Suggested Boson-Lepton Pair Couplings and the Anomalous Magnetic Moment of the Muon*

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The contributions to the anomalous magnetic moment of the muon due to possible couplings of scalar and vector bosons to lepton pairs are calculated to second order. From the results obtained and the comparison between the experimental and theoretical values of $\frac{1}{2}(g-2)_\mu$, we find new stringent tests of several theories which have been recently put forward.

THERE have been recent developments in the measurement of the anomalous magnetic moment of the muon. The most precise experimental result which has been reported is[1]

$$\kappa_\mu \equiv \left(\frac{g-2}{2}\right)_{\mu^-} = (11666 \pm 5) \times 10^{-7}. \qquad (1)$$

It is the purpose of this note to discuss how this measurement yields new stringent tests on several theories which have been recently put forward.

The possible contributions to κ_μ have been summarized recently by Kinoshita.[2] The theoretical prediction is[3–7]

$$(\kappa_\mu)_{\rm th} = (11656.3 \pm 2.0) \times 10^{-7} \text{ (limit of error)}. \qquad (2)$$

The error here is a combined upper limit from uncertainties in the value of the fine structure constant α,[4] the sixth-order radiative corrections[5]; hadronic contributions to the photon propagator[6]; and the weak contribution.[7]

We have calculated the second-order contribution to κ_μ due to a possible vector or scalar exchange of mass M and minimal coupling constant f to lepton pairs. Two different methods were used: the usual Feynman rules and a dispersion-relation technique. In the latter case, we write a dispersion relation for the magnetic moment form factor $F_2(q^2)$ and calculate the absorptive amplitude from the unitarity condition, using the $\mu^+\mu^-$ intermediate state only. The result is[8] ($\lambda \equiv 4m_\mu^2/M^2$)

$$\delta\kappa_\mu \equiv F_2(0) = \frac{f^2}{8\pi^2}\int_0^1 dz \frac{P(z)}{z^2+(1-z)M^2/m_\mu^2} \equiv \frac{f^2}{8\pi^2}A(\lambda), \qquad (3)$$

with

$$P(z) = z^2(2-z) \qquad \text{(scalar)}$$
$$= 2z^2(1-z), \quad \text{(vector)}$$

$$A(\lambda) = \tfrac{3}{2} - 4/\lambda + (2/\lambda^2)(3\lambda-4)\ln(\tfrac{1}{4}\lambda) + (2/\lambda^2)(\lambda^2-5\lambda+4)\phi(\lambda) \quad \text{(scalar)}$$
$$= 1 - 8/\lambda + (8/\lambda^2)(\lambda-2)\ln(\tfrac{1}{4}\lambda) + (2/\lambda^2)(\lambda^2-8\lambda+8)\phi(\lambda), \quad \text{(vector)} \qquad (4)$$

$$\phi(\lambda) = \frac{-1}{(1-\lambda)^{1/2}} \ln\frac{1+(1-\lambda)^{1/2}}{1-(1-\lambda)^{1/2}} \quad (\lambda<1)$$
$$= \frac{1}{(\lambda-1)^{1/2}} \cos^{-1}\left(\frac{2-\lambda}{\lambda}\right) \quad (\lambda>1).$$

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[1] J. Bailey *et al.*, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Acceleration Center, SLAC, 1967 (unpublished). The previous value was $\kappa_\mu = (1165 \pm 3) \times 10^{-6}$. See F. J. M. Farley *et al.*, Nuovo Cimento **44**, 281 (1966). The error limits correspond to 1 standard deviation.

[2] T. Kinoshita, in the Summer School of Theoretical Physics, Cargèse, 1967 (unpublished).

[3] J. Schwinger, Phys. Rev. **75**, 1912 (1949); A. Peterman, Helv. Phys. Acta **30**, 407 (1957); C. M. Sommerfield, Phys. Rev. **107**, 328 (1957); Ann. Phys. (N. Y.) **5**, 26 (1958); H. Suura and E. H. Wichman, Phys. Rev. **105**, 1930 (1957); A. Peterman, *ibid.* **105**, 1931 (1957); H. H. Elend, Phys. Letters **21**, 720 (1966); M. V. Terent'ev, Zh. Eksperim. i Teor. Fiz. **43**, 619 (1962) [English transl.: Soviet Phys.—JETP **16**, 444 (1963)].

