Pion-Nucleon Total Cross Sections from 0.5 to 2.65 GeV/c

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Total cross sections of π^+ and π^- mesons on protons and deuterons have been measured in a transmission experiment to relative accuracies of $\pm0.2\%$ over the laboratory momentum range 0.46–2.67 GeV/c. The systematic error is estimated to be about $\pm0.5\%$ over most of the range, increasing to about $\pm2\%$ near both ends. Data have been obtained at momentum intervals of 25–50 MeV/c with a momentum resolution of $\pm0.6\%$. No new structure is observed in the $\pi^\pm p$ total cross sections, but results differ in several details from previous experiments. From 1–2 GeV/c, where systematic erros are the smallest, the total cross section of π^- mesons on deuterons is found to be consistently higher than that of π^+ mesons by $(1.3\pm0.3)\%$; about half of this difference may be understood in terms of Coulomb-barrier effects. The πd and πN total cross sections are used to check the validity of the Glauber theory. Substantial disagreements (up to 2 mb) are observed, and the conclusion is drawn that the Glauber theory is inadequate in this momentum range.

1. INTRODUCTION

SEVERAL measurements[1–4] have been made within the last six years of π^+p and π^-p total cross sections over the momentum range covered in this experiment. They show systematic differences, varying slowly with momentum, of 5–10%. In the present experiment, statistical errors have been reduced to an insignificant level (0.1%), and a determined effort has been made to keep systematic errors at or below 1%. In particular, the form of the extrapolation to zero solid angle has been examined in detail. Many sets of data have been repeated at intervals of up to a few months and under varying beam conditions, and the reproducibility of the measurements has been observed to be about $\pm0.2\%$.

The πd total cross sections have also been measured as a check on (a) charge independence and (b) the Glauber theory.

A. Notation

The following conventional notation is used throughout the text; other notation required at specific points will be explained as it arises. $\sigma(\pi t)$ is the total cross section of π mesons on target t, p is the laboratory momentum of the incident particle, β is the laboratory velocity of the incident particle, θ is the scattering angle, $f(\theta)$ is the amplitude for nuclear scattering through angle θ, $p_T=2p\sin\frac{1}{2}\theta$ is the momentum transfer, $t=-p_T{}^2$ is the 4-momentum transfer squared, k is the wave number in the c.m. system, and α is the fine-structure constant.

2. EXPERIMENTAL DETAILS

Total cross sections have been measured by the conventional transmission technique, using targets of liquid hydrogen and deuterium. The general layout of the experiment is shown in Fig. 1. It is very similar to that used in a previous experiment on nucleon-nucleon total cross sections,[5] to which reference will be made for many details of technique and for full explanation of corrections to the data. This paper will be referred to as I.

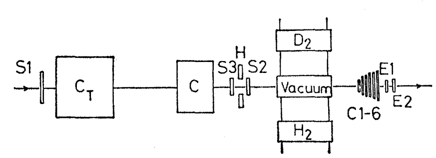

FIG. 1. The experimental layout. $S1$–$S3$ and H are scintillation counters defining the beam. C_T and C are the threshold and DISC Čerenkov counters, respectively. $C1$–6 are the transmission counters and $E1$ and $E2$, the efficiency counters.

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[4] T. J. Devlin, J. Solomon, and G. Bertsch, Phys. Rev. Letters **14**, 1031 (1965).

[5] D. V. Bugg, D. C. Salter, G. H. Stafford, R. F. George, K. F. Riley, and R. J. Tapper, Phys. Rev. **146**, 980 (1966).

A. Beam

The experiment was done in a 17-m unseparated beam, produced near 0° from a 10-cm external copper target at Nimrod.[6] It was a conventional two-stage beam with an acceptance of 1 msr. The final focus was at the defining counter S_2 (Fig. 1). The flux of particles varied with momentum from about 10^5 to 3×10^5 particles/pulse, in an effective spill time of about 80 msec. The momentum resolution, determined at the intermediate focus, varied slightly with momentum and was typically $\pm0.6\%$. This compares with an energy loss of about 12 MeV over the length of the target.

The fields of the three bending magnets which defined the momentum of the beam and its position in the experimental area were continuously monitored by Hall probes. This was essential because the total cross section varies extremely rapidly with momentum over much of the range covered in this experiment; for example, $d\sigma(\pi^-p)/dp$ rises to 0.2 mb/(MeV/c) near 925 MeV/c. Momentum settings were reproducible to ±2 MeV/c. Absolute momenta were determined by floating a current-carrying wire through the bending magnets; the uncertainty in this calibration is estimated to be $\pm0.5\%$.

B. Čerenkov Counters

The π mesons were identified by suitable combinations of a threshold Čerenkov counter, set to reject muons and electrons, and a DISC differential Čerenkov counter[7] using a liquid radiator.

The threshold counter was of conventional design. It was filled with ethylene gas at pressures up to 440 psi, and was viewed by a 58 AVP photomultiplier. The active length was 50 cm. At every momentum, the pressure was set as nearly as possible at the π threshold so as to achieve maximum rejection against muons and electrons. At a pressure of 80 psi, the inefficiency for vetoing electrons was 10^{-4}. This high efficiency was necessary because of the large electron contamination in the beam; this varied from 5 to 40% of the total negative beam and 5 to 35% of the total positive beam, and was sharply peaked near 700 MeV/c. The muon contamination was measured to be 4% at 750 MeV/c. The threshold counter reduced the electron and muon contamination in the beam to a negligible level below 1.4 GeV/c. Above this momentum, possible inefficiencies in this counter begin to contribute to systematic errors on the total cross section. These could have been as much as 0.3% at 1.9 GeV/c and could have risen rapidly thereafter and reached 1.5% at the highest momentum.

The DISC Čerenkov counter and its performance is described in an accompanying paper on K-nucleon total cross sections.[8] It was used primarily to separate π^+ from protons, although by virtue of its small angular acceptance it also provided some discrimination against muons from π decay after the last bending magnet. Measurements of $\sigma(\pi^-p)$ with and without it in the beam agreed within the expected errors.

