as k and k' go simultaneously to zero in all their components. The third part, consisting of the remainder of (D7), is the additional term which must be added to maintain gauge invariance. In perturbation theory, it corresponds to the "sea-gull" term pictured in Fig. 3.

The expression (D7) may be simplified. For this we shall need the differential form of (D3):

$$\Gamma^{\epsilon\varphi}(p,p)=p^{\epsilon}\frac{\partial}{\partial p_{\varphi}}D^{-1}(p)-g^{\epsilon\varphi}D^{-1}(p)\,. \tag{D8}$$

[This is not manifestly symmetric in ϵ and φ. Symmetry is established by recalling that $D^{-1}(p)$, for spin-0 particles, is a function only of p^2, so that $p^{\epsilon}(\partial/\partial p_{\varphi})\times D^{-1}(p)$ is proportional to $p^{\epsilon}p^{\varphi}$.] Next, we recall that the scattering amplitude is given by contracting the above with $\epsilon^{\mu}\epsilon^{\nu}$ and $\epsilon^{*\alpha}\epsilon^{*\beta}$, with $\epsilon^{\mu}\epsilon_{\mu}=\epsilon^{*\alpha}\epsilon^{*}{}_{\alpha}=0$. Also, p may be taken to be in its rest frame, and the polarization tensors may be chosen without a time component. Hence terms in (D7) proportional to p^{μ}, p^{ν}, p^{α}, p^{β}, $g^{\mu\nu}$, and $g^{\alpha\beta}$ do not contribute. It is seen that the first two terms in (D7) do not contribute. The third term *does* contribute. In the remainder, the only terms that are not proportional to a momentum are those that involve, e.g., $g^{\nu\beta}\Gamma^{\mu\alpha}$. However, from (D8) it is seen that $\Gamma^{\mu\alpha}$ is proportional either to a momentum, $p^{\mu}p^{\alpha}$, or to D^{-1}, which vanishes on the mass shell. Thus in the zero-energy limit we may take for $S_{\mu\nu,\alpha\beta}$

$$S_{\mu\nu,\alpha\beta}=\Gamma_{\gamma\delta}(p,p)F^{\gamma\delta}{}_{,\mu\nu,\alpha\beta}\,, \tag{D9}$$

i.e., only the graviton-exchange term contributes. We insert (D8); the portion of $\Gamma_{\gamma\delta}$ proportional to $g^{\gamma\delta}D^{-1}$ does not contribute, since D^{-1} vanishes on the mass shell. Thus

$$S_{\mu\nu,\alpha\beta}=p_{\gamma}\frac{\partial D^{-1}(p)}{\partial p^{\delta}}F^{\gamma\delta}{}_{,\mu\nu,\alpha\beta}\,. \tag{D10}$$

D^{-1} has the form $(p^2-m^2)(1+\Sigma(p^2))$, where $\Sigma(p^2)$ vanishes on the mass shell. Therefore the final result for the zero-energy scattering amplitude for gravitons off spin-0 particles is

$$S_{\mu\nu,\alpha\beta}=2p_{\gamma}p_{\delta}F^{\gamma\delta}{}_{,\nu\mu,\alpha\beta}=2m^2F^{00}{}_{,\mu\nu,\alpha\beta}\,. \tag{D11}$$

The cross section which follows from (D11) is then given by (3.11).

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$SU(3)$ Symmetry-Breaking Interactions*

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It is proposed that all symmetry-breaking interactions H^i are of the current-current form, and a calculational method suitable for obtaining sum rules is developed. The method essentially is to express the matrix elements of H^i in terms of the matrix elements of the double commutator of H^i with $SU(3)$ generators. When the method is combined with the technique of reducing a pion, it can be shown that the S-wave amplitudes of both the triangle relation and the Lee-Sugawara relation of nonleptonic decays correspond to sum rules of photonic decay amplitudes and semileptonic decay amplitudes of hyperons.

1. INTRODUCTION

VARIOUS consequences of broken $SU(3)$ symmetry do not depend on the specific form of the symmetry-breaking interaction but only on its transformation properties or on the postulate that the symmetry-violating processes are dominated by tadpole diagrams.[1] Nevertheless, the description of nonleptonic decays in terms of the weak Hamiltonian of the current-current form[2]

$$H\sim d_{6jk}J_{\mu}{}^{j}J_{\mu}{}^{k}\,, \tag{1}$$

has been useful in correlating the experimental data, where $J_{\mu}{}^{k}$ is expressible in terms of quark fields q as

$$J_{\mu}{}^{k}=i\bar{q}\gamma_{\mu}(1+\gamma_5)\lambda_k\tfrac{1}{2}q=V_{\mu}{}^{k}+A_{\mu}{}^{k}\,, \tag{2}$$

and $\mu=1,\cdots,4$, $k=1,\cdots,8$.

