)=-31/105$, which gives a rather important repulsion. Hence, although the isospin properties of the N^*, N, π and $N^{*\prime}$, N', π systems are alike, the spin properties contain enough difference to destroy what seems at first to be a straightforward analogy. One might argue that for consistency, the model should include not only the separate N, N^*, π and N', $N^{*\prime}$, π systems described by P-wave (static) kinematics, but also the coupling between these systems, via the P-wave vertex $N'N^*\pi$. Although this greatly enlarges the bootstrap system, we have looked into this problem and have found no self-consistent solutions at all. There is a very simple reason for this. The presence of N^* leads mainly to rather large repulsions for N', hence pushing its mass even further above $N^{*\prime}$. The mutual antagonism between N^* and N' is borne out empirically by the small branching ratio of the P-wave $N'\to N^*\pi$ decay relative to the F-wave $N'\to N\pi$.[9] Since, in terms of number of particles, the model described in this section contains the greatest degree of simplicity, it is apparent that further effort must be accompanied by the inclusion of more channels.

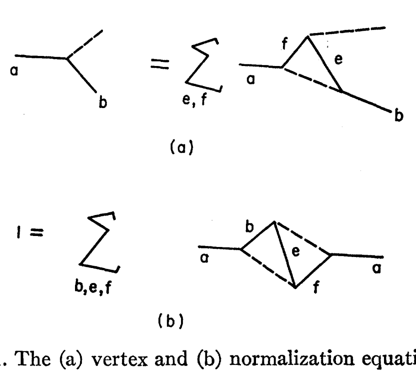

FIG. 1. The (a) vertex and (b) normalization equations.

3. MODEL WITH $SU(n)\times O(3)$ SYMMETRY

In a bootstrap model containing many particles, it is useful to seek a group whose irreducible representations correspond to particle multiplets (at least in some approximate manner) and which, at the same time, is consistent with the dynamics used. Otherwise the bootstrap machinery is too cumbersome to manipulate and any calculation becomes a game of numbers. Now, in both the current algebra and quark descriptions of the baryon spectrum, excited states may appear according to the group $SU(n)\times O(3)$.[10] (The value of n depends upon the particular model one chooses. In the work described below, we take $n=2$ for simplicity, noting however, that our conclusions generalize trivially to $n=3$ and 6.) This group has also been used in the phenomenological effort to classify the baryon ground state by Gyuk and Tuan,[11] and the very interesting work of Capps[12] who studies the dynamics of the baryon ground state, particularly that of the negative-parity baryons. (Also see Sec. 5 of this paper.)

Motivated by the above considerations, we investigate the excitation spectrum expected for positive-parity baryons whose coupling constants and masses are classified according to $SU(2)\times O(3)$, basing our study on bootstrap dynamics as in Sec. 2. First, we describe this symmetry, according to which the excited states of N and N^* have spin $\mathbf{J}=\mathbf{1/2}+\mathbf{K}$ and $\mathbf{J}=\mathbf{3/2}+\mathbf{K}$, respectively, where $K=2, 4, \cdots$,[13] and so the chain of particles arising from the nucleon ground state has spin-parity $(\frac{1}{2}^+)$, $(\frac{3}{2}^+,\frac{5}{2}^+)$, $(\frac{7}{2}^+, \frac{9}{2}^+)$, $\cdots$. We have used the

[9] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[10] For instance, see M. Gell-Mann, Phys. Rev. Letters **14**, 77 (1965) and R. H. Dalitz, in *Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 215.

[11] I. P. Gyuk and S. F. Tuan, Phys. Rev. **151**, 1253 (1966).

[12] R. H. Capps, Phys. Rev. **158**, 1343 (1967), and references cited therein.

[13] Values of $K=1, 3, 5, \cdots$ would lead to an obvious disagreement with experiment. There are several possible reasons for dismissing odd values of K, the most promising being that in bootstrap dynamics, states with even values of K tend to repel those with odd K, implying that it is possible to have a consistent model with all K even or odd, but not both. The author is indebted to Professor R. E. Cutkosky for several enlightening conversations on this point and related subjects.

