with the quark-model value of α and vanishing mixing angle at $t=0$. In addition, the particularly simple version of the theory discussed in Sec. IV suggests that there may be zeros of G_E^n at experimentally accessible values of t. Although the form factor may be too small to permit the predicted changes of sign to be seen, nevertheless, it should be possible to distinguish between this theory and the energy-independent one.

Finally, the theory predicts

$$F_{K_0}'(0)=G_E^{n\prime}(0)\,,$$

whereas the energy-independent theory yields

$$F_{K_0}'(0)=0.6\ G_E^{n\prime}(0)\,.$$

It would be of interest if this quantity could be measured.

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Quark Model with Factorizability Assumption

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Characteristic features of the hadron-hadron elastic scattering at high energies are explained in the quark model with a factorizability assumption. The factorizability assumption is that the elastic amplitude $F(\theta)$ is assumed to be proportional to a product of the probability amplitude of each constituent quark scattered at an angle θ (and π-θ, if possible). Various relations for the differential cross sections are obtained, together with those for the total cross sections. For example, we have relations such as $\sigma_{K^-p}{}^T=\sigma_{K^-n}{}^T$, $\sigma_{\bar{p}p}{}^T-\sigma_{pp}{}^T\cong 5(\sigma_{pn}{}^T-\sigma_{pp}{}^T)$, $(d\sigma_{\pi^+p}/dt)(d\sigma_{K^-p}/dt)(d\sigma_{pp}/dt)=(d\sigma_{\pi^-p}/dt)(d\sigma_{K^+p}/dt)(d\sigma_{pn}/dt)$, and the conclusion that the ratio $(d\sigma_{\pi^+p}/dt)/(d\sigma_{K^+p}/dt)$ is θ-independent; these are essentially different from the results obtained from the usual quark model with the additivity assumption. At $\theta=90°$, the relations become especially simple, and experimental tests of our model seem to be feasible. For the backward scattering, our model also predicts several relations which are qualitatively in good agreement with experiments on pion-, kaon-, and nucleon-nucleon scattering.

I. INTRODUCTION

VARIOUS features of the hadron reactions at high energy have been explained to a considerable extent by the quark model. In particular, the so-called additivity assumption proposed by Lipkin and others succeeded in giving many interesting relations among the total cross sections for various hadron scatterings at high energies.[1] Some of these relations are indeed in good agreement with experimental data. Note, however, that the additivity assumption seems to be legitimate only in the forward direction, and the relations for the total cross sections are derived from those for the forward amplitudes by virtue of the optical theorem.

As for the differential cross sections at a finite angle, on the other hand, the results are not so remarkable, though a few phenomenological analyses are made by extending the additivity assumption to a finite but small angle.[2] In the additivity assumption, when a quark is scattered by some finite angle, accompanying quarks are assumed to be dragged along. It seems, however, to be difficult to extend this idea to the hadron scattering at an arbitrary angle without any new drastic assumption.

Recently, we introduced a new assumption, the factorizability assumption, to the usual quark model.[3] In the factorizability assumption, the probability amplitude of the hadron scattering at a c.m. scattering angle θ is described as a product of respective probability amplitude $G(\theta)$ of the consituent quark scattered at θ. When the exchange process of quarks is possible, the resultant probability amplitude is expressed as a summation over all such possibilities. Here $G(\theta)$ is the probability amplitude of an individual quark scattered at θ in the force field which is produced by all of the residual quarks participating in the reaction. $G(\theta)$ is, in general, different for baryon-baryon, meson-baryon, and meson-meson scattering. In this paper, we assume that $G(\theta)$ is the same for these processes as a first approximation.

As an example, let us consider proton-neutron (p-n) scattering. The basic quarks are denoted as usual as

[1] E. M. Levin and L. L. Frankfurt, Zh. Eksperim i Teor. Fiz. Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966); H. J. Lipkin, *ibid.* 16, 1015 (1966); C. H. Chan, Phys. Rev. 152, 1244 (1966).

[2] M. Bando, I. Fukui, Y. Takada, S. Wakaizumi, and T. Yoshida, Progr. Theoret. Phys. (Kyoto) 37, 128 (1967); H. J. Lipkin, F. Scheck, and H. Stern, Phys. Rev. 152, 1375 (1966).

[3] M. Kawaguchi, Y. Sumi, and H. Yokomi, Progr. Theoret. Phys. (Kyoto) 38, 1178 (1967); 38, 1183 (1967).

