New πN Superconvergent Sum Rules and Determination of P'-Trajectory Parameters*

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New superconvergent dispersion relations for the forward, crossing-even πN scattering amplitude have been derived and compared with experiment. The essential point is the assumption of dominant Regge behavior at high energy and the inclusion of the real part of the scattering amplitude. Sum rules for the P and P' residues involving only experimentally accessible quantities are computed, using data of Höler and Lindenbaum. The Regge P' parameters are found as $\alpha_{P'}\approx 0.39$, $\gamma_{P'}\approx 1.82\mu^{-1}$.

In an earlier paper[1] we derived a new type of superconvergent dispersion relations for the forward pion-nucleon charge-exchange scattering amplitude $C^{(-)}(\nu) = A^{(-)}(\nu)+\nu B^{(-)}(\nu)$.[2] Tests of such sum rules have been found to be satisfactory[3] because only experimentally accessible quantities are involved. An application of such dispersion sum rules to determine the S-wave πN scattering lengths was noted by Gilbert[4] a long time ago; Olsson[5] was also able to determine the Regge ρ parameters within the same formalism.

In this paper we apply the same technique to the πN crossing-even forward-scattering amplitude $C^{(+)}(\nu) = A^{(+)}(\nu)+\nu B^{(+)}(\nu)$. We shall obtain three superconvergent dispersion sum rules, and shall demonstrate how a better determination of a_1 and a_3 as well as the P' Regge parameters can be obtained.

We start by considering the function

$$F(\nu)=\frac{e^{\pi\beta i}}{(\nu^2-\mu^2)^\beta}\frac{1}{\nu}C^{(+)}(\nu)\,, \tag{1}$$

where ν and μ are the laboratory energy and the mass of the incident pion, $\mathrm{Im}C^{(+)}(\nu)=\frac{1}{2}(\nu^2-\mu^2)^{1/2}\times[\sigma_-(\nu)+\sigma_+(\nu)]$, and β is an arbitrary real constant less than unity. We normalize $(\nu^2-\mu^2)^\beta$ to be real positive as $\nu\to b+i0^+$, $b>\mu$. In this way, $\mathrm{Im}F(\nu)=0$ for $|\nu|<\mu$, except for the pole contributions at $\nu=\mp\nu_B =\pm\mu^2/2m$ (m= nucleon mass) and at $\nu=0$. Accepting the Regge behavior for $C^{(+)}(\nu)$ as $\nu\to\infty$, i.e.,

$$C^{(+)}(\nu)\underset{\nu\to\infty}{\longrightarrow} -\gamma_P\frac{e^{-\frac{1}{2}i\pi\alpha_P}}{\sin\frac{1}{2}\pi\alpha_P}\nu^{\alpha_P}-\gamma_{P'}\frac{e^{-\frac{1}{2}i\pi\alpha_{P'}}}{\sin\frac{1}{2}\pi\alpha_{P'}}\nu^{\alpha_{P'}}\,, \tag{2}$$

where γ_i and α_i are the residue and the intercept of the ith Regge trajectory at $t=0$ (P= Pomeranchukon, $P'=f^0$ meson), we obtain the following superconvergent dispersion relations:

$$\int_{-\infty}^{+\infty} d\nu\ \mathrm{Im}F(\nu+i\epsilon)=0$$

or

$$\int_\mu^\infty d\nu\frac{1}{(\nu^2-\mu^2)^\beta}\frac{1}{\nu}[\cos(\pi\beta)\ \mathrm{Im}C^{(+)}(\nu) +\sin(\pi\beta)\ \mathrm{Re}C^{(+)}(\nu)]=-\frac{g^2}{2m}\frac{\pi}{(\mu^2-\nu_B{}^2)^\beta} +\tfrac{1}{2}\pi[C^{(+)}(0)/\mu^{2\beta}]\,, \tag{3}$$

with $\frac{1}{2}\alpha_P=\frac{1}{2}<\beta<1$. When $\beta\to 1-0$, Eq. (3) reduces to

$$-\int_\mu^\infty d\nu\frac{1}{\nu^2-\mu^2}\frac{1}{\nu}\ \mathrm{Im}C^{(-)}(\nu)+\tfrac{1}{2}\pi\frac{C^{(+)}(\mu)}{\mu^2} =-\frac{g^2}{2m}\frac{\pi}{\mu^2-\nu_B{}^2}+\tfrac{1}{2}\pi\frac{C^{(+)}(0)}{\mu^2}\,, \tag{4}$$