notation (J^P) and have put particles belonging to a given multiplet within the same parentheses. Our $SU(2)\times O(3)$ model contains only baryons with isospins $T=\frac{1}{2}$, $\frac{3}{2}$, so for each value of K there are just three independent P-wave couplings, which we call g_0, g_1, g_2, corresponding, respectively, to coupling two $T=\frac{1}{2}$, one $T=\frac{1}{2}$ and $T=\frac{3}{2}$, and two $T=\frac{3}{2}$ baryons at a vertex. For instance, if $K=0$, $g_0=g(NN\pi)$, $g_1=g(N^*N\pi)$, $g_2=g(N^*N^*\pi)$. Suppose for some value of K, we wish to express the coupling of a specific baryon of spin J to one of spin J' (together with a P-wave pion) in terms of the g_i, $i=0, 1, 2$. As derived in the Appendix, the appropriate formula is given in terms of a $6j$ symbol

$$g_i(J',J)=(-)^{1-2J-J'+K+S}[(2S+1)(2J'+1)]^{1/2} \times\begin{Bmatrix}1 & S' & S\\ K & J & J'\end{Bmatrix} g_i(S',S)\,,\quad i=0,1,2\,, \tag{5}$$

where S, S' are defined by $\mathbf{J}=\mathbf{S}+\mathbf{K}$, $\mathbf{J}'=\mathbf{S}'+\mathbf{K}$. As an example, for $K=2$, g_0 relates the following couplings involving two $T=\frac{1}{2}$ excited nucleon states: $g(\frac{5}{2}^+,\frac{5}{2}^+,\pi)=(7/15)^{1/2}g_0$; $g(\frac{5}{2}^+,\frac{3}{2}^+,\pi)=(8/15)^{1/2}g_0$; $g(\frac{3}{2}^+,\frac{3}{2}^+,\pi)=-(\frac{1}{5})^{1/2}g_0$, all of which follows simply from Eq. (5).

With the above description of $SU(2)\times O(3)$ in hand, we are ready to construct a model of positive-parity baryon excited states. We extend the model described in Sec. 2 by allowing two $SU(2)\times O(3)$ multiplets of a given K with isospins $\frac{1}{2}$, $\frac{3}{2}$ to form a reciprocal P-wave bootstrap. The question is then: Is the use of the $SU(2)\times O(3)$ symmetry consistent with our P-wave dynamical model of interacting high-spin baryons? That the answer is evidently affirmative follows from this remarkable property of the $SU(2)\times O(3)$ [and in general $SU(n)\times O(3)$] model: *The bootstrap equations (1a), (1b) are independent of the parameter* K. Before proving this statement, we emphasize that it immediately implies that when two $SU(2)\times O(3)$ multiplets of a given K corresponding to excited N, N^* states form a reciprocal P-wave bootstrap, the excited nucleon multiplet has a mass lower than the excited N^* multiplet by the same amount as the ground state ($K=0$) $N(938)$, $N^*(1236)$ mass difference. This provides the physically plausible extension of the N-N^* model we futilely sought in Sec. 2. Now onto the proof, which for reasons for space and simplicity we limit to a vertex equation (1a). Consider the situation depicted in Fig. 1(a), in which two baryons of spin J and J' with the same value of K and a P-wave meson combine to form a vertex. We show below that the vertex equation defined by this diagram is actually independent of K. Since this information is contained in the coupling-constant structure of the process and in particular the crossing coefficient, we make no mention of the particle propagators. The triangle diagram in Fig. 1(a) (right-hand side of the equation) then has the form

$$\langle J_2M_2|\pi|JM\rangle\langle J_2M_2|\pi|J_3M_3\rangle\langle J'M'|\pi|J_3M_3\rangle, \tag{6}$$

where we sum over repeated symbols in (6) and hereafter. The vector relation $\mathbf{J}=\mathbf{K}+\mathbf{S}$ implies, in ket notation,

$$|JM\rangle=|Ss\rangle|Kk\rangle(KkSs|JM)\,, \tag{7}$$

where the term in parentheses represents an $SU(2)$ Clebsch-Gordan coefficient. Putting this information into (6) for each of the baryons, we have

$$\begin{aligned}(J_2M_2|K_2k_2S_2s_2)&\langle S_2s_2|\pi|Ss\rangle\langle K_2k_2|Kk\rangle(KkSs|JM)\\ &\times(J_2M_2|K_2k_3S_2s_3)\langle S_2s_3|\pi|S_3s_4\rangle\langle K_2k_3|K_3k_4\rangle\\ &\times(K_3k_4S_3s_4|J_3M_3)(J'M'|K'k'S's')\langle S's'|\pi|S_3s_5\rangle\\ &\qquad\times\langle K'k'|K_3k_5\rangle(K_3k_5S_3s_5|J_3M_3)\,.\end{aligned} \tag{8}$$