$\mathcal{P}$, $\mathcal{N}$, and λ, where $\mathcal{P}$ and $\mathcal{N}$ belong to the isodoublet with the strangeness $S=0$ and λ to the isosinglet with $S=-1$. In Fig. 1 are shown the possible diagrams for p-n scattering and the probability amplitudes associated with each diagram. Then, the elastic p-n scattering amplitude $F_{pn}(\theta)$ is proportional to $G^6(\theta)+4G^4(\theta)G^2(\pi-\theta)+G^2(\theta)G^4(\pi-\theta)$ in our model. For other elastic processes the scattering amplitude can be calculated in the same way, and various interesting relations for the differential cross sections are obtained as well as those for the total cross sections.

Under this assumption, we do not take into consideration the process is which a quark-antiquark pair is once annihilated and another pair subsequently created. Such a process may be considered to contribute dominantly only in the inelastic processes in which some quantum numbers other than the energy momentum are transferred, but the present experimental data seem to show that the cross sections for these inelastic processes are small and fall rapidly with increasing s, compared with those for the elastic processes.[4] Thus we may neglect the quark-pair annihilation-creation process to first order, and consequently we confine ourselves mainly to the elastic scattering of hadrons. Further approximation is made that all possible differences between quarks and antiquarks of different kinds are neglected. Moreover, spins of the particles concerned are entirely ignored, for simplicity. Hence we have only a single complex function $G(\theta)$ of s and t for a whole description of the elastic scattering of hadrons, apart from certain kinematical factors.

Expressions of the scattering amplitudes are given in Sec. II for the typical elastic processes. Section III deals with the relations for the total cross sections and the real-to-imaginary ratios of the forward amplitudes. The phase angle of $G(\theta)$ and the magnitude of the ratio $r(\theta)=G(\pi-\theta)/G(\theta)$ in the forward direction are determined. Relations among the differential cross sections are given in Sec. IV, together with discussions of the small-angle elastic scattering. Comparison with experimental data is also made in detail. In Sec. V, we concentrate our effort on the behavior of large-angle scattering. Our model predicts especially simple relations for the differential cross sections at $\theta=90°$. It is shown that some characteristic features of the backward scattering are easily explained in our model. Further remarks and discussions are given in Sec. VI.

II. EXPRESSIONS FOR ELASTIC SCATTERING AMPLITUDE

For p-p scattering at an angle θ, we have, first, one diagram in which all the quarks are scattered at θ;

[4] Strong suppression of the quark-pair creation process was already pointed out by J. Iizuka, Progr. Theoret. Phys. (Kyoto) Suppl. 37–38, 21 (1966); J. Iizuka, K. Okada, and O. Shito, Progr. Theoret. Phys. (Kyoto) 35, 1061 (1966). See also D. R. O. Morrison, Phys. Letters 22, 528 (1966); S. Okubo, *ibid.* 5, 165 (1963); M. Imachi, T. Matsuoka, K. Ninomiya, and S. Sawada, Progr. Theoret. Phys. (Kyoto) 38, 1198 (1967).

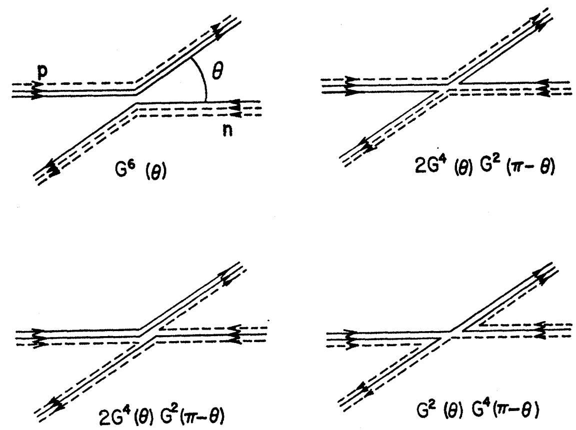

FIG. 1. Possible diagrams of the p-n elastic scattering in our model. The solid line shows the $\mathcal{P}$ quark and the dashed line the $\mathcal{N}$ quark. The contribution to the scattering amplitude from each diagram is also shown.

