We comment on our result. Concerning the S-wave scattering lengths, Hamilton's[10] new values are $a_3 = -0.091\pm0.005$ and $2a_1+a_3=0.270\pm0.008$, which yield $a_1+2a_3=-0.002\pm0.008$. As regards the P' parameters, a diversity of results appears in the literature. Barger and Olsson[11] gave $\alpha_{P'}=0.39\pm0.24$; Rarita *et al.*[12] gave $\alpha_{P'}=0.57$ with $\gamma_{P'}=14.8$ mb BeV or $\alpha_{P'}=0.73$ with $\gamma_{P'}=21$ mb BeV; Meshcheriakov *et al.*,[13] imposing $\alpha_{P'}=0.50$, obtained $\gamma_{P'}=13.86$ mb BeV; while Igi,[14] who was the first one to propose the existence of P', gave $\gamma_{P'}=3.05\mu^{-1}=18.3$ mb BeV for $\alpha_{P'}=0.4$ ($\gamma_P=21.6$ mb). Although the experimental uncertainties in $\mathrm{Re}C^{(+)}(\nu)$ may still be large, we believe that our method provides a better method for the determination of all these parameters, since all the integrals [Eqs. (5), (7), and (9)] are superconvergent.

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Field-Current Identities and Intermediate Bosons*

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The observed total weak-interaction current $\mathcal{J}_\mu^{\mathrm{wk}}(x)$ and the observed total electromagnetic current $\mathcal{J}_\mu^{\gamma}(x)$ are assumed to be, respectively, the same local operators, apart from constant multiplicative factors, as the hypothetical charged intermediate boson field $W_\mu(x)$ and the corresponding neutral intermediate boson field $W_\mu^0(x)$. The field algebra satisfied by these current operators is discussed. It is shown that, neglecting higher-order weak-interaction effects, one can obtain finite higher-order electromagnetic corrections for the known hadrons and leptons, such as the electromagnetic mass shifts of p, π, e, μ, etc., and the radiative corrections to the weak decays of these particles.

I. INTRODUCTION

THE purpose of this paper is to show that within the general framework of field-current identities[1–3] it is possible to derive finite higher-order electromagnetic corrections for the known hadrons and leptons, provided one identifies the observed weak and electromagnetic current operators, $\mathcal{J}_\mu^{\mathrm{wk}}$ and $\mathcal{J}_\mu^{\gamma}$, as proportional to some weakly coupled fields such as the (hypothetical) intermediate boson fields. In order to show that such field-current identities are indeed possible, let us first examine the definitions of these observed current operators.

The total electromagnetic current operator $\mathcal{J}_\mu^{\gamma}$ is, by definition, related to the electromagnetic field A_μ by[4]

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu}=e_0\mathcal{J}_\nu^{\gamma}, \tag{1.1}$$

where e_0 is the unrenormalized charge of the electron ($e_0<0$), and

$$F_{\mu\nu}=\frac{\partial}{\partial x_\mu}A_\nu-\frac{\partial}{\partial x_\nu}A_\mu.$$

To give a precise definition of the total observed weak-interaction current operator $\mathcal{J}_\mu^{\mathrm{wk}}(x)$, we assume that the bilinear product

$$s_\lambda^{\mathrm{wk}}(l)=i\nu_l(x)^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)l(x) \tag{1.2}$$

is an observable, where $l(x)$ and $\nu_l(x)$ represent field operators of the particles l^- and ν_l, respectively, $l=e$ or μ, and the dagger denotes the Hermitian conjugate. The observed total weak-interaction current $\mathcal{J}_\mu^{\mathrm{wk}}(x)$ is then defined to be proportional to the derivative of the S matrix with respect to $s_\lambda^{\mathrm{wk}}(e)$ or $s_\lambda^{\mathrm{wk}}(\mu)$. We have

$$i\frac{G_F}{\sqrt{2}}\mathcal{J}_\lambda^{\mathrm{wk}}(x)=\frac{\partial S}{\partial s_\lambda^{\mathrm{wk}}(e)}=\frac{\partial S}{\partial s_\lambda^{\mathrm{wk}}(\mu)}, \tag{1.3}$$

where

$$G_F\cong10^{-5}m_N^{-2}, \tag{1.4}$$

which denotes the Fermi coupling constant of the weak interaction, and m_N is the nucleon mass. In (1.3), the equality $[\partial S/\partial s_\lambda^{\mathrm{wk}}(e)]=[\partial S/\partial s_\lambda^{\mathrm{wk}}(\mu)]$ expresses the $\mu-e$ symmetry property of the weak interaction.

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[4] Throughout the paper, the subscript μ denotes the space-time index, $\mu=4$ is the time component, $x_4=it$, and $\mu=i$ (or j, or k) denotes the space component. All repeated indices are to be summed over.

Throughout the following discussions, we assume the following:

(i) All presently known leptonic and semileptonic weak processes such as

$$\nu_l + A \rightarrow l^- + B \qquad (1.5)$$

and

$$A \rightarrow B + l^- + \bar{\nu}_l \qquad (1.6)$$

are transmitted by the hypothetical charged intermediate boson field $W_\mu(x)$.

(ii) There exists a neutral intermediate boson field W_μ^0 which is the source of the electromagnetic field; i.e.,

$$\mathcal{J}_\mu^\gamma = \kappa W_\mu^0,$$

where κ is a constant.

(iii) In the absence of all other fields, these intermediate boson fields satisfy a set of non-Abelian field algebra and the Lagrangian is invariant with respect to the corresponding symmetry group, the simplest possibility being the SU_2 symmetry.

This last assumption is needed to distinguish between the electromagnetic field A_μ and other possible linear combinations of the neutral spin-1 fields such as $A_\mu' = A_\mu + (\text{const}) W_\mu^0$. The field algebra, or the corresponding symmetry, is satisfied when one sets $A_\mu = 0$, but not $A_\mu' = 0$.

According to (i), the transition matrix elements of the weak-interaction Hamiltonian for (1.5) and (1.6) are given by

$$f\langle B | W_\lambda(x) | A \rangle [s_\lambda^{\text{wk}}(e) + s_\lambda^{\text{wk}}(\mu)], \qquad (1.7)$$

where $W_\lambda(x)$ is the usual charged intermediate boson field, f is related to G_F by

$$f^2/m_W^2 = G_F/\sqrt{2}, \qquad (1.8)$$

and m_W is the mass of the charged intermediate boson. Comparison between (1.3) and (1.7) leads to the current-field identity

$$\mathcal{J}_\mu^{\text{wk}}(x) = -(m_W^2/f) W_\mu(x). \qquad (1.9)$$

For all presently known weak reactions (1.5) and (1.6), the matrix element $\langle B | W_\lambda | A \rangle$ is $O(f)$. Therefore, $\langle B | \mathcal{J}_\mu^{\text{wk}} | A \rangle$ is $O(1)$.

In Sec. II we discuss an SU_2-triplet model of intermediate bosons, W^+, W^0, and W^-. These intermediate boson fields are related to the observed current operators by (1.9) and

$$\mathcal{J}_\mu^\gamma(x) = -(m_W^2/f_0) W_\mu^0(x), \qquad (1.10)$$

where f_0 may differ from f by a numerical factor, depending on the details of the model. For example, as we shall see, a particularly simple choice for the SU_2 model discussed in Sec. II is

$$f_0 = 2\sqrt{2} f. \qquad (1.11)$$

Different symmetry requirements could lead to different choices of the ratio (f_0/f); those will be given in Appendix A. In general, we assume that f_0 is of the same order of magnitude as f; i.e.,

$$f_0 = O(f)$$

and, therefore,

$$f_0^2/m_W^2 = O(G_F). \qquad (1.12)$$

From these field-current identities, one finds that, to all orders of e^2 and G_F, the total electromagnetic current operator $\mathcal{J}_\mu^\gamma$ satisfies, among others, the following field algebra:

$$[\mathcal{J}_i^\gamma(r,t), \mathcal{J}_j^\gamma(r',t)] = [\mathcal{J}_4^\gamma(r,t), \mathcal{J}_4^\gamma(r',t)] = 0, \qquad (1.13)$$

$$[\mathcal{J}_4^\gamma(r,t), \mathcal{J}_j^\gamma(r',t)] = (m_W^2/f_0^2) \nabla_j \delta^3(r-r'), \qquad (1.14)$$

and

$$\begin{aligned}[(\partial/\partial t)\mathcal{J}_j^\gamma(r,t) - i\nabla_j \mathcal{J}_4^\gamma(r,t), \mathcal{J}_k^\gamma(r',t)] \\ = -i(m_W^2/f_0^2) m_0^2 \delta_{jk} \delta^3(r-r') \\ -i(f^2/m_W^2)[\mathcal{J}_j^{\text{wk}}(r,t)\mathcal{J}_k^{\text{wk}}(r,t)^\dagger \\ + \mathcal{J}_j^{\text{wk}}(r,t)^\dagger \mathcal{J}_k^{\text{wk}}(r,t)]\delta^3(r-r'), \qquad (1.15)\end{aligned}$$

where m_0 is the bare mass of the neutral intermediate boson. It is important to note that in (1.15) the coefficient of the quadratic term $\mathcal{J}_j^{\text{wk}} \mathcal{J}_k^{\text{wk}\dagger}$ is $-i(G_F/\sqrt{2})$; it can be neglected if one is not interested in higher-order weak-interaction effects. The commutator between $\partial \mathcal{J}_j^\gamma(r,t)/\partial t$ and $\mathcal{J}_k^\gamma(r',t)$ becomes, then, a c number.

Note added in proof. This commutator can be a c number to all orders in e and G_F, provided that one does not apply the Yang-Mills theory to the intermediate boson system; otherwise, this commutator is a c number only if higher-order weak-interaction effects are neglected.

By introducing additional boson fields, the relevant group can be easily enlarged to, e.g., $SU_2 \times SU_2$, or $SU_3 \times SU_3$. The corresponding field algebra can also be derived. Furthermore, if one wishes, the interaction between the hadrons and the intermediate bosons can be made to preserve the usual isospin symmetry. Thus, for example, there is no $O(f^2)$ term in the mass difference between different hadrons of the same isospin multiplet. Such a general treatment is given in Appendix A. The usual theory of vector and axial-vector dominance in strong interactions assumes a different form in the present case; these will be discussed in Appendix B.

In all these cases, independently of the particular group structure assumed, the electromagnetic interactions between all known particles, hadrons and leptons, are mediated only through W^0 via the sequence

$$\text{known particles} \rightleftarrows W^0 \rightleftarrows \gamma \rightleftarrows W^0 \rightleftarrows \text{known particles}. \qquad (1.16)$$

In this sequence, the interaction between W^0 and the known particles is $O(f)$ and that between W^0 and γ is $O(e/f)$. One may combine this chain of neutral spin-1 boson propagators into a single term called the *effective* *photon propagator* $D_\gamma(q^2)$. As $q^2\to 0$, $D_\gamma(q^2)\to q^{-2}$, but, as we shall see,

$$\text{as } q^2\to\infty,\quad D_\gamma(q^2)\to O(q^{-6}), \tag{1.17}$$

provided all higher-order weak-interaction effects are neglected. The general properties of such an electromagnetic interaction are discussed in Sec. III, and the detailed behavior of this effective photon propagator $D_\gamma(q^2)$ is given in Sec. IV. The q^{-6} dependence of $D_\gamma(q^2)$ at large q^2 makes it possible to obtain finite values for many previously divergent radiative correction calculations which were evaluated in a theory without W^0. The present theory gives a convergent result if the previous integral diverges like either $O(\ln q^2)$, or $O(q^2)$, as the square of the virtual photon momentum $q^2\to\infty$. This includes the electromagnetic mass shifts of any spin-$\frac{1}{2}$ and spin-0 charged particles, such as e, μ, p, π, etc.; it also applies to the radiative corrections to the weak decays of these particles. The details of these radiative corrections will be given in a separate paper.

In contrast, the higher-order weak-interaction effects remain divergent, at least in the perturbation series expansion. In this sense, the present formulation cannot, as yet, be regarded as a fundamental theory, but one that provides unambiguous rules to obtain finite radiative corrections within the general framework of the local field theory.