But

$$\langle Kk|K'k'\rangle=\delta_{KK'}\delta_{kk'}\,, \tag{9}$$

so (8) becomes

$$\begin{aligned}(J_2M_2|KkS_2s_2)&\langle S_2s_2|\pi|Ss\rangle(KkSs|JM)(J_2M_2|Kk_3S_2s_3)\\ &\times\langle S_2s_3|\pi|S_3s_4\rangle(Kk_3S_3s_4|J_3M_3)(J'M'|Kk'S's')\\ &\qquad\times\langle S's'|\pi|S_3s_5\rangle(Kk'S_3s_5|J_3M_3)\,.\end{aligned} \tag{10}$$

We reduce (10) by using the completeness property of $SU(2)$ Clebsch-Gordan coefficients, first multiplying by $(Kk_1Ss_1|JM)$ and $(J'M'|Kk_1S's_1')$. Now, since

$$\begin{aligned}(KkSs|JM)(Kk_1Ss_1|JM)&=\delta_{kk_1}\delta_{ss_1}\,,\\ (J'M'|Kk_1S's_1')(J'M'|Kk'S's')&=\delta_{k_1k'}\delta_{s_1's'}\,,\end{aligned} \tag{11}$$

Eq. (10) becomes

$$\begin{aligned}(J_2M_2|KkS_2s_2)&\langle S_2s_2|\pi|Ss_1\rangle(J_2M_2|Kk_3S_2s_3)\\ &\times\langle S_2s_3|\pi|S_3s_4\rangle(Kk_3S_3s_4|J_3M_3)\langle S's_1'|\pi|S_3s_5\rangle\\ &\qquad\times(Kk_1'S_3s_5|J_3M_3)\,.\end{aligned} \tag{12}$$

But

$$\begin{aligned}(J_2M_2|Kk_3S_2s_3)(J_2M_2|KkS_2s_2)&=\delta_{s_3s_2}\delta_{kk_3}\,,\\ (Kk_3S_3s_4|J_3M_3)(Kk_1S_3s_5|J_3M_3)&=\delta_{s_5s_4}\delta_{k_1k_3}\,,\end{aligned} \tag{13}$$

which leaves us with the final form of (12),

$$\langle S_2s_2|\pi|Ss_1\rangle\langle S_2s_2|\pi|S_3s_4\rangle\langle S's_1'|\pi|S_3s_4\rangle\,. \tag{14}$$

The left-hand side of Fig. 1(a) presents a much less formidable task of reduction. Starting with $\langle J'M'|\pi|JM\rangle$, we have in the S, K basis, using relations implied by (7),

$$\langle S's'|\pi|Ss\rangle\langle K'k'|Kk\rangle(J'M'|K'k'S's')(KkSs|JM) \tag{15}$$

and from (9), Eq. (15) becomes

$$\langle S's'|\pi|Ss\rangle(J'M'|KkS's')(KkSs|JM)\,. \tag{16}$$

Since we mutliplied the triangle diagram part (i.e., right-hand side) of this equation by $(Kk_1Ss_1|JM)\times(J'M'|Kk_1S's_1')$, we must do the same to (16), which then becomes

$$(KkSs|JM)(Kk_1Ss_1|JM)(J'M'|Kk_1S's_1')\times(J'M'|KkS's')\langle S's'|\pi|Ss\rangle\,, \tag{17}$$

and from orthogonality, (17) immediately reduces to the desired form,

$$\langle S's_1'|\pi|Ss_1\rangle\,. \tag{18}$$

Equations (14) and (18) constitute the right- and left-hand sides of an equation,

$$\langle S'{s_1}'|\pi|Ss_1\rangle=\langle S_2s_2|\pi|Ss_1\rangle\langle S_2s_2|\pi|S_3s_4\rangle \times\langle S'{s_1}'|\pi|S_3s_4\rangle, \quad (19)$$

which is just the vertex equation (1a), for $K=0$, and this realization completes our proof.

