# Energy Levels of $Er^{167}$, $Er^{169}$, and $Er^{171}$ from $(d,p)$ Reactions*

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High-resolution $(d,p)$-reaction spectroscopy has been used to study the energy levels of $Er^{167}$, $Er^{169}$, and $Er^{171}$ to excitation energies greater than 2.5 MeV. The energy resolution for proton groups observed in a magnetic spectrograph was $\approx 0.08\%$. The measured $Q$ values are $4209\pm10$, $3773\pm10$, and $3450\pm10$ keV for $Er^{166}(d,p)Er^{167}$, $Er^{168}(d,p)Er^{169}$, and $Er^{170}(d,p)Er^{171}$ reactions, respectively. A large number of the observed energy levels have been assigned to specific Nilsson orbitals and their superimposed rotational bands. Generally, the energy systematics of the Nilsson bands qualitatively follow expectations of the Nilsson model. The energy systematics, decoupling parameters, and cross sections suggest considerable mixing between $\gamma$-vibrational bands and certain Nilsson states with the same spin and parity. The strongest indications in these data for mixings of collective and intrinsic states involve the $\frac{1}{2}-[510]$ band. In $Er^{167}$ and $Er^{169}$, this band appears to be considerably lowered from the position expected by energy systematics, and appears to have a lower cross section than expected for a pure $\frac{1}{2}-[510]$ band.

## I. INTRODUCTION

In the region of mass numbers from $\approx 150$ to $\approx 190$, nuclear energy-level schemes have characteristics which suggest these nuclei have permanent deformations. Energy levels in these nuclei follow collective-model[1,2] systematics. Reviews[3–5] concerning deformed odd-mass nuclei indicate a high degree of regularity of many of the properties of these nuclei. The purpose of the present research is to provide information on the positions of energy levels predicted by the Nilsson model[6] in $Er^{167}$, $Er^{169}$, and $Er^{171}$, and to imply information about excitations having collective character.

The lower excited states of $Er^{167}$ have been well studied by a variety of techniques, and results[7] and references[7,8] have been summarized. A considerable amount of our understanding of $Er^{167}$ levels has come from decay-scheme[9] and neutron-capture[10] experiments. Information about $Er^{169}$ levels is less extensive, and available decay schemes[11,12] leading to $Er^{169}$ levels disagree in some details. Lower excited states of $Er^{171}$ have been less intensively investigated.

Isoya[13] performed coarse-resolution $(d,p)$ experiments on Er isotopes using natural-Er targets and reported a few levels in the nuclei of interest here. Experimental levels reported in the present paper from high-resolution $(d,p)$ experiments on enriched Er targets have previously been discussed,[14] and this earlier interpretation[14] has been extended considerably in the present paper. We have attempted to use the available data[7–10] on $Er^{167}$, especially energy separations and comparisons of the $(d,p)$ cross sections of analogous levels in neighboring odd-mass Dy, Yb, Hf, and W nuclei,[15–17] to identify the intrinsic and collective states in $Er^{167}$, $Er^{169}$, and $Er^{171}$.

## II. EXPERIMENTAL PROCEDURES

Targets were prepared[14] by evaporating $Er_2O_3$ obtained from Oak Ridge National Laboratory (ORNL) enriched in the various target nuclei onto 20–40-$\mu g/cm^2$ carbon backings. The enriched $Er_2O_3$ sample analyses as reported by ORNL are given in Table I. Because

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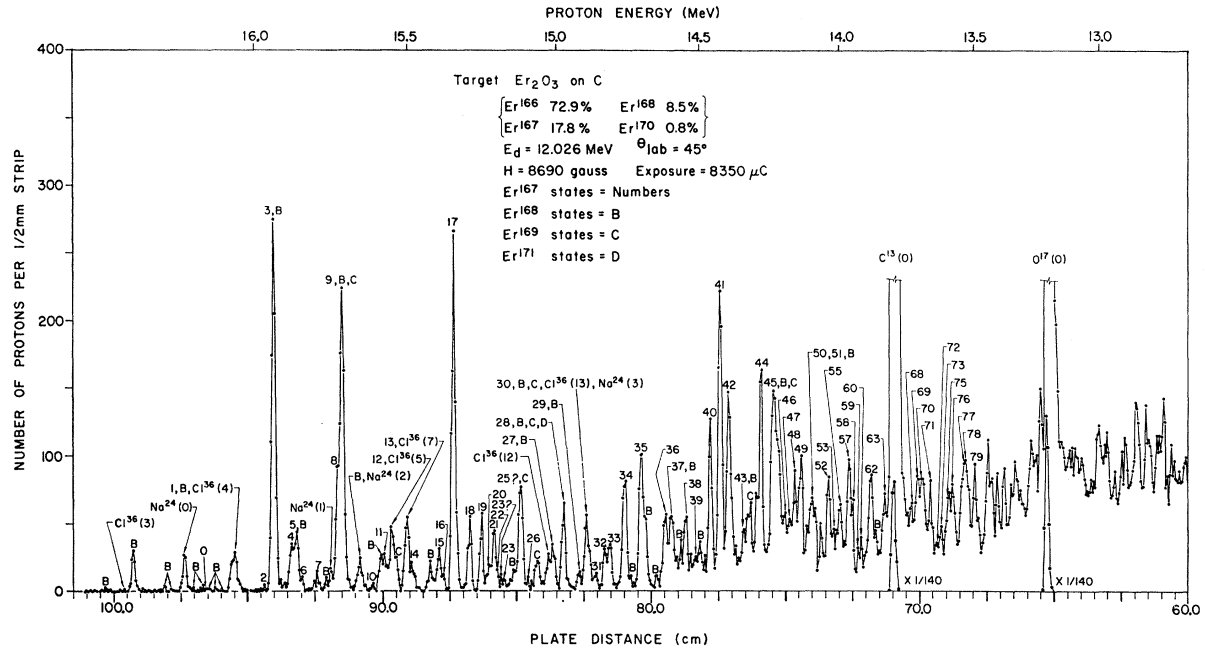


FIG. 1. Proton spectrum at 45° for the reaction $Er^{166}(d,p)Er^{167}$. The target contained approximately 200 (μg Er)/cm². The numbers indicate $Er^{167}$ levels. Letters indicate proton groups, or a contribution to a numbered proton group, from other Er isotopes as indicated. Positions of some levels of various light-mass impurities are also indicated.

there is reason[14,18,19] to believe the target material was not necessarily deposited as stoichiometric $Er_2O_3$, target thicknesses were determined by scattering 4-MeV deuterons from the targets. In calculating the amounts of Er in the targets, the observed intensity of the elastically scattered deuterons were assumed to be given by the Rutherford scattering law.

Targets were prepared for each isotope investigated with thicknesses ranging from ≈200 to ≈1000 μg/cm². Protons emitted from the targets were analyzed with a 60-cm Browne-Buechner-type[20] spectrograph. The exposure and calibration procedure has been previously described.[14,21] A total of 20 exposures were made using two targets each of $Er^{166}$ and $Er^{168}$ and using one target of $Er^{170}$. Data reported herein were obtained with exposures varying from ≈3500 to ≈8500 μC at laboratory angles of 35°, 45°, 60°, and 65° for $Er^{166}$, of 45°, 65°, and 133° for $Er^{168}$, and of 45°, 65°, and 133° for $Er^{170}$ targets, with incident deuteron energies of ≈12.0 MeV. One of the $Er^{166}(d,p)Er^{167}$ exposures at 45° was taken with 12.5-MeV incident energy. With the exception of some Na levels and Cl levels, no light impurities were consistently identified in the proton-energy spectra, although there were indications of small amounts of Ca and Si. No heavy impurities with mass differing significantly from ≈170 amu were identified by varying the spectrograph angle.