C. Scintillation Counters and Electronics

Two circular scintillation counters S_1 and S_2, 5 and 2.5 in. in diam, defined the beam. A halo counter H, 6 in. in diam and with a 2.4-in.-diam hole in its center was used in anticoincidence to veto any particles missing S_2.

The transmission counters $C1$–6 were six concentric scintillation counters, each $\frac{1}{2}$ in. thick, and with diameters $4\sqrt{x}$ in., where $x=2$, 4, 6, 9, 12, and 16. Their sizes were such that (a) they encompassed the unscattered beam, (b) losses due to single and multiple Coulomb scattering were small, and (c) they fell well within the range of nuclear-elastic and inelastic scattering, so that the extrapolation to zero solid angle was approximately linear. The five successive pairs of transmission counters were taken in threefold coincidence BC_iC_{i+1} with the beam pulse B, in order to minimize counts from Čerenkov light produced in their light guides, as discussed in I. The rear edges of the BCC resolution curves were adjusted to be the same within about 0.1 nsec; this ensured that they all counted the same spectrum of scattered particles.

The small counters E_1 and E_2 were used in coincidence to record continuously the efficiencies of the transmission counters. By putting BE_1E_2 into coincidence with BCC, the efficiency of the complete system, scintillator+photomultiplier+electronics, was monitored. This monitoring was essential since the targets scattered about 10% of the beam and, in consequence, an undetected change of 0.01% in the efficiency of the transmission counters between measurements with target full and target empty would have led to an error of 0.1% in the total cross section. The efficiencies were noticeably rate-dependent for beam intensities much above 10^5 per pulse, mainly because of small residual deadtime effects in the electronics.

First-order deadtime effects in the electronics were eliminated by paralyzing the system whenever two particles traversed S_2 or H within the deadtime (30 nsec) of the BCC coincidence units. The photomultiplier pulse from S_2 was fanned out, and after being delayed by different amounts the pulses so formed were added to generate a continuous pulse P, 35 nsec long; the amplitude of P was defined by a limiter. A second pulse in S_2 within 35 nsec had the effect of stretching P to a time 35 nsec after the arrival of the second pulse.

[6] J. D. Davies, J. D. Dowell, P. M. Hattersley, R. J. Homer, A. W. O'Dell, M. L. Sproul, A. A. Carter, K. F. Riley, R. J. Tapper, D. V. Bugg, D. C. Salter, and E. J. N. Wilson, Nuovo Cimento (to be published).

[7] R. Meunier, J. P. Stroot, B. Leontic, A. Lundby, and P. Duteil, Nucl. Instr. Methods **17**, 20 (1962).

[8] D. V. Bugg, R. S. Gilmore, K. M. Knight, D. C. Salter, G. H. Stafford, E. J. N. Wilson, J. D. Davies, J. D. Dowell, P. M. Hattersley, R. J. Homer, A. W. O'Dell, A. A. Carter, K. F. Riley, and R. J. Tapper, following paper, Phys. Rev. **168**, 1466 (1968).

Then P was applied as a delayed veto to the S_1S_2 coincidence unit. This device was proof against any sequence of pulses in S_2 and ensured that no beam pulse was preceded within 35 nsec by another particle. The halo veto pulse H was stretched in a similar manner and applied in parallel with P.

The electronics also eliminated accidental coincidences in the transmission counters caused by two particles traversing the beam line within the resolving time τ of the BCC coincidences. Suppose a particle counting in S_2 is followed within time τ by another in either S_3 or H, and that the first scatters in the target but the second does not. The first generates a beam pulse B, and the transmission counters register the second as being in coincidence with B. The halo counter was used to veto such events straightforwardly. Twofold accidentals in S_2 were eliminated by the device described in detail in I: A fast gate recognized two particles separated by a time interval between 5 and 15 nsec, and the heights of the S_2 and S_3 pulses were used to discriminate with $>98\%$ efficiency against two particles traversing these counters within about 5 nsec of one another. Let us denote the output of this unit by D, and that of the DISC Čerenkov counter by C. Then a beam particle was defined by the logic $B=S_1S_2C\bar{H}\bar{C}_T\bar{P}\bar{D}$.

D. Targets

Three identical targets 55 cm long were used; one was filled with liquid hydrogen, a second with liquid deuterium, and the third was evacuated. The total cross section is obtained from the ratio t_e/t_f of the transmission through evacuated and full targets according to the relation

$$\sigma=(1/Nl)\ln(t_e/t_f).$$

Here l is the length of the target and N the number of atoms per unit volume in it. The targets are described in detail in I.

3. COLLECTION AND TREATMENT OF DATA

Data were taken at momentum intervals of about 25 MeV/c within the range 0.46–2.65 GeV/c, the measurements being made at intervals over a period of 6 months. At many momenta, measurements were repeated several times during this period, and it was found possible to reproduce results with an accuracy of $\pm0.2\%$ and ±2 MeV/c.

The treatment of the data has been discussed in detail in I, and it will suffice here to list the steps, together with the orders of magnitude of the corrections for the present experiment.

(i) The efficiency of each pair of transmission counters C_iC_{i+1} was determined from the ratio $BE_1E_2C_iC_{i+1}/BE_1E_2$. Typically this was 99%.

(ii) Corrections were applied to each counter for single and multiple Coulomb scattering, taking into account the divergence and distribution of particles in the beam. The correction was much the largest for the smallest transmission counter, where it was typically 0.1 mb. A formula for this correction is given in I, but in the present experiment a significant fraction of the incident beam was not contained by the smaller counters and a more appropriate formula is

$$d\sigma(\text{Coul.})=\frac{-4\pi(\alpha\hbar c)^2}{p^2\theta_i^2\beta^2}\left\{\left\langle\left(1-\frac{r^2}{R_i^2}\right)^{-2}\right\rangle+\frac{1}{2}\frac{\langle\theta^2\rangle}{\theta_i^2}\times\left\langle\left(3+\frac{8r^2}{R_i^2}+\frac{r^4}{R_i^4}\right)\left(1-\frac{r^2}{R_i^2}\right)^{-4}\right\rangle\right\}.$$

Here θ_i is the half-angle subtended at the target by counter i of radius R_i. The mean-square angle of multiple scattering is $\langle\theta^2\rangle$, and the braces indicate averages over the distribution in radius r of the beam at the counter surfaces. The second term was only significant for the smallest counter.