4. ASPECTS OF THE $SU(2)\times O(3)$ MODEL

Although we have been successful in showing that the excited states of the nucleon will have a lower mass than those of the N^* in a model containing plausible dynamical ingredients, we have no estimate for the excitation energy. In addition, the correspondence between the model and existing data is not clear in that we predict mutliplets of particles only parts of which have apparently been verified experimentally[14] (for more on this subject, see Sec. 6). Both of these situations can be clarified by allowing each excited state to couple to a ground-state meson-baryon composite with the appropriate orbital angular momentum. To carry this program out in its entirety is beyond our capability at present, particularly in the determination of the momentum structure of the Bethe-Salpeter amplitude for different partial waves. Therefore, we forego any attempt to calculate the excitation energy, and limit ourselves here to a qualitative discussion of how the ground-excited state coupling is expected to affect the $SU(2)\times O(3)$ symmetric results. For convenience, we focus our attention on the $K=0$ members of the N, N^* systems. With the notation (T,J^P), we enumerate the quantum numbers of the relevant states: $K=0$ $(\frac{1}{2},\frac{1}{2}^+)$ and $(\frac{3}{2},\frac{3}{2}^+)$; $K=2$ $(\frac{1}{2},\frac{5}{2}^+)$, $(\frac{1}{2},\frac{3}{2}^+)$ and $(\frac{3}{2},\frac{7}{2}^+)$, $(\frac{3}{2},\frac{5}{2}^+)$, $(\frac{3}{2},\frac{3}{2}^+)$, $(\frac{3}{2},\frac{1}{2}^+)$. In Table I we give the *signs* of several long-range forces occurring in $N\pi$, $N^*\pi$ channels which couple to the above $K=2$ excited states [$+$ ($-$) stands for attraction (repulsion) in Table I]. These forces give us insight into the question of excited-state $\rightarrow$ ground-state decay widths and perhaps also of the mass splitting within an $SU(2)\times O(3)$ multiplet. The only states in which the forces are clearly attractive are $(\frac{1}{2},\frac{5}{2}^+)$ and $(\frac{3}{2},\frac{7}{2}^+)$, which we associate with the $N'(1688)$ and $N^{*\prime}(1924)$, whereas all other $K=2$ states have repulsions of varying degrees. Therefore, we expect the decay widths of $N'(1688)$ and $N^{*\prime}(1924)$ into $N\pi$, the most easily observed final-state configuration, to exceed those of the other $K=2$ particles. This is apparently the present empirical situation.[14] It is a much more difficult problem to estimate the mass splitting in an $SU(2)\times O(3)$ multiplet because there may be important effects in addition to the ground–excited state coupling we have been discussing. We have in mind processes similar to those which lead to the mass difference between the negative-parity baryons $Y_0^*(1405)$ and $Y_0^*(1520)$ which are evidently members of the same $SU(2)\times O(3)$ multiplet.[12] All we can say at this point is that it is not likely that $N'(1688)$, $N^{*\prime}(1924)$ should lie substantially above the average mass of their respective multiplets.

TABLE I. Signs of forces coupling ground and first excited states.[a] Our notation for the exchanged particles is as follows: N = nucleon, $N^*=N^*(1236)$, $\rho^{V,T}$ = vector and tensor coupling of the ρ meson. Plus and minus represent attraction and repulsion, respectively.

Reaction	Exchanged particle \ T, J^P	$\frac{1}{2}$, $\frac{3}{2}+$	$\frac{1}{2}$, $\frac{5}{2}+$	$\frac{3}{2}$, $\frac{1}{2}+$	$\frac{3}{2}$, $\frac{5}{2}+$	$\frac{3}{2}$, $\frac{7}{2}+$
$N+\pi\rightarrow N+\pi$	N	$-$	$+$	$-$	$-$	$+$
	N^*	$+$	$+$	$+$	$+$	$+$
	ρ^V	$+$	$+$	$-$	$-$	$-$
	ρ^T	$-$	$+$	$-$	$-$	$+$
$N^*+\pi\rightarrow N^*+\pi$	N	$-$	$+$	$-$	$-$	$+$
	N^*	$-$	$-$	$-$	$+$	$+$
$N+\pi\rightarrow N^*+\pi$	N	$-$	$+$	$-$	$-$	$+$
	N^*	$+$	$-$	$+$	$-$	$+$

[a] We do not include the $T=\frac{3}{2}$, $J^P=\frac{3}{2}^+$, particle because $N^*(1238)$ already has these quantum numbers, and so effectively acts as a repulsion.