TABLE I. Target analyses of enriched Er targets.[a]

| Isotope enriched | Percentages of Er isotopes | | | | | |
|---|---|---|---|---|---|---|
| | $Er^{162}$ | $Er^{164}$ | $Er^{166}$ | $Er^{167}$ | $Er^{168}$ | $Er^{170}$ |
| 166[b] | <0.1 | 0.1 | 72.9 | 17.7 | 8.5 | 0.8 |
| 167[c] | <0.1 | <0.1 | 14.0 | 58.8 | 25.3 | 1.93 |
| 168[d] | <0.05 | <0.05 | 3.4 | 11.4 | 76.9 | 8.3 |
| 170[d] | <0.05 | <0.05 | 1.68 | 2.1 | 9.0 | 87.3 |

[a] Obtained from the Stable Isotopes Division, Oak Ridge National Laboratory, Oak Ridge, Tenn.
[b] Other rare earths reported <0.2%; Si present, but <0.02%.
[c] Other rare earths reported <0.3%; Ca present, but <0.02%.
[d] Other rare earths reported <0.2%; Ca present, but <0.02%.

Relatively poor enrichments of the target materials, together with high energy-level densities in final nuclei, caused considerable difficulty in making isotopic assignments of the observed proton groups. Isotopic assignments of observed proton groups were made on the basis of their relative intensities observed in four targets of widely varying isotopic compositions (see Table I). An internal normalization was performed for a set of four exposures at 45° leading to levels in $Er^{167}$, $Er^{168}$, $Er^{169}$, and $Er^{171}$. The $Er^{168}$ energy-level and cross-section data have been reported earlier.[21] A proton group corresponding[21] to the 1094-keV excited state in $Er^{168}$ was intense enough to appear in the spectra giving $Er^{167}$ and $Er^{169}$ states. This group was used to normalize data taken at 45° in other isotopes to an $Er^{166}(d,p)Er^{167}$ run made on a thin (≈200 μg/cm²), uniform $Er^{166}$ target. Because the $Er^{168}$ group was too weak in $Er^{171}$ spectra for a reliable normalization, a

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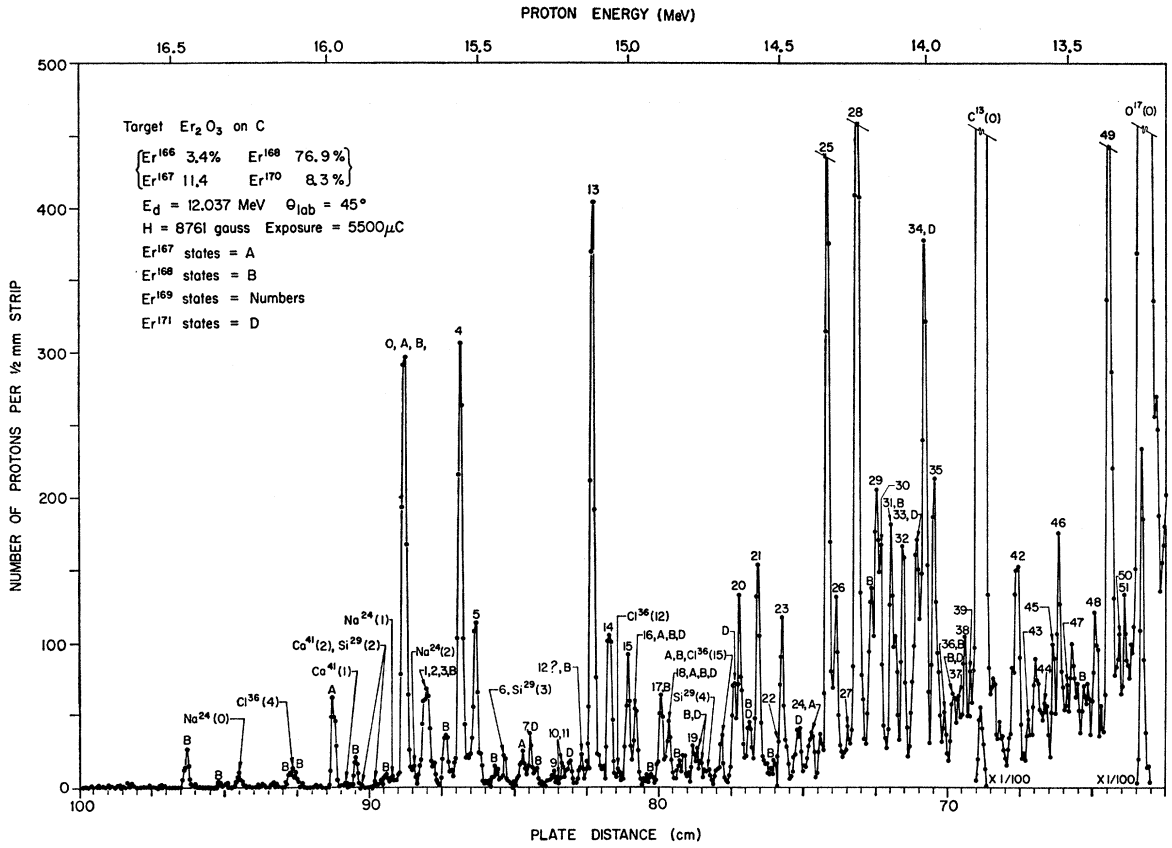


FIG. 2. Proton-energy spectrum at 45° for the reaction $Er^{168}(d,p)Er^{169}$. The target contained approximately 500 ($\mu$g Er)/cm$^2$. The group-labeling scheme is analogous to that of Fig. 1.

secondary standard was chosen to relate $Er^{171}$ data to the other Er data. This secondary-standard group with a $Q$ value of 2711 keV was relatively uncontaminated by Er-isotope impurity groups. Relative errors in the normalization were determined mainly by counting statistics, and are $\pm 15\%$ for most groups. For the smallest groups, containing fewer than 50 counts, statistical counting error becomes $>15\%$. An error of $\approx \pm 60\%$ is assigned to absolute cross sections, because a discrepancy of $\approx 40\%$ was discovered in the beam-integrating circuit between the time target-thickness data and some of the energy-level data were taken.

## III. RESULTS

Energy levels observed in $Er^{167}$, $Er^{169}$, and $Er^{171}$ are given in Table II. The excitation energies reported in this paper are believed accurate within $\pm 3$ keV, except as indicated in the footnotes in Table II. A few groups not listed in Table II are considered individually in Sec. IV, which follows this section.

Proton-energy spectra are shown in Figs. 1, 2, and 3, with numbers above the proton groups indicating levels in $Er^{167}$, $Er^{169}$, and $Er^{171}$, respectively. Capital letters are used in Figs. 1–3 to indicate proton groups arising from "isotopic-impurity" groups. The letters A, B, C, and D indicate proton groups corresponding to states in $Er^{167}$, $Er^{168}$, $Er^{169}$, and $Er^{171}$, respectively. A rough criterion for using one or more of the letters A, B, C, or D is that an isotopic-impurity group contributes $>10\%$ of the counts in the lettered group. At plate distances greater than that of the ground-state group of interest in a particular spectrum only the stronger isotopic impurities are indicated, to avoid needless detail in Figs. 1, 2, and 3. There are a few instances where letters are indicated (for emphasis) with a numbered group, even though the contribution is somewhat less than 10%. These generally occur when a group appears in three or all four of the Er nuclei investigated[22] within 10 keV of the same $Q$ value. Whenever a light impurity might contribute $\gtrsim 10\%$ of the total intensity of a group, the final state in the light nucleus is indicated, e.g., "$Na^{24}(0)$" would indicate the ground-state proton group for the $Na^{23}(d,p)Na^{24}$ reaction, "$Na^{24}(3)$" would indicate the third excited state of $Na^{24}$, etc. Light-impurity-group positions are not indicated in Figs. 1–3 if addition of such labels would not materially add to information presented in Figs. 1, 2, and 3. Such a light-impurity group would not be indicated, e.g., if the position at

[22] See Ref. 21 for $Er^{168}$ levels.