(iii) A correction was applied for decay of pions into muons in and after the threshold counter C_T. The correction was largest for the smallest transmission counter, since S_2 and the halo counter determined the range of decay angles accepted by the electronics. The correction was calculated by a Monte Carlo method, using the measured beam distribution and divergence. It ranged from a minimum of 2.5% at 2.5 GeV/c to a maximum of 4.5% at 0.75 GeV/c for the largest counter.

(iv) Energy loss in the full targets caused more pions to decay after these targets than after the vacuum target. Because of the narrow cone of $\pi\to\mu\nu$ decays, this caused very few particles to miss the transmission counters, but it somewhat modified the beam distribution over them. The effect was calculated by Monte Carlo methods; typically its magnitude was 0.05 mb.

(v) The data were corrected for Coulomb-nuclear interference, using the formulas given in I. The description given there of the correction to be applied to the deuterium data has given rise to some confusion and needs amplifying. There are two terms,

$$(d\sigma/d\Omega)(CN)=2g\{\text{Re}[f_{\pi p}(\mathbf{q})C_{\pi p}^{\dagger}(\mathbf{q})]+\text{Re}[S(\mathbf{q})C_{\pi p}^{\dagger}(\mathbf{q})f_{\pi n}(\mathbf{q})]\},$$

where $\mathbf{q}$ is the 3-momentum transfer, g is a factor allowing for the shadowing of one nucleon by the other, and

$$S(\mathbf{q})=\int\exp(i\mathbf{q}\cdot\mathbf{r})\ |\psi_{\text{D}}(\mathbf{r})|^2d\mathbf{r},$$

where $\mathbf{r}$ is the neutron-proton separation. We have approximated $S(\mathbf{q})$ by $\exp(-18\,q^2)$. Both terms have been included in the calculations both here and in I.

Values for Re$f(0)$ were taken from dispersion-relation calculations.[9] The angular dependence of Re$f(\mathbf{q})_{\pi p}$ was

[9] A. A. Carter, Ph.D. thesis, University of Cambridge (unpublished).

taken from phase-shift analyses[10] below 1 GeV/c; above that, it was taken to be the same as the angular dependence of Im$f(\mathbf{q})$. The effect of Fermi motion was folded into Re$f_{\pi p}$ and Re$f_{\pi n}$ before they were used for correcting the deuterium data.

At low momenta, the difference between the cross sections obtained with and without the Coulomb-nuclear interference correction was as much as 0.44 mb for $\pi^- p$, 1.75 mb for $\pi^+ p$, and 1.30 mb for πd data; above 1.4 GeV/c it was $\leqslant 0.3$ mb.

(vi) The partial cross sections σ_i recorded by transmission counters i were then fitted to a quadratic in the solid angle Ω_i;

$$\sigma_i = A + B\Omega_i + C\Omega_i^2.$$

In a preliminary stage of the experiment a careful study was made of the form of the extrapolation to zero solid angle. Data obtained at one momentum are plotted in Fig. 2 as a function of solid angle. It is obvious that the extrapolation is not linear. Indeed, unless the range of transverse momentum is confined below 250 MeV/c one needs cubic terms. The lower limit which can be used is governed by the beam size and by Coulomb scattering. At momenta below 1.2 GeV/c the transmission counters were fixed in position at a distance of about 100 cm from the center of the target; they could not be moved any closer. At higher momenta, they were moved back along a railway in such a way as to keep constant the range of p_T covered by the counters.

Preliminary values of A, B, and C were determined from a least-squares fit to the experimental data. Values of C/p^4, when plotted against p, were statistically compatible with a smoothly varying function. A smooth curve was drawn through them, and final values of A and B were determined from a least-squares fit using the smoothed values of C.

(vii) The mean length of each target was evaluated by averaging over the beam distribution and allowing

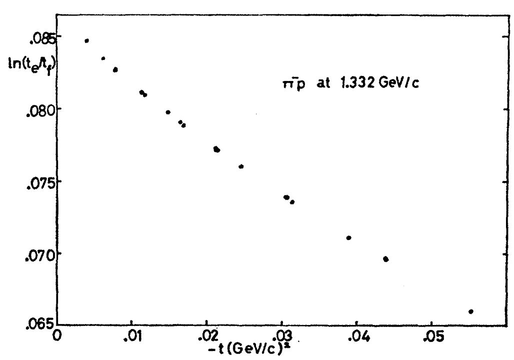

FIG. 2. The form of the extrapolation to zero solid angle over an extended range at 1.332 GeV/c. A small correction has been applied to the value obtained from each counter to allow for the absorption of scattered particles in preceding counters.

[10] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965); A. Donnachie, CERN Report No. 66/1042/5-TH 690 (unpublished).

for the curvatures of the end windows. The correction to the over-all length was typically 2 mm.

(viii) Deuterium data were corrected for contamination of the deuterium by hydrogen. The composition was determined by emptying the targets and analyzing samples from the gas cylinders in a mass spectrograph. The composition varied slightly through the experiment from 98.0 to 98.6% deuterium. There is an over-all systematic error in the deuteron cross sections of $\pm 0.2\%$ due to uncertainties in the deuterium density and composition.

(ix) At peaks and valleys in the total cross sections, corrections have been applied for the momentum resolution. The correction is largest for $\sigma(\pi^- p)$ at the peak of the $N^*(1688)$, where it is $+0.17$ mb.

4. RESULTS

Statistical errors are $\leqslant \pm 0.1\%$, and the reproducibility of results over periods of months is $\pm 0.2\%$. However, although the statistical accuracy obtainable with a transmission experiment such as the present one can be made very high, the systematic errors are much more difficult to reduce. The systematic error is estimated to be about $\pm 0.5\%$ over the central part of the momentum range covered, but increases at both the high- and low-momentum ends to about $\pm 2\%$. The principal contribution to the systematic errors are as follows.