The rest of this section is devoted to a qualitative discussion of how our ideas graft onto a Regge-type theory in which the basic entity is a trajectory.[15] We think of our static-model calculations as a first approximation to a complete angular momentum analysis. To be specific, consider first, the N, N^* reciprocal bootstrap system. Let us study the nucleon trajectory, concentrating on its coupling to the $N\pi$ channel, where the nucleon is a positive-signature particle lying on the $J=L-\frac{1}{2}$ trajectory. In this model the most important attractive force is due to $N^*(1236)$ exchange. For fixed energy above threshold, this force (constructed by performing a direct-channel partial-wave analysis on an appropriate linear combination of the one-sided functions[16] $A^\pm$, $B^\pm$) decreases monotonically[17] for half-integer $J>\frac{1}{2}$. In Sec. 1 of this paper, we argued that meson-baryon composites with large orbital angular momentum cannot indefinitely support a rising trajectory. We make this thought manifest in Fig. 2(a) where we imply that the $T=\frac{1}{2}$, $J=L-\frac{1}{2}$ positive-signature trajectory (trajectory A) must eventually turn over (just where, is naturally a delicate dynamical question). The N^* trajectory, driven by nucleon exchange, behaves in a similar manner. The next step is to study the $K=2$ P-wave reciprocal bootstrap described in Sec. 3. We expect the same qualitative behavior—trajectories rise to give posi-

[14] However, see C. Lovelace [CERN Report TH. 837 (unpublished)] for a discussion of the latest πN phase-shift analyses. Lovelace's analysis indicates several new resonances, some of which would completely fill the slots in the $K=2$ excited states of N and N^*, and with essentially the correct properties.

[15] The author's knowledge of the matter discussed in this section benefited greatly from a discussion with Professor G. F. Chew.

[16] The one-sided functions are necessary to treat the presence of exchange forces in a manner consistent with angular momentum analyticity. See G. F. Chew, *The Analytic S-Matrix* (W. A. Benjamin, Inc., New York, 1967). The amplitudes A, B are the usual invariant amplitudes of πN scattering. See S. C. Frautschi and J. D. Walecka [Phys. Rev. **120**, 1486 (1960)] both for their definition and their relation to partial-wave amplitudes.

[17] The angular momentum dependence of the force for half-integer J is given by the functions $Q_L(y)$, $Q_{L+1}(y)$, where $J=L+\frac{1}{2}$ and $y>1$.

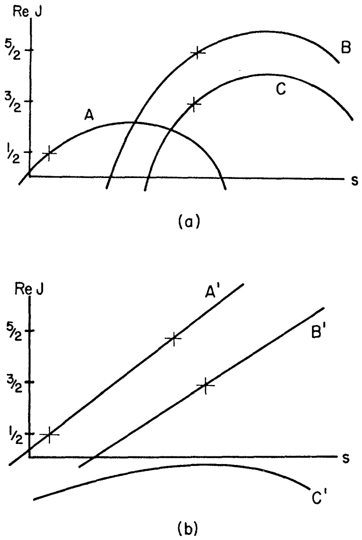

FIG. 2. (a) Trajectories with isospin $\frac{1}{2}$ in the decoupled $SU(2)\times O(3)$ model. Trajectory A corresponds to $K=0$; B, C to $K=2$. Crosses denote particles. (b) Trajectories with isospin $\frac{1}{2}$ in the coupled $K=0$, 2 $SU(2)\times O(3)$ model. Trajectory A' contains the particles $N(938)$ and $N'(1688)$.

tive-parity baryons with quantum numbers $T=\frac{1}{2}$, $J=\frac{3}{2}$, $\frac{5}{2}$ and $T=\frac{3}{2}$, $J=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, but eventually the trajectories fall. The isospin-$\frac{1}{2}$, $K=2$ trajectories (trajectories B, C) are shown in Fig. 2(a). Of course, there is no way to determine the relative mass of the $K=0$, 2 baryons, since at this stage the models are decoupled. What we are doing in Fig. 2(a) is setting the stage for turning on the coupling between the isospin-$\frac{1}{2}$ $K=0$, 2 baryons, the result of which is shown in Fig. 2(b). There we see that the highest-lying trajectory (trajectory A') can continue to rise, fed by attractive forces of nearby singularities. The nucleon and $N'(1688)$ lie on this trajectory. The other $K=2$ baryon, with quantum numbers $T=\frac{1}{2}$, $J^P=\frac{3}{2}^+$ lies on trajectory B' (signature does not allow trajectory B' to have a particle at $J=\frac{1}{2}$), whose behavior is more difficult to predict because of the repulsions in the $N\pi$, $N^*\pi$ channels. Conceivably B' can be linear in s if attractive forces exist in the $K=4$ link in the chain of excited states. This is the situation depicted in Fig. 2(b) and is to be regarded as but a conjecture. It is quite possible that only the highest-lying trajectories exhibit the observed linear relation between ReJ and s.

