Scalar-Meson Candidates for Tadpoles*

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It is proposed that members of a recently observed scalar octet with particle masses around 1 GeV could play the role of tadpoles. If the anomalous branching ratio of $2\pi/\bar{K}K$ of the isosinglet member of the octet is accounted for by SU_3 violation, the resulting octet-dominance-violating contribution to the mass formula very much improves the Gell-Mann–Okubo mass relations for pseudoscalar mesons.

ACCORDING to the most recently published table of elementary particles,[1,2] the lowest-mass scalar mesons for which there is reasonably strong evidence are the isosinglet[3] η_V(1050 MeV) and the isotriplet π_V (1003 MeV), both of vanishing strangeness. The existence of the remaining member of the octet K_V (1080 MeV) is less well established. Both the η_V and π_V have been seen as peaks in the $\bar{K}K$ scattering amplitudes near threshold rather than as particles. However, the evidence for other scalar mesons, the κ (725 MeV) and σ (410 MeV), is, if anything, even more tenuous than that for the η_V and π_V.

For the purposes of this note, we will assume that the η_V and π_V are in fact particles. As such, they possess a number of unusual properties:

(a) If the K_V (1080) is indeed found to exist, η_V, π_V, and K_V form an almost degenerate octet.

(b) The observed width of the η_V and π_V is of the order of 100 MeV or less. This width is considerably smaller than that which could be expected, assuming coupling strengths typical of other resonances and taking into consideration the larger available phase space for the π decays of π_V and η_V, in comparison with those, say, of the ρ.

(c) The $\pi\pi$ decay mode of η_V appears to be suppressed. No known selection rules forbid $\eta_V\to\pi\pi$, yet the data[1] yield only an upper limit $R_{\eta_V}(\pi\pi/\bar{K}K)\leq 2.5$. On the basis of an $SU(3)$-symmetric interaction and use of physical masses to estimate phase space, this ratio should be 8.8 rather than 2.5.[4]

One possible way to account for the anomalous branching ratio is to claim the existence of an $SU(3)$ singlet scalar which is mixed with the η_V, analogously to an ω-ϕ mixture for the vector mesons. There are two objections to this procedure. In the first place, the almost degenerate mass spectrum of the octet implies either a small mixing of a singlet which is of very different mass or a singlet essentially degenerate in mass with the octet. No such neighboring resonance is observed, however. Secondly, even if singlet-octet mixing could occur, an additional selection rule would still have to be imposed to account for the suppression of the $\eta_V\to\pi\pi$ channel (see Ref. 4).

We prefer to make the somewhat simpler assumption that the suppression of the $\pi\pi$ channel is due to $SU(3)$-symmetry violation in the coupling constants of the SPP term in the effective Hamiltonian. We write the part of H containing η_V as

$$H\eta_V=f\eta_V[\eta\eta+\alpha\bar{K}K-(2-\alpha)\boldsymbol{\pi}\cdot\boldsymbol{\pi}], \tag{1}$$

where we obtain the $SU(3)$-symmetric limit for $\alpha=1$, and where we have chosen the $SU(3)$ violation to be minimal by making the deviations symmetric for the $\bar{K}K$ and $\pi\pi$ coupling. Choosing $R_{\eta_V}\cong 2.5$, we obtain $\alpha\cong 1.3$. Taking the quoted value[1] for the η_V width, $\Gamma_{\eta_V}=50$ MeV, and the above value of α, we obtain the dimensionless coupling

$$f^2/4\pi M_K^2=0.31. \tag{2}$$

This is to be compared with a typical meson coupling, that obtained from ρ decay;

$$f_{\rho\pi\pi}^2/4\pi=2.15. \tag{3}$$

Expressions (2) and (3) serve to justify our remark (b). The $SU(3)$-symmetric couplings of the scalar mesons to the pseudoscalars are anomalously low. It is, therefore, not surprising that $SU(3)$-violating couplings are of the order of 30%, rather than the expected 10% of the $SU(3)$-symmetric couplings, and, therefore, play a more important role in the present case than they normally do.

The relative importance of $SU(3)$-violating interactions makes the η_V and π_V particularly good candidates for tadpoles.[5] One possible set of tadpole terms could arise from the SPP interaction. (Of course, many other virtual processes could contribute to the tadpole.) The

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[1] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Soding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **39**, 1 (1967).

[2] J. Alitti *et al.*, Phys. Letters **15**, 69 (1959).

[3] We use the notation of Ref. 1 for these particles.

[4] For the π_V, we would expect a large $\pi_V\to\eta\pi$ rate. Here the situation is confused by the fact (see Ref. 1) that no $(\eta\pi)$ mass spectra exist for $\bar{p}p$ reactions. On the other hand, there is some evidence in $\pi^{\pm}p$ experiments [J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964)] for an $(\eta\pi)$ peak around 1 GeV. In these latter experiments there are no $(\bar{K}K)$ mass spectra.