II. SU_2-TRIPLET MODEL OF INTERMEDIATE BOSONS

We first discuss a simple system consisting of the electromagnetic field A_μ, the three intermediate boson fields $W_\mu{}^0(x)$,

$$W_\mu(x)=(1/\sqrt{2})[W_\mu{}^1(x)-iW_\mu{}^2(x)] \tag{2.1}$$

and

$$W_\mu{}^*(x)=(1/\sqrt{2})[W_\mu{}^1(x)+iW_\mu{}^2(x)], \tag{2.2}$$

the usual four lepton fields $e(x)$, $\mu(x)$, $\nu_e(x)$, and $\nu_\mu(x)$, but, for clarity of presentation, only two hadron fields q_1 and q_2. If one uses the Sakata model[5] of the usual triplet p^+, n^0, and λ^0, then

$$q_1=p$$

and

$$q_2=n\cos\theta+\lambda\sin\theta, \tag{2.3}$$

where θ is the Cabibbo angle.[6]

It is convenient to define

$$\psi_l(x)=\begin{pmatrix}\nu_l(x)\\ l(x)\end{pmatrix}\quad\text{and}\quad \psi_h(x)=\begin{pmatrix}q_1(x)\\ q_2(x)\end{pmatrix}, \tag{2.4}$$

where the neutrino fields $\nu_e(x)$ and $\nu_\mu(x)$ are described by the two-component theory

$$\gamma_5\nu_l(x)=\nu_l(x). \tag{2.5}$$

Thus, in the familiar decomposition[7]

$$\psi_a(x)=\psi_a{}^L(x)+\psi_a{}^R(x), \tag{2.6}$$

where $a=l$ or h,

$$\psi_a{}^L(x)=\tfrac{1}{2}(1+\gamma_5)\psi_a(x) \tag{2.7}$$

and

$$\psi_a{}^R(x)=\tfrac{1}{2}(1-\gamma_5)\psi_a(x), \tag{2.8}$$

one has for the lepton field

$$\psi_l{}^R(x)=\frac{1}{2}\begin{pmatrix}0\\ (1-\gamma_5)l(x)\end{pmatrix}. \tag{2.9}$$

The total electric charge operator Q can be written as

$$Q=I_3+\tfrac{1}{2}Z, \tag{2.10}$$

where Z is related to the usual baryon number N and the usual total leptonic number L (defined to be $+1$ for l^- and ν_l) by

$$Z=N-L, \tag{2.11}$$

and I_3 is related to the Pauli matrix τ_3 by $I_3=\frac{1}{2}\tau_3$ for ψ_l and ψ_h. For the intermediate boson field, $Z=0$, and $I_3=1$, 0, and -1, respectively, for W_μ, $W_\mu{}^0$, and $W_\mu{}^*$. It is important to note that I_3 is *not* the usual third component isospin operator T_3. For the hadrons, $I_3=T_3+\frac{1}{2}S$, and S is the strangeness.

In accordance with assumptions (i), (ii), and (iii), the total Lagrangian for this simple model is a sum of three parts, the spin-1 boson field part $\mathcal{L}_{\text{field}}$, the matter part $\mathcal{L}_{\text{matter}}$, and the interaction part $\mathcal{L}_{\text{int}}$:

$$\mathcal{L}=\mathcal{L}_{\text{field}}+\mathcal{L}_{\text{matter}}+\mathcal{L}_{\text{int}}, \tag{2.12}$$

$$\mathcal{L}_{\text{field}}=-\tfrac{1}{4}F_{\mu\nu}{}^2-\tfrac{1}{4}(1+\eta)(\hat{W}_{\mu\nu})^2-\tfrac{1}{2}m_W{}^2(\mathbf{W}_\nu)^2, \tag{2.13}$$

$$\mathcal{L}_{\text{matter}}=-\psi_a{}^\dagger\left(\gamma_4\gamma_\mu\frac{\partial}{\partial x_\mu}+M_a\right)\psi_a, \tag{2.14}$$

and

$$\mathcal{L}_{\text{int}}=-i\sqrt{2}f[(\psi_a{}^L)^\dagger\gamma_4\gamma_\mu(\tau_1W_\mu{}^1+\tau_2W_\mu{}^2)\psi_a{}^L] -i\tfrac{1}{2}f_0[\psi_a{}^\dagger\gamma_4\gamma_\mu(\tau_3+Z)\psi_a]\hat{W}_\mu{}^0, \tag{2.15}$$

[5] Z. Maki, M. Nakagawa, Y. Ohnuki, and S. Sakata, Progr. Theoret. Phys. (Kyoto) **23**, 1174 (1960). If one uses the quark model [M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, Conseil Européen pour la Recherche Nucléarie Report (unpublished)], then it is necessary also to include the interaction between $\hat{W}_\mu{}^0$ and $q_3=-n\sin\theta+\lambda\cos\theta$ because q_3 is charged.

[6] N. Cabibbo, Phys. Rev. Letters **10**, 513 (1963).

[7] Such decomposition has been used since the early days of parity nonconservation. We note that the total Lagrangian (2.12), except for the mass term, is invariant under a $U_2\times U_2$ symmetry group which transforms $\mathbf{W}_\mu\to\mathbf{W}_\mu$, $\psi_h\to\psi_h$, $A_\mu\to A_\mu$, but

$$\begin{pmatrix}\psi_e{}^L\\ \psi_\mu{}^L\end{pmatrix}\to u\begin{pmatrix}\psi_e{}^L\\ \psi_\mu{}^L\end{pmatrix}\quad\text{and}\quad\begin{pmatrix}\psi_e{}^R\\ \psi_\mu{}^R\end{pmatrix}\to v\begin{pmatrix}\psi_e{}^R\\ \psi_\mu{}^R\end{pmatrix},$$

where u and v are two arbitrary (2×2) unitary matrices. In addition, it is also invariant under the usual $U_1\times U_1$ gauge group generated by the total charge Q and the total baryon number N.

The total lepton number L is one of the particular generators of the $U_2\times U_2$ group. From (2.10) and (2.11), one sees that $I_3=Q-\frac{1}{2}N+\frac{1}{2}L$ is always conserved. This $U_2\times U_2$ group has been discussed by G. Feinberg and F. Gürsey [Phys. Rev. **128**, 378 (1962)] and T. D. Lee [Nuovo Cimento **35**, 945 (1965)].

where $\mathbf{W}_\mu$ denotes the SU_2 triplet $W_\mu{}^1$, $W_\mu{}^2$, and $W_\mu{}^0$, the components of $\hat{W}_\mu=(\hat{W}_\mu{}^1,\hat{W}_\mu{}^2,\hat{W}_\mu{}^0)$ are related to those of $\mathbf{W}_\mu$ by

$$\hat{W}_\mu{}^0=W_\mu{}^0+(e_0/f_0)A_\mu$$

and

$$\hat{W}_\mu{}^i=W_\mu{}^i, \quad \text{for} \quad i=1 \quad \text{and} \quad 2,$$

$$\hat{W}_{\mu\nu}=\frac{\partial}{\partial x_\mu}\hat{W}_\nu-\frac{\partial}{\partial x_\nu}\hat{W}_\mu-f_0(\hat{W}_\mu\times\hat{W}_\nu),$$

$\boldsymbol{\tau}=(\tau_1,\tau_2,\tau_3)$ denotes the usual set of three Pauli matrices, all repeated indices are to be summed over, the subscript a varies over e, μ, and h, and M_a is the appropriate mass matrix. In (2.13), the parameter η is a constant which, as we shall discuss in Sec. III, is important for the renormalization problem of the intermediate boson.

The choice of the numerical factor between f and f_0 depends on the approximate symmetry properties of the model. For example, a particularly simple choice is

$$f_0=2\sqrt{2}f. \tag{2.16}$$

Therefore,

$$f_0{}^2/m_W{}^2=4\sqrt{2}G_F, \tag{2.17}$$

and (2.15) becomes

$$\mathcal{L}_{\text{int}}=-i\tfrac{1}{2}f_0(\psi_a{}^L)^\dagger\gamma_4\gamma_\mu\boldsymbol{\tau}\psi_a{}^L\cdot\hat{W}_\mu -i\tfrac{1}{2}f_0[\psi_a{}^\dagger\gamma_4\gamma_\mu Z\psi_a+(\psi_a{}^R)^\dagger\gamma_4\gamma_\mu\tau_3\psi_a{}^R]\hat{W}_\mu{}^0. \tag{2.18}$$

In this case, the SU_2 symmetry is violated by the electromagnetic field A_μ, the mass matrix M_a, and the term inside the square bracket in $\mathcal{L}_{\text{int}}$ given by (2.18). Thus, in the absence of $\psi_a{}^R$ and A_μ, the Lagrangian $\mathcal{L}$ transforms like an SU_2 scalar, provided it operates only on states with $Z=N-L=0$. Other choices[8] of the numerical factor (f_0/f) are also possible, which would lead to different symmetry requirements for the model. All our discussions are, however, valid independently of the particular value of (f_0/f).

This simple model can be easily generalized to include arbitrary varieties of other hadron fields ψ_h. In the following, ψ_h represents all hadron fields which can be either the quarks, or the known baryons, or the strongly interacting spin-1 mesons, etc. In the general case, the total Lagrangian $\mathcal{L}$ can in place of (2.12), be, written as

$$\mathcal{L}=\mathcal{L}_{\text{field}}+\mathcal{L}_l+\mathcal{L}_h, \tag{2.19}$$

where $\mathcal{L}_{\text{field}}$ is given by (2.13) and $\mathcal{L}_l$ denotes the leptonic part of the previous $(\mathcal{L}_{\text{matter}}+\mathcal{L}_{\text{int}})$; i.e.,

$$\mathcal{L}_l=(2.14)+(2.15), \tag{2.20}$$

but summing over only $a=e$ and μ. The hadronic part $\mathcal{L}_h$ now also includes the usual strong interactions; it is assumed to have the general functional form

$$\mathcal{L}_h=\mathcal{L}_h(\psi_h,D_\nu\psi_h,\hat{W}_{\mu\nu}). \tag{2.21}$$

Under the infinitesimal transformation of the triplet field

$$\begin{pmatrix}q_1\\q_2\end{pmatrix}\to\begin{pmatrix}q_1\\q_2\end{pmatrix}+i2^{-1/2}(f/f_0)(\tau_1\delta\theta_1+\tau_2\delta\theta_2)(1+\gamma_5)\begin{pmatrix}q_1\\q_2\end{pmatrix}$$

and

$$q_3\to q_3,$$

where q_1 and q_2 are given by (2.3) and

$$q_3=-n\sin\theta+\lambda\cos\theta,$$

we assume that the general hadron field ψ_h transforms as

$$\psi_h\to\psi_h+(T_1\delta\theta_1+T_2\delta\theta_2)\psi_h. \tag{2.22}$$

The operator D_ν in (2.21) is given by

$$D_\nu\psi_h=\left(\frac{\partial}{\partial x_\nu}+f_0\mathbf{T}\cdot\hat{W}_\nu\right)\psi_h. \tag{2.23}$$

The matrix $\mathbf{T}$ has three components, $\mathbf{T}=(T^1,T^2,T^0)$, where T^1 and T^2 are determined by (2.22), and T^0 is related to the total charge operator Q by

$$T^0=iQ. \tag{2.24}$$

Since in the general form (2.21) $\mathcal{L}_h$ may also depend on $\hat{W}_{\mu\nu}$, we assume

$$\mathcal{L}_h=0 \quad \text{if all} \quad \psi_h=0. \tag{2.25}$$

This eliminates the possibility of an isolated term such as $-\frac{1}{4}\eta\hat{W}_{\mu\nu}{}^2$ in $\mathcal{L}_h$.