5. COMMENT ON THE NEGATIVE-PARITY BARYON SPECTRUM

The occurrence of roughly equal numbers of positive- and negative-parity baryon states implies the possibility of a unified description of both. We have seen how an $SU(n)\times O(3)$ multiplet structure can arise from a particular dynamical model of the positive-parity baryon states. Capps, in several publications,[12] has developed an interesting model of negative-parity baryons as follows. He has shown how the ground state of these particles may correspond to a $\mathbf{70}^-\times\mathbf{1}$ structure, the content of the $SU(6)$ multiplet $\mathbf{70}$ being $(8,2)+(1,2)+(8,4)+(10,2)$ while $\mathbf{70}\times\mathbf{1}$ contains $(8,2)_2$, $(8,4)_2$ $(1,2)$ $(1,4)$ $(8,2)_4$, $(8,4)_4$, $(8,6)_4$, $(10,2)$, $(10,4)$. The subscripts indicate which submultiplet of $\mathbf{70}$ is used, e.g., $(8,2)_4$, $(8,4)_4$, $(8,6)_4$ all come from $(8,4)\times\mathbf{1}$, and the entries in parentheses give the $SU(3)$, $SU(2)$ multiplicities, respectively. The particles in $\mathbf{70}\times\mathbf{1}$ are generated as S- and D-wave bound states and resonances of baryons ($\mathbf{56}$) and mesons ($\mathbf{35}$) which are mutually attracted by meson exchange forces. Appealing features of this model are the natural explanation of the $Y_0^*(1520)$ unitary singlet and the agreement in scale between the experimental and predicted coupling constants.[12] However, it also provides some unobserved states, particularly with quantum numbers (8,4) and (10,4). The complexity of the baryon spectrum between 1.5 and 2.5 GeV is such that it may take some time and a great deal of effort to correctly unravel the data and hence accept or reject the Capps model on this basis (however, see Ref. 14).

We wish to point out here the existence of a piece of data which directly involves tentative members of the $\mathbf{70}\times\mathbf{1}$ multiplet, namely, the P-wave decay processes $Y_1^*(1765)\to Y_0^*(1520)\pi$ and $Y_1^*(1660)\to Y_0^*(1405)\pi$. The obvious $\mathbf{70}\times\mathbf{1}$ assignments are $Y_1^*(1761)\in(8,6)$, $Y_0^*(1520)\in(1,4)$ and $Y_0^*(1405)\in(1,2)$. The $Y_1^*(1660)$ particle could belong to either $(8,4)_4$ or $(8,4)_2$ or to some linear combination thereof. An additional and particularly attractive feature about these decays is that the strangeness quantum number does not change, i.e., $SU(3)$ is not needed in obtaining theoretical values for the rates so *$SU(3)$ breaking effects do not vitiate the strength of the predictions made.* Let us first extract phenomenological coupling constants from the observed decay widths. We use the following formulas[18] for the decays $J^P\to J'^P\pi$:

$$D_{3/2}\to S_{1/2}\pi\,,\quad \frac{g^2}{4\pi}=\frac{3\Gamma}{q^3}\frac{M_R}{E+M}\,,$$
$$D_{5/2}\to D_{3/2}\pi\,,\quad \frac{g^2}{4\pi}=\frac{3\Gamma}{q^3}\frac{M_R}{E+M+(4/15)(q^2/M^2)(E+2M)}\,, \tag{20}$$

where M_R is the resonance mass, q is the decay momentum in the center-of-mass system, M and E are the mass and energy of the decay baryon, and Γ is the decay width. The kinematics are remarkably similar for the two decay processes, and we find

$$\frac{g^2(Y_1^*(1765)\to Y_0^*(1520)\pi)}{g^2(Y_1^*(1660)\to Y_0^*(1405)\pi)}=1.05\frac{\Gamma_{1765}}{\Gamma_{1660}}\,, \tag{21}$$

[18] J. G. Rushbrooke, Phys. Rev. **143**, 1345 (1966).