Then, the following equal-time commutation relation holds for

$$\begin{aligned}&H^i\sim d_{ijk}J_{\mu}{}^{j}J_{\mu}{}^{k}\,,\\ &[Q^i,H^j]=[Q_5{}^i,H^j]=if_{ijk}H^k\,,\\ &[Q^i,\pi^j]=if_{ijk}\pi^k\,,\end{aligned} \tag{3}$$

where

$$Q^i(x_0)=-i\int d^3x\;V_4{}^i(\mathbf{x},x_0)\,,$$

$$Q_5{}^i(x_0)=-i\int d^3x\;A_4{}^i(\mathbf{x},x_0)\,,$$

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[1] S. Coleman and S. L. Glashow, Phys. Rev. **134B**, 671 (1964).

[2] Y. Hara, T. Nambu, and J. Schechter, Phys. Rev. Letters **16**, 380 (1966).

TABLE. I. Operators $I^{\pm}=(Q^1\pm iQ^2)/\sqrt{2}$, $V^{\pm}=(Q^4\pm iQ^5)/\sqrt{2}$, $U^{\pm}=(Q^6\pm iQ^7)/\sqrt{2}$, Q^3, and Q^8, on meson states. To obtain those on baryons, substitute $\pi\to\Sigma$, $K\to N$, $\bar{K}\to\Xi$, and $\eta\to\Lambda$.

	I^+	I^-	V^+	V^-	U^+	U^-	Q^3	Q^8
π^+	0	$-\pi^0$	0	$\bar{K}^0/\sqrt{2}$	$-K^+/\sqrt{2}$	0	π^+	0
π^0	$-\pi^+$	π^-	$-\frac{1}{2}K^+$	$\frac{1}{2}K^-$	$\frac{1}{2}K^0$	$-\frac{1}{2}\bar{K}^0$	0	0
π^-	π^0	0	$-K^0/\sqrt{2}$	0	0	$K^-/\sqrt{2}$	$-\pi^-$	0
K^+	0	$K^0/\sqrt{2}$	0	$-\frac{1}{2}(\pi^0+\sqrt{3}\eta)$	0	$-\pi^+/\sqrt{2}$	$\frac{1}{2}K^+$	$\frac{1}{2}\sqrt{3}K^+$
K^0	$K^+/\sqrt{2}$	0	0	$-\pi^-/\sqrt{2}$	0	$\frac{1}{2}(\pi^0-\sqrt{3}\eta)$	$-\frac{1}{2}K^0$	$\frac{1}{2}\sqrt{3}K^0$
$\bar{K}^0$	0	$-K^-/\sqrt{2}$	$\pi^+/\sqrt{2}$	0	$\frac{1}{2}(-\pi^0+\sqrt{3}\eta)$	0	$\frac{1}{2}\bar{K}^0$	$-\frac{1}{2}\sqrt{3}\bar{K}^0$
K^-	$-\bar{K}^0/\sqrt{2}$	0	$\frac{1}{2}(\pi^0+\sqrt{3}\eta)$	0	$\pi^-/\sqrt{2}$	0	$-\frac{1}{2}K^-$	$-\frac{1}{2}\sqrt{3}K^-$
η	0	0	$-\frac{1}{2}\sqrt{3}K^+$	$\frac{1}{2}\sqrt{3}K^-$	$-\frac{1}{2}\sqrt{3}K^-$	$\frac{1}{2}\sqrt{3}K^0$	0	0

π represents the meson field or the corresponding baryon field,[3] and the time dependence is suppressed. Equation (3) is also valid when π is replaced by Q.

Since it appears desirable to put all symmetry-breaking interactions on the same footing, we propose that all symmetry-breaking Hamiltonians H^i are of the current-current form and transform as the ith member of an octet and examine the resulting relations among observables. In particular, the Hamiltonian H^i responsible for mass breaking is H^8, electromagnetic mass splitting is $H^3+(H^8/\sqrt{3})$, nonleptonic decay and photonic decay is given by H^6, and semileptonic decays by H^4.