which is nothing but the ordinary dispersion relation at $\nu=0$. Numerical evaluation shows that $C^{(+)}(0)$ is a large quantity of the same order as the Born term $(g^2/2m)\pi/(\mu^2-\nu_B{}^2)$; consequently, we combine (3) and (4) to eliminate $C^{(+)}(0)$, obtaining

$$\int_1^\infty\frac{d\nu}{(\nu^2-1)^\beta}\frac{1}{\nu}\{[\cos\pi\beta+(\nu^2-1)^{\beta-1}]\ \mathrm{Im}C^{(+)}(\nu) +\sin\pi\beta\ \mathrm{Re}C^{(+)}(\nu)\}=-\frac{g^2}{2m}\frac{\pi}{1-\nu_B{}^2} \times[(1-\nu_B{}^2)^{1-\beta}-1]+\tfrac{1}{2}\pi C^{(+)}(\mu)\,, \tag{5}$$

where we have set $\mu=1$ for simplicity. As $\beta\to\frac{1}{2}\alpha_P+0$, we obtain a sum rule for the residue of the Pomer-

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[1] Yu-Chien Liu and S. Okubo, Phys. Rev. Letters **19**, 190 (1967).

[2] We follow the notation of K. Nishijima, *Fundamental Particles* (W. A. Benjamin, Inc., New York, 1964).

[3] See Ref. 1. In that reference we used an S-wave scattering-length approximation to evaluate the integral near $\nu\approx\mu$. Results are slightly improved on using $\mathrm{Re}C^{(-)}(\nu)=\mathrm{Re}C^{(-)}(\mu)+C(\nu^2-\mu^2)$, with C determined by plotting $\mathrm{Re}C^{(-)}(\nu)$ versus $(\nu^2-\mu^2)$. (Theoretically, the constant C is related to P-wave scattering lengths.) Such a modification, however, is important in this calculation, because in the range from 0 to 0.02 BeV/c, $\mathrm{Re}C^{(+)}(\nu)/\mathrm{Re}C^{(+)}(\mu)\approx 3$ [while $\mathrm{Re}C^{(-)}(\nu)/\mathrm{Re}C^{(-)}(\mu)\approx 1$].

[4] W. Gilbert, Phys. Rev. **108**, 1078 (1957).

[5] M. G. Olsson, Phys. Rev. Letters **19**, 550 (1967).

anchukon:

$$\tfrac{1}{2}\pi\sigma_{\pm}(\infty)\equiv\tfrac{1}{2}\pi\gamma_P=\int_1^\infty \frac{d\nu}{\nu^2-1}\frac{1}{\nu}$$
$$\times[\mathrm{Im}C^{(+)}(\nu)+(\nu^2-1)^{1/2}\,\mathrm{Re}C^{(+)}(\nu)]+\frac{g^2}{2m}\frac{\pi}{1-\nu_B{}^2}$$
$$\times[(1-\nu_B{}^2)^{1/2}-1]-\tfrac{1}{2}\pi C^{(+)}(\mu). \quad (6)$$

In principle, this sum rule serves to determine γ_P if $\alpha_P=1$ is assumed. However, using the same experimental data as in Ref. 1, and neglecting the contribution of P' as compared to P above 20 BeV/c, we found $\gamma_P=\sigma_{\pm}(\infty)\approx 26$ mb. If one takes into account the contribution from the P' trajectory, with values of $\alpha_{P'}$ and $\gamma_{P'}$ to be determined below, one can pull this value down to the experimental one: $\gamma_P=22.12\pm 0.94$ mb.[6]

To this end we consider subtracted forms of $F(\nu)$, i.e.,

$$G(\nu)=\frac{e^{\pi\beta i}}{(\nu^2-\mu^2)^\beta}\frac{1}{\nu}[C^{(+)}(\nu)-C^{(+)}(\mu)]$$

and

$$H(\nu)=\frac{e^{\pi\beta i}}{(\nu^2-\mu^2)^\beta}\frac{1}{\nu}[C^{(+)}(\nu)-i\gamma_P(\nu^2-\mu^2)^{1/2}].$$