secondly, the number of processes in which one of the quark pairs is scattered at $\pi-\theta$ and the others at θ is five (four for the $\mathcal{P}$-quark pair and one for the $\mathcal{N}$-quark pair). Replacing θ by $\pi-\theta$, we have the same number: one for the process where all the quarks are scattered at $\pi-\theta$ and five where two of the quark pairs are scattered at $\pi-\theta$. Then the scattering amplitude $F_{pp}(\theta)$ is written as

$$F_{pp}(\theta)\propto G^6(\theta)[1+5r^2(\theta)+5r^4(\theta)+r^6(\theta)]$$
$$=G^6(\theta)[1+r^2(\theta)][1+4r^2(\theta)+r^4(\theta)], \quad (2.1)$$

where we put

$$r(\theta)=G(\pi-\theta)/G(\theta). \quad (2.2)$$

For p-n scattering, $F_{pn}(\theta)$ is already given in Sec. I:

$$F_{pn}(\theta)\propto G^6(\theta)[1+4r^2(\theta)+r^4(\theta)]. \quad (2.3)$$

Since, under our assumptions, any quark-pair annihilation and creation should never happen in $\bar{p}$-p scattering, we have

$$F_{\bar{p}p}(\theta)\propto G^6(\theta). \quad (2.4)$$

To obtain the exact amplitude F, a certain kinematical factor should be needed on the right-hand side of Eqs. (2.1), (2.3), and (2.4). It may, however, be regarded as common to all the cases of nucleon-nucleon and antinucleon-nucleon scattering. These circumstances may also be the case in meson-nucleon scattering.

In a similar way, one obtains the following expressions for meson-nucleon scattering:

$$F_{\pi^+p}(\theta)\propto G^5(\theta)[1+2r^2(\theta)], \quad (2.5)$$

$$F_{\pi^-p}(\theta)\propto G^5(\theta)[1+r^2(\theta)], \quad (2.6)$$

$$F_{K^+p}(\theta)\propto G^5(\theta)[1+2r^2(\theta)], \quad (2.7)$$

$$F_{K^-p}(\theta)\propto G^5(\theta), \quad (2.8)$$

$$F_{K^+n}(\theta)\propto G^5(\theta)[1+r^2(\theta)], \quad (2.9)$$

$$F_{K^-n}(\theta)\propto G^5(\theta). \quad (2.10)$$

From Eqs. (2.7)–(2.10) one can immediately see the following identities for the kaon-nucleon scattering:

$$F_{K^-p}=F_{K^-n}, \tag{2.11}$$

$$2F_{K^+n}=F_{K^+p}+F_{K^-p}. \tag{2.12}$$

Note that, for instance, for the pion-nucleon scattering one has the following relations in terms of the isotopic-spin states:

$$F_3\propto G^5(\theta)[1+2r^2(\theta)], \tag{2.13}$$

$$F_1\propto G^5(\theta)[1+\tfrac{1}{2}r^2(\theta)], \tag{2.14}$$

where the subscripts 3 and 1 denote the total isotopic spin $I=\frac{3}{2}$ and $\frac{1}{2}$, respectively.

III. RELATIONS FOR TOTAL CROSS SECTIONS

One can easily obtain the following relations for the total cross section by virtue of the optical theorem:

$$\sigma_{K^-p}{}^T-\sigma_{K^-p}{}^T=0, \tag{3.1}$$

$$\sigma_{K^-p}{}^T-\sigma_{K^+n}{}^T=\sigma_{K^+n}{}^T-\sigma_{K^+p}{}^T=\sigma_{K^-n}{}^T-\sigma_{K^+n}{}^T, \tag{3.2}$$

$$\sigma_{\pi^+p}{}^T/\sigma_{\pi^-p}{}^T=\sigma_{K^+p}{}^T/\sigma_{K^+n}{}^T. \tag{3.3}$$

The higher order of $r^2(0°)$ is safely neglected, as will be estimated below; then we get a few additional relations:

$$5(\sigma_{pn}{}^T-\sigma_{pp}{}^T)=\sigma_{\bar{p}p}{}^T-\sigma_{pp}{}^T, \tag{3.4}$$

$$\sigma_{K^+n}{}^T/\sigma_{K^-n}{}^T=\sigma_{\pi^+p}{}^T/\sigma_{\pi^-p}{}^T. \tag{3.5}$$

Some of these relations are in disagreement with experiment well outside experimental errors. In particular, the first two differences in (3.2) differ consistently by an order of magnitude from 6 to 20 BeV/c. The difference $\sigma_{K^-p}{}^T-\sigma_{K^+n}{}^T$ is characteristically about 4 mb, while the difference $\sigma_{K^+n}{}^T-\sigma_{K^+p}{}^T$ is a few tenths of a millibarn.[5]