In Sec. 2, the method of calculation based on the commutation relations between H^i and charges Q^i is obtained. In Sec. 3, the mass breaking, electromagnetic mass splittings, and magnetic moments are considered. Sections 4–6 are devoted to nonleptonic decays, weak electromagnetic decay (photonic decay) of baryons, and semileptonic decays, respectively. Section 7 deals with off-mass-shell amplitudes in which various sum rules in nonleptonic, photonic, and semileptonic decays are shown to be related. A summary is given in Sec. 8.

2. METHOD OF CALCULATION

The method is based on the assumption of the existence of octets of mesons and baryons, the $SU(3)$ algebra given in Eq. (3), and the symmetry-breaking Hamiltonian H^i of the current-current form. We express the Hamiltonian H^i in terms of its double commutators with the $SU(3)$ generators $Q^{\pm}$ and make use of the Jacobi identity

$$[Q^+,[Q^-,H^i]]-[Q^-,[Q^+,H^i]]=[[Q^+,Q^-],H^i], \quad (4)$$

where $Q^{\pm}$ represent

$$I^{\pm}=(Q^1\pm iQ^2)/\sqrt{2}, \quad V^{\pm}=(Q^4\pm iQ^5)/\sqrt{2}, \quad U^{\pm}=(Q^6\pm iQ^7)/\sqrt{2}. \quad (5)$$

Equation (4) is sandwiched between the initial state α and the final state β; then the commutator is opened up so that $Q^{\pm}$ operate on the states. Then one obtains relations among the matrix elements of the form $\langle\beta|H^i|\alpha\rangle$ with the aid of Table I. This method is a convenient way of obtaining sum rules, when the Hamiltonian is of the current-current form.[4] One notes that to get sum rules all one needs is the operations of $Q^{\pm}$, Q^3, and Q^8 on the states tabulated in Table I.

3. MASS RELATIONS

We assume that the baryon mass breaking is due to the Hamiltonian $H^8=d_{8ij}V^iV^j$, and obtain from Eqs. (3) and (5)

$$[U^+,[U^-,H^8]]=[U^-,[U^+,H^8]]=\tfrac{3}{4}H^8-\tfrac{1}{4}\sqrt{3}H^3. \quad (6)$$

The same expression is used for the Hamiltonian even when the currents are not of the $V_\mu+A_\mu$ form (when it does not lead to confusion). In order to get matrix elements of the form $\langle\alpha|H^8|\alpha\rangle$, Eq. (6) is applied to $\alpha=\Lambda$ and Σ^0 so that the H^3 on the right-hand side of Eq. (6) gives no contribution;

$$\langle\alpha|H^8|\alpha\rangle=\tfrac{4}{3}[\langle U^+U^-\alpha|H^8|\alpha\rangle+\langle\alpha|H^8|U^-U^+\alpha\rangle -\langle U^+\alpha|H^8|U^+\alpha\rangle-\langle U^-\alpha|H^8|U^-\alpha\rangle]. \quad (7)$$

After the U^+ and U^- operations are carried out and H^8 is suppressed, one gets for Λ and Σ^0, respectively,

$$3\langle\Lambda|\Lambda\rangle-3\langle\Xi^0|\Xi^0\rangle-3\langle n|n\rangle=\sqrt{3}[\langle\Sigma^0|\Lambda\rangle+\langle\Lambda|\Sigma^0\rangle], \quad (8)$$

$$\langle\Sigma^0|\Sigma^0\rangle+\langle\Xi^0|\Xi^0\rangle+\langle n|n\rangle=-\sqrt{3}[\langle\Sigma^0|\Lambda\rangle+\langle\Lambda|\Sigma^0\rangle]. \quad (9)$$

Equations (8) and (9) yield

$$3\langle\Lambda|\Lambda\rangle+\langle\Sigma^0|\Sigma^0\rangle=2\langle\Xi^0|\Xi^0\rangle+2\langle n|n\rangle, \quad (10)$$

which is the Gell-Mann–Okubo mass formula.

