Discrete Flow Rates of the Helium Film*

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Measurements of the helium-film flow rates over a stainless steel substrate have been made using the heat-transport method of Bowers, Brewer, and Mendelssohn. The moving film from a reservoir is vaporized inside a copper box at the top of the stainless steel capillary in a closed system, by electrical heating. The vapor recondenses in the isothermal reservoir. On any day's run, the rate was constant and reproducible to within 1%. The complete range of flow rates observed at 1.2°K on separate days was about 50%. Seven discrete rates were in fact observed in 18 runs nearly equally spaced at intervals of 0.83×10^{-5} cm^2 sec^{-1}. At present, no theory predicts discrete flow rates.

RECENT reports[1,2] of film flow have shown that different rates are possible at a given temperature. In general, the separation between values was observed to be a constant which depended upon temperature. We wish to report some measured variations in flow rates over stainless steel which may be related to the effects reported in Refs. 1 and 2, using a "heat-transport" method first used by Bowers, Brewer, and Mendelssohn.[3] Most other workers have measured flow rates by observing the change of the helium level in a small beaker with time. We felt that the heat-transport method would provide an independent assessment which is necessary in view of the many different results available.

The form of the measuring device is shown schematically in Fig. 1. It consists of a small cylindrical copper box A closed except for the thin-walled stainless steel tube B leading to another smaller copper box C. Tube B and box C are surrounded by a good vacuum 10^{-6} Torr, while the lower end and walls of A are immersed in a constant-temperature helium bath. Helium is condensed into box A until it is about half full, through a fine capillary tube not shown. The film forms on the inside of the stainless steel tube B and inside the copper box C. The latter has a small heater H and carbon resistance thermometer T_1 mounted on it externally. When current flows through H, the joule heat is conducted through the copper head and causes the film in this region to evaporate. The vaporized film passes down the tube and condenses in the bulk helium in A and is immediately replaced in C by liquid-helium film flow. In the steady state, the power dissipated in the heater is given by $P=(L+ST)\dot{M}$, where P is the power, L the latent heat of bulk helium per gram, S the entropy per gram of liquid at temperature T, and $\dot{M}$ the mass flow per sec. Since the mass flow is given by $\dot{M}=\pi D\rho R_s$, where D is the inside diameter of the tube, ρ the density of bulk liquid, and R_s the film flow rate in cm^2 sec^{-1}, the flow rate can be expressed as $R_s=P/\pi D\rho(L+ST)$. As the applied electrical power increases, the film flow rate R_s increases and the temperature T of the copper box C remains constant except for a small temperature rise due to the Kapitza resistance. There is a critical power P_c and a corresponding critical film flow rate R_c, above which the thermometer T_1 on box C shows a marked temperature rise. Since P may be increased towards P_c arbitrarily slowly, it is easy to obtain precise values of R_c. It should be noted that this method permits flow rates between zero and R_c to be "switched on," contrasting with beaker methods[1,2] where the flow rates are critical or zero.

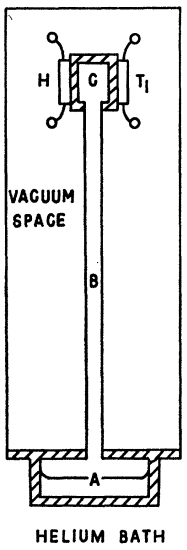

FIG. 1. ^{4}He film tube assembly. A: Bulk ^{4}He copper reservoir. B: Stainless steel tube. C: Copper head. H: Heater. T_1: Carbon resistance thermometer.

The first observation of interest was the extreme stability of the measured critical flow rate at a given temperature. Any variation in the flow rate was less than about $\pm1\%$. On no occasions were metastable flow rates or transitions to other levels recorded during a run, as reported by Harris-Lowe *et al.*[1] and Allen and Armitage.[2] This difference might be due to either the metal substrate or the method of measurement, although we know of no theoretical reason why the former should be responsible. In making the measurement, there will presumably be a third-sound transient generated as the power P to the heater H (Fig. 1)

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approaches the critical value P_c. It is conceivable that during this time any metastable film which may be present is reduced to a "basic" film having a minimum and constant flow rate. Furthermore, when the cryostat was cycled from a temperature below the λ point to a temperature above the λ point and back, the measured flow rate remained constant to within ±0.5% after each excursion. This was also the case when cycling to 4.2°K.