By an argument similar to that above we obtain the following superconvergent dispersion sum rules:

$$\int_1^\infty d\nu\,\frac{1}{(\nu^2-1)^\beta}\frac{1}{\nu}\{[\cos\pi\beta+(\nu^2-1)^{\beta-1}]\,\mathrm{Im}C^{(+)}(\nu)$$
$$+\sin\pi\beta[\mathrm{Re}C^{(+)}(\nu)-\mathrm{Re}C^{(+)}(\mu)]\}$$
$$=-\frac{g^2}{2m}\frac{\pi}{1-\nu_B{}^2}[(1-\nu_B{}^2)^{1-\beta}-1];\quad (1<\beta<\tfrac{3}{2}) \quad (7)$$

and

$$\int_1^\infty d\nu\,\frac{1}{(\nu^2-1)^\beta}\frac{1}{\nu}\{[\cos\pi\beta(\mathrm{Im}C^{(+)}(\nu)-\gamma_P(\nu^2-1)^{1/2})$$
$$+(\nu^2-1)^{\beta-1}\,\mathrm{Im}C^{(+)}(\nu)]+\sin\pi\beta\,\mathrm{Re}C^{(+)}(\nu)\}$$
$$=-\frac{g^2}{2m}\frac{\pi}{1-\nu_B{}^2}[(1-\nu_B{}^2)^{1-\beta}-1]$$
$$+\tfrac{1}{2}\pi[C^{(+)}(\mu)+\gamma_P];\quad (\tfrac{1}{2}>\beta>\alpha_{P'}/2), \quad (8)$$

where again $C^{(+)}(0)$ has been eliminated by using Eq. (3). When $\beta\to\frac{3}{2}-0$, Eq. (7) reduces to the Gilbert[4] type of inverse dispersion relation

$$-\tfrac{1}{2}\pi\lim_{\nu\to 1^+}\left[\frac{\mathrm{Im}C^{(+)}(\nu)}{(\nu^2-1)^{1/2}}\right]+\int_1^\infty \frac{d\nu}{(\nu^2-1)^{3/2}}\frac{1}{\nu}$$
$$\times\{(\nu^2-1)^{1/2}\,\mathrm{Im}C^{(+)}(\nu)-[\mathrm{Re}C^{(+)}(\nu)-\mathrm{Re}C^{(+)}(\mu)]\}$$
$$=-\frac{g^2}{2m}\frac{\pi}{(1-\nu_B{}^2)}[(1-\nu_B{}^2)^{-1/2}-1], \quad (9)$$

TABLE I. Test of Eq. (5), with $f^2/4\pi=(\mu/2m)^2g^2/4\pi=0.081$.

β	Left-hand side	Right-hand side
1.0	0.0076	0.0076
0.9	0.016	0.016
0.8	0.040	0.040
0.7	0.062	0.064
0.6	0.085	0.088
0.5	0.11	0.11

while as $\beta\to\frac{1}{2}\alpha_{P'}+0$, Eq. (8) reduces to

$$\int_1^\infty d\nu\,\frac{1}{(\nu^2-1)^{\alpha_{P'}/2}}\frac{1}{\nu}\{[\cos\tfrac{1}{2}\pi\alpha_{P'}(\mathrm{Im}C^{(+)}(\nu)$$
$$-\gamma_P(\nu^2-1)^{1/2})+(\nu^2-1)^{\frac{1}{2}\alpha_{P'}-1}\,\mathrm{Im}C^{(+)}(\nu)]$$
$$+\sin\tfrac{1}{2}\pi\alpha_{P'}\,\mathrm{Re}C^{(+)}(\nu)\}-\tfrac{1}{2}\pi\frac{\gamma_P{}'}{\sin\frac{1}{2}\pi\alpha_{P'}}=-\frac{g^2}{2m}\frac{\pi}{1-\nu_B{}^2}$$
$$\times[(1-\nu_B{}^2)^{1-\frac{1}{2}\alpha_{P'}}-1]+\tfrac{1}{2}\pi[C^{(+)}(\mu)+\gamma_P] \quad (10)$$

by using the validity of Eq. (2) for large ν. Equation (10) is a sum rule for the P' residue, if $\alpha_{P'}$ is known.