[4] We have used the solid-state value $\alpha^{-1} = 137.0359$ [see W. H. Parker, B. N. Taylor, and D. N. Langenberg, Phys. Rev. Letters **18**, 287 (1967)] but have allowed for a 25 parts-per-million error to cover values obtained from other experiments. For a review see D. R. Yennie, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Acceleration Center, SLAC, 1967 (unpublished).

[5] The radiative corrections have been calculated through orders $(\alpha/\pi)^3[\ln(m_\mu/m_e)]^2$ and $(\alpha/\pi)^3[\ln(m_\mu/m_e)]$ but not $(\alpha/\pi)^3$; T. Kinoshita, Nuovo Cimento **51B**, 141 (1967); A. Peterman (to be published); S. D. Drell and J. S. Trefil (unpublished).

[6] C. Bouchiat and L. Michel, J. Phys. Radium **22**, 121 (1961); see also L. Durand, Phys. Rev. **127**, 441 (1962); **129**, 2935 (1963); and more recently, T. Kinoshita and R. J. Oakes, Phys. Letters **25B**, 143 (1967).

[7] S. J. Brodsky and J. D. Sullivan, Phys. Rev. **156**, 1644 (1967); T. Burnett and M. J. Levine, Phys. Letters **24B**, 467 (1967); R. A. Shaffer, Phys. Rev. **135**, B187 (1964). Since these corrections are logarithmically dependent on a cutoff, we have allowed for a factor of 2 in estimating the weak contribution to κ_μ.

[8] The expression for vector exchange has been given by Berestetskii, Krokhin, and Khlebnikov (see Ref. 11). However, there appears to be an error in their final expression corresponding to our $A(\lambda)$. Our result for scalar exchange disagrees with a previous calculation by W. S. Cowland, Nucl. Phys. **8**, 397 (1958).

The asymptotic behavior of $\delta\kappa_\mu$ for $M^2 \gg m_\mu^2$ is

$$\delta\kappa_\mu = \frac{f^2}{8\pi^2}\frac{m_\mu^2}{M^2}\times\begin{cases}\ln(M^2/m_\mu^2) - 7/6 + (3m_\mu^2/M^2) \\ \quad \times\ln(M^2/m_\mu^2) + (57/4)m_\mu^2/M^2 \\ \quad\quad + \cdots \quad \text{(scalar)} \\ \frac{2}{3} - (2m_\mu^2/M^2)\ln(M^2/m_\mu^2) \\ \quad + (25/6)m_\mu^2/M^2 + \cdots \quad \text{(vector)};\end{cases} \quad (5)$$

and for $M^2 \ll m_\mu^2$

$$\delta\kappa_\mu = \frac{f^2}{4\pi}\times\begin{cases}\dfrac{3}{4\pi} + \dfrac{1}{(4m_\mu^2/M^2 - 1)^{1/2}} + \cdots \quad \text{(scalar)} \\ \dfrac{1}{2\pi} + \dfrac{1}{(4m_\mu^2/M^2 - 1)^{1/2}} + \cdots \quad \text{(vector)}.\end{cases} \quad (6)$$

The standard use of the comparison between the experimental value of κ_μ and $(\kappa_\mu)_{\text{th}}$ has been to check the validity of quantum electrodynamics at high momentum transfers. Assuming a modification in the photon propagator of the form[9,10]

$$1/q^2 \to 1/q^2 - 1/(q^2 - \Lambda^2) = (1/q^2)\Lambda^2/(\Lambda^2 - q^2), \quad (7)$$

one is led to a negative correction[11]

$$\delta\kappa_\mu/\kappa_\mu = -\tfrac{2}{3}m_\mu^2/\Lambda^2 + O(m_\mu^4/\Lambda^4). \quad (8)$$

On the other hand, from the comparison between (1) and (2), we have

$$(\kappa_\mu - (\kappa_\mu)_{\text{th}})/\kappa_\mu = (0.8 \pm 0.4)\times 10^{-3}; \quad (9)$$

hence a lower limit on Λ is obtained:

$$\Lambda > 40m_\mu \sim 4.5 \text{ BeV}, \quad \text{(3 standard-deviation effect)} \quad (10)$$

which is the best limit to be placed yet on the validity of quantum electrodynamics.