(a) Uncertainties arise in the corrections for Coulomb scattering [Sec. 3 (ii)] and pion decay [Sec. 3 (iii)], because of uncertainties in the beam distribution. These uncertainties become large for the smaller transmission counters at low momenta. It was found necessary to omit counter 1 in the extrapolation to zero solid angle below 1.06 GeV/c, and to omit counter 2 as well below 0.77 GeV/c. These omissions increased the uncertainties in the form of the extrapolation to zero solid angle in this momentum range. As a check, measurements were made at a few momenta with a small beam defined by a $\frac{1}{4}$-in.$\times\frac{1}{4}$-in. counter instead of S_2; in this case, corrections for beam size are extremely small. These measurements lend considerable confidence to the form used for the extrapolation to zero solid angle, particularly at low momenta.

(b) There were residual uncertainties in the C coefficient in the extrapolation to zero solid angle.

(c) At high momenta, the gas pressure in the threshold counter was so low that the efficiency for vetoing electrons and muons in the beam may have been impaired. At these momenta, the muon contamination is unknown, but is believed to have been $<4\%$. No correction has been applied for these contaminations. However, the systematic error has been evaluated from the most pessimistic estimate of the efficiency of the counter and the magnitude of the contamination. This estimate is probably too large. In particular, on one or two occasions where the pressure of gas in the threshold counter was slightly misset, the measured total cross section was unaffected.

Tables I and II list the measurements of π^+p, π^+d, π^-p, and π^-d total cross sections. The estimated systematic error is included in the tables, together with the principal source: a, b, or c. At momenta where several measurements have been made, the average values are presented.

A. $\sigma(\pi^+p)$ and $\sigma(\pi^-p)$

Figures 3 and 4 show results of this experiment and the trends of previous ones. The errors shown are the estimated systematic errors.

Even with an improvement in statistical accuracy of one order of magnitude, no new features are apparent in the cross sections, and the gross features are common to all sets of measurements. There are, however, some differences in the heights, positions, and widths of the known structures. Apart from different over-all normalizations all the data on $\sigma(\pi^+p)$ are compatible, but for $\sigma(\pi^-p)$ the differences are more marked. Except for a momentum shift, our values of $\sigma(\pi^-p)$ agree well with those of Princeton[4] over the $N^*(1688)$ peak, but we find a much narrower and smaller peak at the $N^*(1512)$ mass, in agreement with the Berkeley[2] and Saclay[1] data there. All sets of data give similar widths for the

TABLE I. Total cross sections of π^- on protons and deuterons.

p[a] (MeV/c)	$\sigma(\pi^-p)$[b] (mb)	Systematic error (mb)	$\sigma(\pi^-d)$[b] (mb)	Systematic[c] error (mb)	Major source of error[d]
474	26.36	±0.67	67.49	±1.68	a
499	27.04	0.59	62.15	1.30	a, b
513	27.49	0.54	60.42[e]	1.24	a, b
527	⋯	⋯	58.24	1.20	a, b
550	28.52	0.50	55.35	1.10	a, b
565	29.36[e]	0.45	54.36[e]	0.90	b
576	29.55	0.42	53.16	0.87	b
595	30.96[e]	0.40	52.61	0.83	b
622	32.80[e]	0.30	52.32[e]	0.70	a, b
632	34.24	0.29	52.74	0.65	a, b
651	36.65[e]	0.27	53.72[e]	0.60	a, b
664	39.59	0.24	54.77	0.55	a, b
673	40.69	0.21	55.46	0.40	a, b
682	43.56[e]	0.24	56.48[e]	0.52	a, b
693	45.18	0.24	57.39	0.52	a, b
702	45.68	0.24	57.85	0.50	a, b
711	46.31[e]	0.24	58.37[e]	0.50	a, b
718	46.00	0.24	58.24	0.50	a, b
743	44.10	0.24	58.25	0.50	a, b
774	40.38[e]	0.23	57.32	0.50	a, b
804	36.67	0.21	56.24	0.50	a, b
812	36.48[e]	0.21	56.36[e]	0.50	a, b
818	36.17	0.21	56.40	0.50	a, b
824	35.80	0.21	56.38	0.50	a, b
845	36.10	0.21	57.53	0.50	a, b
873	38.45	0.20	59.88[e]	0.50	a, b
883	40.23	0.19	61.51	0.50	a, b
907	44.41	0.19	⋯	⋯	a, b
931	49.58[e]	0.19	68.51[e]	0.50	a, b
939	51.58[e]	0.19	70.05	0.45	a, b
971	57.78[f]	0.17	74.41[e]	0.40	a, b
988	59.88[e]	0.17	75.65	0.35	a, b
996	60.09	0.18	76.33	0.33	a, b
1002	60.58[e]	0.18	⋯	⋯	a, b
1016	59.75[e]	0.18	76.68[e]	0.32	a, b
1027	57.80	0.18	76.57	0.30	a, b
1050	53.80	0.18	75.24	0.29	a, b
1075	49.75	0.18	73.51	0.28	a
1089	46.69[g]	0.19	72.46[g]	0.28	a
1101	44.41[e]	0.19	71.72	0.28	a
1127	41.02[f]	0.19	69.90[f]	0.29	a
1151	39.29	0.19	68.85	0.28	a
1166	38.15[e]	0.19	68.32[e]	0.28	a
1176	38.14	0.19	68.36	0.28	a
1195	37.19[e]	0.19	68.19[e]	0.28	a
1201	37.18	0.19	68.40	0.28	a
1226	36.76	0.19	68.61	0.28	a, b
1251	36.57	0.19	69.24	0.28	a, b
1275	36.45[e]	0.19	69.78[e]	0.28	b
1281	36.42	0.19	70.13	0.28	b
1312	36.56	0.19	70.91	0.28	b
1332	36.63[f]	0.19	71.86[g]	0.28	b
1349	36.64	0.19	72.51	0.28	b
1355	36.61	0.19	72.41	0.27	b
1380	36.68	0.19	73.28	0.27	b
1399	36.65[e]	0.19	73.31[e]	0.26	b
1409	36.62	0.18	73.53	0.26	b
1433	36.54[h]	0.18	73.56[h]	0.25	b, c
1451	36.49	0.18	73.63	0.25	b, c
1476	36.07[h]	0.18	73.10[h]	0.25	b, c
1494	36.06	0.18	72.84	0.25	b, c
1531	35.54	0.18	71.93	0.25	b, c
1572	35.08[g]	0.18	70.73[e]	0.25	b, c
1604	34.75[h]	0.19	69.64[h]	0.25	b, c
1622	34.62[e]	0.19	69.19[e]	0.25	b, c
1672	34.64	0.20	67.93	0.26	b, c
1719	34.70	0.20	66.92	0.26	c
1773	35.08	0.21	66.05	0.30	c
1785	35.11	0.22	66.17	0.34	c
1821	35.51	0.23	65.41	0.38	c
1851	35.49[h]	0.26	65.39[h]	0.42	c
1869	35.58	0.26	65.08	0.42	c
1916	36.03	0.24	64.91	0.42	c
1951	36.08	0.25	64.71	0.43	c
1967	36.18	0.25	64.67	0.44	c
2016	36.38	0.27	64.63	0.44	c
2050	36.42[h]	0.30	64.54[h]	0.48	c
2067	36.34	0.30	64.46	0.49	c
2102	36.10[e]	0.30	64.52	0.54	c
2168	36.06	0.30	64.30	0.48	c
2217	35.77	0.39	64.05	0.66	c
2267	35.44	0.40	64.02	0.67	c
2366	34.63	0.40	63.62	0.67	c
2414	34.35	0.45	63.42	0.78	c
2470	33.80	0.80	63.10	1.40	c
2522	33.53[e]	0.85	63.18[e]	1.40	c
2568	33.32	0.70	62.84	1.30	c
2614	32.95	0.70	62.39	1.30	c
2665	32.89	0.80	62.12	1.30	c