Now we estimate the magnitude of $r^2(0°)$ under the assumption $\alpha_{\pi^+p}=\alpha_{\pi^-p}$ at high energy, for simplicity, where α is the real-to-imaginary ratio of the forward-scattering amplitude. Then the imaginary part of $r^2(0°)$ disappears, and one gets

$$\sigma_{\pi^+p}{}^T/\sigma_{\pi^-p}{}^T=(1+2r^2)/(1+r^2). \tag{3.6}$$

Using the experimental results, we get $r^2(0°)=-0.07$ to -0.05 for incident pion momenta 10 to 20 BeV/c.[5] Under the same assumption, more relations for the total cross sections are obtained as follows:

$$\sigma_{pp}{}^T/\sigma_{pn}{}^T=\sigma_{K^+n}{}^T/\sigma_{K^-n}{}^T, \tag{3.7}$$

$$(\sigma_{pn}{}^T/\sigma_{pp}{}^T)(\sigma_{K^+p}{}^T/\sigma_{K^-p}{}^T)=\sigma_{\pi^+p}{}^T/\sigma_{\pi^-p}{}^T. \tag{3.8}$$

The above assumption also leads to simple relations for the real-to-imaginary ratios of the various forward-scattering amplitudes as follows:

$$\alpha_{\pi^-p}=\alpha_{\pi^+p}=\cot 5\delta, \tag{3.9}$$

$$\alpha_{pp}=\alpha_{pn}=\alpha_{\bar{p}p}=\cot 6\delta, \tag{3.10}$$

where δ is the phase of $G(0°)$. If δ is chosen as 19°, the values of α_{π^-p} and α_{pp} are -0.09 and -0.45, respectively. These values explain the qualitative features of the experimental data for $\pi^\pm$-p and p-p scattering, but disagree with $\bar{p}$-p experiments.[6,7]

In our model, the difference between different meson (nucleon)-nucleon total cross sections is entirely due to the quantity $r^2(0°)$, but agreement with experiment is not satisfactory, as seen from the relation (3.2). A reasonable procedure to save this situation would be to take into account the difference in quark and antiquark scatterings.

Incidentally, the Pomeranchuk theorem is satisfied in our model, as is easily seen from (2.1)–(2.10), because $r^2(0°)$ would tend to zero at the high-energy limit [this will be shown explicitly later; see (5.11) in Sec. V]. The isotopic-spin independence of strong interactions at the high-energy limit is also realized [see, for example, Eqs. (2.13) and (2.14)].

IV. RELATIONS AMONG DIFFERENTIAL CROSS SECTIONS

Detailed behavior of the differential cross sections is experimentally known only at small angles in the high-energy region. All cases of hadron elastic scattering, $A+B\to A+B$, show the diffraction-type behavior, $d\sigma_{AB}/dt=\exp(a_{AB}+b_{AB}t)$, up to about $t=-1$ (BeV/c)^2. In order to examine our results experimentally, let us rewrite the formulas in Sec. II to give the elastic differential cross sections:

$$\frac{d\sigma_{K^-p}}{dt}=\frac{d\sigma_{K^-n}}{dt}, \tag{4.1}$$

$$\frac{d\sigma_{\pi^+p}}{dt}\frac{d\sigma_{K^+n}}{dt}=\frac{d\sigma_{\pi^-p}}{dt}\frac{d\sigma_{K^+p}}{dt}, \tag{4.2}$$

$$\frac{d\sigma_{K^+n}}{dt}\frac{d\sigma_{pn}}{dt}=\frac{d\sigma_{K^-p}}{dt}\frac{d\sigma_{pp}}{dt}, \tag{4.3}$$

$$\frac{d\sigma_{\pi^-p}}{dt}\frac{d\sigma_{K^+p}}{dt}\frac{d\sigma_{pn}}{dt}=\frac{d\sigma_{\pi^+p}}{dt}\frac{d\sigma_{K^-p}}{dt}\frac{d\sigma_{pp}}{dt}. \tag{4.4}$$

Unfortunately, there are no neutron-target data on kaon scattering, so we cannot directly check Eqs. (4.1)–(4.3) experimentally. Instead, these relations may be

[5] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontić, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. 138, B913 (1965); K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters 19, 330 (1967).

[6] S. J. Lindenbaum, paper presented at Coral Gables Conference on Symmetry Principle, 1967 (unpublished).