TABLE II. Energy levels of $Er^{167}$, $Er^{169}$, and $Er^{171}$.

| Group No. | $Er^{167}$ $Q$ (keV)[a,b] | Excitation energy (keV) | $Er^{169}$ $Q$ (keV)[a,c] | Excitation energy (keV) | $Er^{171}$ $Q$ (keV)[a,d] | Excitation energy (keV) |
|---|---|---|---|---|---|---|
| 0 | (4209) | 0.0 | 3773 | 0.0 | 3450 | 0.0 |
| 1 | 4131 | 78 | 3704 | 69 | 3374 | 76 |
| 2 | (4030) | 179 | 3694±5 | 79±5 | 3274 | 176 |
| 3 | 4001 | 208 | 3685 | 88 | 3256 | 194 |
| 4 | 3947 | 262 | 3594 | 179 | 3177 | ≈273[e] |
| 5 | 3929 | 280 | 3546 | 227 | (3141) | (309) |
| 6 | 3909 | 300 | 3454 | 319 | 3032 | 418 |
| 7 | 3859 | 350 | 3378 | 395 | (2996) | (454) |
| 8 | 3793 | 416 | 3353 | 420 | 2923 | 527 |
| 9 | 3780 | 429 | 3301 | 472 | 2805±5 | 645±5 |
| 10 | 3671 | 538 | (3288) | (485) | 2749 | 701 |
| 11 | 3636 | 573 | (3269) | (504) | 2710 | 740 |
| 12 | 3614 | 595 | ((3218)) | (555) | 2660 | 790 |
| 13 | 3555 | 654 | 3173 | 600 | 2575 | 875 |
| 14 | 3535 | 674 | 3118 | 655 | 2550 | 900 |
| 15 | 3459 | 750 | 3058 | 715 | 2484 | 966 |
| 16 | 3442 | 767 | (3030) | (743) | 2394±4 | 1066±4 |
| 17 | 3406 | 803 | 2949 | 824 | 2340±4 | 1110±4 |
| 18 | 3357 | 852 | 2923 | 850 | 2234 | 1216 |
| 19 | 3313 | 896 | 2846 | 927 | 2195 | 1255 |
| 20 | 3282 | 927 | 2693 | 1082 | 2163 | 1287 |
| 21 | 3269 | 940 | 2626 | 1147 | 2152 | 1298 |
| 22 | 3256 | 953 | 2556 | 1217 | 2123 | 1327 |
| 23 | ((3237))[f] | ((972))[f] | 2540 | 1233 | 2078 | 1372 |
| 24 | ((3219))[g] | ((990))[g] | 2452 | 1321 | 2051 | 1399 |
| 25 | ((3176)) | ((1033)) | 2387±5 | 1386±5 | 1984 | 1466 |
| 26 | 3153 | 1056 | 2362±7 | 1411±7 | 1951 | 1499 |
| 27 | 3071 | 1138 | 2324 | 1452 | 1918 | 1532 |
| 28 | 3036 | 1173 | 2284 | 1489 | 1888 | 1562 |
| 29 | (2982) | (1227) | 2214 | 1559 | 1835 | 1615 |
| 30 | (2954) | (1255) | 2201 | 1572 | 1801 | 1649 |
| 31 | 2926 | 1283 | 2168 | 1605 | 1771 | 1679 |
| 32 | 2900 | 1309 | 2128 | 1654 | 1731 | 1719 |
| 33 | 2879 | 1330 | 2077 | 1696 | 1692 | 1758 |
| 34 | 2824 | 1385 | 2051 | 1722 | 1661 | 1789 |
| 35 | 2766 | 1443 | 2017 | 1756 | 1597 | 1853 |
| 36 | 2683 | 1526 | 1975 | 1798 | 1534 | 1916 |
| 37 | 2663 | 1546 | 1928 | 1845 | 1498 | 1952 |
| 38 | 2614 | 1595 | 1909 | 1864 | 1483 | 1967 |
| 39 | 2578 | 1631 | 1890 | 1883 | 1468 | 1982 |
| 40 | 2523 | 1686 | 1866 | 1907 | 1436 | 2014 |
| 41 | 2489 | 1720 | 1816 | 1957 | 1419 | 2031 |
| 42 | 2460 | 1749 | 1735 | 2038 | 1397 | 2053 |
| 43 | 2409 | 1800 | 1682 | 2091 | 1364 | 2086 |
| 44 | 2342 | 1867 | 1618 | 2155 | 1334 | 2116 |
| 45 | 2296 | 1913 | 1592 | 2181 | 1319 | 2131 |
| 46 | 2248 | 1961 | 1575 | 2198 | 1294 | 2156 |
| 47 | 2233 | 1976 | 1530 | 2243 | 1282 | 2168 |
| 48 | 2214 | 1995 | 1445 | 2328 | 1260 | 2190 |
| 49 | 2193 | 2016 | 1394 | 2379 | 1237 | 2213 |
| 50 | 2159 | 2050 | 1354 | 2419 | 1197 | 2253 |
| 51 | 2142 | 2067 | 1335 | 2438 | 1174 | 2276 |
| 52 | 2096 | 2113 | 1289 | 2484 | 1130 | 2320 |
| 53 | 2080 | 2129 | 1251 | 2522 | 1097 | 2353 |
| 54 | 2071 | 2138 | 1191 | 2582 | 1071 | 2379 |
| 55 | 2053 | 2156 | | | 1045 | 2405 |
| 56 | 2040 | 2169 | | | 1017 | 2433 |
| 57 | 2019 | 2190 | | | 979 | 2471 |
| 58 | 2008 | 2201 | | | 945 | 2505 |
| 59 | 1984 | 2225 | | | 900 | 2550 |
| 60 | 1971 | 2238 | | | 873 | 2577 |
| 61 | 1960 | 2249 | | | 848 | 2602 |
| 62 | 1940 | 2269 | | | 834 | 2616 |
| 63 | 1890 | 2319 | | | 804 | 2646 |
| 64 | 1873 | 2336 | | | 785 | 2665 |
| 65 | 1848 | 2361 | | | 767 | 2683 |
| 66 | 1825 | 2384 | | | 745 | 2705 |
| 67 | 1799 | 2408 | | | 723 | 2727 |
| 68 | 1787 | 2422 | | | 686 | 2764 |
| 69 | 1762 | 2447 | | | 663 | 2787 |
| 70 | 1747 | 2462 | | | 636 | 2814 |
| 71 | 1720 | 2489 | | | 606 | 2844 |
| 72 | 1691 | 2518 | | | 561 | 2889 |

TABLE II. (*continued*).

| Group No. | $Er^{167}$ $Q$ (keV)[a,b] | Excitation energy (keV) | $Er^{169}$ $Q$ (keV)[a,c] | Excitation energy (keV) | $Er^{171}$ $Q$ (keV)[a,d] | Excitation energy (keV) |
|---|---|---|---|---|---|---|
| 73 | 1679 | 2530 | | | 518 | 2932 |
| 74 | 1657 | 2552 | | | 493 | 2957 |
| 75 | 1647 | 2562 | | | 467 | 2983 |
| 76 | 1633 | 2576 | | | 440 | 3010 |
| 77 | 1599 | 2610 | | | 415 | 3035 |
| 78 | 1573 | 2633 | | | 398 | 3052 |
| 79 | 1553 | 2656 | | | 376 | 3074 |
| 80 | | | | | 342 | 3108 |
| 81 | | | | | 307 | 3143 |
| 82 | | | | | 290 | 3160 |
| 83 | | | | | 260 | 3190 |
| 84 | | | | | 234 | 3216 |
| 85 | | | | | 219 | 3231 |
| 86 | | | | | 195 | 3255 |
| 87 | | | | | 149 | 3301 |
| 88 | | | | | 105 | 3345 |
| 89 | | | | | 67 | 3383 |
| 90 | | | | | 36 | 3414 |
| 91 | | | | | −40 | 3490 |

[a] Parentheses indicate a level for which the evidence is weak because of (1) background problems (applies to very small groups) or (2) insufficient resolution from other Er levels or impurities. Double parentheses indicate both problems contribute to this uncertainty.
[b] The $Q$ values and excitation energies reproduced within ±3 keV of the mean through group No. 42. Above No. 42, the level density causes 4–10-keV deviations. The estimated absolute error is approximately ±10 keV for Nos. 0–42 and approximately ±15 keV for Nos. 43–79 in the $Q$ values.
[c] The $Q$ values and excitation energies deviating more than 4 keV from the mean are noted for levels up through No. 35. Above No. 35, the level density causes a somewhat larger error of approximately 5–10 keV. Absolute errors of approximately ±10 keV in $Q$ values are estimated for levels 0–35 and of approximately ±15 keV for levels 36–54.
[d] The $Q$ values and excitation energies deviating more than 3 keV from the mean are reported through level No. 40. Above No. 40, level density causes 4–10-keV deviations. The estimated absolute error in $Q$ values is approximately ±10 keV for Nos. 0–40 and approximately ±15 keV for Nos. 43–91.
[e] This group in $Er^{171}$ spectra is probably a doublet, with excitation energies of ≈268 and ≈278 keV. See the text.
[f] In Fig. 1, labeled as "23?".
[g] In Fig. 1, labeled as "23".