From the general Lagrangian (2.19), one finds that the Maxwell equation becomes

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu}=-(e_0m_W{}^2/f_0)W_\nu{}^0. \tag{2.26}$$

The equation of motion of the intermediate boson field is

$$(1+\eta)\frac{\partial}{\partial x_\mu}\hat{W}_{\mu\nu}-m_W{}^2\mathbf{W}_\nu =f_0[\mathbf{s}_\nu(h)+\mathbf{s}_\nu(e)+\mathbf{s}_\nu(\mu)] +f_0(1+\eta)(\hat{W}_\mu\times\hat{W}_{\mu\nu}), \tag{2.27}$$

where $\mathbf{s}_\nu(e)$, $\mathbf{s}_\nu(\mu)$, and $\mathbf{s}_\nu(h)$ are, respectively, the source functions due to ψ_e, ψ_μ, and ψ_h. By using (2.15), we find that, for the simple case that ψ_h is given by (2.4), the components of $\mathbf{s}_\nu(a)$ are given by

$$s_\nu{}^0(a)=i\tfrac{1}{2}\psi_a{}^\dagger\gamma_4\gamma_\nu(\tau_3+Z)\psi_a \tag{2.28}$$

[8] Another simple choice is $f_0=\sqrt{2}f\cos\theta$, where θ is the Cabibbo angle. In this case, the $T=1$ part of the vector interaction in $\mathcal{L}_{\text{int}}$ between the hadrons and the intermediate bosons preserves the usual isospin symmetry. Thus, for example, to first order in f, the magnitudes of the (virtual) transition matrix elements for $\pi^0\to\pi^0+W^0$, $\pi^0\to\pi^++W^-$, $\pi^0\to\pi^-+W^+$, $\pi^+\to\pi^++W^0$, and $\pi^+\to\pi^0+W^+$ are in the ratio 0:1:1:1:1. For a discussion of the general relation between f_0 and f, see Appendix A, especially Eq. (A22).

and

$$s_\nu{}^k(a)=i2^{-1/2}(f/f_0)\psi_a{}^\dagger\gamma_4\gamma_\nu(1+\gamma_5)\tau_k\psi_a\,, \quad (2.29)$$

where $k=1$, or 2 and a can be e, or μ, or h. For the general case (2.19), $\mathbf{s}_\nu(e)$ and $\mathbf{s}_\nu(\mu)$ remain the same, but the components of $\mathbf{s}_\nu(h)$ become

$$s_\nu{}^i(h)=\epsilon^{ijk}[(\partial\mathcal{L}_h/\partial\hat{W}_{\mu\nu}{}^j)-(\partial\mathcal{L}_h/\partial\hat{W}_{\nu\mu}{}^j)]\hat{W}_\mu{}^k +(\partial/\partial x_\mu)[(\partial\mathcal{L}_h/\partial\hat{W}_{\mu\nu}{}^i)-(\partial\mathcal{L}_h/\partial\hat{W}_{\nu\mu}{}^i)] -\sum[\partial\mathcal{L}_h/\partial D_\nu\psi_h]T^i\psi_h\,, \quad (2.30)$$

where the sum extends over all hadron fields ψ_h and ϵ^{ijk} is $+1$, -1, or 0 depending on whether (ijk) is an even permutation of (012), an odd permutation, or otherwise.

In the Coulomb gauge, the electromagnetic field A_μ is given by

$$A_j=A_j{}^{\mathrm{tr}} \quad\text{and}\quad A_4=i\phi\,, \quad (2.31)$$

where the transverse part $A_j{}^{\mathrm{tr}}$ is divergence free,

$$\nabla_j A_j{}^{\mathrm{tr}}=0\,. \quad (2.32)$$

and ϕ is determined by the Laplace equation

$$\nabla^2\phi=i(e_0 m_W{}^2/f_0)W_4{}^0\,. \quad (2.33)$$

It is convenient to choose ψ_l, ψ_h, and the spatial components $A_j{}^{\mathrm{tr}}$ and $\hat{W}_j$ as the general coordinates; their conjugate momenta are, respectively,

$$P_l=i\psi_l{}^\dagger\,, \quad (2.34)$$

$$P_h=-i(\partial\mathcal{L}_h/\partial D_4\psi_h)\,, \quad (2.35)$$

$$\Pi_j{}^{\mathrm{tr}}=-E_j{}^{\mathrm{tr}}=(\partial A_j{}^{\mathrm{tr}}/\partial t)\,, \quad (2.36)$$

and

$$(\mathbf{P}_W)_j=i[(1+\eta)\hat{W}_{4j}-(\partial\mathcal{L}_h/\partial\hat{W}_{4j}) +(\partial\mathcal{L}_h/\partial\hat{W}_{j4})]\,. \quad (2.37)$$

Thus, for $\nu=4$, (2.27) becomes

$$\mathbf{W}_4=m_W{}^{-2}[i\nabla_j(\mathbf{P}_W)_j+if_0(\mathbf{P}_W)_j\times\hat{W}_j +if_0\textstyle\sum_a P_a\mathbf{T}\psi_a]\,, \quad (2.38)$$

where the sum extends over $a=e$, μ, and all hadron fields. For the leptons, $a=e$, or μ, similarly to (2.22) and (2.24), we define $\mathbf{T}=(T^1,T^2,T^0)$ to be $T^1=i2^{-1/2}(f/f_0)\times\tau_1(1+\gamma_5)$, $T^2=i2^{-1/2}(f/f_0)\tau_2(1+\gamma_5)$, and $T^0=iQ=i\frac{1}{2}\times(\tau_3+Z)$.

By following the usual derivation of field algebra[2,3] and by using (1.9) and (1.10), one finds that independently of the detailed structure of $\mathcal{L}_h$, the total observed current operators $\mathcal{J}_\mu{}^\gamma$ and $\mathcal{J}_\mu{}^{\mathrm{wk}}$ satisfy, in addition to (1.13) and (1.14),

$$[\mathcal{J}_i{}^{\mathrm{wk}}(r,t),\mathcal{J}_j{}^\gamma(r',t)]=[\mathcal{J}_i{}^{\mathrm{wk}}(r,t),\mathcal{J}_j{}^{\mathrm{wk}}(r',t)] =[\mathcal{J}_i{}^{\mathrm{wk}}(r,t),\mathcal{J}_j{}^{\mathrm{wk}}(r',t)^\dagger]=0 \quad (2.39)$$

and

$$[\mathcal{J}_4{}^{\mathrm{wk}}(r,t),\mathcal{J}_\mu{}^\gamma(r',t)]=-[\mathcal{J}_4{}^\gamma(r,t),\mathcal{J}_\mu{}^{\mathrm{wk}}(r',t)] =i\mathcal{J}_\mu{}^{\mathrm{wk}}(r,t)\delta^3(r-r')\,. \quad (2.40)$$

If $\mathcal{L}_h$ satisfies

$$[\partial\mathcal{L}_h/\partial\hat{W}_{4i}{}^0(r,t)-\partial\mathcal{L}_h/\partial\hat{W}_{i4}{}^0(r,t),W_j{}^0(r',t)]=0\,, \quad (2.41)$$

then by using the results given in Appendix A of Ref. 3, it can be readily verified that the total electromagnetic hadron current $\mathcal{J}_\mu{}^\gamma$ satisfies (1.15), where the bare mass m_0 is given by

$$m_0{}^2=(1+\eta)^{-1}m_W{}^2+(e_0/f_0)^2 m_W{}^2\,. \quad (2.42)$$

All these commutation relations (1.13)–(1.15), (2.39), and (2.40) are valid to all orders in e and G_F. As an example of (2.41), $\mathcal{L}_h$ can be of the form

$$M_{\mu\nu}{}^a(\psi_h)\hat{W}_{\mu\nu}{}^a+\mathcal{L}_h{}'(\psi_h,D_\nu\psi_h)\,,$$

where $M_{\mu\nu}{}^0=\partial\mathcal{L}_h/\partial\hat{W}_{\mu\nu}{}^0$ depends only on ψ_h.

Remarks

(1) Recently, several authors have shown[9–11] that in a theory without intermediate bosons the application of field algebra leads to an infinite mass difference between $\pi^\pm$ and π^0. In the present case, if one neglects the higher-order weak-interaction effects, the equal-time commutator between $(\partial/\partial t)\mathcal{J}_j{}^\gamma-i\nabla_j\mathcal{J}_4{}^\gamma$ and $\mathcal{J}_k{}^\gamma$ is, according to (1.15), a c number. By following the same argument used in Refs. 9 and 10 (which is, in turn, based on the general method developed by Bjorken[12]), it is easy to show that the second-order electromagnetic mass shift of the pion must be finite.

(2) In the absence of other fields, the equal-time commutator between the charged intermediate boson fields $W_4(r,t)$ and $W_4(r',t)^\dagger$ is simply proportional to $W_4{}^0(r,t)\times\delta^3(r-r')$. From (2.28) and (2.29), one sees that this simple relation is no longer valid in the presence of the matter field. Thus, the equal-time commutator between $\mathcal{J}_4{}^{\mathrm{wk}}$ and $\mathcal{J}_4{}^{\mathrm{wk}\dagger}$ leads to a new operator which is *different* from the observed current $\mathcal{J}_\mu{}^\gamma$. This new operator cannot be exactly proportional to an intermediate boson field in the theory. This is of course a general feature of only intermediate boson theory, reflecting only the well-known fact that, except for

$$ie(x)^\dagger\gamma_4\gamma_\lambda e(x)+i\mu^\dagger(x)\gamma_4\gamma_\lambda\mu(x)\,,$$

all other neutral bilinear combinations of the lepton fields are not coupled to the hadrons. Nevertheless, it is possible to extend the field-current identity to the commutator between $\mathcal{J}_4{}^{\mathrm{wk}}$ and $\mathcal{J}_3{}^{\mathrm{wk}\dagger}$, provided the universality between hadrons and leptons is somewhat altered. For example, if one wishes, one may assume that the field-current identity can be applied to the commutator between $\mathcal{J}_4{}^{\mathrm{wk}}$ and $\mathcal{J}_4{}^{\mathrm{wk}\dagger}$ and that such a field-current identity is violated not by the hadrons, but

[9] G. C. Wick and Bruno Zumino, Phys. Letters **25B**, 479 (1967).
[10] M. B. Halpern and G. Segrè, Phys. Rev. Letters **19**, 611 (1967); **19**, 1000 (1967).
[11] B. W. Lee and N. T. Nieh, Phys. Rev. **166**, 1507 (1968); see also, J. Schwinger, Phys. Rev. Letters **19**, 1154 (1967).
[12] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

only by the leptons. As we shall see, this necessitates the introduction of additional intermediate boson fields. This generalization is given in Appendix A.

(3) The usual idea of vector and axial-vector dominance by ρ, ϕ, ω, A_1, etc., can be incorporated into the present theory; they correspond to the identification that the hadronic source function $\mathbf{s}_\nu(h)$ in (2.27) is proportional to a linear sum of these strongly interacting spin-1 meson fields. The details are given in Appendix B. It will be shown that (2.41) is satisfied, commutation relations (1.15) holds, and therefore the $\pi^\pm$, π^0 mass difference remains finite. Furthermore, in such a case, the usual isovector part of the hadronic electromagnetic form factor $[F_{AB}{}^\gamma(q^2)]_{T=1}$ for any real or virtual photon transition

$$A \to B+\gamma$$

is related to the correspondingly defined form factor $F_{AB}{}^\rho(q^2)$ of the transition

$$A \to B+\rho^0$$

at the same 4-momentum transfer q_μ by

$$[F_{AB}{}^\gamma(q^2)]_{T=1}=\left(\frac{m_W{}^2}{q^2+m_W{}^2}\right)\left(\frac{m_\rho{}^2}{q^2+m_\rho{}^2}\right)F_{AB}{}^\rho(q^2). \quad (2.43)$$

Such an identity holds for all hadron states A and B, if higher-order electromagnetic and weak-interaction effects are neglected. Similar conclusions hold also for the isocsalar part $(J_\mu{}^\gamma)_{T=0}$ provided one replaces $\rho_\mu{}^0$ by an appropriate mixture of the ϕ^0 and ω^0 meson fields.