[7] K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, T. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters 14, 74 (1965); G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillethum, J. Pahl, J. P. Scanlon, J. J. Walters, A. M. Wetherell, and P. Zanella, Phys. Letters 19, 341 (1965).

regarded as predictions for $K^{\pm}$-n scattering. However, if the higher order of r^2 is neglected at small angles, since $|r^2(0°)|$ is sufficiently small, as estimated in Sec. III [see also (4.10)], the differential cross section for K^+-n scattering is written as

$$2\left(\frac{d\sigma_{K^+n}}{dt}\right)\cong\frac{d\sigma_{K^+p}}{dt}+\frac{d\sigma_{K^-p}}{dt}. \tag{4.5}$$

Therefore we can calculate the parameters for K^+-n scattering from the $K^{\pm}$-p experiments, using (4.5). Then at small angles, (4.2) means that

$$\begin{pmatrix}a\\b\end{pmatrix}_{\pi^+p}+\begin{pmatrix}a'\\b'\end{pmatrix}_{K^+n}=\begin{pmatrix}a\\b\end{pmatrix}_{\pi^-p}+\begin{pmatrix}a\\b\end{pmatrix}_{K^+p}, \tag{4.6}$$

where a_{K^+n}' and b_{K^+n}' denote the calculated parameters from the $K^{\pm}$-p data through (4.5). Values of a' and b' are 3.12 ± 0.13 and 7.27 ± 0.70 $(\text{BeV}/c)^{-2}$, respectively, at about 12 BeV/c incident kaon momenta.[8] The comparison of (4.6) with experiments yields

$$\begin{pmatrix}6.69\pm0.17\\16.20\pm0.97\end{pmatrix}=\begin{pmatrix}6.59\pm0.13\\15.55\pm0.68\end{pmatrix},$$

which seems to show good agreement of our model with experiments.[9]

Similarly, we have the following relations from (4.3) and (4.4) by analogy with (4.6):

$$\begin{pmatrix}a'\\b'\end{pmatrix}_{K^+n}+\begin{pmatrix}a\\b\end{pmatrix}_{pn}=\begin{pmatrix}a\\b\end{pmatrix}_{K^-p}+\begin{pmatrix}a\\b\end{pmatrix}_{pp}, \tag{4.7}$$

$$\begin{pmatrix}a\\b\end{pmatrix}_{\pi^-p}+\begin{pmatrix}a\\b\end{pmatrix}_{K^+p}+\begin{pmatrix}a\\b\end{pmatrix}_{pn}=\begin{pmatrix}a\\b\end{pmatrix}_{\pi^+p}+\begin{pmatrix}a\\b\end{pmatrix}_{K^-p}+\begin{pmatrix}a\\b\end{pmatrix}_{pp}. \tag{4.8}$$

In the above cases, we must perform the comparison at very low momenta, since the p-n data are known only up to about 7 BeV/c.[10] From the experiments around 7 BeV/c, (4.7) and (4.8) yield[8–10]

$$\begin{pmatrix}7.86\pm0.35\\14.72\pm1.37\end{pmatrix}=\begin{pmatrix}8.24\pm0.32\\17.98\pm1.41\end{pmatrix}$$

and

$$\begin{pmatrix}11.14\pm0.34\\23.14\pm1.52\end{pmatrix}=\begin{pmatrix}11.83\pm0.36\\28.56\pm1.64\end{pmatrix}.$$

Thus the agreement of the relations for the a's with the data is excellent, whereas there seems to exist a discrepancy between the predictions and the data for the b's. Nevertheless, we may conclude that the agreement is satisfactory if we remember the appreciable experimental errors contained and the crudeness of our model.

Now let us estimate the magnitude of r^2 at the forward direction from the differential cross sections. This is an independent method to determine r^2 from that with the total cross sections in Sec. III. In this way we can examine the self-consistency of our model. In terms of the parameters for $\pi^{\pm}$-p scattering, we have

$$\ln\left|\frac{1+2r^2}{1+r^2}\right|=\tfrac{1}{2}[a_{\pi^+p}-a_{\pi^-p}+(b_{\pi^+p}-b_{\pi^-p})t], \tag{4.9}$$

which implies that r^2 is sufficiently small compared with unity, because the right-hand side of (4.9) is nearly equal to zero when the experimental values of the a's and b's are used. In fact, it is almost impossible to determine r^2 from (4.9), as the $\pi^{\pm}$-p data are too close to each other, with rather large experimental errors. On the other hand, we obtain the following relation for p-p and $\bar{p}$-p scattering by neglecting the higher order of r^2:

$$\ln|1+5r^2|\cong5\,\mathrm{Re}r^2\cong\tfrac{1}{2}[a_{pp}-a_{\bar{p}p}+(b_{pp}-b_{\bar{p}p})t]. \tag{4.10}$$

This gives $\mathrm{Re}r^2(\theta=0°)=-(0.053\pm0.030)$ at about 12 BeV/c.[8,9] Note that the imaginary part of r^2 does not appear to this order, and hereafter we shall ignore the imaginary part of r^2. This value is quite consistent with that estimated in Sec. III.