Our second observation of interest concerns the lack of reproducibility of the critical flow rate at a given temperature from run to run. Between runs, the apparatus was allowed to come to room temperature while the flow tube system shown in Fig. 1 was pumped continuously and maintained at a pressure of 10^{-6} Torr. The variations in flow rate from run to run can be seen in Fig. 2, showing the results at 1.2°K for 18 runs. This temperature has been chosen because the rate of change of flow rate with temperature is a minimum. Below 1.2°K the flow rate rises quite steeply, and above 1.2°K it falls off in proportion to the superfluid density. A

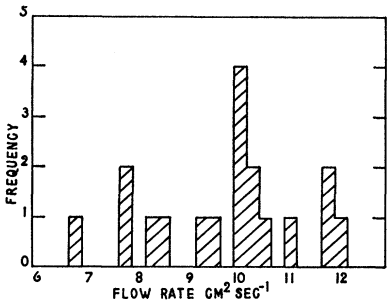

FIG. 2. Histogram of critical flow rates measured at 1.2°K; each frequency unit corresponds to one day's mean of ten observations.

surprising feature of the data is the wide range of flow rate observed (5.1×10^{-5} cm^2 sec^{-1}), considering the reproducibility within any given run. The mean flow rate is 9.9×10^{-5} cm^2 sec^{-1} and the most frequently occurring flow rate is 10.3×10^{-5} cm^2 sec^{-1}. Furthermore, the 18 runs gave seven discrete flow rates, roughly evenly spaced.

This may be more apparent in Fig. 3 where the same flow rates are plotted against an integer corresponding to the suspected flow rate level. Although the number of data points is small, it should be noted that only one flow rate is available per day's run and that each is the mean of many measurements and can be stated with confidence level of ±0.3%. The slope of the least-squares-fitted straight line in Fig. 3 gives an estimate of the spacing size ΔR_c, since the fitted line is testing the equation $R_{cni}=R_0+n\Delta R_c$, where n is the level number, R_0 a constant, and R_{cni} the ith critical flow rate associated with the nth level. The value found for ΔR_c was $(0.83\pm0.02)\times10^{-5}$ cm^2 sec^{-1}.

In order to see to what extent the flow rates cluster

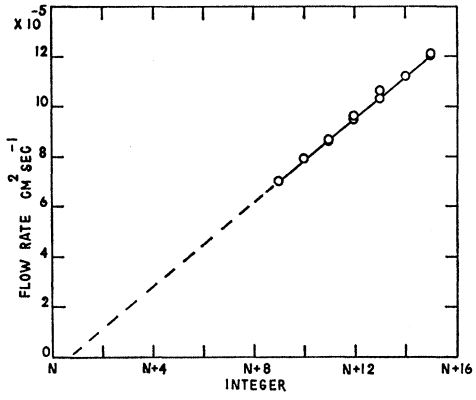

FIG. 3. Critical flow rates versus an integer corresponding to the suspected flow-rate level.

about the n values we formed a histogram of the deviations of all the observed flow rates R_{cni} from the least-squares-fitted straight-line flow-rate value $\bar{R}_{cn}$ at the nth level, the histogram including all seven levels. The tendency for the flow rates to peak about the selected n values is measured by the standard deviation σ_{expt} of this histogram. We found σ_{expt} to be 0.141×10^{-5} cm^2 sec^{-1}. In order to test the significance of this value the following statistical tests were made, to see if the observed flow rates could possibly be randomly distributed about a single mean flow rate.

Using the mean and standard deviations of the observed flow rates we generated sets of artificial data each containing 18 *random* flow rates. This task was done on an IBM 1620 computer. Each set of data was treated in exactly the same way as the experimental data and the standard deviations about the least-squares fits calculated. The value obtained was $\sigma_{\text{random}}=(0.198\pm0.022)\times10^{-5}$ cm^2 sec^{-1}. This is compared with $\sigma_{\text{expt}}=0.141\times10^{-5}$ cm^2 sec^{-1}.

We now made the hypothesis H_0: $\sigma_{\text{random}}=0.198\times10^{-5}$ cm^2 sec^{-1} and test it against the hypothesis H_1: $\sigma_{\text{random}}<0.198\times10^{-5}$ cm^2 sec^{-1}. In making these hypotheses we take the viewpoint that σ_{expt} is obtained by taking a sample of size $N=18$ runs from a population where the standard deviation is given by σ_{random}. Rejection of the hypothesis H_0 will therefore determine that σ_{expt} is significantly less than σ_{random} which in turn means that there is a marked grouping of the 18 flow rates about the chosen integer values.