III. NEUTRAL INTERMEDIATE BOSON AND ELECTROMAGNETIC INTERACTION

In order to examine more clearly the properties of the electromagnetic interaction in the present theory, it is useful to exhibit in the Lagrangian only its explicit dependence on $W_\mu{}^0$ and A_μ; the charged intermediate boson field can be grouped together with other particles. We note that for either the SU_2-triplet model of the intermediate bosons, or the more general case discussed in Appendix A, the total Lagrangian (2.19) or (A36) of the entire system can also be rewritten in the form

$$\mathfrak{L}=-\tfrac{1}{4}(1+\eta)\hat{G}_{\mu\nu}{}^2-\tfrac{1}{2}m_W{}^2(W_\mu{}^0)^2-\tfrac{1}{4}F_{\mu\nu}{}^2 +\mathfrak{L}_\psi(\psi,D_\nu{}^0\psi,\hat{G}_{\mu\nu}), \quad (3.1)$$

where $W_\mu{}^0$ is the neutral intermediate boson field, ψ represents ψ_h, ψ_l, and all other intermediate boson fields except $W_\mu{}^0$,

$$\hat{G}_{\mu\nu}=\frac{\partial}{\partial x_\mu}\hat{W}_\nu{}^0-\frac{\partial}{\partial x_\nu}\hat{W}_\mu{}^0, \quad (3.2)$$

$$\hat{W}_\mu{}^0=W_\mu{}^0+(e_0/f_0)A_\mu, \quad (3.3)$$

$$D_\nu{}^0\psi=\left(\frac{\partial}{\partial x_\nu}+if_0Q\hat{W}_\nu{}^0\right)\psi, \quad (3.4)$$

and Q is the total charge operator.

The Lagrangian (3.1) is clearly invariant under the electromagnetic gauge transformation. It follows from either the Maxwell equation (2.26) or the equation of motion of $W_\nu{}^0$ that

$$\frac{\partial W_\nu{}^0}{\partial x_\nu}=0. \quad (3.5)$$

In (3.1) the interaction of the electromagnetic field A_μ with ψ occurs only through the combined field $\hat{W}_\mu{}^0$. The usual minimal electromagnetic interaction corresponds to the requirement that $\mathfrak{L}_\psi$ is independent of $\hat{G}_{\mu\nu}$; this would be the case[13] if in (2.21) the function $\mathfrak{L}_h$ is independent of $\hat{W}_{\mu\nu}{}^0$. In the following, unless explicitly stated, it is not necessary to assume the validity of the minimal electromagnetic interaction.

We consider the spectral representation

$$\langle \text{vac}|[W_\mu{}^0(x),W_\nu{}^0(0)]|\text{vac}\rangle = \int \sigma_W(M^2)\left[\delta_{\mu\nu}-M^{-2}\left(\frac{\partial^2}{\partial x_\mu \partial x_\nu}\right)\right]\Delta_W(x)dM^2, \quad (3.6)$$

where

$$\Delta_M(x)=-i(2\pi)^{-3}\int(\mathbf{q}^2+M^2)^{-1/2} \times\sin[(\mathbf{q}^2+M^2)^{1/2}t]\exp(i\mathbf{q}\cdot\mathbf{r})d^3q \quad (3.7)$$

and

$$\sigma_W(M^2)\geqq 0.$$

By using the equal-time commutator

$$[W_4{}^0(r,t),W_j{}^0(r',t)]=m_W{}^{-2}\nabla_j\delta^3(r-r'), \quad (3.8)$$

one has[14]

$$\int M^{-2}\sigma_W dM^2=m_W{}^{-2}. \quad (3.9)$$

By using (1.15), or (A41), and setting the vacuum expectation value of the second term on its right-hand side to zero,[15] one derives the additional sum rule

$$\int \sigma_W dM^2=(m_0/m_W)^2, \quad (3.10)$$

---

[13] That the term $-\frac{1}{4}(1+\eta)(\hat{W}_{\mu\nu})^2$ in (2.13), or the term $-\frac{1}{4}(1+\eta)(\hat{W}_{\mu\nu})^2$ in (A13), can be cast into the minimal interaction form follows from the general discussions given by T. D. Lee [Phys. Rev. **140**, B967 (1965)].

[14] K. Johnson, Nucl. Phys. **25**, 435 (1961).

[15] Relative to the first term on the right-hand side of (1.15), or (A41), the second term is formally smaller by a factor $G_F{}^2$. As already noted in Ref. 2, the vacuum expectation of the covariant tensor $\langle \text{vac}|\mathfrak{J}_\mu{}^{\text{wk}}\mathfrak{J}_\nu{}^{\text{wk}*}+\mathfrak{J}_\mu{}^{\text{wk}*}\mathfrak{J}_\nu{}^{\text{wk}}|\text{vac}\rangle$, where $\mathfrak{J}_\mu{}^*=\mathfrak{J}_\mu{}^\dagger$ for $\mu\neq 4$ and $-\mathfrak{J}_4{}^\dagger$ for $\mu=4$, or $\langle \text{vac}|\xi^a C^{alm}\xi^b C^{bln}J_\mu{}^m J_\nu{}^n|\text{vac}\rangle$ for the more general model discussed in Appendix A, has opposite signs for $\mu=\nu$ spacelike and timelike. Such a term can be explicitly shown to be zero if one adopts a suitable regularization procedure, such as the ξ-limiting process discussed by T. D. Lee and C. N. Yang [Phys. Rev. **128**, 885 (1962)] and T. D. Lee [*ibid.* **128**, 899 (1962)].

where m_0 is given by (2.42). To see the physical meaning of m_0, let us define the bare mass of W^0 to be the mass of W^0 determined by the spectrum of (3.1) in the absence of $\mathfrak{L}_\psi$. By setting $\mathfrak{L}_\psi=0$, it can be readily verified that the square of the bare W^0 mass is

$$m_0^2=(1+\eta)^{-1}m_W^2+(e_0/f_0)^2 m_W^2,$$

which is (2.42). Similarly, by examining the spectrum of the total Lagrangian (2.19) or (A36) in the limit of $f_0=0$, but keeping (e_0/f_0) finite, one can conclude

$$(1+\eta)^{-1/2}m_W=\text{bare mass of } W^\pm, \qquad (3.11)$$

which is the same as m_0 only if the $W^0-\gamma$ coupling (e_0/f_0) is zero.

We note that

$$m_0^2=\int \sigma_W dM^2 \Big/ \int M^{-2}\sigma_W dM^2 \qquad (3.12)$$

is independent of the wave-function normalization of $W_\mu{}^0$. For convenience, we will fix the normalization[16] of $W_\mu{}^0$ by requiring that the spectrum of the quadratic expression

$$-\tfrac{1}{4}\hat{G}_{\mu\nu}{}^2-\tfrac{1}{2}m_W{}^2(W_\mu{}^0)^2-\tfrac{1}{4}F_{\mu\nu}{}^2 \qquad (3.13)$$

is the same as the physical spectrum. This choice relates the parameter m_W to the physical mass of W^0. We have

$$\bar{m}_W=\text{physical mass of } W^0=m_W[1+(e_0/f_0)^2]^{1/2}. \quad (3.14)$$

This particular convention has the advantage that in a perturbation series expansion in which the total Lagrangian is

$$\mathfrak{L}=\mathfrak{L}_0+\mathfrak{L}_1, \qquad (3.15)$$

the unperturbed Lagrangian is

$$\mathfrak{L}_0=(3.13)+\mathfrak{L}_{\text{free}}(\psi), \qquad (3.16)$$

and the perturbation is

$$\mathfrak{L}_1=-\tfrac{1}{4}\eta\hat{G}_{\mu\nu}{}^2+\mathfrak{L}_\psi(\psi,D_\nu{}^0\psi,\hat{G}_{\mu\nu})-\mathfrak{L}_{\text{free}}(\psi), \quad (3.17)$$

where $\mathfrak{L}_{\text{free}}(\psi)$ denotes the free-field part of $\mathfrak{L}_\psi$ (with the observed mass of ψ), the unperturbed spectrum is already adjusted to be the same as the physical spectrum; in addition, as we shall see, the expression $-\frac{1}{4}\eta\hat{G}_{\mu\nu}{}^2$ is particularly convenient for the cancellation of infinities in the perturbation series expansion.

If we limit ourselves to the case that ψ consists of only spin-0 and spin-$\frac{1}{2}$ particles, then the Lagrangian (3.15) gives a renormalizable theory; the S matrix is finite to any given order of its perturbation series expansion of $\mathfrak{L}_1$. For example, in the propagator of W^0 the vacuum polarization term due to pair creations of ψ and $\bar{\psi}$ is infinite; because of the conservation law (3.5), it must be of the form

$$(q^2\delta_{\mu\nu}-q_\mu q_\nu)F(q^2), \qquad (3.18)$$

where $F(q^2)$ contains the usual logarithmically divergent integrals. These infinities can be cancelled by the counter term $-\frac{1}{4}\eta\hat{G}_{\mu\nu}{}^2$ in (3.17) by choosing

$$\eta=-\text{Re}F(q^2=-\bar{m}_W{}^2), \qquad (3.19)$$

where Re denotes the real part. The resulting W^0 propagator depends only on the difference

$$(q^2\delta_{\mu\nu}-q_\mu q_\nu)[F(q^2)-\text{Re}F(q^2=-\bar{m}_W{}^2)],$$

and is, therefore, finite. In (3.1), the coupling constant f_0 is already the *renormalized* (i.e., finite) coupling constant, and $W_\mu{}^0$ the *renormalized* field operator. The details of such perturbation series have been analyzed in the literature.[17] Furthermore, it has been shown[1] that the unrenormalized charge e_0 is finite. If one neglects $O(e_0{}^2)$ and $O(f_0{}^2)$, but keeps all orders in (e_0/f_0), then one has the approximate relation that m_W is the physical mass of $W^\pm$; i.e.,

$$\frac{\text{physical mass of } W^0}{\text{physical mass of } W^\pm}\cong[1+(e_0/f_0)^2]^{1/2}. \quad (3.20)$$

From (3.19), one sees that the perturbation series expansion of η has divergent terms. Yet, if one sums over the perturbation series, η must be finite. This follows directly from the two sum rules (3.9), (3.10), and $\sigma_W\geqq 0$. The integral $\int M^{-2}\sigma_W dM^2=m_W{}^{-2}$ must be finite. If the unrenormalized theory of W^0 is divergent, then $\int\sigma_W dM^2=\infty$, and one has $m_0=\infty$ but $\eta=-1$.

In the present theory, owing to the necessary inclusion of the spin-1 charged intermediate boson, the complete Lagrangian (3.1) cannot be renormalized by using the usual perturbation series expansion. Nevertheless, we will assume that the spectral representation (3.6) does *exist* for *finite* values of f_0 and m_W, and that the sum rules (3.9) and (3.10) hold. Consequently, η is finite. The failure of the power-series expansion is attributed to the presence of possible singularities, such as $f_0{}^2\ln f_0$, etc., in the theory.

IV. PROPAGATORS

The covariant propagator of $W_\mu{}^0$ is defined to be

$$\mathfrak{D}_{\mu\nu}{}^W(q)=\frac{-i}{(2\pi)^4}\int\frac{\sigma_W}{q^2+M^2-i\epsilon}\left[\delta_{\mu\nu}+\frac{q_\mu q_\nu}{M^2}\right]dM^2, \quad (4.1)$$

where σ_W is given by (3.6), ϵ is a positive infinitesimal quantity, q_μ denotes the 4-momentum $(\mathbf{q},iq_0)$, and $q^2=\mathbf{q}^2-q_0{}^2$. The Fourier transform of $\mathfrak{D}_{\mu\nu}{}^W(q)$ is related to the vacuum expectation value of the time-ordered

[16] See Ref. 1 for a detailed discussion of the various possible choices of the wave-function renormalization for a spin-1 meson field.

[17] T. D. Lee and Bruno Zumino, Nuovo Cimento (to be published).

product $T(W_\mu{}^0(x),W_\nu{}^0(0))$ by[18]

$$\langle \text{vac}|T(W_\mu{}^0(x),W_\nu{}^0(0))|\text{vac}\rangle = \int \mathfrak{D}_{\mu\nu}{}^W(q)\exp(iq_\lambda x_\lambda)d^3q dq_0 + i\int M^{-2}\sigma_W dM^2\delta_{\mu4}\delta_{\nu4}\delta^4(x), \quad (4.2)$$

where $\delta^4(x)=\delta^3(\mathbf{r})\delta(t)$.