A similar argument, neglecting the higher orders of r^2, leads to the following relations:

$$\begin{pmatrix}a\\b\end{pmatrix}_{\pi^+p}+\begin{pmatrix}a\\b\end{pmatrix}_{pn}=\begin{pmatrix}a\\b\end{pmatrix}_{\pi^-p}+\begin{pmatrix}a\\b\end{pmatrix}_{pp}, \tag{4.11}$$

$$\begin{pmatrix}a\\b\end{pmatrix}_{pn}=\frac{1}{5}\begin{pmatrix}a\\b\end{pmatrix}_{\bar{p}p}+\frac{4}{5}\begin{pmatrix}a\\b\end{pmatrix}_{pp}. \tag{4.12}$$

Again the experimental agreement is good for the relations among the a's, but it gets worse for the b's at about 7 BeV/c. This may be ascribed partly to the fact that the energy in question is too low for our approximations to be valid.

Finally, it is interesting to observe that the ratios

$$\left(\frac{d\sigma_{\pi^+p}}{dt}\right)\Big/\left(\frac{d\sigma_{K^+p}}{dt}\right)\quad\text{and}\quad\left(\frac{d\sigma_{\pi^-p}}{dt}\right)\Big/\left(\frac{d\sigma_{K^+n}}{dt}\right)$$

should be θ-independent in our model. Over-all angular distributions are known only in the region of a few BeV, and the presently available data are not inconsistent

[8] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 503 (1963); K. J. Foley, R. S. Gilmore, S. J. Lindenbaum, W. A. Love, S. Ozaki, R. H. Willen, R. Yamada, and L. C. L. Yuan, *ibid.* 15, 45 (1965).

[9] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 425 (1963); D. O. Caldwell, B. Elsner, D. Harting, A. C. Helmholz, W. C. Middelkoop, B. Zacharov, P. Dalpeaz, S. Focardi, G. Giacomelli, L. Monari, J. A. Beaney, R. A. Donald, P. Mason, and J. W. Jones, Phys. Letters 8, 288 (1964).

[10] M. N. Kreisler, F. Martin, M. L. Perl, M. J. Longo, and S. T. Powell, III, Phys. Rev. Letters 16, 1217 (1966).

with this conclusion; the K^+-p angular distribution, even at 3.55 BeV/c, shows a resemblance to that for the π^+-p scattering at that momentum.[11]

V. LARGE-ANGLE SCATTERING

Let us first discuss the scattering at $\theta=90°$, where all the quarks considered are scattered through the same angle. Obviously, one has $r(90°)=1$, and therefore the relations for the differential cross sections become especially simple. For the nucleon- and antinucleon-nucleon scattering, one obtains

$$\left(\frac{d\sigma_{\bar{p}p}}{dt}\bigg/\frac{d\sigma_{pp}}{dt}\right)_{90°}=\frac{1}{144} \tag{5.1}$$

and

$$\left(\frac{d\sigma_{pn}}{dt}\bigg/\frac{d\sigma_{pp}}{dt}\right)_{90°}=\frac{1}{4}. \tag{5.2}$$

These relations serve as a simple experimental test of our model; the present experimental data around 7 BeV/c show that $(d\sigma_{pp}/dt)_{90°}\approx(d\sigma_{pn}/dt)_{90°}\approx0.002$ mb $(\mathrm{BeV}/c)^{-2}$,[10,12] but the large experimental error contained prevents us from drawing a definite conclusion. Experimentally, it may be easier to check (5.1) if a high-energy antiproton beam is available.

Similarly, for the meson-nucleon scattering, one has

$$\left(\frac{d\sigma_{\pi^-p}}{dt}\bigg/\frac{d\sigma_{\pi^+p}}{dt}\right)_{90°}=\frac{4}{9} \tag{5.3}$$

and

$$\left(\frac{d\sigma_{K^-p}}{dt}\bigg/\frac{d\sigma_{K^+p}}{dt}\right)_{90°}=\frac{1}{9}, \tag{5.4}$$

which will also be easily checked experimentally in the near future.

Next, in order to investigate the backward scattering, let us replace θ by $\pi-\theta$ in the amplitudes in Sec. III; then we have for the nucleon- and antinucleon-nucleon scattering

$$F_{pn}(\pi-\theta)=r^2(\theta)F_{pn}(\theta), \tag{5.5}$$

$$F_{\bar{p}p}(\pi-\theta)=r^6(\theta)F_{\bar{p}p}(\theta), \tag{5.6}$$

besides the self-evident relation $F_{pp}(\pi-\theta)=F_{pp}(\theta)$.