45° would be directly under a strong Er group, or if it occurs in regions of high energy-level density.

In the case of $Er^{167}$, the ground-state proton group could not definitely be observed, because $Er^{168}$ excited-state proton groups fall in this region. Theoretical calculations discussed later indicate the ground-state group should not be observed in our experiments. However, an upper limit on the cross section for the $Er^{167}$ ground state was obtained using a few counts which appear to be above the general background. The $Er^{166}(d,p)Er^{167}$ ground-state $Q$ value of 4209±10 keV was obtained by adding the $Q$ value for group No. 3 (4001±10 keV) in Fig. 1, to the excitation energy of the well-known[7,10] $\frac{1}{2}$-state occurring at 207.8 keV in $Er^{167}$. Cross sections discussed later indicate group 3 corresponds to the 207.8 keV excited state of $Er^{167}$. The ground-state groups of $Er^{169}$ and $Er^{171}$ were observed and the $Q$ values for the $(d,p)$ reaction leading to these nuclei were determined as 3773±12 and 3450±10 keV, respectively.

The group labeled 4 in the $Er^{171}$ spectrum shown in Fig. 3 is believed to be a doublet, but an especially strong contribution from $Er^{169}$-group No. 13 prevents an accurate determination of the energy separation, estimated to be ≈10 keV. If it is a doublet, the excitation energies would be ≈268 and ≈278 keV, whereas the excitation energy is ≈273 keV if the group corresponds to a single state. Except for $Er^{171}$-group 4, impurities did not present severe problems in interpreting the lower-energy region of the energy levels in $Er^{171}$.

Differential cross sections at 45° are given in Table III for populating levels in the three nuclei, $Er^{167}$, $Er^{169}$, and $Er^{171}$. The cross sections are reported as relative values for each isotope in Table III; the group-3 intensity ≡1.00 for $Er^{167}$, the group-0 intensity ≡1.00 for $Er^{169}$, and the group-1 intensity ≡1.00 for $Er^{171}$. Footnotes to Table III give factors to obtain a set of cross sections which are internally consistent within the normalization error of approximately ±15%.

## IV. DISCUSSION

### A. General

In attempting to identify the intrinsic states and their associated bands in $Er^{167}$, $Er^{169}$, and $Er^{171}$, two complementary methods were used. One method involves identification of characteristic patterns in cross sections of individual band members. These characteristic patterns might be described as band "fingerprints" or band "signatures." Reasonable values for rotational-energy separations of band members provide the second, complementary identification method. Both criteria are explained below.

The differential cross section for a $(d,p)$ reaction on an even nucleus may be written[23] as

$$d\sigma_{I,l,\Omega,N}/d\omega = 2C_{I,l}(\Omega,N)^2\phi_l U_i^2. \quad (1)$$

In Eq. (1) the quantities $C_{I,l}(\Omega,N)$ are the expansion coefficients[23] for the Nilsson orbitals[3,6] in terms of the

[23] G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

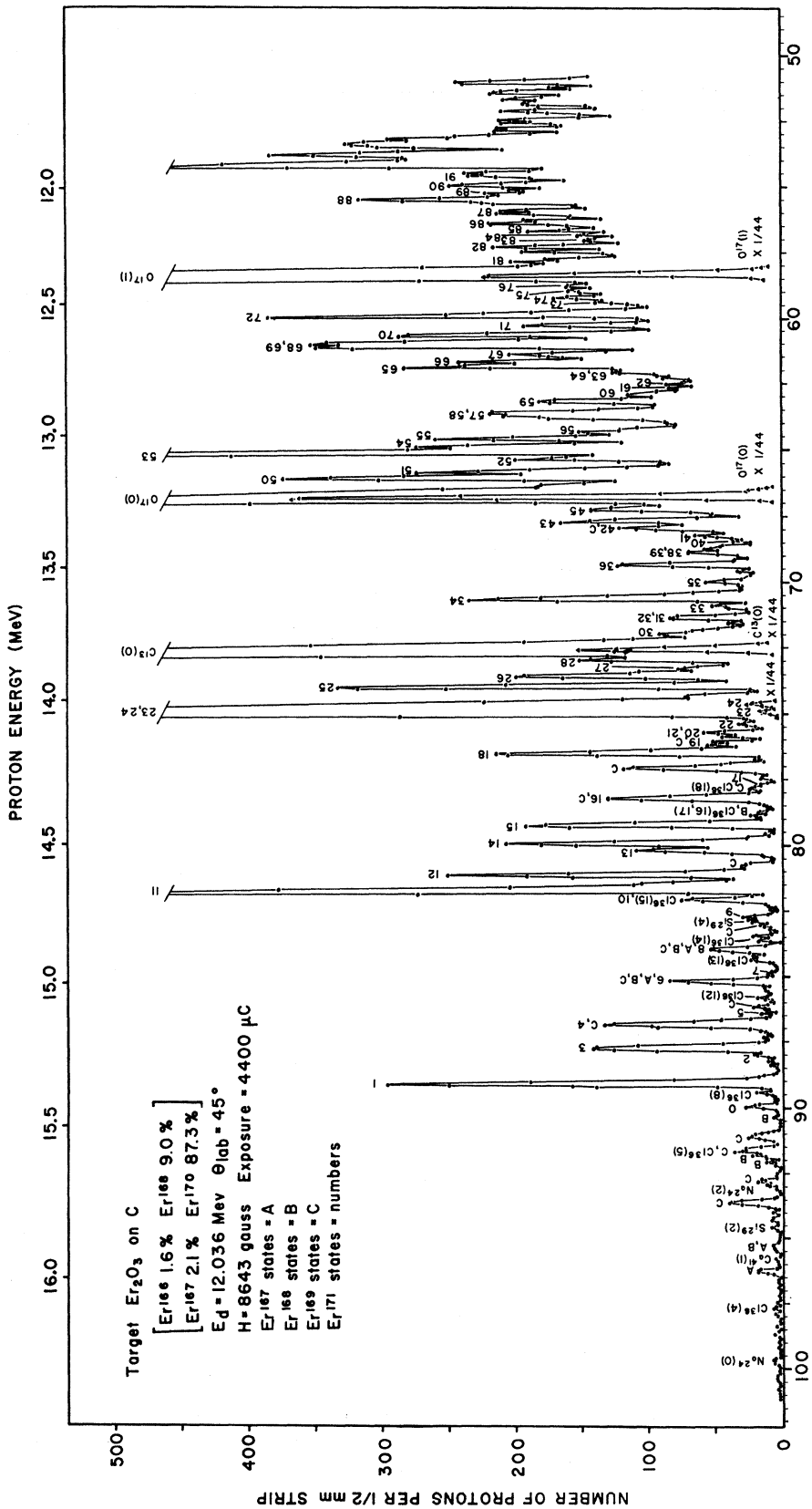


FIG. 3. Proton spectrum at 45° for the reaction $Er^{170}(d,p)Er^{171}$. The target thickness was approximately 700 μg Er/cm². The group-labeling scheme is analogous to that of Fig. 1.