In practice, sum rules (7) and (9), as contrasted with Eq. (5), are insensitive to reasonable choices of the P' parameters, because in such Gilbert-type sum rules, even the dominant P contribution is very small at high energies. Therefore, they serve to determine the low-energy parameters a_1 and a_3 accurately. Using the data from Höhler and Strauss[7] and from Lindenbaum,[6] we found that the best values are $a_1\approx 0.173\mu^{-1}$, $a_3\approx -0.087\mu^{-1}$, with $a_1+2a_3=-0.001\mu^{-1}$.[8] On the other hand, Eqs. (8) and (10) are insensitive to reasonable a_1 and a_3, and offer a better determination of $\alpha_{P'}$ and $\gamma_{P'}$: $\alpha_{P'}\approx 0.39$, $\gamma_{P'}\approx 1.82\mu^{-1}=10.9$ mb BeV.[9] These so-obtained low- and high-energy parameters for $C^{(+)}(\nu)$ are then used to test the sum rule (5), with the result displayed in Table I. The agreement of the left- and the right-hand side is satisfactory.

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[6] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 330 (1967); **19**, 193 (1967).

[7] G. Höhler and R. Strauss (private communication). Also for an earlier report of such data in the c.m. system, see G. Höhler, G. Ebel, and J. Giesecke, Z. Physik **180**, 430 (1964).

[8] Another independent way using the broad area subtraction method of Adler [S. L. Adler, Phys. Rev. **137**, B1022 (1965)] enables us to obtain $a_1+2a_3\approx 0.000\mu^{-1}$ and $a_1-a_3\approx 0.290\mu^{-1}$. This yields $a_1\approx 0.197\,\mu^{-1}$ and $a_3\approx -0.093\,\mu^{-1}$. What are essential in our calculation are the combination a_1+2a_3 for $C^{(+)}(\nu)$ and a_1-a_3 for $C^{(-)}(\nu)$ (as in Ref. 1), but not a_1 and a_3 separately.

[9] A first guess for these values is obtained by several authors, using the nonsuperconvergent finite-energy sum rules. See, M. Olsson, Phys. Letters **26B**, 310 (1968); and R. Dolen *et al.*, Phys. Rev. Letters, **19**, 402 (1967). The normalization of γ_p (or $\gamma_p{}'$) is conventionally specified by either of the following asymptotic conditions:

$$C^{(+)}(\nu)\gamma_p\cdot(\nu/\mu)^{\alpha_p}\quad\text{or}\quad C^{(+)}(\nu)\sim\gamma_p(\nu/E_0)^{\alpha_p},$$

where $E_0=1$ BeV. Thus, it is convenient to adopt normalizations of γ_p (similarly of $\gamma_p{}'$) so that it has the dimension μ^{-1} for the first case (i.e., pion natural unit) and mb BeV for the second case (i.e., we use a scaling factor $E_0=1$ BeV). Unfortunately, the conversion formula from one unit to the other requires us to specify values of $\alpha_p{}'$.

We comment on our result. Concerning the S-wave scattering lengths, Hamilton's[10] new values are $a_3=-0.091\pm0.005$ and $2a_1+a_3=0.270\pm0.008$, which yield $a_1+2a_3=-0.002\pm0.008$. As regards the P' parameters, a diversity of results appears in the literature. Barger and Olsson[11] gave $\alpha_{P'}=0.39\pm0.24$; Rarita *et al.*[12] gave $\alpha_{P'}=0.57$ with $\gamma_{P'}=14.8$ mb BeV or $\alpha_{P'}=0.73$ with $\gamma_{P'}=21$ mb BeV; Meshcheriakov *et al.*,[13] imposing $\alpha_{P'}=0.50$, obtained $\gamma_{P'}=13.86$ mb BeV; while Igi,[14] who was the first one to propose the existence of P', gave $\gamma_{P'}=3.05\mu^{-1}=18.3$ mb BeV for $\alpha_{P'}=0.4$ ($\gamma_P=21.6$ mb). Although the experimental uncertainties in $\mathrm{Re}C^{(+)}(\nu)$ may still be large, we believe that our method provides a better method for the determination of all these parameters, since all the integrals [Eqs. (5), (7), and (9)] are superconvergent.