For the electromagnetic splitting, we get from Eqs. (3) and (5)

$$[U^+,[U^-,H^3]]=[U^-,[U^+,H^3]]=-\tfrac{1}{4}\sqrt{3}H^8+\tfrac{1}{4}H^3, \quad (11)$$

which together with Eq. (6) leads to

$$[U^+,[U^-,H^3+(H^8/\sqrt{3})]]=0. \quad (12)$$

Taking the matrix elements of Eq. (12) with respect to

[3] M, Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[4] The expression $\pi^+=(\pi_1-i\pi_2)/\sqrt{2}$, for example, is an annihilation operator of π^+, and all the states are defined with a positive phase. Table I is then constructed by using Eq. (3).

the baryons, rearranging, and suppressing $H^3+(H^8/\sqrt{3})$, we obtain

$$\begin{aligned} \langle p|p\rangle &= \langle\Sigma^+|\Sigma^+\rangle,\\ \langle n|n\rangle &= \langle\Xi^0|\Xi^0\rangle,\\ \langle\Sigma^-|\Sigma^-\rangle &= \langle\Xi^-|\Xi^-\rangle,\\ 2\langle n|n\rangle+\langle\Sigma^0|\Sigma^0\rangle &= 3\langle\Lambda|\Lambda\rangle,\\ \langle\Lambda|\Sigma^0\rangle+\langle\Sigma^0|\Lambda\rangle &= 2\sqrt{3}(\langle\Lambda|\Lambda\rangle-\langle n|n\rangle). \end{aligned} \tag{13}$$

Equation (13) yields for the electromganetic baryon mass splitting $M(\alpha)=M_0{}^\alpha+\langle\alpha|\alpha\rangle$, where $M_0{}^\alpha$ is the common mass of the isomultiplet

$$M(\Xi^-)-M(\Xi^0)=M(\Sigma^-)-M(\Sigma^+)+M(p)-M(n), \tag{14}$$

which is the Coleman-Glashow relation.[5]

The medium-strong and electromagnetic mass splittings have been considered by Chiu and Schechter[6] in the current-current picture, into which they introduce spurions. The spurions are the expectation values with respect to one-particle states of the products of vector-current octets, and also those of pseudovector-current octets, and are decomposed into the $SU(3)$ representations **27**, **8**s, **8**a, and **I**. We have assumed that the Hamiltonian transforms as a member of an octet so that our result corresponds to neglecting spurions in the **27** (and **I**) representation. The present method relies only on the $SU(3)$ transformation property of the Hamiltonian and avoids the use of $SU(3)$ coefficients which give the amount of spurion in the representations.

4. NONLEPTONIC DECAYS OF HYPERONS

The Lee-Sugawara (L-S) relation, $\Lambda_--\sqrt{3}\Sigma_0{}^++2\Xi_-{}^-=0$, has been derived on the basis of the invariance of the weak Hamiltonian under R conjugation.[7] The L-S relation for S waves is obtained here in the framework of the Hamiltonian H^6 given in Eq. (1) without R conjugation.[8] Equation (4) reproduced with H^6 is

$$[Q^+,[Q^-,H^6]]-[Q^-,[Q^+,H^6]]=[[Q^+,Q^-],H^6]. \tag{15}$$

Take the matrix elements of Eq. (15) with respect to the initial baryon A, final baryon B, and meson π, and open up the commutator so that $Q^\pm$ operates on the states.

When the change in strangeness S satisfies $S(A)-S(B)=-1$, as in nonleptonic decay of hyperons, comparison of the matrix elements of both sides of Eq. (15) for specific particles A, B, and π yield with the aid of Table I

$$\langle B\pi|H^6|A\rangle=\langle B\pi|iH^7|A\rangle. \tag{16}$$

This is an interesting result because H^6 has $CP=1$, whereas iH^7 has $CP=-1$, and yet the matrix elements are equal. This is possible because the states are not eigenstates of CP having baryon number 1. We obtain the following relations for $Q^\pm=U^\pm$, $I^\pm$, and $V^\pm$, using Eqs. (3) and (16):

$$\begin{aligned} \langle B\pi|[U^+,[U^-,H^6]]|A\rangle &= \tfrac{1}{2}\langle B\pi|H^6+iH^7|A\rangle\\ &= \langle B\pi|H^6|A\rangle, \end{aligned} \tag{17}$$

$$\langle B\pi|[I^+,[I^-,H^6]]|A\rangle=\tfrac{1}{4}\langle B\pi|H^6-iH^7|A\rangle=0, \tag{18}$$

$$\begin{aligned} \langle B\pi|[V^+,[V^-,H^6]]|A\rangle &= \tfrac{1}{4}\langle B\pi|H^6+iH^7|A\rangle\\ &= \tfrac{1}{2}\langle B\pi|H^6|A\rangle. \end{aligned} \tag{19}$$