[a] This column refers to $\sigma(\pi p)$; momenta for $\sigma(\pi d)$ are all 1 MeV/c lower. Momenta are corrected to the center of the target. There is an uncertainty of ±2 MeV/c in every momentum, and an over-all systematic error of ±0.5% affecting all momenta.
[b] Uncorrected for Coulomb-barrier effect.
[c] This does not include a systematic error of ±0.2% common to all deuterium data and arising from the accuracy with which the density of liquid deuterium is presently known.
[d] Notation: a, Coulomb and decay corrections; b, the form of the extrapolation to zero solid angle; c, possible inefficiencies in the threshold counter.
[e] Averages within a range of 5 MeV/c of two measurements.
[f] Averages within a range of 5 MeV/c of five measurements.
[g] Averages within a range of 5 MeV/c of three measurements.
[h] Averages within a range of 5 MeV/c of four measurements.

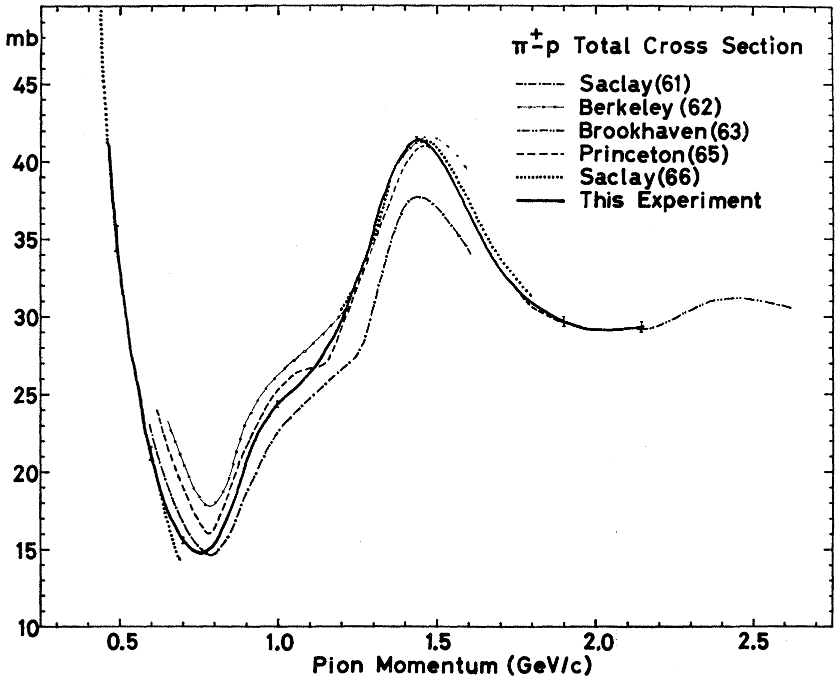

FIG. 3. Values of $\sigma(\pi^+p)$ from this experiment and from Refs. 1–4. Error bars on our value indicate the estimated systematic errors. Relative errors between neighboring points are ±0.2%.

$N^*(1688)$ of about 100 MeV, except that from the earlier Saclay experiment, which yields a rather smaller value. One new, but expected, feature of the present data is the establishment of an actual maximum in $\sigma(\pi^-p)$ at the position of the $I=\frac{3}{2}$ $N^*(1920)$. Data at the upper end of the momentum range covered in this experiment are in satisfactory agreement with that obtained previously at Brookhaven.[3]

Figure 5 shows the $I=\frac{1}{2}$ total cross section.

B. $\sigma(\pi^-d)-\sigma(\pi^+d)$

The principle of charge independence of nuclear forces leads us to expect the total nuclear cross sections for π^+ and π^- mesons on deuterium to be equal. Results obtained in this experiment, which are shown on Fig. 6, disagree with this prediction by about 1%. Many of the systematic errors cancel in the difference $\{\sigma(\pi^-d)-\sigma(\pi^+d)\}$. Table III gives a representative set of