Let

$$\hat{D}_{\mu\nu}{}^W(q)=\text{covariant propagator of } \hat{W}_\mu{}^0. \quad (4.3)$$

Similarly to (4.2), the Fourier transform of $\hat{D}_{\mu\nu}{}^W(q)$ differs from $\langle\text{vac}|T(\hat{W}_\mu{}^0(x),\hat{W}_\nu{}^0(0))|\text{vac}\rangle$ only in the appropriate contact term proportional to $\delta_{\mu4}\delta_{\nu4}\delta^4(x)$. It is convenient to express $\hat{D}_{\mu\nu}{}^W(q)$ as

$$\hat{D}_{\mu\nu}{}^W(q)=-i(2\pi)^{-4}[\hat{D}_W(q^2)\delta_{\mu\nu}+\hat{E}_W(q^2)q_\mu q_\nu]. \quad (4.4)$$

From the definition (3.3) of $\hat{W}_\mu{}^0$, one sees that $\hat{E}_W(q^2)\times q_\mu q_\nu$ depends on the gauge of $A_\mu(x)$. The gauge invariance of the theory ensures that this longitudinal part $\hat{E}_W(q^2)q_\mu q_\nu$ does not contribute to the evaluation of the transition matrix elements for any physical processes.

In (3.1), the electromagnetic field A_μ is coupled to ψ only through $f_0\hat{W}_\mu{}^0$. Thus, we can define an *effective photon propagator* $\mathfrak{D}_{\mu\nu(q)}{}^\gamma$:

$$e^2\mathfrak{D}_{\mu\nu}{}^\gamma(q)\equiv f_0{}^2\hat{D}_{\mu\nu}{}^W(q)-[f_0{}^2\hat{D}_{\mu\nu}{}^W(q)]_{e_0=0}, \quad (4.5)$$

where e is the renormalized charge and e_0 the unrenormalized charge. In this subtraction, all other parameters, such as η, f_0, and m_W, are kept *fixed*; i.e., $[\hat{D}_{\mu\nu}{}^W(q)]_{e_0=0}$ is the covariant $W_\mu{}^0$ propagator for the problem in which the Lagrangian is the same (3.1), except that $e_0=0$. Similarly to (4.4), we may write

$$\mathfrak{D}_{\mu\nu}{}^\gamma(q)=-i(2\pi)^{-4}[D_\gamma(q^2)\delta_{\mu\nu}+E_\gamma(q^2)q_\mu q_\nu]. \quad (4.6)$$

In (4.5), the renormalized charge e^2 is determined by the requirement that

$$D_\gamma(q^2)\to q^{-2} \quad \text{as} \quad q^2\to 0. \quad (4.7)$$

We will now show that as q^2 approaches infinity, $D_\gamma(q^2)$ goes to zero much faster than q^{-2}.

Theorem 1:

$$\hat{D}_W(q^2)=N^2\int\frac{\sigma_W}{q^2+M^2-i\epsilon}dM^2+N\left(\frac{e_0}{f_0}\right)^2\left(\frac{1}{q^2}\right) -\left(\frac{e_0}{f_0}\right)^2 m_W{}^2\left[2N+\left(\frac{e_0}{f_0}\right)^2 m_W{}^2\left(\frac{1}{q^2}+\frac{1}{m_0{}^2}\right)\right] \times\int\frac{\sigma_W}{q^2+M^2-i\epsilon}\left(\frac{1}{M^2}-\frac{1}{m_0{}^2}\right)dM^2, \quad (4.8)$$

where

$$N=[1+(e_0/f_0)^2(1+\eta)]^{-1}. \quad (4.9)$$

This theorem can be proved by following exactly the arguments used in Ref. 17. For completeness, the details are given in Appendix C.

Theorem 2:

$$\lim_{q^2\to\infty} q^2D_\gamma=0. \quad (4.10)$$

Proof: The sum rules (3.9) and (3.10) imply that

$$\int\sigma_W\left(\frac{1}{M^2}-\frac{1}{m_0{}^2}\right)dM^2=0. \quad (4.11)$$

Furthermore, by using (4.8), one finds that as $q^2\to\infty$

$$\hat{D}_W(q^2)\to(1+\eta)^{-1}q^{-2},$$

which is independent of e_0. Theorem 2 then follows on account of (4.5).

Theorem 3: At fixed values of $(e_0/f_0)^2$ and $(1+\eta)$, but setting $f_0=0$, one has

$$D_\gamma(q^2)=\frac{m_W{}^2m_0{}^2}{q^2[q^2+(1+\eta)^{-1}m_W{}^2][q^2+m_0{}^2](1+\eta)}, \quad (4.12)$$

where $m_0{}^2$ is given by (2.42).

Proof: In the limit $f_0=0$, the field $\hat{W}_\mu{}^0$ is not coupled to any ψ; i.e., in the Lagrangian (3.1), $\mathfrak{L}_\psi$ becomes independent of $\hat{W}_\mu{}^0$. [As $f_0\to 0$, the minimal interaction between $\hat{W}_\mu{}^0$ and ψ certainly vanishes. We assume here that the same also holds for the nonminimal interaction, if it exists.] Theorem 3 can be readily proved by considering the explicit dynamical solution for the Lagrangian (3.1) in the absence of $\mathfrak{L}_\psi$.

We note that in the limit $f_0=0$,

$$\sigma_W(M^2)=(m_0/m_W)^2\delta(M^2-m_0{}^2) \quad (4.13)$$

and

$$e^2=e_0{}^2[1+(e_0/f_0)^2(1+\eta)]^{-1}. \quad (4.14)$$

If one sets, in addition to $f_0=0$, also $\eta=0$, then m_0 reduces to

$$\bar{m}_W=m_W[1+(e_0/f_0)^2]^{1/2},$$

and D_γ becomes simply

$$D_\gamma(q^2)=\frac{m_W{}^2\bar{m}_W{}^2}{q^2(q^2+m_W{}^2)(q^2+\bar{m}_W{}^2)}. \quad (4.15)$$

Theorem 2 states that without any approximation, as $q^2\to\infty$, $D_\gamma(q^2)$ approaches zero faster than any constant times q^{-2}. Theorem 3 states that at fixed (e_0/f_0), but neglecting $O(f_0{}^2)$, $D_\gamma(q^2)\to(\text{constant})q^{-6}$ as $q^2\to\infty$. We recall that in the perturbation series expansion in $\mathfrak{L}_1$, which is given by (3.17), the counter term η, according to (3.19), can be regarded as $O(f_0{}^2)$. *Thus,*

[18] See, e.g., Appendix A of T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962); K. Johnson, Nucl. Phys. **25**, 431 (1961).

neglecting $O(f_0^2)$ *but keeping* (e_0/f_0) *fixed* [therefore, $O(e^2)$ is also neglected], *one finds that* $D_\gamma(q^2)$ *is given by (4.15).*

Theorem 4: The renormalized charge e is related to the unrenormalized charged e_0 by

$$e^2=e_0^2\left\{N-(e_0^2/f_0^2)m_W^4 \times\int\sigma_W\left[\frac{1}{M^2}-\frac{1}{m_0^2}\right]\frac{1}{M^2}dM^2\right\}, \quad (4.16)$$

where N is given by (4.9).

Proof: This theorem can be easily derived by using (4.8) and taking the limit $q^2f_0^2\hat{D}_W(q^2)$ at $q^2=0$.

From Theorem 4, it follows that

$$Ne_0^2=e^2y^{-1}[1(1-2y)^{1/2}], \quad (4.17)$$

where

$$y=2N^{-2}(e^2/f_0^2)\int m_W^4M^{-2}(M^{-2}-m_0^{-2})\sigma_W dM^2. \quad (4.18)$$

As $f_0\rightarrow 0$, but at fixed values of (e_0/f_0) and $(1+\eta)$, $y\rightarrow 0$ and, therefore, (4.17) reduces to (4.14). This limit requires us to choose in (4.17) the negative branch $-(1-2y)^{1/2}$, instead of the positive one. By using (4.9) and (4.17), one finds that

$$e_0^2=e^2\{y[1-(1-2y)^{1/2}]^{-1}-(e^2/f_0^2)(1+\eta)\}^{-1}. \quad (4.19)$$

According to (4.11), one has

$$\int M^{-2}(M^{-2}-m_0^{-2})\sigma_W dM^2 =\int(M^{-2}-m_0^{-2})^2\sigma_W dM^2, \quad (4.20)$$

which implies that

$$y\geqq 0. \quad (4.21)$$

The requirement that e_0^2 should be real and positive (which is also the necessary condition that there is no ghost state) leads to the following inequalities:

$$(f_0^2/e^2)>(1+\eta)y^{-1}[1-(1-2y)^{1/2}]\geqq(1+\eta) \quad (4.22)$$

and

$$y\leqq\tfrac{1}{2}, \quad (4.23)$$

i.e.,

$$(f_0^2/e^2)\geqq 4N^{-2}\int m_W^4(M^{-2}-m_0^{-2})^{-2}\sigma_W dM^2 \geqq 4\int m_W^4(M^{-2}-m_0^{-2})^{-2}\sigma_W dM^2. \quad (4.24)$$

From (4.19), one sees that

$$e_0^2\geqq e^2. \quad (4.25)$$

Furthermore, at finite values of e^2, f_0^2, and m_W^2, the unrenormalized charge e_0^2 is also finite.[1] As y varies from 0 to $\frac{1}{2}$, $N(e_0/e)^2$ varies from 1 to 2, and $(e_0/e)^2$ from

$$[1-(1+\eta)(e/f_0)^2]^{-1}\geqq 1 \quad (4.26)$$

to

$$2[1-2(1+\eta)(e/f_0)^2]^{-1}\geqq 2. \quad (4.27)$$

If the integral $\int\sigma_W\, dM^2$ diverges [*Note added in proof.* It turns out that the inequalities (4.28) and (4.30) are actually valid independently of whether the integral $\int\sigma_W\, dM^2$ is divergent or not] then $(1+\eta)=0$, $N=1$,

$$1\leqq(e_0^2/e^2)\leqq 2, \quad (4.28)$$

(4.16) reduces to

$$e^2=e_0^2\left[1-(e_0^2/f_0^2)m_W^4\int M^{-4}\sigma_W dM^2\right], \quad (4.29)$$

(4.24) becomes

$$(f_0^2/e^2)\geqq 4m_W^4\int M^{-4}\sigma_W dM^2, \quad (4.30)$$

and the inequality (4.22) imposes no limitation of f_0^2. On the other hand, in the mathematical limit of $f_0=0$, but at fixed (e_0/f_0) and $(1+\eta)$, one finds, by using (4.13), that the inequality (4.24) imposes no restriction on (f_0^2/e^2) and only the inequality (4.22) remains.

For the realistic case, it is important to know the values of the right-hand sides of (4.22) and (4.24), or (4.30) if $\int\sigma_W dM^2$ diverges. At present, it remains an open question whether f_0^2 can be much smaller than $e^2=(4\pi/137)$, or f_0^2 should be of the same order of magnitude as e^2. In the latter case, one sees that, by using (1.8), m_W would be comparable to $(e^2/4\sqrt{2}G_F)^{1/2}\cong 40m_N$.

As already noted in the Introduction, the present theory requires the electromagnetic interaction between all known particles to occur only through the chain (1.16). Thus, neglecting all higher-order weak-interaction effects, the $W^0-\gamma-W^0$ sequence gives rise to the effective photon propagator $D_\gamma(q^2)$ which is given by (4.15) and is $O(q^{-6})$ as $q^2\rightarrow\infty$. As a result, the electromagnetic shifts and the radiative corrections of weak decays of spin-0 and spin-$\frac{1}{2}$ particles can become finite. The details of these calculations will be given in a subsequent paper.