In the same way, we obtain similar relations for meson-nucleon scattering, as follows:

$$F_{\pi^-p}(\pi-\theta)=r^3(\theta)F_{\pi^-p}(\theta), \tag{5.7}$$

$$F_{\pi^+p}(\pi-\theta)\propto r^3(\theta)G^5(\theta)[2+r^2(\theta)], \tag{5.8}$$

$$F_{K^-p}(\pi-\theta)=r^5(\theta)F_{K^-p}(\theta), \tag{5.9}$$

$$F_{K^+p}(\pi-\theta)\propto r^3(\theta)G^5(\theta)[2+r^2(\theta)]. \tag{5.10}$$

Using (5.7) and (5.8), we can roughly estimate the s dependence of $r(0°)$ from the experimental forward-backward ratio of the $\pi^\pm$-p scattering. Recent experiment shows that[13]

$$(d\sigma_{\pi^-p}/d\Omega)_{180°}\propto s^{-1.10\pm0.12}$$

and

$$(d\sigma_{\pi^+p}/d\Omega)_{180°}\propto s^{-1.40\pm0.10}.$$

This gives the s dependence of $|r^2(0°)|$ as

$$|r^2(0°)|\propto s^{-(0.7\sim0.8)}. \tag{5.11}$$

From (5.5) and (5.11), and the value of $r^2(0°)$ estimated previously at 12 BeV/c, we expect that the forward-backward ratio $(d\sigma_{pn}/dt)_{180°}/(d\sigma_{pn}/dt)_{0°}$ is of the order of 10^{-2} at about 7 BeV/c, which conforms to the data very well.[10] In the same way, from (5.6) one must have $(d\sigma_{\bar{p}p}/dt)_{180°}/(d\sigma_{\bar{p}p}/dt)_{0°}\lesssim10^{-6}$ in the 10-BeV region. This serves as one of the experimental tests of our model.

For pion-nucleon scattering, (5.7) implies that the forward-backward ratio of the π^--p differential cross section is of the order of 10^4 at 12 BeV/c, which is in agreement with the data.[13] Furthermore, from (5.8) we observe that

$$F_{\pi^+p}(180°)\approx2r^3(0°)F_{\pi^-p}(0°)=2F_{\pi^-p}(180°), \tag{5.12}$$

since $r^2(0°)$ is very small compared with unity. Then we have the ratio of the backward $\pi^\pm$-p scattering $[(d\sigma_{\pi^+p}/dt)/(d\sigma_{\pi^-p}/dt)]_{180°}\approx4$ in agreement with experiments.

The forward-backward ratios for kaon-nucleon scattering are known only in the low-energy region. When Eq. (5.11) is extrapolated to a low incident momentum such as 3.55 BeV/c, where experimental data are available, we obtain

$$\left(\frac{d\sigma_{K^-p}}{dt}\right)_{180°}\bigg/\left(\frac{d\sigma_{K^-p}}{dt}\right)_{0°}\approx10^{-5}$$

and

$$\left(\frac{d\sigma_{K^+p}}{dt}\right)_{180°}\bigg/\left(\frac{d\sigma_{K^-p}}{dt}\right)_{180°}\approx4|r(0°)|^{-4}\approx10^2\text{–}10^3.$$

Again we see that these numerical values are consistent with the data.[11]

Finally, note that the ratio $F(\pi-\theta)/F(\theta)$ is a function of only $r^2(\theta)$, and consequently one sees that the backward peak of the cross section becomes sharper than the forward one, since $|r^2(\theta)|$ is expected to decrease with increasing θ. Of course, this is only a

[11] J. Banaigs, J. Berger, C. Bonnel, J. Duflo, L. Goldzahl, F. Plouin, W. F. Baker, P. J. Carlson, V. Chabaud, and A. Lundby, Phys. Letters **24B**, 317 (1967).

[12] J. V. Allaby, G. Cocconi, A. N. Diddens, A. Klovning, G. Matthiae, E. J. Sacharidis, and A. M. Wetherell, Phys. Letters **25B**, 156 (1967).