It has been recently shown that the possible identity between hadronic currents and corresponding vector and axial-vector field operators[12] leads to a specific set of commutation relations, the so-called algebra of fields,[13] which is simpler than the usual current algebra.[14] The possibility that the leptonic part of the electromagnetic current also satisfies the algebra of fields has been discussed by Lee and Zumino.[15] Such a possibility has been demonstrated, explicitly, within the context of a model which postulates the existence of a direct coupling of lepton pairs to a new (hypothetical) neutral vector boson (B^0). This coupling has precise experimental consequences. Thus, e.g., it has been shown by Lee and Zumino[15] that from its possible contribution to the scattering of charged leptons, and the present knowledge on electron-electron scattering experiments,[16] one expects

$$\left(\frac{f_B^2}{4\pi}\right)\Big/ M_B^2 < \frac{\alpha}{(0.76\text{ BeV})^2} \sim \frac{1}{(9\text{ BeV})^2}, \quad (11)$$

(95% confidence)

where M_B denotes the mass of the B^0 boson and f_B its coupling constant to lepton pairs.

A more stringent upper limit on $(f_B^2/4\pi)/M_B^2$ can be obtained from the possible contribution of B^0 exchange to the anomalous magnetic moment of the muon. Indeed, from the result obtained in Eq. (5) (vector case) and using Eq. (9), we are led to the following 95% confidence limit:

$$\frac{f_B^2/4\pi}{M_B^2} < \frac{2\pi\times 2.0\times 10^{-6}}{m_\mu^2[\frac{2}{3} + O(m_\mu^2/M_B^2)]} \sim \frac{1}{(24\text{ BeV})^2}. \quad (12)$$

On the other hand, the discrepancy between the values in Eqs. (1) and (2) could be regarded as indicative of the existence of the B^0 boson, with mass and coupling constant such that $(f_B^2/4\pi)/M_B^2 \simeq 1/(33\text{ BeV})^2$.

The above analysis ignores possible modifications of lepton form factors due to the existence of the B^0. In one possible model,[15] which is analogous to the theory of Ref. 12 for the isovector current, the lepton current $J_\nu^\gamma \equiv (M_B^2/f_B)B_\nu$ obeys the (renormalized) equation of motion of a vector field:

$$\partial^\mu(\partial_\mu B_\nu - \partial_\nu B_\mu) + M_B^2 B_\nu = J_\nu^B. \quad (13)$$

The lepton form factors are then spin projections of

$$F_\nu \equiv \langle p|J_\nu^\gamma|p+q\rangle = [M_B^2/(M_B^2 - q^2)] \times\langle p|J_\nu^B|p+q\rangle, \quad (14)$$

where $|p\rangle$ and $|p+q\rangle$ are electron (or muon) states. In an alternative model,[15] photons do not couple directly to the B field. The source of the electromagnetic field and also the unrenormalized B field is $S_\nu = \bar\psi_e\gamma_\nu\psi_e + \bar\psi_\mu\gamma_\nu\psi_\mu$. In this case, one finds for the lepton form factor

$$F_\nu \equiv \langle p|S_\nu|p+q\rangle = [M_B^2/(M_B^2 - q^2)] \times(1 - q^2/M_0^2)\langle p|J_\nu^B|p+q\rangle, \quad (15)$$

where M_0 is the bare mass of the B. It is reasonable to assume that $\langle p|J_\nu^B|p+q\rangle$ does not increase for large spacelike q^2. Also, we shall take $M_0^2 \gg M_B^2$; (14) and (15) then lead to experimental restrictions on M_B *independent*

[9] We should remember, however, that this type of modification is not totally uncontroversial because of the ghostlike nature of the modified propagator.

[10] We work in the metric $g^{00} = 1$, $g^{ij} = -\delta_{ij}$, $i, j = 1, 2, 3$.

[11] V. B. Berestetskii, O. N. Krokhin, and A. K. Khelbnikov, Zh. Eksperim. i Teor. Fiz. **30**, 788 (1956) [English transl.: Soviet Phys.—JETP **3**, 761 (1956)].

[12] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

[13] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[14] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[15] T. D. Lee and B. Zumino, Phys. Rev. **163**, 1667 (1967).

[16] W. C. Barber, B. Gittelman, G. K. O'Neill, and B. Richter, Phys. Rev. Letters **16**, 1127 (1966).

of the magnitude of f_B:

(i) $e-e$ scattering: The one-photon exchange matrix element is modified by a factor $[M_B^2/(M_B^2-q^2)]^2$. From Ref. 16, $M_B>0.76$ BeV (95% confidence).

(ii) $\frac{1}{2}(g-2)_\mu$: From Eqs. (7)-(9), $M_B>6$ BeV (99% confidence).[17]

(iii) $e-p$ scattering: The presence of an electron form factor leads to a $1/q^6$ (or faster) fall off for the observed form factors when $q^2 \gtrsim M_B^2$. The present experimental results[18] imply a limit on $M_B \gtrsim 3$ BeV.