Apply Eq. (17) to Λ_- and $\Xi_-{}^-$, where the superscripts denote the charge of the parent and subscripts that of the decay meson. Then, suppressing H^6, one has

$$\begin{aligned} \sqrt{3}\Lambda_--\Sigma_-{}^0 &= -\sqrt{2}\langle\Sigma^+\pi^-|\Xi^0\rangle+\sqrt{2}\langle pK^-|\Xi^0\rangle,\\ \sqrt{3}\Xi_-{}^-+\sqrt{2}\Sigma_-{}^- &= \langle\Sigma^0\pi^-|\Xi^-\rangle+\sqrt{2}\langle nK^-|\Xi^-\rangle. \end{aligned} \tag{20}$$

Next use Eq. (18) for the amplitudes that appear below and obtain

$$\begin{aligned} \Sigma_0{}^+ &= (\Sigma_+{}^+-\Sigma_-{}^-)/\sqrt{2}=-\Sigma_-{}^0,\\ \langle\Sigma^-\pi^+|\Xi^0\rangle-\langle\Sigma^+\pi^-|\Xi^0\rangle &+ \sqrt{2}\langle\Sigma^0\pi^-|\Xi^-\rangle=0,\\ \langle pK^-|\Xi^0\rangle+\langle nK^-|\Xi^-\rangle &= \langle n\bar{K}^0|\Xi^0\rangle, \end{aligned} \tag{21}$$

and then apply Eq. (19) to $\langle n\pi^+|\Sigma^+\rangle$ and get

$$\langle n\pi^+|\Sigma^+\rangle+\langle\Sigma^-\pi^+|\Xi^0\rangle=\langle n\bar{K}^0|\Xi^0\rangle. \tag{22}$$

The combination of Eqs. (20)–(22) yields

$$\Lambda_--\sqrt{3}\Sigma_0{}^++2\Xi_-{}^-=(\sqrt{\tfrac{2}{3}})[\langle nK^-|\Xi^-\rangle-\langle n\pi^+|\Sigma^+\rangle]. \tag{23}$$

Returning to Eqs. (1) and (16), H^6 is invariant under U-spin transformation $1\leftrightarrow 4$, $2\leftrightarrow 5$, $6\to 6$, $7\to -7$, and 3 and 8 going into each other which for baryons correspond to[9]

$$\begin{aligned} &\Sigma^+\leftrightarrow p,\quad \Sigma^-\leftrightarrow\Xi^-,\quad n\leftrightarrow\Xi^0,\\ &\Sigma^0\to\tfrac{1}{2}(\Sigma^0+\sqrt{3}\Lambda),\quad \Lambda\to\tfrac{1}{2}(\sqrt{3}\Sigma^0-\Lambda). \end{aligned} \tag{24}$$

This invariance can be seen from the coefficients d_{6ij} of H^6. On the other hand, iH^7 is invariant under the previous transformation with the following sign changes: $\Sigma^+\to -p$, $\Sigma^-\to -\Xi^-$, and $n\to -\Xi^0$.

Further, time-reversal invariance implies the following crossing relation for P- and S-wave amplitudes:

$$\langle B\pi|A\rangle=\langle A\pi^*|B\rangle. \tag{25}$$

If we assume invariance of H^6 under the product of parity operation P and U-spin transformation Eq. (24) and apply this product $P\times U$ to the S-wave amplitude $\langle n\bar{K}^0|\Xi^0\rangle_S$, we find with the aid of Eq. (25)

$$\langle n\bar{K}^0|\Xi^0\rangle_S=-\langle n\bar{K}^0|\Xi^0\rangle_S=0,$$

so that from Eq. (22), we find for the S-waves,

$$\langle n\pi^+|\Sigma^+\rangle_S=-\langle\Sigma^-\pi^+|\Xi^0\rangle_S=\langle nK^-|\Xi^-\rangle_S. \tag{26}$$

[5] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

[6] Y. T. Chiu and J. Schechter, Nuovo Cimento **47**, 214 (1967).

[7] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964).

[8] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964); S. P. Rosen, Phys. Rev. **137**, B431 (1965).

[9] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963).