[13] H. Brody, R. Lanza, R. Marshall, J. Niederer, W. Selove, M. Schochet, and R. Van Berg, Phys. Rev. Letters **16**, 828 (1966); **16**, 968 (1966); W. R. Frisken, A. L. Read, H. Ruderman, A. D. Krish, J. Orear, R. Rubinstein, D. B. Scarl, and D. H. White, *ibid.* **15**, 313 (1965); A. Ashmore, C. J. S. Damerell, W. R. Frisken, R. Rubinstein, J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan, and D. H. White, *ibid.* **19**, 460 (1967).

qualitative argument, and quantitatively more precise discussion will be needed.

VI. REMARKS AND DISCUSSIONS

We have so far discussed the elastic scattering of mesons and nucleons on the nucleon target. The charge-exchange scattering $\pi^-+p\rightarrow\pi^0+n$ may also be treated under our assumptions. In the same way as in the elastic scattering, the reaction amplitude $F_{\pi^-p}{}^{(\mathrm{ce})}(\theta)$ is written as

$$F_{\pi^-p}{}^{(\mathrm{ce})}(\theta)=2^{-1/2}r^2(\theta)G^5(\theta)\,, \tag{6.1}$$

which clearly satisfies charge independence.[14] Hence we obtain

$$\left(\frac{d\sigma_{\pi^-p}{}^{(\mathrm{ce})}}{dt}\Big/\frac{d\sigma_{\pi^-p}}{dt}\right)_{0°}=\frac{|r(0)|^4}{2|1+r^2(0)|^2}. \tag{6.2}$$

From the value estimated in Sec. V, this ratio is of the order of 10^{-3} in the 10-BeV/c region, which is roughly in accord with experiment.

We should not claim that the above fact is evidence of the validity of our model for general inelastic processes. In fact, the charge-exchange scattering $K^-+p\rightarrow\bar{K}^0+n$ is forbidden under our assumption because of the relation (2.11). Note that the charge-exchange process between quarks or the pair annihilation-creation process must be introduced for the charge-exchange scattering, such as $K^-+p\rightarrow\bar{K}^0+n$ and $\bar{p}+p\rightarrow\bar{n}+n$. This is also the case for general inelastic two-body processes, including the so-called quasi-two-body process. An analysis of two-body reactions along this line will be discussed elsewhere.[15]

Furthermore, for the π^--p charge-exchange scattering, it is usually expected that the spin-flip amplitude is as large as the spin-nonflip amplitude in order to explain the experimental angular distribution in a simple way. Consequently the spin of the quarks must be properly taken into consideration. This is also necessary for quantitative discussions of large-angle elastic scattering.

One of the assumptions made in the present paper is that the differences between quarks is neglected. Under this assumption, the total and differential cross sections should be entirely the same for the π^+-p and K^+-p scattering (and for the π^--p and K^+-n) if the mass difference between pion and kaon is ignored [see (2.5) and (2.9) in Sec. II]. Experimentally, this is not the case, and we may conclude that the behavior of the λ quark is somewhat different from the $\mathcal{P}$ and $\mathfrak{N}$ quark. As far as the results given in the previous sections are concerned, such a difference is eliminated by taking appropriate ratios or products.

From Eqs. (2.5)–(2.10) we can derive the Johnson-Treiman relation which was obtained by the $SU(6)$ symmetry theory and by the quark model with the additivity assumption.[16] We would like to point out that this situation is entirely accidental, and most of the relations derived from the additivity assumption can not be obtained by the factorizability assumption, and vice versa.

As was discussed in Sec. III, the agreement of our results with experiments is certainly poorer than those of the additivity assumption, as far as the total cross sections are concerned. We do not think, however, that this is evidence for discarding our model. We suggest that both the additivity mechanism and the factorizability mechanism can contribute to hadron scattering, and that each of these may be dominant in a different range of momentum transfer.

One may ask a question about the physical meaning of the factorizability assumption, especially about $G(\theta)$. Actually, many of the results in this paper are obtained without using $G(\theta)$ explicitly. One can get these results by assuming that a hadron-hadron scattering is the sum of a direct term and possible exchange terms, and that the ratio of any exchange contribution to the corresponding direct term is given by $r(\theta)$. Our aim, however, is to describe every hadron process in terms of a single quantity $G(\theta)$, even though this is a crude approximation.

In conclusion, it should be emphasized that the factorizability assumption is a possible feature of the quark model, though the physical basis of this assumption is not clear at present. Further experimental information for large-angle scattering is desirable to check our model.

[14] On this point, we are indebted to Professor A. Kanazawa for valuable discussions.

[15] For application of the quark model to inelastic two-body reactions, see, for example, M. Kawaguchi and H. Yokomi, Progr. Theoret. Phys. (Kyoto) 37, 772 (1967); 38, 735 (1967).

[16] K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).