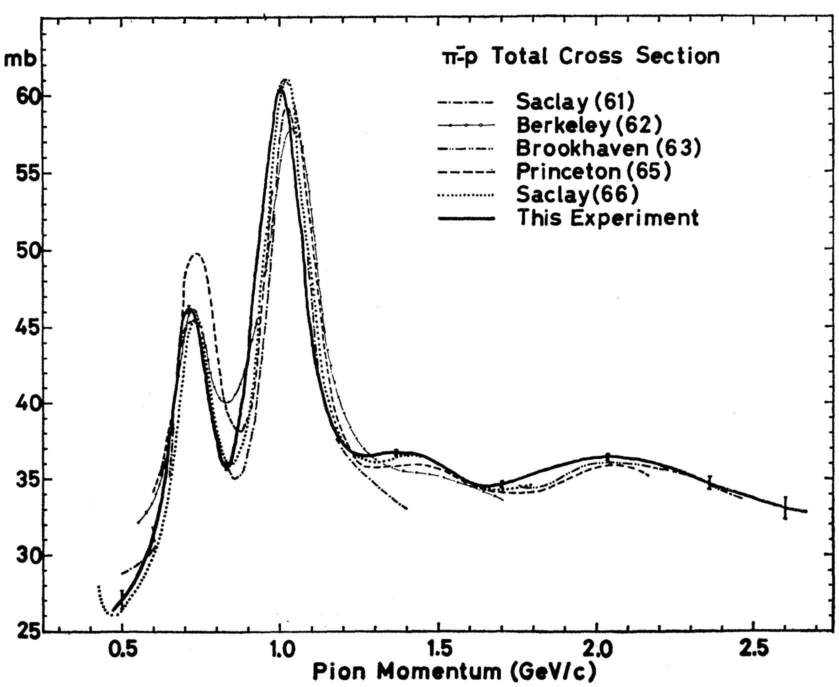

FIG. 4. Values of $\sigma(\pi^-p)$ from this experiment and from Refs. 1–4. Error bars on our values indicate the estimated systematic errors. Relative errors between neighboring points are ±0.2%.

values for this difference as a function of momentum, together with the estimated systematic errors. From 1 to 2 GeV/c, where the systematic errors are the smallest, $\sigma(\pi^- d)$ is consistently higher than $\sigma(\pi^+ d)$ by 1.3±0.3%.

Two possible causes for this discrepancy will be discussed. The first is the Coulomb-barrier penetration effect, familiar in low-energy nuclear physics. The Coulomb potential leads to an attraction of the incident π^- to the deuteron and a repulsion of the π^+, resulting

TABLE II. Total cross sections of π^+ on protons and deuterons.

p[a] (MeV/c)	$\sigma(\pi^+ p)$[b] (mb)	Systematic[c] error (mb)	$\sigma(\pi^+ d)$[b] (mb)	Systematic[c] error (mb)	Major source of error[d]
464	41.23	±0.90	...	...	a
489	35.32	0.75	63.71	±1.68	a
512	31.15[e]	0.60	59.16[f]	1.24	a, b
540	27.11	0.55	55.78	1.16	a, b
565	24.36[e]	0.42	53.44[e]	0.89	b
594	21.45	0.38	52.03	0.83	b
625	18.95	0.27	51.91	0.72	b
654	17.11	0.24	53.35[e]	0.69	b
685	16.01	0.22	55.98	0.66	b
711	15.16[e]	0.21	57.75[e]	0.62	b
730	15.00	0.21	58.04	0.60	b
764	14.71	0.20	57.09	0.58	b
794	15.21	0.18	55.88	0.55	b
812	15.75	0.18	55.59	0.51	b
818	15.96	0.18	55.42	0.51	b
838	16.89	0.18	56.23	0.51	b
868	18.60	0.18	58.71	0.50	b
897	20.38[e]	0.18	62.53[e]	0.50	b
931	22.29	0.18	68.44	0.48	b
962	23.38[e]	0.18	72.65[e]	0.45	b
992	24.10	0.17	75.48	0.40	b
1020	24.58[e]	0.17	75.95[e]	0.35	b
1049	25.30	0.17	74.40	0.33	b
1090	25.97	0.18	71.28	0.33	a, b
1131	27.15[e]	0.18	68.45[e]	0.32	a, b
1170	28.57[e]	0.19	67.09[e]	0.34	a, b
1200	29.94[e]	0.19	67.06[e]	0.34	a, b
1230	31.45[e]	0.19	67.51[e]	0.35	a, b
1259	33.20	0.19	68.17	0.35	a, b
1278	34.54	0.19	69.04	0.35	a, b
1299	35.85	0.19	69.77	0.35	b
1337	38.33[e]	0.18	71.16[e]	0.34	b
1378	40.05	0.16	72.18	0.32	b
1403	40.92[e]	0.16	72.73[e]	0.29	b
1434	41.46[e]	0.16	72.84[e]	0.26	b, c
1476	41.02	0.16	72.85	0.26	b, c
1506	40.25	0.18	71.93	0.28	b, c
1517	40.08	0.18	71.59	0.29	b, c
1556	38.32[e]	0.18	70.50[e]	0.29	b, c
1605	36.05[g]	0.17	68.86[g]	0.29	b, c
1654	34.57[e]	0.20	67.76[e]	0.32	c
1721	32.20	0.23	66.05	0.40	c
1785	31.07	0.25	65.31	0.45	c
1851	30.15[f]	0.26	64.70[e]	0.51	c
1929	29.53	0.28	64.37	0.52	c
1954	29.40	0.29	64.29	0.54	c
2004	29.33	0.30	64.20	0.56	c
2042	29.28	0.32	63.99	0.60	c
2053	29.21	0.32	63.93	0.60	c
2101	29.28	0.34	63.92	0.65	c
2469	30.99	0.42	62.92	0.81	c

[a] See Ref. a of Table I.
[b] See Ref. b of Table I.
[c] See Ref. c of Table I.
[d] See Ref. d of Table I.
[e] See Ref. e of Table I.
[f] See Ref. g of Table I.
[g] See Ref. h of Table I.

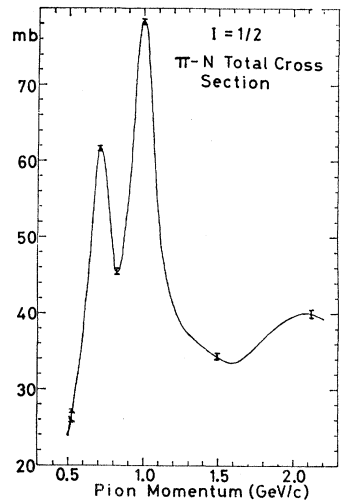

FIG. 5. Values of $\sigma(I=\frac{1}{2})$ from this experiment. Error bars indicate the estimated systematic errors. Relative errors between neighboring points are ±0.2%.

in a relative enhancement of $\sigma(\pi^- d)$. The magnitude of this effect is discussed elsewhere,[11] and the following approximate formula is derived:

$$\partial\sigma = \frac{\sigma(\pi^- d) - \sigma(\pi^+ d)}{\frac{1}{2}[\sigma(\pi^- d) + \sigma(\pi^+ d)]} = \frac{2\alpha}{k\beta}\left(\frac{1}{R_p} + \frac{1}{R_d}\right), \quad (1)$$

$R_p = 1.10 \pm 0.10$ F.