Equations (23) and (26) lead to the L-S relation for S waves (parity-violating case),

$$\Lambda_- - \sqrt{3}\Sigma_0^+ + 2\Xi_-^- = 0. \quad (27)$$

We cannot obtain the L-S relation for the P-wave amplitudes in the current-current form of Hamiltonian because Eq. (26) does not follow from the previous transformations. For the $SU(3)$ amplitudes of the form $\langle B\pi|AH^6\rangle$, Eq. (25) is equivalent to the requirement $A_{8as}=A_{8sa}$, and Eq. (26) to $A_{27}=A_{8s}=A_{8a}=0$ and $A_{10}=-A_{10}{}^*$. The L-S relation is valid with no restrictions on the $SU(3)$ amplitudes except $A_{8as}=A_{8sa}$, so that Eq. (26) is a severe restriction.[10]

5. WEAK ELECTROMAGNETIC DECAY OF HYPERONS

Let us consider the decay mode $A\rightarrow B+\gamma$ and assume that the weak vertex $A-B$ transforms as H^6. We find as in Sec. 4 for $S(A)-S(B)=-1$, the Eqs. (16)–(19) without the pion in the final state.

Equation (17) (without the pion) yields[11]

$$\begin{aligned} 2\langle\Sigma^0\gamma|\Xi^0\rangle &= \langle n\gamma|\Sigma^0\rangle+\sqrt{3}\langle n\gamma|\Lambda\rangle, \\ 2\langle\Lambda\gamma|\Xi^0\rangle &= \sqrt{3}\langle n\gamma|\Sigma^0\rangle-\langle n\gamma|\Lambda\rangle. \end{aligned} \quad (28)$$

When Eq. (19) (without the pion) is applied to $\langle B|H^6|A\rangle$, or rather to $\langle B\gamma|H^6|A\rangle$ which is obtained by coupling the weak vertex to the electromagnetic field operator,[12] we find for $\Sigma^+\rightarrow p+\gamma$ and $\Lambda\rightarrow n+\gamma$, suppressing H^6,

$$\sqrt{2}\langle p\gamma|\Sigma^+\rangle+\langle\Sigma^0\gamma|\Xi^0\rangle+\sqrt{3}\langle\Lambda\gamma|\Xi^0\rangle=0, \quad (29)$$

$$\sqrt{2}\langle\Sigma^-\gamma|\Xi^-\rangle+\langle n\gamma|\Sigma^0\rangle+\sqrt{3}\langle n\gamma|\Lambda\rangle=0. \quad (30)$$

The result of carrying out the U-spin transformation from Eq. (24) to Eqs. (29) and (30) is

$$\langle p\gamma|\Sigma^+\rangle=-\sqrt{2}\langle n\gamma|\Sigma^0\rangle, \quad (31)$$

$$\langle\Sigma^-\gamma|\Xi^-\rangle=-\sqrt{2}\langle\Sigma^0\gamma|\Xi^0\rangle. \quad (32)$$

The relations (28)–(32) are obtained from the assumption that the weak vertex A-B transforms like H^6 and are valid for parity-conserving and parity-violating amplitudes.

6. SEMILEPTONIC DECAYS OF HYPERONS

It is assumed that the baryon part of the semileptonic decays transforms as H^4. Equation (4) reproduced for H^4 is

$$[Q^+,[Q^-,H^4]]-[Q^-,[Q^+,H^4]]=[[Q^+,Q^-],H^4]. \quad (33)$$

We find as in Sec. 4 for $S(A)-S(B)=-1$,

$$\langle B|H^4|A\rangle=\langle B|iH^5|A\rangle, \quad (34)$$

so that

$$\begin{aligned} \langle B|[U^+,[U^-,H^4]]|A\rangle &= \tfrac{1}{4}\langle B|H^4+iH^5|A\rangle \\ &= \tfrac{1}{2}\langle B|H^4|A\rangle, \end{aligned} \quad (35)$$

$$\begin{aligned} \langle B|[I^+,[I^-,H^4]]|A\rangle &= \tfrac{1}{4}\langle B|H^4+iH^5|A\rangle \\ &= \tfrac{1}{2}\langle B|H^4|A\rangle. \end{aligned} \quad (36)$$