The well-known statistical theorem tells us that ns^2/σ^2 possesses a χ^2 distribution with $n-1$ degrees of freedom, where s is the standard deviation of the sample containing n measurements taken from the population of standard deviation σ. In the present case, $ns^2/\sigma^2=9.14$. We find from χ^2 tables that there is a 90% probability that χ^2 would be greater than or equal to 9.31 and therefore we have little ground for retaining the hypothesis H_0. From this small sample test, one concludes that the chances of obtaining the flow-rate pattern shown in Fig. 2 from random measurements of a

single flow rate is about 10%. Furthermore it should be noted that σ_{expt} falls 2.6 standard deviations of σ_{random} from σ_{random} and the probability of this happening if $\sigma_{\text{expt}}=\sigma_{\text{random}}$ is about 1%. This implies that the flow rates are not randomly distributed about a single mean. The former small sample test gives a more pessimistic conclusion than the latter which assumes large sample statistics.

From the foregoing we conclude that there is a regular spacing in the observed flow rates, and from Fig. 3 we see that the value of the spacing is $(0.83\pm0.02)\times10^{-5}$ cm^2 sec^{-1}. Harris-Lowe *et al.*[1] and Allen and Armitage[2] have seen regularly spaced flow rates at a given temperature, changing repeatedly (nearly always to lower values) during single runs using the breaker flow method. They found spacings of 0.64×10^{-5} cm^2 sec^{-1} at 0.99°K and 0.5×10^{-5} cm^2 sec^{-1} at 1.19°K, respectively. (No errors were stated.) This seems to be a different but possibly related phenomenon. At present no theory predicts discrete flow rates. The weight of independent evidence in favor of discrete flow rates should stimulate theoretical activity.

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Mobility of a Charged Impurity in a Fermi Liquid*

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The low-temperature mobility of a charged impurity in a polarizable Fermi liquid is calculated. It is shown that the polarization effect is considerably larger than a typical hard-core effect, leading to a smaller low-temperature mobility.

INTRODUCTION

In this paper we present a calculation of the mobility of a charged impurity in a polarizable Fermi liquid. Such a calculation is appropriate for discussing the motion of ions or electrons in low-temperature liquid He^3. A general analysis of this problem would involve two primary considerations: a description of the Fermi liquid without the impurity, and a method of coupling the impurity to the liquid. We will not concern ourselves with the details of these two considerations, but rather start with a simple model which embodies such details in its parameters.

Under the simplest possible assumptions, we regard the Fermi liquid as a gas of noninteracting quasiparticles obeying Fermi statistics and possessing a spectrum

$$\epsilon(k)=\hbar^2k^2/2m. \tag{1}$$

The effect of the interactions between the particles is considered to be included in the effective mass m, which is not the He^3 mass m_3. From the Landau-Fermi liquid theory[1] and specific-heat measurements[2] on He^3, $m=3.08m_3$. Two other quantities will be of interest. For the zero-temperature Fermi gas, one defines a Fermi wave number k_F related to the particle density ρ through

$$\rho=k_F^3/3\pi^2 \tag{2}$$

and a Fermi energy

$$\epsilon_F=\hbar^2k_F^2/2m. \tag{3}$$

With a mass density[2] of 0.081 g/cc, one finds a number density $\rho=1.7\times10^{22}$ particles/cc, $k_F=0.78\times10^8$ cm^{-1}, and $\epsilon_F/\kappa=5m_3/$m°K, where κ is Boltzmann's constant.

For the impurities, we have the following picture in mind. First of all, their number density $\rho_i=N_i/\Omega$ is to be very small, so that impurities do not interact with each other. The actual impurity may be either an electron or an ion, and hence through its charge will be coupled to the liquid by means of polarizability effects. Several aspects of such mechanisms have been discussed in the literature,[3,4] and the resulting structures resemble bubbles for electrons and "snowballs" for ions. To avoid the difficulties associated with the specific structure of the impurity and its polarization cloud, we approximate the situation as follows. Each impurity will be considered a Boltzmann gas particle of mass M interacting with the Fermi liquid through a repulsive hard-core interaction and an attractive polarization interaction. Hence, part of the interaction between the impurity and the liquid is absorbed in the mass M, and the rest is approximated by a hard-core delta-function pseudopotential and a polarization effect.

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