TABLE III. Relative differential cross sections at 45° for population of energy levels in $Er^{167}$, $Er^{169}$, and $Er^{171}$ by the $(d,p)$ reaction. Parentheses correspond to those in Table II.

| $Er^{167}$ | | $Er^{169}$ | | $Er^{171}$ | |
|---|---|---|---|---|---|
| Group No. in Fig. 1 | Relative cross section[a] | Group No. in Fig. 2 | Relative cross section[a] | Group No. in Fig. 3 | Relative cross section[a] |
| 0 | <0.005 | 0 | 1.00 | 0 | 0.061 |
| 1 | ≤0.09 | 1 | ≈0.042 | 1 | 1.00 |
| 2 | ≤0.008 | 2 | ≈0.21 | 2 | 0.05 |
| 3 | 1.00 | 3 | 0.046 | 3 | 0.58 |
| 4 | 0.15 | 4 | 1.02 | 4 | 0.48[b] |
| 5 | 0.065 | 5 | 0.44 | 5 | 0.024 |
| 6 | 0.025 | 6 | 0.057 | 6 | 0.23 |
| 7 | 0.08 | 7 | 0.046 | 7 | 0.024 |
| 8 | 0.45 | 8 | 0.013 | 8 | 0.018 |
| 9 | 0.84 | 9 | 0.014 | 9 | 0.053 |
| 10 | 0.087 | 10 | (0.012) | 10 | 0.072 |
| 11 | 0.069 | 11 | (0.039) | 11 | 1.85 |
| 12 | ≤0.023 | 12 | ((0.044)) | 12 | 0.77 |
| 13 | ≤0.15 | 13 | 1.26 | 13 | 0.34 |
| 14 | 0.046 | 14 | 0.38 | 14 | 0.61 |
| 15 | 0.11 | 15 | 0.24 | 15 | 0.60 |
| 16 | 0.027 | 16 | (0.11) | 16[c] | 0.047[c] |
| 17 | 1.66 | 17 | 0.19 | 17 | 0.039 |
| 18 | 0.47 | 18 | 0.14 | 18 | 0.75 |
| 19 | 0.45 | 19 | 0.066 | 19 | 0.18 |
| 20 | 0.035 | 20 | 0.28 | 20 | 0.095 |
| 21 | 0.066 | 21 | 0.45 | 21 | 0.135 |
| 22 | 0.045 | 22 | 0.32 | 22 | 0.101 |
| 23? | ((0.003)) | 23 | | 23 | 2.16 |
| 23 | ((0.024)) | 24 | 0.10 | 24 | 2.86 |
| 25? | <0.01[c,d] | 25[c] | 1.62[c] | 25 | 1.03 |
| 26 | 0.056 | 26 | 0.34 | 26 | 0.72 |
| 27 | 0.34 | 27 | 0.12 | 27 | 0.29 |
| 28 | 0.59 | 28 | 2.61 | 28 | 0.49 |
| 29 | 0.12 | 29 | 0.97 | 29 | Obscured by $C^{13}(0)$ |
| 30 | 0.12[e] | 30 | | 30 | 0.32 |
| 31 | 0.22 | 31 | 0.46 | 31 | 0.11 |
| 32 | 0.054 | 32 | 0.49 | 32 | 0.31 |
| 33 | 0.094 | 33 | 0.70 | 33 | 0.17 |
| 34 | 0.88 | 34 | 1.21 | 34 | 0.93 |
| 35 | 0.98 | 35 | 0.80 | 35 | 0.19 |
| 36 | 0.49 | 36 | 0.16 | 36 | 0.42 |
| 37 | 0.52 | 37 | 0.15 | 37 | 0.081 |
| 38 | 0.59 | 38 | 0.26 | 38 | 0.11 |
| 39 | 0.26 | 39 | 0.16 | 39 | 0.17 |
| 40 | 0.88 | 40 | Obscured by $C^{13}(0)$ | 40 | 0.84 |
| 41 | 1.56 | 41 | Obscured by $C^{13}(0)$ | 41 | 0.18 |
| 42 | 1.45 | 42 | 0.69 | 42 | 0.39 |
| 43 | 0.14 | 43 | 0.13 | 43 | 0.38 |
| 44 | 1.75 | 44 | 0.24 | 44 | 0.75 |
| 45 | 1.58 | 45 | 0.27 | 45 | 0.42 |
| 46 | | 46 | 0.47 | 46 | Obscured by $O^{17}(0)$ |
| 47 | | 47 | 0.21 | 47 | Obscured by $O^{17}(0)$ |
| 48 | | 48 | 0.42 | 48 | Obscured by $O^{17}(0)$ |
| 49 | | 49 | 1.4 | 49 | Obscured by $O^{17}(0)$ |
| 50 | | 50 | 0.32 | 50 | 0.91 |
| 51 | | 51 | | 51 | 0.76 |

[a] Absolute values of the cross sections for $Er^{166,168,170}(d,p)Er^{167,169,171}$ may be obtained by multiplying the relative values by 280, 260, and 220 μb/sr, respectively. The estimated error for values so obtained is ±60%, as explained in the text. The absolute cross sections so obtained should be internally consistent within the normalization error of ±15%, as explained in the text.

[b] Believed to be a doublet in $Er^{171}$. A strong $Er^{169}$ isotopic impurity is present. This group is discussed in further detail in the text.

[c] Cross sections were extracted for levels up to the indicated level number. For higher level numbers in the respective odd-mass Er isotopes, the cross sections were only estimated because the increasing frequency of $Q$-value overlap makes the process excessively tedious. This procedure for the higher level numbers should seldom alter the reported relative cross sections more than ≈20%.

[d] Probably entirely isotopic impurities.

[e] Estimated at 60° assuming no angular dependence.

eigenfunctions of a spherical potential. The states are labeled by $N$, the principal quantum number, by $I$, the angular momentum of the final state, by Ω, the projection of $I$ on the nuclear symmetry axis, and by $l$, the orbital angular momentum of the captured neutron. The angular dependence of the cross section is contained in $\phi_l$, which can be calculated[24] with the distorted-wave Born approximation. The quantities $U_i^2$ represent the probability[2] that the intrinsic state is unoccupied in the initial nucleus.

[24] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL 3240, 1962 (unpublished).

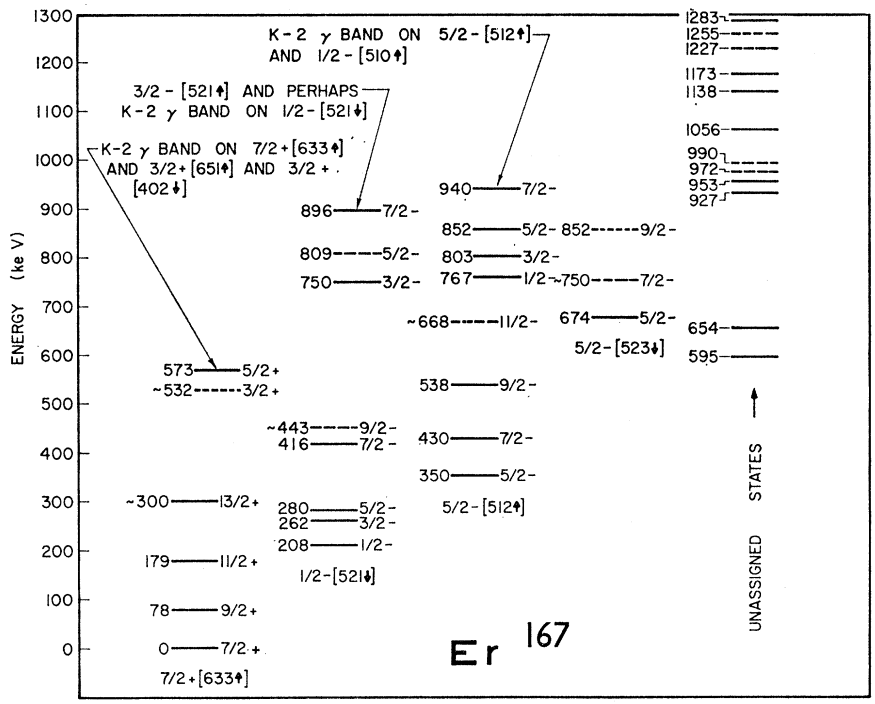


FIG. 4. Level diagram for $Er^{167}$.

The values of $\phi_l$ used in the present cross-section calculations are similar to those used by Kern *et al.*[25] The particular optical-model parameters used can change calculated values for $\phi_l$ by factors of approximately 2, although relative values of cross sections for different $l_n$ values at a given angle are not as sensitive to the parameters chosen for theoretical calculations.[13] Siemssen and Erskine[26] recently made a careful study of the spectroscopic factor $S_l$ for the $W^{182}(d,p)W^{183}$ reaction, in which they conclude theoretical $S_l$ values calculated from Nilsson-model[6] wave functions may be incorrect by a factor of approximately 2. In view of the uncertainties in both experimental and theoretical cross sections, the present data were interpreted by relying more heavily on relative values than on absolute magnitudes when comparing calculations with experimental data.