The anomalous magnetic moment measurement also places limits on two other (hypothetical) bosons which have been recently proposed to explain the $-\frac{1}{4}$-MHz discrepancy between theory and experiment in the Lamb shift[19,20] (the splitting $2S_{1/2}-2P_{1/2}$ in H and D).

As Yennie has emphasized,[4] the Lamb-shift discrepancy could be reconciled by the presence of a new interaction that is weak, repulsive, and long ranged. If the interaction is due to a scalar-meson coupling (S^0), as proposed by Yennie and Farley, then its effect would be of higher order in $e-p$ scattering; also $\pi^0 \to S^0+\gamma$ is forbidden. Assuming a universal coupling of S^0 to all charged particles $f^2/4\pi \equiv \alpha'$, we find from Eq. (6) a possible contribution to the anomalous magnetic moment $\delta\kappa_\mu=(3\alpha'/4\pi)[1+O(M_S/2m_\mu)]$; hence, using Eq. (9), $\alpha'/\alpha<1.2\times10^{-3}$ (95% confidence). On the other hand, the 2-standard-deviation discrepancy between theory and experiment for κ_μ could be regarded as evidence for the existence of the scalar interaction. The values of M_S and α' needed to fit both the Lamb shift ($\alpha' m_e^2/M_S^2 \cong 10^{-8}$) and κ_μ are $M_S \sim 10$ MeV and $\alpha'/\alpha \sim 6\times10^{-4}$.

We notice that a vector interaction with the above couplings and mass is ruled out because of the appreciable Dalitz pair rate which is predicted in $\pi^0 \to \gamma+V^0 \to \gamma+e^+-e^-$.

It is, however, possible that there is a new vector meson V^0 which couples photons only to hadrons (as ρ^0) with mass $\lesssim 70$ MeV and small coupling constant ϵ.[20] In this case, $\pi^0 \to \gamma+e^++e^-$ would display a very narrow resonance in the pair distribution. Also, the form factors obtained from electron-proton scattering would be modified, for small q^2, by a term $\epsilon m_V^2/(m_V^2-q^2)$. As seen in Fig. 1 of Ref. 20, the resulting modifications of the form factors are not excluded by experiment for $m_V \lesssim 70$ MeV; in fact, the data show a hint of a change in slope at small q^2, although this may well be due to a common systematic error in the analysis of the experiments. The effect of the proton electric form factor $G_{Ep}(q^2=0)$ on the Lamb shift in H is

$$\Delta E = \tfrac{1}{2}(Z\alpha)^4 m_e [m_e^2 G_{Ep}'(q^2=0)]. \qquad (16)$$

The Lamb-shift discrepancy would be resolved if $G_{Ep}'(q^2=0)$ were tripled; i.e., if $\epsilon/m_V^2 \cong 0.02/(70\text{ MeV})^2$.

The corresponding contribution of this V^0 coupling to the anomalous magnetic moment of the muon is $\delta\kappa_\mu/\kappa_\mu=\epsilon^2 A_V(\lambda)$, where $\lambda=4m_\mu^2/m_V^2$, and A_V is defined in Eq. (4). For $m_V \sim 70$ MeV, this gives a correction to $(\kappa_\mu)_{\text{th}}$ within the limits given in Eq. (9).

We note that the V^0 coupling also induces a small change in the ground-state hfs splitting in H. It is, however, ~ 7 parts per million, which is not seriously inconsistent with experiment.

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[17] We assume here $f_B^2 \ll 1$ and neglect B exchange. We note that a modification of the muon propagator would not affect the dispersion calculation of $\frac{1}{2}(g-2)_\mu$. See J. A. McClure and S. D. Drell, Nuovo Cimento **57**, 1638 (1965); N. Kroll, *ibid.* **45**, 65 (1966).

[18] R. Taylor, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center SLAC, 1967 (unpublished).

[19] G. W. Erickson and D. R. Yennie Ann. Phys. (N. Y.) **35**, 271 (1965); **35**, 447 (1965); M. F. Soto, Jr., Phys. Rev. Letters **17**, 1153 (1966); A review of the experimental and theoretical work is given in D. R. Yennie, Ref. 4.

[20] R. C. Barrett, S. J. Brodsky, G. W. Erickson, and M. H. Goldhaber, Phys. Rev. **166**, 1589 (1968).