When Eq. (35) is applied to $\Lambda\rightarrow pe^-\bar{\nu}$ and $\Sigma^-\rightarrow ne^-\bar{\nu}$ one obtains, suppressing H^4 and the particles $e^-\bar{\nu}$,

$$\sqrt{3}\langle p|\Lambda\rangle+\sqrt{2}\langle\Sigma^+|\Xi^0\rangle-\langle p|\Sigma^0\rangle=0, \quad (37)$$

$$\sqrt{3}\langle\Lambda|\Xi^-\rangle+\sqrt{2}\langle n|\Xi^-\rangle-\langle\Sigma^0|\Xi^-\rangle=0. \quad (38)$$

Similarly, when Eq. (36) is used, one gets

$$\langle n|\Sigma^-\rangle=\sqrt{2}\langle p|\Sigma^0\rangle, \quad (39)$$

$$\langle\Sigma^+|\Xi^0\rangle=\sqrt{2}\langle\Sigma^0|\Xi^-\rangle. \quad (40)$$

The combination of Eqs. (37)–(40) leads to

$$\sqrt{2}\langle p|\Lambda\rangle+\sqrt{3}\langle n|\Sigma^-\rangle+2\sqrt{2}\langle\Lambda|\Xi^-\rangle=0, \quad (41)$$

which is consistent with the Cabibbo theory[13] when the electron and antineutrino are added to the final state.

7. OFF-MASS-SHELL AMPLITUDE

In the previous sections, all the relations were obtained on the mass shell. The present formalism is now combined with the technique of taking a pion off the mass shell which leads to some new insight among sum rules.

Consider the amplitude $\langle B\pi^-|H^6|A\rangle$ and reduce the π^- of mass μ in conjunction with the equal-time commutators Eq. (3) and the hypothesis of partial conservation of axial-vector current (PCAC)

$$\begin{aligned} \partial_\mu A_\mu{}^i &= c\mu^2\phi^i, \quad i=1,2,3 \\ c &= M_N g_A/g_r. \end{aligned} \quad (42)$$

We obtain, with the aid of Eqs. (3), (34), and (42),

$$\begin{aligned} \lim_{q(\pi^-)\rightarrow 0}\langle B\pi^-|H^6|A\rangle &= (1/\sqrt{2}c)\langle B|[Q_5{}^1+iQ_5{}^2,H^6]|A\rangle \\ &= (1/\sqrt{2}c)\langle B|[Q^1+iQ^2,H^6]|A\rangle \\ &= (1/c)\langle B|[I^+,H^6]|A\rangle \\ &= (1/2\sqrt{2}c)\langle B|H^4+iH^5|A\rangle \\ &= (1/\sqrt{2}c)\langle B|H^4|A\rangle. \end{aligned} \quad (43)$$

The normalization factors and momenta of the particles are suppressed. Note that because of our assumption that H^6 is of the current-current form, Eq. (3) holds and the steps on the right-side hand of Eq. (43) can be taken. Equation (43) gives only the S-wave or parity-violating amplitude for nonleptonic decays which cor-

[10] The amplitudes $\langle n\bar{K}^0|\Xi^0\rangle_S=\langle n\pi^+|\Sigma^+\rangle_S=\langle nK^-|\Xi^-\rangle_S=0$ in Ref. 2, so that Eq. (26) gives no restriction in the model of Ref. 2.

[11] R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965).

[12] K. Tanaka, Phys. Rev. **140**, B463 (1965). A misprint of the coefficient of $\bar{\Lambda}n$ of Eq. (4), $1/\sqrt{3}$, should be corrected to $1/\sqrt{6}$. Equations (28)–(32) are consistent with Eq. (4) of this reference.

[13] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

responds to the parity-conserving amplitude in photonic decays.[2] The P-wave amplitude can also be obtained as in Ref. 2.

We start from the following relations of Eq. (21):

$$\langle p\pi^0|H^6|\Sigma^+\rangle=[\langle n\pi^+|H^6|\Sigma^+\rangle-\langle n\pi^-|H^6|\Sigma^-\rangle]/\sqrt{2}, \tag{44}$$

$$-\langle p\pi^-|H^6|\Sigma^0\rangle=[\langle n\pi^+|H^6|\Sigma^+\rangle-\langle n\pi^-|H^6|\Sigma^-\rangle]/\sqrt{2}; \tag{45}$$

and reduce the pions from the amplitudes using Eq. (43) and Table I:

$$\lim_{q(\pi^0)\to 0}\langle p\pi^0|H^6|\Sigma^+\rangle=(1/c)\langle p|[Q^3,H^6]|\Sigma^+\rangle=-(1/2c)\langle p|H^6|\Sigma^+\rangle,$$

$$\lim_{q(\pi^+)\to 0}\langle n\pi^+|H^6|\Sigma^+\rangle=(1/\sqrt{2}c)\langle p|H^6|\Sigma^+\rangle+(1/c)\langle n|H^6|\Sigma^0\rangle, \tag{46}$$

$$\lim_{q(\pi^-)\to 0}\langle n\pi^-|H^6|\Sigma^-\rangle=-(1/c)\langle n|H^6|\Sigma^0\rangle.$$