TABLE IV. Neutron single-particle energy levels.[a]

| Neutron No. | Assigned orbital $I\pi[Nn_z\Sigma]$ | Energy, $\hbar\omega^0$ I[b] | Energy, $\hbar\omega^0$ II[c] |
|---|---|---|---|
| 93 | $\frac{3}{2}-[521]$ | 0.0 | 0.0 |
| 95 | $\frac{5}{2}+[642]$ | 0.04 | 0.04 |
| 97 | $\frac{5}{2}-[523]$ | 0.07 | 0.07 |
| 99 | $\frac{7}{2}+[633]$ | 0.22 | 0.22 |
| 101 | $\frac{1}{2}-[521]$ | 0.24 | 0.24 |
| 103 | $\frac{5}{2}-[512]$ | 0.29 | 0.29 |
| 105 | $\frac{7}{2}-[514]$ | 0.41 | 0.33 |
| 107 | $\frac{9}{2}+[624]$ | 0.48 | 0.41 |
| 109 | $\frac{1}{2}-[510]$ | | 0.48 |
| 111 | $\frac{3}{2}-[512]$ | | 0.55 |
| 113 | $\frac{7}{2}-[503]$ | | 0.59 |

[a] Calculations of Ref. 28 obtained with an adjusted Nilsson potential and the pairing interaction parameters $G_N=(26/A)$ and $G_Z=(29/A)$ MeV.
[b] This set of energies applies to mass numbers in the range $156\leq A\leq 174$, with atomic numbers $64\leq Z\leq 70$.
[c] This set of energies applies to mass numbers in the range $174<A\leq 186$, with atomic numbers $70<Z<74$.

Complementary criteria of reasonable rotational-energy separations were used to support the band assignments. Energy positions of rotational-band members with spin $I$ built on a band head (where $I\equiv K$) are given[1,2] by the equation

$$W(I)=E_0+A\{[I(I+1)+a(-1)^{I+1/2}(I+\tfrac{1}{2})] -[K(K+1)+a(-1)^{I+1/2}(K+\tfrac{1}{2})]\}, \quad (2)$$

where $E_0$ is the excitation energy of the intrinsic state or band head, $A\equiv\hbar^2/\mathcal{J}$ is often named the inertial parameter, and $a$ is the decoupling parameter which can be nonzero only for the $K=\frac{1}{2}$ bands. Couplings with other bands can cause deviations[1,2,27] from the simple $I$ dependence of Eq. (2). Our measurements are not of sufficient precision to evaluate these higher-order correction terms. We have, therefore, used Eq. (2) only to extract $A$ values from the data when $K\neq\frac{1}{2}$ and to extract $A$ and $a$ values when $K=\frac{1}{2}$.

In the paragraphs which follow, the level structure of $Er^{167}$, $Er^{169}$, and $Er^{171}$ will each be discussed in turn. Finally, the systematics of energy and cross sections

[25] J. Kern, O. Mikoshiba, R. K. Sheline, T. Udagawa, and S. Yoshida, in *Proceedings of the International Conference on Nuclear Physics, Gatlinburg, Tennessee, 1966* (Academic Press Inc., New York, 1967); and Nucl. Phys. **A104**, 642 (1967).
[26] R. H. Siemssen and J. R. Erskine, Phys. Letters **19**, 90 (1967).
[27] A. Bohr and B. R. Mottelson, At. Energ. **14**, 41 (1963).

of the various bands common to more than one nucleus will be considered. The detailed description of $Er^{167}$, $Er^{169}$, and $Er^{171}$ must of necessity involve a careful analysis of the Nilsson states expected at a deformation of approximately 0.3 for neutron orbitals in the vicinity of $N=99$ for $Er^{167}$, $N=101$ for $Er^{169}$, and $N=103$ for $Er^{171}$. The appropriate levels and the approximate energy spacing as given by Soloviev[28] are presented in Table IV.

### B. $Er^{167}$ Energy Levels

Energy levels of $Er^{167}$ have been previously studied in considerable detail. The ground state has been clearly shown[3,7] to be the $\frac{7}{2}+[633]$ state. Rotational members of this band have been observed as high as the $\frac{15}{2}+$ state, utilizing Coulomb excitation.[29] The next Nilsson state is observed[3,7] to be the $\frac{1}{2}-[521]$ band at 207.8 keV. A state at 531.5 keV, which initially was interpreted[9] as an intrinsic Nilsson state, has been established[30] as the $\frac{3}{2}+(K-2)$ $\gamma$ vibration built on the $\frac{7}{2}+[633]$ ground state.

The levels in $Er^{167}$ reported here generally agree with previous interpretations and provide additional information on some of the higher-lying states. Detailed assignments of $Er^{167}$ excited states are shown in Fig. 4. Theoretical calculations predict that we should not observe the $Er^{167}$ ground state. Our data are consistent with this prediction, although an upper limit for the intensity of the ground-state group was obtained. However, the intensity pattern of the rest of the superimposed rotational band with members at 78, 179, and approximately 300 keV leaves no doubt that these are the $\frac{9}{2}+$, $\frac{11}{2}+$, and $\frac{13}{2}+$ members of the $\frac{7}{2}+[633]$ rotational band.

The $\frac{1}{2}-[521]$ band is clearly seen with states at 208, 262, 280, and 416 keV corresponding to spin-parity $\frac{1}{2}-$, $\frac{3}{2}-$, $\frac{5}{2}-$, and $\frac{7}{2}-$, respectively. A small shoulder at an excitation energy of 443-keV on the right-hand side of proton-peak 9 in Fig. 1 may correspond to the $\frac{9}{2}-$ state, which has been observed[10] at 441.98 keV. Because the latter group is unresolved in the presence of the former strong proton group, and because the intense $Er^{169}$ ground-state group occurs approximately at this position, evidence for an $Er^{167}$ group must be considered tentative. Therefore, the smaller of the two groups is not labeled in Fig. 1, and its position is shown dashed in Fig. 4.

Cross sections and energy separations strongly indicate that proton groups 7 and 9 in Fig. 1 correspond to the $\frac{5}{2}-$ and $\frac{7}{2}-$ members of the $\frac{5}{2}-[512]$ band. The $\frac{9}{2}-$ member is calculated to be at 537.3 keV, and indeed, proton group 10 is observed at 538 keV. However, it seems probable that this proton group results from two states. Group 10 is treated more fully in the discussion of the $\frac{3}{2}+$ band. Using Eq. (2), an $\frac{11}{2}-$ state is predicted at approximately 668 keV. This state would be masked by proton groups 13 and 14, and its expected position is shown by a dashed line in Fig. 4.

The $\frac{3}{2}+$ band head at 531.54 keV, which has been interpreted[10,30] as the $(K-2)$ $\gamma$ band built on the ground state, lies within approximately 6.5 keV of the weak proton group labeled 10 in Fig. 1. Furthermore, group 10 is asymmetric on the high proton-energy side. Possibly, a small contribution from a state at 531.5 keV is present in group 10, which was indicated above to arise primarily from another band. Because of its uncertainty, the possible $\frac{3}{2}+$ band head is shown dashed in Fig. 4. The $\frac{5}{2}+$ member of this band may be expected in the approximate position of a level observed at 573 keV in these experiments. We tentatively accept the interpretation[10,30] of this band as a $(K-2)$ $\gamma$-vibrational band built on the ground-state configuration. However, the present $(d,p)$ data can only confirm the presence of some single-particle strength in the position of one state in this band. Calculations[31,32] suggest the most important intrinsic components admixed with this vibrational band observable with the $(d,p)$ reaction should be the $\frac{3}{2}+[651]$ and $\frac{3}{2}+[402]$ bands.

Next in the excitation spectrum are two relatively weak states at 595 and 654 keV. These states are shown as unassigned in Fig. 4. There is a possibility that one of these states is the $\frac{9}{2}+$ member of the $\frac{5}{2}+[642]$ orbital, which is expected in this excitation region. The density of states, together with the expected minute intensity for other members of this band, made any definite assignment impossible. One might also consider the possibility of unidentified impurities in these two positions.