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APPENDIX A: GENERALIZATION OF THE MODEL

In the absence of leptons, the total electromagnetic current $\mathcal{J}_\mu{}^\gamma$ and the total weak-interaction current $\mathcal{J}_\mu{}^{\mathrm{wk}}$

become, respectively, their hadronic components $J_\mu{}^\gamma$ and $J_\mu{}^{\rm wk}$. It is useful to decompose $J_\mu{}^\gamma$ into an isovector part $(J_\mu{}^\gamma)_{T=1}$ and an isoscalar part $(J_\mu{}^\gamma)_{T=0}$,

$$J_\mu{}^\gamma=(J_\mu{}^\gamma)_{T=1}+(J_\mu{}^\gamma)_{T=0}, \tag{A1}$$

and to decompose $J_\mu{}^{\rm wk}$ into a sum of vector parts $(V_\mu{}^{\rm wk})_S$ and axial-vector parts $(A_\mu{}^{\rm wk})_S$,

$$J_\mu{}^{\rm wk}=\cos\theta[(V_\mu{}^{\rm wk})_{S=0}+(A_\mu{}^{\rm wk})_{S=0}] +\sin\theta[(V_\mu{}^{\rm wk})_{S=1}+(A_\mu{}^{\rm wk})_{S=1}], \tag{A2}$$

where θ is the Cabibbo angle and the subscript S denotes the change in strangeness. We assume that these current operators $(J_\mu{}^\gamma)_T$, $(V_\mu{}^{\rm wk})_S$, $(A_\mu{}^{\rm wk})_S$ and their Hermitian conjugates are members of a set of N operators

$$\{J_\mu{}^a\}=\{J_\mu{}^1,J_\mu{}^2,\cdots J_\mu{}^N\}, \tag{A3}$$

called the set of *observed hadron current operators*. In this Appendix, these current operators are assumed, in the absence of leptons, to satisfy the *complete* field algebra related to a symmetry group $\mathcal{G}$, where $\mathcal{G}$ can be either $SU_3\times SU_3$, or some other groups, and N is the total number of generators of $\mathcal{G}$.

For clarity, we first discuss the system without leptons. The field current identities (1.9) and (1.10) imply that there should exist a corresponding set of N intermediate bosons $\{W_\mu{}^a\}$. These intermediate boson fields $W_\mu{}^a$ are related to the observed hadron current operator $J_\mu{}^a$ by

$$J_\mu{}^a(x)=-\tfrac{1}{2}(m_W{}^2/f)W_\mu{}^a(x), \tag{A4}$$

where f is given by (1.8), $f^2/m_W{}^2=G_F/\sqrt{2}$.

For example, if $\mathcal{G}=SU_3\times SU_3$, the set $\{W_\mu{}^a\}$ consists of eight vector intermediate bosons $W_\mu{}^{V,\lambda}$ and eight axial-vector intermediate bosons $W_\mu{}^{A,\lambda}$, where $\lambda=1, 2, \cdots, 8$. In the usual notations of SU_3 indices the charged intermediate boson field W_μ in (1.9) and the neutral intermediate boson field $W_\mu{}^0$ in (1.10) are related to these $W_\mu{}^{V,\lambda}$ and $W_\mu{}^{A,\lambda}$ by

$$W_\mu=\tfrac{1}{2}[(W_\mu{}^{V,1}-iW_\mu{}^{V,2})\cos\theta+(W_\mu{}^{V,4}-iW_\mu{}^{V,5})\sin\theta +(W_\mu{}^{A,1}-iW_\mu{}^{A,2})\cos\theta +(W_\mu{}^{A,4}-iW_\mu{}^{A,5})\sin\theta] \tag{A5}$$

and

$$W_\mu{}^0=(\tfrac{3}{4})^{1/2}[W_\mu{}^{V,3}+3^{-1/2}W_\mu{}^{V,8}]. \tag{A6}$$

The field-current identity (A4) becomes

$$(V_\mu{}^{\rm wk})_{S=0}=-\tfrac{1}{2}(m_W{}^2/f)(W_\mu{}^{V,1}-iW_\mu{}^{V,2}),$$
$$(A_\mu{}^{\rm wk})_{S=0}=-\tfrac{1}{2}(m_W{}^2/f)(W_\mu{}^{A,1}-iW_\mu{}^{A,2}), \text{ etc.}$$

The infinitesimal transformations of the symmetry group $\mathcal{G}$ can be represented by

$$W_\mu{}^a\rightarrow W_\mu{}^a+C^{abc}(\delta\theta)^bW_\mu{}^c, \tag{A7}$$

$$\psi_h\rightarrow\psi_h+T^a(\delta\theta)^a\psi_h, \tag{A8}$$

$$A_\mu\rightarrow A_\mu, \tag{A9}$$

where all superscripts a, b, and c, vary from 1 to N, $\delta\theta^a$ is a set of infinitesimal numbers, C^{abc} is the totally antisymmetric structure constant of the symmetry group $\mathcal{G}$, $-iT^a$ is the matrix representation of its Hermitian generators on ψ_h which satisfies

$$[T^a,T^b]=C^{abc}T^c,$$

A_μ is the electromagnetic field, and ψ_h represents all hadron fields which can be of either half-integer or integer spin.

The usual electromagnetic gauge transformation of the first kind

$$\exp(iQ\alpha)$$

is assumed to be a member of $\mathcal{G}$, where α is a *constant* denoting the angle of rotation and Q is the total charge operator in units of e. Under the transformation $\exp(iQ\alpha)$, the electromagnetic field A_μ is invariant, but

$$W_\mu{}^a\rightarrow W_\mu{}^a+C^{abc}(\xi^b\alpha)W_\mu{}^c \tag{A10}$$

and

$$\psi_h\rightarrow\psi_h+T^a(\xi^a\alpha)\psi_h, \tag{A11}$$

where $\xi^1\cdots\xi^N$ are constants depending on $\mathcal{G}$.

The total Lagrangian $\mathfrak{L}$ for a system without leptons can be written as

$$\mathfrak{L}=\mathfrak{L}_{\rm field}+\mathfrak{L}_h, \tag{A12}$$

where, similarly to (2.13) and (2.21),

$$\mathfrak{L}_{\rm field}=-\tfrac{1}{2}m_W{}^2(W_\mu{}^a)^2-\tfrac{1}{4}(1+\eta)(\hat{W}_{\mu\nu}{}^a)^2-\tfrac{1}{4}F_{\mu\nu}{}^2, \tag{A13}$$

$$\mathfrak{L}_h=\mathfrak{L}_h(\psi_h,D_\nu\psi_h,\hat{W}_{\mu\nu}{}^a), \tag{A14}$$

$$\hat{W}_\mu{}^a=W_\mu{}^a+\tfrac{1}{2}(e_0/f)\xi^aA_\mu, \tag{A15}$$

$$\hat{W}_{\mu\nu}{}^a=\frac{\partial}{\partial x_\mu}\hat{W}_\nu{}^a-\frac{\partial}{\partial x_\nu}\hat{W}_\mu{}^a+2fC^{abc}\hat{W}_\nu{}^b\hat{W}_\nu{}^c, \tag{A16}$$

and

$$D_\nu\psi_h=\left(\frac{\partial}{\partial x_\nu}+2fT^a\hat{W}_\nu{}^a\right)\psi_h. \tag{A17}$$

The function $\mathfrak{L}_h$ can be an arbitrary function of ψ_h, $D_\nu\psi_h$, and $\hat{W}_{\mu\nu}{}^a$. Furthermore, except for the electromagnetic gauge transformation, $\mathfrak{L}_h$ is *not* required to be invariant under $\mathcal{G}$. The factors 2 and $\frac{1}{2}$ in these expressions are due to the corresponding factor $\frac{1}{2}$ in (A4) and (A5), so that f satisfies (1.8).

From (A12), it follows that the Maxwell equation becomes

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu}=e_0J_\nu{}^\gamma, \tag{A18}$$

where

$$J_\nu{}^\gamma=\xi^aJ_\nu{}^a=-\tfrac{1}{2}(m_W{}^2/f)\xi^aW_\nu{}^a. \tag{A19}$$

In the absence of leptons, (1.10) becomes

$$J_\nu{}^\gamma=-(m_W{}^2/f_0)W_\nu{}^0. \tag{A20}$$

Thus, one identifies

$$W_\nu{}^0(x)=(\xi^a/\xi)W_\nu{}^a(x) \tag{A21}$$

and

$$f_0=(2f/\xi), \tag{A22}$$

where

$$\xi=(\xi^a\xi^a)^{1/2}. \tag{A23}$$

For the case of $\mathcal{G}=SU_3\times SU_3$, by using (A6), one sees that

$$\xi=2/\sqrt{3}, \quad f_0=\sqrt{3}f \tag{A24}$$

and, therefore,

$$(f_0{}^2/m_W{}^2)=3G_F/\sqrt{2}. \tag{A25}$$

Similarly to (2.27), the intermediate boson field $W_\nu{}^a$ satisfies

$$(1+\eta)\frac{\partial}{\partial x_\mu}\hat{W}_{\mu\nu}{}^a-m_W{}^2W_\nu{}^a =2f(1+\eta)C^{abc}\hat{W}_{\mu\nu}{}^b\hat{W}_\mu{}^c+2fs_\nu{}^a(h), \tag{A26}$$

where

$$s_\nu{}^a(h)=-C^{abc}[(\partial\mathcal{L}_h/\partial\hat{W}_{\mu\nu}{}^b)-(\partial\mathcal{L}_h/\partial\hat{W}_{\nu\mu}{}^b)]\hat{W}_\mu{}^c +(\partial/\partial x_\mu)[(\partial\mathcal{L}_h/\partial\hat{W}_{\mu\nu}{}^a)-(\partial\mathcal{L}_h/\partial\hat{W}_{\nu\mu}{}^a)] -\sum_h(\partial\mathcal{L}_h/\partial D_\nu\psi_h)T^a\psi, \tag{A27}$$

where the sum extends over all hadron fields.

Just as in Sec. II, for convenience, we use the Coulomb gauge and choose ψ_h, $\hat{W}_j{}^a$ and the transverse electromagnetic field $A_j{}^{\text{tr}}$ as the generalized coordinates; their conjugate momenta are, respectively,

$$P_h=-i(\partial\mathcal{L}_h/\partial D_4\psi_h), \tag{A28}$$

$$(P_W{}^a)_j=i[(1+\eta)\hat{W}_{4j}{}^a-(\partial\mathcal{L}_h/\partial\hat{W}_{4j}{}^a)+(\partial\mathcal{L}_h/\partial\hat{W}_{j4}{}^a)], \tag{A29}$$

and

$$\Pi_j{}^{\text{tr}}=-E_j{}^{\text{tr}}=(\partial A_j{}^{\text{tr}}/\partial t). \tag{A30}$$

Thus, for $\nu=4$, (A26) becomes

$$W_4{}^a=m_W{}^{-2}[i\nabla_j(P_W{}^a)_j-2ifC^{abc}(P_W{}^b)_j\hat{W}_j{}^c +2if\sum_h P_hT^a\psi_h]. \tag{A31}$$

By following the usual derivation of field algebra[2,3] and by using (A4), one finds that independently of the detailed structure of $\mathcal{L}_h$, the hadron currents $J_\mu{}^a$ satisfy

$$[J_i{}^a(r,t),J_j{}^b(r',t)]=0, \tag{A32}$$

$$[J_4{}^a(r,t),J_4{}^b(r',t)]=C^{abc}J_4{}^c(r,t)\delta^3(r-r'), \tag{A33}$$

and

$$[J_4{}^a(r,t),J_i{}^b(r',t)]=C^{abc}J_i{}^c(r,t)\delta^3(r-r')+(2\sqrt{2}G_F)^{-1} \times\{\delta^{ab}\nabla_i-eC^{abc}\xi^c[A_i{}^{\text{tr}}(r,t)]_{\text{ren}}\}\delta^3(r-r'), \tag{A34}$$

where e is the renormalized electric charge and $(A_i{}^{\text{tr}})_{\text{ren}}$ the renormalized field, related to the unrenormalized quantities e_0 and $A_i{}^{\text{tr}}$ by

$$e(A_i{}^{\text{tr}})_{\text{ren}}=e_0A_i{}^{\text{tr}}. \tag{A35}$$

Equations (A32)–(A34) are valid to all orders in e and G_F, provided that there is no lepton. We note that the coefficient of the gauge-covariant derivative $\{\delta^{ab}\nabla_i-eC^{abc}\xi^c[A_i{}^{\text{tr}}(r,t)]_{\text{ren}}\}$ in (A29) is

$$(2\sqrt{2}G_F)^{-1}\cong 3.5\times10^4m_N{}^2,$$

instead of $(m_\rho/g_\rho)^2\cong2\times10^{-2}m_N{}^2$ given by Refs. 2 and 3.