Combining (44) and (46), one finds for parity-conserving amplitudes

$$\langle p|H^6|\Sigma^+\rangle=-\sqrt{2}\langle n|H^6|\Sigma^0\rangle \tag{47}$$

or

$$\langle p\gamma|\Sigma^+\rangle=-\sqrt{2}\langle n\gamma|\Sigma^0\rangle,$$

which is Eq. (31) for parity-conserving decays, obtained under the assumption that the weak vertex transforms as H^6. One also notes that Eqs. (46) and (47), that for S waves,[14]

$$\lim_{q(\pi^+)\to 0}\langle n\pi^+|H^6|\Sigma^+\rangle=0. \tag{48}$$

We next express Eq. (45) in terms of the matrix elements of H^4, as in Eq. (43), and obtain

$$\sqrt{2}\langle p|H^4|\Sigma^0\rangle=\langle n|H^4|\Sigma^-\rangle,$$

which is Eq. (39) of semileptonic decays.

In order to relate the L-S relation for S waves to sum rules in photonic decays and semileptonic decays, combine Eqs. (21), (27), and (48);

$$\Lambda_-+(\sqrt{\tfrac{3}{2}})\Sigma_-^-+2\Xi_-^-=0. \tag{49}$$

[14] H. Sugawara, Phys. Rev. Letters **15**, 879, 997 (1965); M. Suzuki, *ibid.* **15**, 986 (1965).

When the pion is reduced in (49) using (43),

$$\langle n|H^6|\Lambda\rangle-\sqrt{3}\langle n|H^6|\Sigma^0\rangle+2\langle\Lambda|H^6|\Xi^0\rangle=0, \tag{50}$$

$$\sqrt{2}\langle p|H^4|\Lambda\rangle+\sqrt{3}\langle n|H^4|\Sigma^-\rangle+2\sqrt{2}\langle\Lambda|H^4|\Xi^-\rangle=0. \tag{51}$$

Equation (50) is the second sum rule (28) of photonic decays for the parity-conserving case when the photon in the final state is suppressed, and Eq. (51) is the sum rule (41) of semileptonic decays for the parity-conserving case.

It has been shown that in the framework of the symmetry-breaking Hamiltonians of the current-current form and pions of zero four-momenta, the triangle relations (44) and (45) of nonleptonic decays for S waves have a common origin with the photonic decay relation (31) and the semileptonic decay relation (39), respectively. Also, the L-S relation for S waves is intimately related to the sum rules (28) of photonic decays and (41) of semileptonic decays.

8. SUMMARY

It is proposed that all symmetry-breaking interactions are of the current-current form. A method of calculation suitable for obtaining sum rules for such Hamiltonians is developed. It relies on the operators $Q^\pm$ that transform one state into another and thus relate various amplitudes.

The procedure is roughly to assume that some observable transforms as a component of the symmetry-breaking Hamiltonian H^i and take its matrix element with respect to initial and final states. The matrix element is expressed in terms of the matrix element of the double commutator of the operators $Q^\pm$ with H^i. All the amplitudes that are related to the original amplitude are obtained with the aid of Table I. The unphysical amplitudes that necessarily appear are re-expressed in terms of physical amplitudes by repeating the procedure above and symmetry property of H^i are fed into the relations. In other words, sum rules among amplitudes which are due to H^i flow from the operations in Table I in a straightforward manner.

The sum rules do not go beyond $SU(3)$ with H^i of the current-current form. There is a possibility that the commutation relations (3) on which the present method is based may be valid even if the $SU(3)$ symmetry of the strong interaction is badly broken.

The method is expounded by showing how various sum rules can be obtained in a simple way. Then, it is combined with the technique of reducing a pion, and it was shown that the triangle relation and the L-S relation of nonleptonic decays correspond to sum rules of photonic decay amplitudes and semileptonic decay amplitudes of hyperons.