TABLE III. The difference between π^- and π^+ total cross sections on the deuteron. No correction has been applied for Coulomb-barrier effects.

p (MeV/c)	$\sigma(\pi^- d) - \sigma(\pi^+ d)$ (mb)	Systematic error (mb)	$\partial\sigma$ (%)
512	1.40±0.17	±0.66	2.4±1.4
564	0.92±0.15	0.62	1.6±1.4
623	0.41±0.15	0.55	0.8±1.3
711	0.62±0.16	0.40	1.1±1.0
814	0.88±0.15	0.33	1.6±0.9
989	0.41±0.21	0.24	0.5±0.6
1017	0.67±0.21	0.23	0.9±0.6
1089	1.12±0.20	0.20	1.5±0.6
1127	1.22±0.20	0.20	1.8±0.6
1167	1.23±0.19	0.21	1.8±0.6
1197	1.23±0.19	0.20	1.8±0.6
1227	1.20±0.19	0.21	1.7±0.6
1276	0.88±0.19	0.20	1.3±0.6
1334	0.88±0.20	0.23	1.3±0.6
1378	1.07±0.20	0.25	1.5±0.6
1400	0.60±0.20	0.26	0.8±0.6
1433	0.72±0.20	0.18	1.0±0.5
1475	0.25±0.20	0.16	0.3±0.5
1604	0.75±0.19	0.17	1.1±0.5
1719	0.83±0.18	0.28	1.2±0.7
1784	0.86±0.18	0.28	1.3±0.7
1850	0.69±0.18	0.23	1.1±0.6
1952	0.41±0.18	0.30	0.6±0.6
2051	0.60±0.18	0.25	0.9±0.7
2100	0.60±0.18	0.31	0.9±0.8
2469	0.18±0.18	0.72	0.3±1.4

[11] D. V. Bugg and N. Cottingham, Rutherford Laboratory Report No. RPP/H/30, 1967 (unpublished).

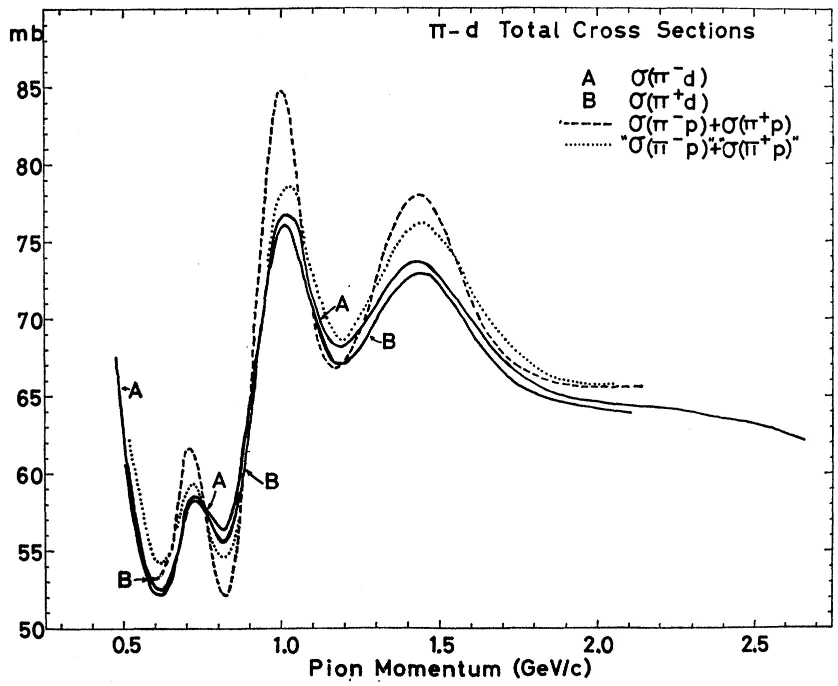

FIG. 6. The total cross sections $\sigma(\pi^-d)$ and $\sigma(\pi^+d)$ observed in this experiment. Relative systematic errors are given in Table III. The dashed curve shows $\sigma(\pi^+p)+\sigma(\pi^-p)$; the dotted curve shows

$$``\sigma(\pi^+p)" + ``\sigma(\pi^-p)",$$

that is, the cross sections with the effect of Fermi motion folded in.

Here k is the wave number for π-nucleon collisions, R_d is 3.26 F, and R_p is a figure parametrizing the nuclear radius of the nucleon and derived from the shape of the angular distribution for elastic scattering. One finds $\partial\sigma = 0.6\%$ at 1 GeV/c and 0.4% at 2 GeV/c, with probable errors of the order of $\pm 0.1\%$. This accounts for only about $\frac{1}{3}$ to $\frac{1}{2}$ of the observed effect.

Thus either charge independence is violated in a systematic way by about 0.75% or there is a residual experimental systematic error of this order. One is inclined to believe the latter. Figure 7 shows a plot of the π^+d and π^-d data as a function of solid angle. If there were to be no discrepancy, an extrapolation such as that shown by the dotted line would be required; the measured data do not justify any polynomial higher than the quadratic fit shown by the dashed line. The slope of $\sigma(\pi^-d)-\sigma(\pi^+d)$ against solid angle is, of course, caused by $\pi^-p \rightarrow$ neutral final states. There is no obvious reason why this slope should steepen near $t=0$. Indeed, one might expect the reverse, since here elastic πp charge exchange is forbidden at $t=0$ by the Pauli principle.