i.e., the ratios of widths and squared couplings are nearly equal. Currently available numbers[9] for the total widths and branching ratios (B.R.) are $Y_1^*(1765)$: $\Gamma_{\text{tot}}=89$ MeV; B.R. $(Y_0^*(1520))=0.2$ and $Y_1^*(1660)$: $\Gamma_{\text{tot}}=50$ MeV; B.R. $(Y_1^*(1660))=$ large. Inserting these values into (21), we have

$$\frac{g^2(Y_1^*(1765)\to Y_0^*(1520)\pi)_{\text{expt}}}{g^2(Y_1^*(1660)\to Y_0^*(1405)\pi)_{\text{expt}}}=\frac{18}{50\times\text{B.R.}(1660\to 1405)}=\frac{0.36}{\text{B.R.}(1660\to 1405)}. \quad (22)$$

If B.R.$(1660\to 1405)>0.36$, then the above ratio of couplings is less than unity. We now proceed to study alternative theoretical descriptions of these modes. All that is really involved is a ratio of squares of appropriate **70**$\times$**1** Clebsch-Gordan coefficients. However, there are two complicating factors present: (i) As mentioned above, the **70**$\times$**1** assignment of $Y_1^*(1660)$ is not unambiguous. (ii) The vertex **70**$\to$**70**$\times$**35** involves an F/D ratio which is not known *a priori*. First, we calculate the simple case, $Y_1^*(1765)\to Y_0^*(1520)\pi$. In **70**$\times$**1** terminology, $Y_1^*(1765)\in(8,6)_4$ and $Y_0^*(1520)\in(1,4)$. The relevant **70**$\times$**1** Clebsch-Gordan coefficient is 0.147 $\cos\theta+0.67\sin\theta$, the angle θ representing the F/D ambiguity in the **70**$\to$**70**$\times$**35** vertex ($\theta=0$ implies pure D). The apparent $Y_1^*(1660)$ ambiguity arising from the possibility of configuration mixing is probably not serious, because the mixing is evidently quite small.[12] Further, the spin part of the couplings $(8,4)_4\to(1,2)$ and $(8,4)_2\to(1,2)$ are $5^{1/2}/3$ and $2/3$, respectively, and are nearly equal. Hence we feel the only important alternatives are to put $Y_1^*(1660)$ into $(8,4)_4$ or $(8,4)_2$. The predictions are shown below.

	$Y_1^*(1660)\in(8,4)_4$	$Y_1^*(1660)\in(8,4)_2$
$\dfrac{g^2(Y_1^*(1765)\to Y_0^*(1520)\pi)}{g^2(Y_1^*(1660)\to Y_0^*(1405)\pi)}$	$\dfrac{9}{5}$	$\dfrac{(0.147\cos\theta+0.16\sin\theta)^2}{(0.18\cos\theta+0.16\sin\theta)^2}$.

The assignment $Y_1^*(1660)\in(8,4)_4$ predicts a ratio independent of the F/D parameter θ, and probably in disagreement with experiment [see statement following (22)]. The assignment $Y_1^*(1660)\in(8,4)_2$, which is the one Capps chooses[12] on the basis of a semiempirical mass formula, involves directly the parameter θ. For $|\theta|<\frac{1}{2}\pi$, a range of θ consistent with the inequality $g^2(1765\to 1520\pi)<g^2(1660\to 1405\pi)$ is almost the entire span of θ. Only when the $Y_1^*(1660)\to Y_0^*(1405)\pi$ branching ratio has been accurately measured can we get an estimate of θ.

6. CONCLUSIONS

In conclusion we stress the following points:

(1) The most important idea introduced in this paper is that the occurrence in nature of very-high-spin baryons forces one to describe these particles as *composites of high-spin baryons and mesons with low orbital angular momentum*. Consequently, there appears to be no limit on the extent to which the baryon spectrum may continue. Previous attempts to explain the baryon spectrum have truncated the problem to one of a finite set of channels, an outlook which in many essentials is equivalent to potential theory. Our approach manifestly requires infinitely many channels to describe the entire baryon spectrum, and so certain results of potential models, such as trajectories necessarily turning over, need not hold here.