Above this point in the energy-level spectrum of $Er^{167}$, we have been able to find a consistent account for the observed data only by assuming there are several unresolved levels. The next intrinsic state proposed on the basis of our data is the $\frac{5}{2}-[523]$ band beginning at 674 keV. Candidates for rotational members are observed at $\approx 750$ and 852 keV; however, the intensities of these states are more consistent with their interpretation as members of other bands. Because these states are probably experimentally unresolved doublets, they are shown dashed in Fig. 4.

The state at 750 keV seems somewhat too large to be the $\frac{7}{2}-$ rotational state of the $\frac{5}{2}-[523]$ band. We, therefore, propose a contribution from the $\frac{3}{2}-[521]$ Nilsson state. The cross section expected for the $\frac{5}{2}-$ rotational state is very small. Its expected position at $\approx 809$ keV would lie under the very strong proton group at 803 keV. Therefore, the level position is

[28] V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **1**, No. 11 (1961).
[29] J. DeBoer, Bull. Am. Phys. Soc. **9**, 108 (1964).
[30] Y. P. Gangrskii and I. Kh. Lemberg, Izv. Akad. Nauk SSSR Ser. Fiz. **26**, 1027 (1962) [English transl.: Bull. Acad. Sci. USSR, Phys. Ser. **26**, 1034 (1962)].
[31] V. G. Soloviev and P. Vogel, Nucl. Phys. **A92**, 449 (1967).
[32] D. R. Bes and Cho Yi-Chung, Nucl. Phys. **86**, 581 (1966).

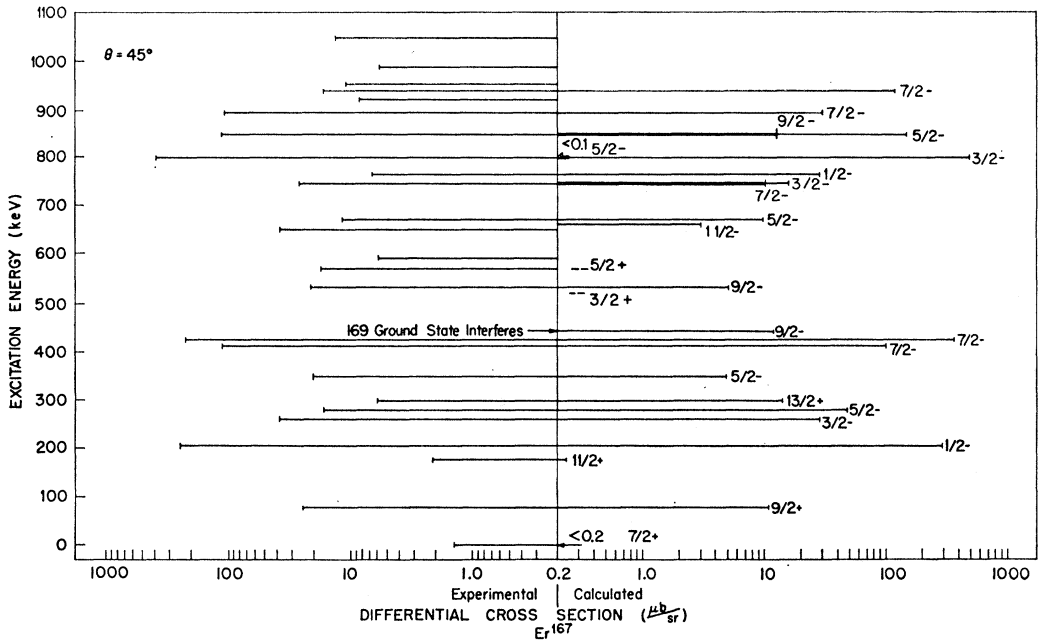


FIG. 5. Comparison of experimental and theoretical differential cross sections at 45° in the reaction $Er^{166}(d,p)Er^{167}$.

dashed in Fig. 4. However, we believe the $\frac{7}{2}-$ rotation corresponds to an observed state at 896 keV. The data are, therefore, consistent with the $\frac{3}{2}-$, $\frac{5}{2}-$, and $\frac{7}{2}-$ members of the $\frac{3}{2}-[521]$ band at 750, ≈809, and 896 keV. This band is expected[31,32] to contain admixtures of the $(K-2)$ $\gamma$-vibrational band associated with the $\frac{1}{2}-[521]$ Nilsson state.

The last band we have assigned to $Er^{167}$ begins at 767 keV. Relative cross sections of the states at 767, 803, 852, and 940 keV show the practically unmistakable "fingerprint" of the $\frac{1}{2}-[510]$ band. Consequently, we have assigned spin parities of $\frac{1}{2}-$, $\frac{3}{2}-$, $\frac{5}{2}-$, and $\frac{7}{2}-$, respectively, to these four states. Systematics[5] of intrinsic states indicate the $\frac{1}{2}-[510]$ band would

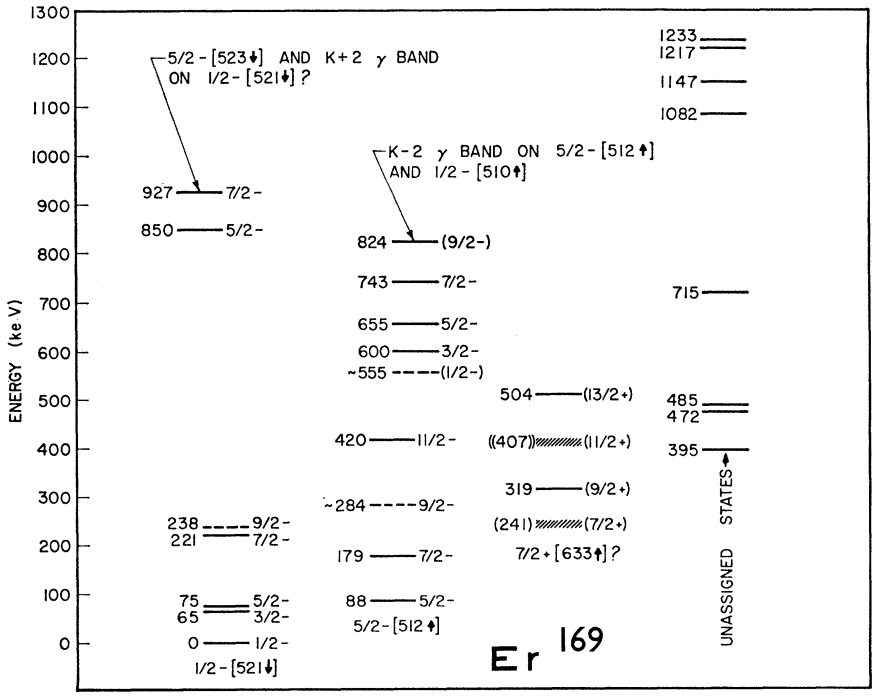


FIG. 6. Level diagram for $Er^{169}$.

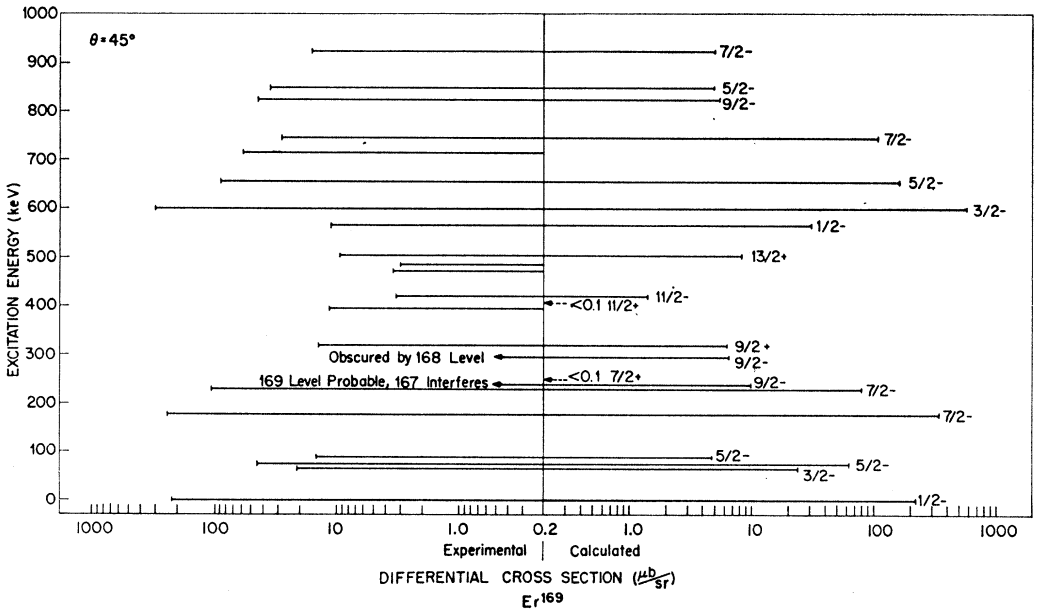


FIG. 7. Comparison of experimental and theoretical differential cross sections at 45° in the reaction $Er^{168}(d,p)Er^{169}$.