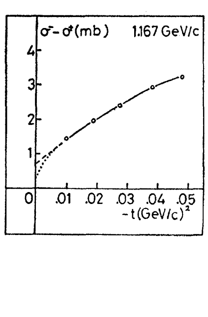

FIG. 7. The observed values of $\{\sigma(\pi^-d)-\sigma(\pi^+d)\}$ as a function of solid angle subtended by the transmission counters. The dashed curve is a least-squares quadratic fit. The dotted curve shows the sort of extrapolation which would be required to agree with charge independence, after allowance for Coulomb-barrier effects discussed in the text.

One concludes that there may be an unknown systematic error of the order of 0.75% in the total cross sections obtained here. The π^+ beam differs from the π^- by virtue of the accompanying flux of protons. These will be heavily ionizing at low momenta and could cause some malfunctioning of the scintillation counters or electronics. If so, the effect should disappear at low rates. During the experiment, data were taken over a range of intensities of about a factor of 10, but no such rate dependence of the results was observed.

C. Glauber Correction

Glauber[12] has proposed that the shadowing of one nucleon by the other in the deuteron may be approximated by the formula

$$\sigma(\pi d) = \sigma(\pi p) + \sigma(\pi n) - \langle r^{-2}\rangle Q/4\pi,$$

where, in the impulse approximation,[13]

$$Q = \sigma(\pi p)\sigma(\pi n) - (4\pi/k)^2 \,\mathrm{Re} f_{\pi P}(0)\, \mathrm{Re} f_{\pi N}(0),$$

and

$$\langle r^{-2}\rangle = \int_0^\infty d^2\mathbf{q}\, S(\mathbf{q})\, \mathrm{Re}\{f_{\pi P}(\mathbf{q}) f_{\pi N}(\mathbf{q})\} / \mathrm{Re}\{f_{\pi P}(0) f_{\pi N}(0)\}, \quad (2)$$

where $S(\mathbf{q})$ is the form factor of the deuteron. Wilkin[14] has pointed out that Glauber's form for Q needs modi-

[12] R. J. Glauber, Phys. Rev. **100**, 242 (1955).
[13] V. Franco and R. J. Glauber, Phys. Rev. **142**, 1195 (1966).
[14] C. Wilkin, Phys. Rev. Letters **17**, 561 (1966).

fication to take charge independence into account. He gives

$$Q=\sigma(\pi p)\sigma(\pi n)[1-\gamma_p\gamma_n]-\tfrac{1}{4}\{[\sigma(\pi p)-\sigma(\pi n)]^2 -[\gamma_p\sigma(\pi p)-\gamma_n\sigma(\pi n)]^2\},$$

where

$$\gamma_{p,n}=\mathrm{Re}f(0)_{\pi p,n}/\mathrm{Im}f(0)_{\pi p,n}.$$

The data of the present experiment have been used assuming charge independence to evaluate $\langle r^{-2}\rangle$ as a function of momentum. In this calculation, the effects of the Fermi motion of the struck nucleons in the deuteron has been included using the methods described in I; the wave function

$$\psi_r=Ne^{-ar}(1-e^{-cr})/r \qquad (3)$$

has been used for the deuteron with $a=0.232$ F^{-1} and $c=0.970$ F^{-1}. The mean of $\sigma(\pi^+d)$ and $\sigma(\pi^-d)$ has been used for the deuteron total cross section.

The result is displayed in Fig. 8. The dashed lines indicate estimated limits of systematic errors. Again some cancellation occurs between the systematic errors in the quantity

$$\sigma(\pi d)-\sigma(\pi p)-\sigma(\pi n).$$

The parameter $\langle r^{-2}\rangle$ shows considerable variations, which coincide with structure in $\sigma(\pi d)$. Similar variations have been seen in the recent experiments at Brookhaven[15] and in those reported by Leray[16] at the Sienna Conference. Equation (2) does not account for these variations. Changing the wave function of the deuteron does not help; if c is varied over a large range or even set to ∞, $\langle r^{-2}\rangle$ changes by less than the indicated errors. The parameter a is given by the binding energy of the deuteron, and cannot be changed. One concludes that the simple Glauber theory is inadequate in the resonance region, and further theoretical understanding is required before deuterium targets can be used to obtain unambiguous information about scattering from neutrons. Empirically, what is needed is something which smooths out structure in $\sigma(\pi d)$ even more than the Fermi motion.

[15] R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, J. Teiger, and C. Wilkin (to be published); G. Giacomelli (private communication).

[16] T. Leray, A. Berthon, M. Crozon, and J. L. Narjoux, in *Proceedings of the Sienna International Conference on Elementary Particles and High-Energy Physics, 1963*, edited by G. Bernardini and G. P. Puppi (Società Italiana de Fisica, Bologna, Italy, 1963), Vol. 1, p. 102; College de France, Rapport Interne, No. PAM 6310, 1963 (unpublished).

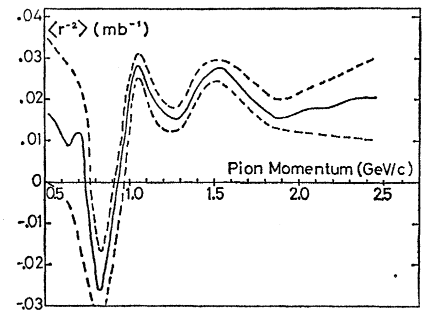

FIG. 8. The parameter $\langle r^{-2}\rangle$ of the Glauber theory as a function of momentum. The dashed lines indicate estimated limits of experimental error.

The Hulthen wave function (3) predicts a value of $\langle r^{-2}\rangle$ of about 0.031 mb^{-1}. Above 1 BeV/c, the mean value of $\langle r^{-2}\rangle$ observed in this experiment is about 0.02 mb^{-1}. This value is rather lower than one would expect although not absurdly so. Previous determinations by Baker *et al.*[17] and Galbraith *et al.*[18] have given 0.024 and 0.042 mb^{-1}, respectively, both with systematic errors of the order of ± 0.01 mb^{-1}.

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[17] W. F. Baker, E. W. Jenkins, T. F. Kycia, R. H. Phillips, A. L. Read, K. F. Riley, and H. Ruderman, in *Proceedings of the Sienna International Conference on Elementary Particles and High-Energy Physics, 1963*, edited by G. Bernardini and G. P. Puppi (Società Italiana de Fisica, Bologna, Italy, 1963), Vol. 1, p. 634.

[18] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontić, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).