(2) We implement the basic idea mentioned above with dynamical models. A class of models, exhibiting $SU(n)\times O(3)$ symmetry, has a natural description in bootstrap theory. According to the latest phase-shift analysis,[14] the positive-parity baryons predicted by our $SU(2)\times O(3)$ model described in Secs. 3 and 4 have been found and with roughly the correct properties, e.g., the $T=\frac{3}{2}$ baryons have average mass greater than the $T=\frac{1}{2}$ baryons, and the decay widths of $N'(1688)$ and $N^{*\prime}(1924)$ into πN exceed those of the other states in the $SU(2)\times O(3)$ multiplets.[19]

Our results generalize trivially to $SU(3)\times O(3)$ and $SU(6)\times O(3)$, each introducing more particles into the model at the expense of treating their masses and couplings more approximately. Because of the unclear experimental situation with the meson spectrum, we have restricted our efforts to studying only the baryons. It is not clear how well the $SU(n)\times O(3)$ model *per se* will describe nature for arbitrarily high-spin objects, since we do not include arbitrarily high-spin mesons in the calculation. However, for moderately low-spin baryons, say $K=0$, 2, and perhaps 4, our model, with pseudoscalar and vector mesons, should be realistic, as is apparently the case.[14]

[19] Due to the enormous difficulty of doing more detailed work both in our theoretical model and in the phase-shift analyses, a precise confrontation of the results of each is not immediately forthcoming. However, the approximate features of the positive-parity baryon spectrum are in accord. In response to a remark made in Ref. 14, a bootstrap model of the P_{11} "Roper" resonance is given by E. Golowich, Phys. Rev. **153**, 1466 (1967). An additional P_{11} resonance with mass 1700 MeV is predicted in the same reference.

(3) It is possible that the excited states of negative-parity baryons behave in an entirely analogous manner to the positive-parity systems described here. Capps already has made plausible in a dynamical model that the negative-parity baryon ground state corresponds to a **70**$\times$**1** multiplet structure.[12] Conceivably a unified dynamical description of positive- and negative-parity baryons is at hand, at least at energies where it makes empirical sense to study the properties of single-particle states.

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APPENDIX

In this Appendix we derive an expression describing the coupling of a baryon of spin J to a composite of a baryon of spin J' and a P-wave pion. The two baryons occur in the chains of excited states generated by ground-state baryons of spin S and S' according to $\mathbf{J}=\mathbf{S}+\mathbf{K}$, $\mathbf{J}'=\mathbf{S}'+\mathbf{K}$. We begin with the formulas inferred by the vector addition just mentioned;

$$\begin{aligned}|JM\rangle &= |Kk\rangle|Ss\rangle(KkSs|JM)\,,\\ |J'M'\rangle &= |Kk\rangle|S's'\rangle(KkS's'|J'M')\,,\end{aligned} \tag{A1}$$

where the quantities in parentheses denote ordinary $SU(2)$ Clebsch-Gordan coefficients. Upon forming the vertex, we have

$$\langle J'M'|\pi|JM\rangle=\langle S's'|\pi|Ss\rangle(KkS's'|J'M')\times(KkSs|JM)\,, \tag{A2}$$

where we have used

$$\langle K'k'|Kk\rangle=\delta_{KK'}\delta_{kk'}\,. \tag{A3}$$

But, by the Wigner-Eckart theorem,

$$\begin{aligned}\langle J'M'|\pi|JM\rangle &= (1lJ'M'|JM)g_i(J',J)\,,\\ \langle S's'|\pi|Ss\rangle &= (1lS's'|Ss)g_i(S',S)\,,\end{aligned} \tag{A4}$$

where $i=0$, 1, 2 exhibits the isospin nature of the coupling, and the "1" in the Clebsch-Gordan coefficient is provided by the presence of a P-wave pion. From (A2)–(A4),

$$(1lJ'M|JM)g_i(J',J)=\sum_{s,s'}(1lS's'|Ss)(S's'Kk|J'M')\times(SsKk|JM)(-)^{S'+K+S+K-J'-J}g_i(S',S)\,, \tag{A5}$$

where we have permuted certain of the Clebsch-Gordan labels. Multiplying by $(-)^{1+1}(-)^{J-J}$ (i.e., by unity), we have

$$(1lJ'M'|JM)g_i(J',J)=\sum_{s,s'}(1lS's'|Ss)(S's'Kk|JM)(SsKk|JM)\times(-)^{1+S'+K+J}(-)^{1-2J-J'+K+S}g_i(S'S) \tag{A6}$$

and upon noting the expansion of a $6j$ symbol in terms of $SU(2)$ Clebsch-Gordan coefficients,[20] we finally obtain

$$g_i(J',J)=\begin{Bmatrix}1 & S' & J\\ K & J & J'\end{Bmatrix}(-)^{1-2J-J'+K+S}\times[(2S+1)(2J'+1)]^{1/2}g_i(S',S)\,. \tag{A7}$$

[20] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, N. J., 1960), p. 95.