The presence of the leptons requires the total Lagrangian (A12) to be replaced by

$$\mathcal{L}=(\text{A12})+\mathcal{L}_l \tag{A36}$$

and

$$\mathcal{L}_l=-\sum l(x)^\dagger\gamma_4\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-if_0\hat{W}_\mu{}^0\right)+(m_l+\delta m_l)\right]l(x) -\tfrac{1}{2}\sum \nu_l(x)^\dagger\gamma_4\gamma_\mu\frac{\partial}{\partial x_\mu}(1+\gamma_5)\nu_l(x) -f\sum[i\nu_l(x)^\dagger\gamma_4\gamma_\mu(1+\gamma_5)l(x)W_\mu+\text{H.c.}], \tag{A37}$$

where W_μ is given by (A5), and, similarly to (A21), $\hat{W}_\mu{}^0$ is related to $\hat{W}_\mu{}^a$ by

$$\hat{W}_\mu{}^0(x)=(\xi^a/\xi)\hat{W}_\mu{}^a(x). \tag{A38}$$

From (A36), it follows that the Maxwell equation is the same as (2.26),

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu}=-(e_0m_W{}^2/f_0)W_\nu{}^0,$$

and that the total electromagnetic current operator $\mathcal{J}_\nu{}^\gamma$ satisfies an almost identical set of equal-time commutation relations as in the case of the SU_2 model of $W^\pm$ and W^0 discussed in Sec. II. Equations (1.13) and (1.14) remain valid. Equation (1.15) has to be slightly modified, since f_0 is now given by (A22). We have

$$[\mathcal{J}_i{}^\gamma(r,t),\mathcal{J}_j{}^\gamma(r',t)]=[\mathcal{J}_4{}^\gamma(r,t),\mathcal{J}_4{}^\gamma(r',t)]=0, \tag{A39}$$

$$[\mathcal{J}_4{}^\gamma(r,t),\mathcal{J}_j{}^\gamma(r',t)]=(2\sqrt{2}G_F)^{-1}\xi^2\nabla_j\delta^3(r-r'), \tag{A40}$$

and

$$[(\partial/\partial t)J_j{}^\gamma(r,t)-i\nabla_jJ_4{}^\gamma(r,t),J_k{}^\gamma(r',t)] =-i(2\sqrt{2}G_F)^{-1}\xi^2m_0{}^2\delta_{jk}\delta^3(r-r')-i(2\sqrt{2}G_F) \times\xi^aC^{alm}\xi^bC^{bln}J_j{}^m(r,t)J_k{}^n(r,t)\delta^3(r-r'), \tag{A41}$$

provided that (2.41) is satisfied. These commutation relations of $\mathcal{J}_\mu{}^\gamma$ are valid, with the inclusion of leptons, to all orders in e^2 and G_F. It is important to note that the results derived in Secs. III and IV also apply to the present general case.

In this generalization, one finds that without leptons the observed hadron currents can satisfy the complete field algebra of any symmetry group $\mathcal{G}$. According to (A14), the interaction between the hadrons and the intermediate bosons can occur both through $D_\nu\psi_h$ and the possible dependence of $\mathfrak{L}_h$ on $\hat{W}_{\mu\nu}{}^a$. We note that if $\mathcal{G}$ is the chiral $SU_3\times SU_3$ group, then the $D_\nu\psi_h$ term by itself preserves the usual isospin symmetry. The violation of isospin symmetry for the nonleptonic weak processes must, then, be either due to the $\hat{W}_{\mu\nu}{}^a$-dependent term in $\mathfrak{L}_h$, or due to some other terms in $\mathfrak{L}_h$ not directly involving the intermediate boson fields. Similar conclusions also apply to violations of other symmetries, such as parity conservation, particle-antiparticle conjugation, etc.

APPENDIX B: VECTOR AND AXIAL-VECTOR DOMINANCE IN STRONG INTERACTION

We continue the discussions given in Appendix A and assume that the hadron fields ψ_h consist of a set of N strongly interacting spin-1 meson fields $\phi_\mu{}^1,\cdots,\phi_\mu{}^N$ and some other fields, denoted by ψ_h', which can be of either integer of half-integer spin. Under the infinitesimal transformations of $\mathcal{G}$, the fields $\phi_\mu{}^a$ transform in the same way as $W_\mu{}^a$; i.e., similarly to (A7),

$$\phi_\mu{}^a\rightarrow\phi_\mu{}^a+C^{abc}(\delta\theta)^b\phi_\nu{}^c. \tag{B1}$$

For the simple SU_2-triplet model discussed in Sec. II, there are only three relevant strongly interacting fields $\phi_\mu{}^1$, $\phi_\mu{}^2$, and $\phi_\mu{}^0$, each of which is an appropriate linear sum of the known spin-1 fields ρ, ϕ, ω, A_1, $K_A{}^*$, etc. The corresponding transformations (B1) for these three fields are determined by (2.22) and $(1+iQ\delta\theta^0)$, where Q is the total charge operator and $\delta\theta^0$ is the corresponding infinitesimal angle of transformation.

In either the general case, or the simple SU_2-triplet model, we formulate the idea of vector and axial-vector dominance by $\phi_\mu{}^a$ to mean simply that the entire hadronic source function which generates the intermediate boson field is proportional to $\phi_\mu{}^a$. Such a relation can be derived by assuming the function $\mathfrak{L}_h$ in the Lagrangian (A14) to be of the form

$$\mathfrak{L}_h=-\tfrac{1}{2}m_\phi{}^2(\phi_\mu{}^a)^2+\mathfrak{L}_h'(\psi_h',D_\nu'\psi_h',\phi_{\mu\nu}{}^a)\,, \tag{B2}$$

where

$$D_\nu'\psi_h'=\left(\frac{\partial}{\partial x_\nu}+gT^a\hat{\phi}_\nu{}^a\right)\psi_h'\,, \tag{B3}$$

$$\hat{\phi}_\nu{}^a=\phi_\nu{}^a+(2f/g)\hat{W}_\nu{}^a\,, \tag{B4}$$

$$\hat{\phi}_{\mu\nu}{}^a=\frac{\partial}{\partial x_\mu}\hat{\phi}_\nu{}^a-\frac{\partial}{\partial x_\nu}\hat{\phi}_\mu{}^a+gC^{abc}\hat{\phi}_\mu{}^b\hat{\phi}_\nu{}^c\,, \tag{B5}$$

and $\mathfrak{L}_h'$ can be an arbitrary function of ψ_h', $D_\nu'\psi_h'$, and $\phi_{\mu\nu}{}^a$. It is important to note that we may choose, for convenience,

$$m_\phi=m_\rho=\text{observed neutral }\rho\text{-meson mass}. \tag{B6}$$

In this choice, the above field operator $\phi_\mu{}^a$ becomes already the renormalized field[19] and g the renormalized ρ coupling constant, where

$$(g^2/4\pi)\cong 2.4\,. \tag{B7}$$

From (A17), it follows that

$$D_\nu'\psi_h'=D_\nu\psi_h'+gT^a\phi_\nu{}^a\psi_h'\,. \tag{B8}$$

Furthermore, by setting ψ_h to be the column matrix

$$\phi_\mu=\begin{bmatrix}\phi_\mu{}^1\\ \vdots\\ \phi_\mu{}^N\end{bmatrix}, \tag{B9}$$

one obtains

$$(D_\nu\phi_\mu)^a=\frac{\partial}{\partial x_\nu}\phi_\mu{}^a+2fC^{abc}\hat{W}_\nu{}^b\phi_\mu{}^c\,, \tag{B10}$$

and therefore

$$\hat{\phi}_{\mu\nu}{}^a=(2f/g)\hat{W}_{\mu\nu}{}^a+(D_\mu\phi_\nu)^a-(D_\nu\phi_\mu)^a +gC^{abc}\phi_\mu{}^b\phi_\nu{}^c. \tag{B11}$$

Thus, the expression (B2) satisfies the condition required by (A14) that $\mathfrak{L}_h$ is a function only of ψ_h, $D_\nu\psi_h$, and $\hat{W}_{\mu\nu}{}^a$, where ψ_h denotes both ϕ_μ and ψ_h'.

By using (A12) and (B2), one can easily verify that (A26) can now be written in a simpler form

$$(1+\eta)\frac{\partial}{\partial x_\mu}\hat{W}_{\mu\nu}{}^a-m_W{}^2W_\nu{}^a=2f(1+\eta)C^{abc}\hat{W}_{\mu\nu}{}^b\hat{W}_\mu{}^c -(2f/g)m_\phi{}^2\phi_\nu{}^a, \tag{B12}$$

and $\phi_\nu{}^a$ satisfies

$$\frac{\partial}{\partial x_\mu}\left(-\frac{\partial\mathfrak{L}_h'}{\partial\hat{\phi}_{\mu\nu}{}^a}+\frac{\partial\mathfrak{L}_h'}{\partial\hat{\phi}_{\nu\mu}{}^a}\right)-m_\phi{}^2\phi_\nu{}^a=gS_\nu{}^a, \tag{B13}$$

where

$$S_\nu{}^a=C^{abc}\left(-\frac{\partial\mathfrak{L}_h'}{\partial\hat{\phi}_{\mu\nu}{}^b}+\frac{\partial\mathfrak{L}_h'}{\partial\hat{\phi}_{\nu\mu}{}^b}\right)\hat{\phi}_\mu{}^c-\sum{}'\frac{\partial\mathfrak{L}_h'}{\partial D_\nu'\psi_h'}T^a\psi_h'\,,$$

where the sum $\sum'$ extends over all hadron fields, except ϕ_μ. Identical results can also be obtained for the simple SU_2 model discussed in Sec. II. According to (B12), the entire hadronic source function (A27), or (2.30), that generates the intermediate boson field $W_\nu{}^a$, is simply

$$s_\nu{}^a(h)=-(m_\phi{}^2/g)\phi_\nu{}^a. \tag{B14}$$

It is convenient to choose $A_j{}^{\text{tr}}$, $\hat{W}_j{}^a$, $\phi_j{}^a$, and ψ_h' as generalized coordinates. Their conjugate momenta are, respectively, $\Pi_j{}^{\text{tr}}=-E_j{}^{\text{tr}}$,

$$(P_W{}^a)_j=i(1+\eta)\hat{W}_{4j}{}^a-i(2f/g) \times[(\partial\mathfrak{L}_h'/\partial\hat{\phi}_{4j}{}^a)-(\partial\mathfrak{L}_h'/\partial\hat{\phi}_{j4}{}^a)], \tag{B15}$$

$$(P_\phi{}^a)_j=-i[(\partial\mathfrak{L}_h'/\partial\hat{\phi}_{4j}{}^a)-(\partial\mathfrak{L}_h'/\partial\phi_{j4}{}^a)], \tag{B16}$$

[19] This corresponds to the choice $\phi_\mu{}^a=(m_\rho{}^0/m_\rho)(\phi_\mu{}^a)^0$, where $(\phi_\mu{}^a)^0$ is the unrenormalized field and $m_\rho{}^0$ the unrenormalized mass. See Ref. 1 for further details.

and

$$P_h' = -i(\partial \mathcal{L}_h'/\partial D_4'\psi_h'). \qquad \text{(B17)}$$

By combining (B15) and (B16), one finds

$$i(1+\eta)\hat{W}_{4j}^a = (P_W{}^a)_j - (2f/g)(P_\phi{}^a)_j.$$

Furthermore, since $(P_\phi{}^a)_i$ commutes with $W_j{}^a = \hat{W}_j{}^a - \frac{1}{2}(e_0/f)A_j{}^{\text{tr}}$ at equal time, condition (2.41) is fulfilled. Thus, the observed total electromagnetic current $\mathcal{J}_\mu{}^\gamma = -(m_W{}^2/f_0)W_\mu{}^0$ satisfies (1.15) or (A41), as well as all other equations of field algebra, such as (1.13) and (1.14), or (A39) and (A40).