[5] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

SPP tadpole diagram gives a nonzero contribution (assuming a degenerate *P* octet mass) only in virtue of $SU(3)$ violation in the coupling constants. Thus, significant $SU(3)$- symmetry violation is required for the very existence of the tadpole.

We now propose to use the same symmetry-breaking interaction in a self-consistent way to help evaluate mass formulas for the pseudoscalar octet. We take as mass operator

$$M_P = m_0(\eta^2 + 2\bar{K}K + \boldsymbol{\pi}\cdot\boldsymbol{\pi}) + m_1[\eta^2 + \alpha\bar{K}K - (2-\alpha)\boldsymbol{\pi}\cdot\boldsymbol{\pi}], \quad (4)$$

and obtain the modified Gell-Mann–Okubo (G-O) mass formula, including improved average masses for isomultiplets[6]:

$$A(\alpha) = (4-\alpha)m_\eta^2 + (2-\alpha)(2m_{\pi^+}^2 - m_{\pi^0}^2) - (3-\alpha)(m_{K^+}^2 + m_{K^0}^2) = 0, \quad (5)$$

which reduces to the usual G-O selection for $\alpha = 1$. To demonstrate the improved fit of the masses to the modified formula, we consider the error

$$E(\alpha) = A(\alpha)/\Delta, \quad \Delta = m_\eta^2 - m\pi^2 \quad (6)$$

and find

$$E(1) = 21\%, \quad E(1.3) = 3.1\%. \quad (7)$$

It must be emphasized that the present approach is an alternative one to the usual way of dealing with the discrepancies in the G-O mass formula for the pseudoscalars, by considering η-X^0 mixing.[7] This may be just as well since there is at present some difficulty in finding suitable candidates[8] for X^0.

The couplings of η_V to vector mesons and to baryons can be assumed to be $SU(3)$ symmetric in view of the accuracy of the G-O relations (with ϕ-ω mixing added for vector mesons) for these particles. There is no evidence at present which can be adduced to contradict this assumption.

We conclude with a few remarks concerning the electromagnetic mass relations for pseudoscalars. If one includes a π_V tadpole [for simplicity, assumed to have $SU(3)$-symmetric couplings] as well as the non-tadpole contributions calculated by Coleman and Schnitzer,[9] one obtains two expressions for the transition mass[10] $m_{\eta\pi^0}{}^2$:

$$m_{\eta\pi^0}{}^2 = \tfrac{1}{3}\sqrt{3}(m_{K^+}^2 - m_{K^0}^2 + m_{\pi^0}^2 - m_{\pi^+}^2 - 0.06m_\pi^2), \quad (8)$$

$$m_{\eta\pi^0}{}^2 = \frac{1}{\sqrt{3}(2\alpha-5)}[(8-3\alpha)m_{K^0}^2 - (2-\alpha)(m_{K^+}^2 - 0.12m_\pi^2) - (4-\alpha)m_\eta^2 - (2-\alpha)m_{\pi^0}^2]. \quad (9)$$

The term $m_{\pi^0}^2 - m_{\pi^+}^2$ in Eq. (8) does not strictly arise in the case of the *P*, where, in the tadpole approximation, $m_{\pi^0}^2 = m_{\pi^+}^2$. It is introduced by analogy with the baryons, where it occurs naturally if one uses the "parallelogram rule."[6]

The requirement that Eqs. (8) and (9) give a unique answer leads to $\alpha = 1.32$, consistent with our previous determination. The value of the transition mass which is obtained is

$$m_{\eta\pi^0}{}^2 = -3730 \text{ MeV}^2,$$

and is of the same order but slightly larger than previous determinations.[10]

A discussion of the role played by the scalar-meson octet in a pole-dominant-model calculation of $K \to 2\pi$ and $K \to 3\pi$ decays will be considered in a subsequent communication.

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[6] G. Feldman and P. T. Matthews, Ann. Phys. (N. Y.) **31**, 469 (1965).

[7] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965).

[8] G. Goldhaber, *Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester* (Interscience Publishers, Inc., New York, 1967), p. 57. According to this review, there appears to be some question whether the X^0 (960 MeV) is not confused with the $\delta_{I=1}$ (964 MeV). The next possible candidate for X^0, *E* (1420 MeV), has such a large mass that the results for *P* mass fits become extremely sensitive to the (rather small) mixing parameter.

[9] S. Coleman and H. J. Schnitzer, Phys. Rev. **136**, B223 (1964).

[10] R. H. Dalitz and F. Von Hippel, Phys. Letters **10**, 153 (1964); S. Okubo, *ibid.* 4, 14 (1963).