[10] J. Hamilton, Phys. Letters **20**, 687 (1966).
[11] V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).
[12] W. Rarita, R. J. Riddell, Jr., C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968).
[13] V. A. Meshcheriakov *et al.*, Phys. Letters **25B**, 341 (1967).
[14] K. Igi, Phys. Rev. **130**, 820 (1963).

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Field-Current Identities and Intermediate Bosons*

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The observed total weak-interaction current $\mathcal{J}_\mu^{\mathrm{wk}}(x)$ and the observed total electromagnetic current $\mathcal{J}_\mu^{\gamma}(x)$ are assumed to be, respectively, the same local operators, apart from constant multiplicative factors, as the hypothetical charged intermediate boson field $W_\mu(x)$ and the corresponding neutral intermediate boson field $W_\mu^0(x)$. The field algebra satisfied by these current operators is discussed. It is shown that, neglecting higher-order weak-interaction effects, one can obtain finite higher-order electromagnetic corrections for the known hadrons and leptons, such as the electromagnetic mass shifts of p, π, e, μ, etc., and the radiative corrections to the weak decays of these particles.

I. INTRODUCTION

THE purpose of this paper is to show that within the general framework of field-current identities[1–3] it is possible to derive finite higher-order electromagnetic corrections for the known hadrons and leptons, provided one identifies the observed weak and electromagnetic current operators, $\mathcal{J}_\mu^{\mathrm{wk}}$ and $\mathcal{J}_\mu^{\gamma}$, as proportional to some weakly coupled fields such as the (hypothetical) intermediate boson fields. In order to show that such field-current identities are indeed possible, let us first examine the definitions of these observed current operators.

The total electromagnetic current operator $\mathcal{J}_\mu^{\gamma}$ is, by definition, related to the electromagnetic field A_μ by[4]

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu}=e_0\mathcal{J}_\nu^{\gamma}, \tag{1.1}$$

where e_0 is the unrenormalized charge of the electron ($e_0<0$), and

$$F_{\mu\nu}=\frac{\partial}{\partial x_\mu}A_\nu-\frac{\partial}{\partial x_\nu}A_\mu.$$

To give a precise definition of the total observed weak-interaction current operator $\mathcal{J}_\mu^{\mathrm{wk}}(x)$, we assume that the bilinear product

$$s_\lambda^{\mathrm{wk}}(l)=i\nu_l(x)^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)l(x) \tag{1.2}$$

is an observable, where $l(x)$ and $\nu_l(x)$ represent field operators of the particles l^- and ν_l, respectively, $l=e$ or μ, and the dagger denotes the Hermitian conjugate. The observed total weak-interaction current $\mathcal{J}_\mu^{\mathrm{wk}}(x)$ is then defined to be proportional to the derivative of the S matrix with respect to $s_\lambda^{\mathrm{wk}}(e)$ or $s_\lambda^{\mathrm{wk}}(\mu)$. We have

$$i\frac{G_F}{\sqrt{2}}\mathcal{J}_\lambda^{\mathrm{wk}}(x)=\frac{\partial S}{\partial s_\lambda^{\mathrm{wk}}(e)}=\frac{\partial S}{\partial s_\lambda^{\mathrm{wk}}(\mu)}, \tag{1.3}$$

where

$$G_F\cong10^{-5}m_N^{-2}, \tag{1.4}$$

which denotes the Fermi coupling constant of the weak interaction, and m_N is the nucleon mass. In (1.3), the equality $[\partial S/\partial s_\lambda^{\mathrm{wk}}(e)]=[\partial S/\partial s_\lambda^{\mathrm{wk}}(\mu)]$ expresses the $\mu-e$ symmetry property of the weak interaction.

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[1] N. M. Kroll, T. D. Lee, and Bruno Zumino, Phys. Rev. **157**, 1376 (1967).
[2] T. D. Lee, S. Weinberg, and Bruno Zumino, Phys. Rev. Letters **18**, 1029 (1967).
[3] T. D. Lee and Bruno Zumino, Phys. Rev. **163**, 1667 (1967).
[4] Throughout the paper, the subscript μ denotes the space-time index, $\mu=4$ is the time component, $x_4=it$, and $\mu=i$ (or j, or k) denotes the space component. All repeated indices are to be summed over.