In the following we will briefly discuss several consequences of the present formulation of vector and axial-vector dominance:

(1) The matrix elements of $W_\mu{}^a$, A_μ, $W_\mu{}^a W_\nu{}^b$, etc., between any states $|A\rangle$ and $|B\rangle$ which consist of only hadrons (and leptons) but *without* photons or intermediate bosons can be classified according to their minimum power dependence on e and f. For example,

$$\langle B|W_\mu{}^a|A\rangle = O(f),$$
$$\langle B|A_\mu|A\rangle = O(e), \qquad \text{(B18)}$$
$$\langle B|W_\mu{}^a W_\nu{}^b|A\rangle - \langle \text{vac}|W_\mu{}^a W_\nu{}^b|\text{vac}\rangle = O(f^2),$$

etc. By using (B12), setting

$$e_0 = 0, \quad \eta = 0, \text{ and neglecting } O(f^2), \qquad \text{(B19)}$$

one obtains

$$\langle B|W_\mu{}^a|A\rangle = 2(f/g)\frac{m_\phi{}^2}{q^2+m_W{}^2} \times\left[\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m_W{}^2}\right]\langle B|\phi_\nu{}^a|A\rangle, \qquad \text{(B20)}$$

where q_μ is the 4-momentum difference between the states A and B, and $q^2 = (q_\mu)^2$. To the same approximation, the corresponding matrix element of the observed hadron current $J_\nu{}^a = -\frac{1}{2}(m_W{}^2/f)W_\nu{}^a$ is, then, given by

$$\langle B|J_\mu{}^a|A\rangle = -g^{-1}\frac{m_\phi{}^2 m_W{}^2}{q^2+m_W{}^2} \times\left[\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m_W{}^2}\right]\langle B|\phi_\nu{}^a|A\rangle. \qquad \text{(B21)}$$

Thus, the form factor relation (2.43) follows; i.e.,

$$(F_{AB}{}^\gamma)_{T=1} = \left(\frac{m_W{}^2}{q^2+m_W{}^2}\right)\left(\frac{m_\rho{}^2}{q^2+m_\rho{}^2}\right)F_{AB}{}^\rho,$$

where

$$(F_{AB}{}^\gamma)_{T=1} = \langle B|(J_\mu{}^\gamma)_{I=1}|A\rangle,$$
$$F_{AB}{}^\rho = -g^{-1}(q^2+m_\rho{}^2)\langle B|\rho_\mu{}^0|A\rangle, \qquad \text{(B22)}$$

$\rho_\mu{}^0$ denotes the neutral ρ-meson field, and it is a member of the set $\{\phi_\mu{}^a\}$.

(2) By using (B13)–(B17), one finds that

$$\phi_4{}^a = m_\phi{}^{-2}[i\nabla_j(P_\phi{}^a)_j - igC^{abc}(P_\phi{}^b)_j\hat{\phi}_j{}^c + ig\textstyle\sum{}' P_h' T^a \psi_h']. \qquad \text{(B23)}$$

The fields $\phi_\mu{}^a$, therefore, satisfy[2,3]

$$[\phi_i{}^a(r,t), \phi_j{}^b(r',t)] = 0, \qquad \text{(B24)}$$

$$[\phi_4{}^a(r,t), \phi_4{}^b(r',t)] = -(g/m_\rho{}^2) \times C^{abc}\phi_4{}^c(r,t)\delta^3(r-r'), \qquad \text{(B25)}$$

and

$$[\phi_4{}^a(r,t), \phi_j{}^b(r',t)] = -(g/m_\rho{}^2)C^{abc}\hat{\phi}_j{}^c(r,t)\delta^3(r-r') + m_\rho{}^{-2}\delta^{ab}\nabla_j\delta^3(r-r'), \qquad \text{(B26)}$$

where, on account of (B6), m_ϕ is set to be m_ρ. All these commutation relations hold to all orders in e and G_F. [See Appendix A of Ref. 3 for other commutation relations.]

(3) Let us consider the spectral representation of the vacuum expectations of the commutators between the observed hadron currents and those between $\phi_\mu{}^a$. By choosing all space components $J_j{}^a$ and $\phi_j{}^a$ to be Hermitian, we may write

$$\langle \text{vac}|[J_\mu{}^a(x), J_\nu{}^b(0)]|\text{vac}\rangle = \int_0^\infty \left[\sigma_J{}^{ab}(M^2)\delta_{\mu\nu} - \sigma_J{}'^{ab}(M^2)\left(\frac{\partial^2}{\partial x_\mu \partial x_\nu}\right)\right]\Delta_M(x)dM^2, \qquad \text{(B27)}$$

$$\langle \text{vac}|[\phi_\mu{}^a(x), \phi_\nu{}^b(0)]|\text{vac}\rangle = \int_0^\infty \left[\sigma_\phi{}^{ab}(M^2)\delta_{\mu\nu} - \sigma'_\phi{}^{ab}(M^2)\left(\frac{\partial^2}{\partial x_\mu \partial x_\nu}\right)\right]\Delta_M(x)dM^2, \qquad \text{(B28)}$$

where $\Delta_M(x)$ is given by (3.7). From CPT invariance, the $(N\times N)$ matrices $\sigma_J = (\sigma_J{}^{ab})$, $\sigma_J' = (\sigma_J'{}^{ab})$, $\sigma_\phi = (\sigma_\phi{}^{ab})$, and $\sigma_\phi' = (\sigma_\phi'{}^{ab})$ are all real and symmetric. Furthermore, $(\sigma_J' - M^{-2}\sigma_J)$ and $(\sigma_\phi' - M^{-2}\sigma_\phi)$ are positive, and σ_J and σ_ϕ are positive definite.

The commutation relations (A34) and (B26) imply, respectively, that

$$\int_0^\infty \sigma_J{}'^{ab}(M^2)dM^2 = (2\sqrt{2}G_F)^{-1}\delta^{ab} \qquad \text{(B29)}$$

and

$$\int_0^\infty \sigma_\phi{}'^{ab}(M^2)dM^2 = m_\rho{}^{-2}\delta^{ab}. \qquad \text{(B30)}$$

For $M^2 < m_W{}^2$, the relevant states cannot contain any intermediate bosons. By using (B21), one finds that, in the approximation (B19) and for $M^2 < m_W{}^2$,

$$\sigma_J \cong (m_\rho{}^2/g)^2[1-(M^2/m_W{}^2)]^{-2}\sigma_\phi \qquad \text{(B31)}$$

and

$$\sigma_J' \cong (m_\rho^2/g)^2\{\sigma_\phi' + m_W^{-2}[1-(M^2/m_W^2)]^{-2} \times [2-(M^2/m_W^2)]\sigma_\phi\}. \quad \text{(B32)}$$

In the integral (B30), the main contribution should be due to hadron states. Thus, one may approximate this convergent integral by

$$\int_0^\infty \sigma_\phi' dM^2 \cong \int_0^{M_{\rm st}} \sigma_\phi' dM^2, \quad \text{(B33)}$$

where $M_{\rm st}$ is a characteristic mass determined by the strong interaction. In the case where $M_{\rm st}$ turns out to be much less than m_W,

$$m_W \gg M_{\rm st} > m_N, \quad \text{(B34)}$$

then the matrix σ_J' satisfies, besides the exact equation (B29), also the approximate equation,

$$\int_0^{M_{\rm st}} \sigma_J' dM^2 \cong (m_\rho/g)^2. \quad \text{(B35)}$$

APPENDIX C: PROOF OF THEOREM 1

To prove Theorem 1 in Sec. IV, we introduce the transformation

$$A_\mu' = N^{-1/2} A_\mu + N^{1/2}(1+\eta)(e_0/f_0) W_\mu^0, \quad \text{(C1)}$$

where

$$N = [1+(1+\eta)(e_0/f_0)^2]^{-1}. \quad \text{(C2)}$$

The Lagrangian (3.1) becomes

$$\mathcal{L} = -\tfrac{1}{4}(1+\eta) N G_{\mu\nu}^2 - \tfrac{1}{2} m_W^2 (W_\mu^0)^2 - \tfrac{1}{4} F_{\mu\nu}'^2 + \mathcal{L}_\psi(\psi, D_\nu^0\psi, \hat{G}_{\mu\nu}), \quad \text{(C3)}$$

where

$$G_{\mu\nu} = \frac{\partial}{\partial x_\mu} W_\nu^0 - \frac{\partial}{\partial x_\nu} W_\mu^0, \quad \text{(C4)}$$

$$\hat{G}_{\mu\nu} = \frac{\partial}{\partial x_\mu} \hat{W}_\nu^0 - \frac{\partial}{\partial x_\nu} \hat{W}_\mu^0, \quad \text{(C5)}$$

$$\hat{W}_\mu^0 = N W_\mu^0 + (e_0/f_0) N^{1/2} A_\mu', \quad \text{(C6)}$$

$$D_\nu^0 \psi = \left[\frac{\partial}{\partial x_\mu} + i(f_0' W_\mu^0 + e_0' A_\mu') Q\right]\psi, \quad \text{(C7)}$$

$$F_{\mu\nu}' = \frac{\partial}{\partial x_\mu} A_\nu' - \frac{\partial}{\partial x_\nu} A_\mu', \quad \text{(C8)}$$

$$f_0' = N f_0, \quad \text{and} \quad e_0' = N^{1/2} e_0. \quad \text{(C9)}$$

Let $D_{\mu\nu}^{WW}(q)$, $D_{\mu\nu}^{AA}(q)$, $D_{\mu\nu}^{AW}(q)$, and $D_{\mu\nu}^{WA}(q)$ be, respectively, the sum of all Feynman-propagator graphs in which the (initial, final) fields are (W_μ^0, W_ν^0), (A_μ', A_ν'), (A_μ', W_ν^0), and (W_μ^0, A_ν'). These are, by definition, covariant functions and $D_{\mu\nu}^{WW}(q)$ is the same $\mathfrak{D}_{\mu\nu}^W(q)$ given by (4.1). We have

$$D_{\mu\nu}^{WW}(q) = \mathfrak{D}_{\mu\nu}^W(q) = \frac{-i}{(2\pi)^4} \int \frac{\sigma_W}{q^2+M^2-i\epsilon} \times \left[\delta_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}\right] dM^2. \quad \text{(C10)}$$

By using the theorem proved in Ref. 17, we can readily establish the following relations:

$$D_{\mu\nu}^{AA} = \frac{-i}{(2\pi)^4}\left(\frac{1}{q^2}\right)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left[1 - \left(\frac{e_0'}{f_0'}\right)^2 m_W^4 \times \int \frac{\sigma_W}{q^2+M^2-i\epsilon}\left(\frac{1}{M^2} - \frac{1}{m_0^2}\right)\left(1 + \frac{q^2}{m_0^2}\right) dM^2\right] \quad \text{(C11)}$$

and

$$D_{\mu\nu}^{AW} = D_{\mu\nu}^{WA} = \frac{-i}{(2\pi)^4}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left(\frac{e_0'}{f_0'}\right) m_W^2 \times \int \frac{\sigma_W}{q^2+M^2-i\epsilon}\left(\frac{1}{m_0^2} - \frac{1}{M^2}\right) dM^2, \quad \text{(C12)}$$

where, apart from some trivial substitutions, (C11) is the same as Eq. (26) in Ref. 17. For convenience, we have adopted the Landau gauge in the above expressions.

By using (C6), one finds that the propagator $\hat{D}_{\mu\nu}^W$, defined by (4.3), is

$$\hat{D}_{\mu\nu}^W = N^2 D_{\mu\nu}^{WW} + 2(e_0/f_0) N^{3/2} D_{\mu\nu}^{WA} + (e_0/f_0)^2 N D_{\mu\nu}^{AA}. \quad \text{(C13)}$$

Theorem 1 then follows.

We note that as $q^2 \to 0$, $D_{\mu\nu}^{WW}$ is finite, but $\hat{D}_{\mu\nu}^W$ carries the photon pole.