have been expected approximately 0.6 MeV higher in excitation. Also, the cross sections for this band are significantly lower than expected for the $\frac{1}{2}-[510]$ band. We interpret this dilution of the single-particle strength of the band to indicate the presence of a component not excited by the $(d,p)$ reaction. Calculations[31,32] indicate the $(K-2)$ $\gamma$ vibration on the $\frac{5}{2}-[512]$ Nilsson state should mix strongly with the $\frac{1}{2}-[510]$ band. The apparent energy displacement and the cross sections are entirely compatible with mixing of these two types of bands.

Attempts to proceed to higher energies in the interpretation are restricted by the experimental resolution. The density of states begins to increase substantially. Furthermore, as one proceeds to higher excitation energies, the mixing of states prohibits confident use

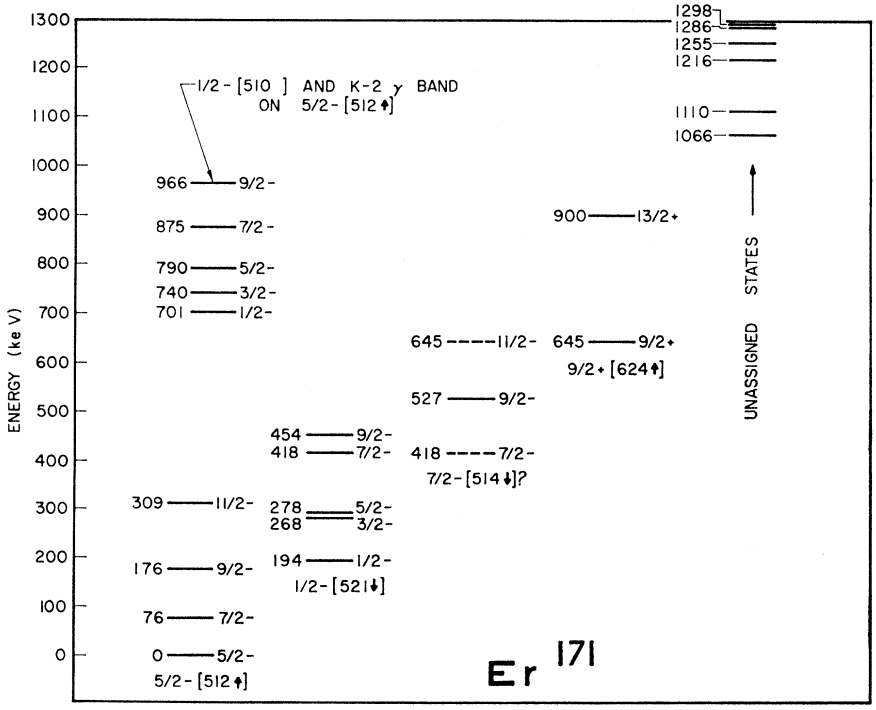


FIG. 8. Level diagram for $Er^{171}$.

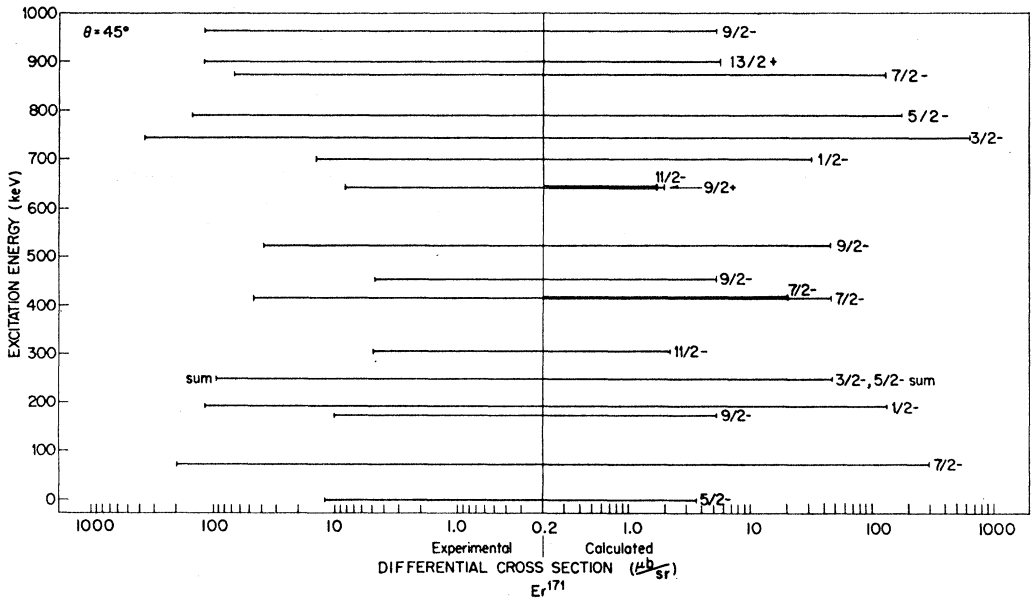


FIG. 9. Comparison of experimental and theoretical differential cross sections at 45° in the reaction $Er^{170}(d,p)Er^{171}$.

of "fingerprint" patterns. With the exception of the states at 595 and 654 keV, all observed states have been assigned up to 940 keV.

In Fig. 5 a plot of the experimental and theoretical intensities for the reaction $Er^{166}(d,p)Er^{167}$ is presented. The $U_i^2$ values used in the theoretical estimates are those used by Isoya,[13] when applicable. For the $\frac{5}{2}-[523]$, $\frac{3}{2}-[521]$, and $\frac{1}{2}-[510]$ bands, $U_i^2$ values of $\approx 0.1$ and $\approx 0.1$ and $\approx 1.0$, respectively, were assumed in calculating theoretical cross sections. In the case of mixed $(K-2)$ $\gamma$-vibrational bands, we have plotted the most prominent Nilsson component through which $(d,p)$ population can be expected. In the case of the $\frac{3}{2}+$ band, beginning at 532 keV, no comparison with theory was made, since no dominant intrinsic component was easily identified. It is immediately evident that generally good agreement between experiment and theory is observed for intense bands. However, the $\frac{1}{2}-$ band beginning at 767 keV has a relatively lower cross section than expected for a pure $\frac{1}{2}-[510]$ band. As discussed above, we believe this observation may be explained by an admixture of the $\frac{1}{2}-[510]$ single-particle structure with a $\gamma$-vibrational component.

### C. $Er^{169}$ Energy Levels

In contrast to the considerable previous work which is available to aid the study of $Er^{169}$, fewer data[7,8,11–13] exist about excited states in $Er^{169}$. Isoya[13] suggested the existence of a $\frac{1}{2}-[521]$ ground state and $\frac{5}{2}-[512]$ band at 80 keV with one $\frac{7}{2}-$ rotational member at approximately 160 keV from his $(d,p)$ measurements.

In Fig. 6 the energy levels observed in the reaction $Er^{168}(d,p)Er^{169}$ are interpreted in terms of rotational bands built on Nilsson states. The ground-state configuration has the Nilsson assignment $\frac{1}{2}-[521]$, with rotational states at 65 and 75 keV having spins and parities $\frac{3}{2}-$ and $\frac{5}{2}-$, respectively. A relatively strong state observed at 227 keV is probably the $\frac{7}{2}-$ member of the $\frac{1}{2}-[521]$ band. A weak group at $\approx 238$ keV is visible in the "tail" of group 5 in Fig. 2. It is unlabeled, but its position is approximately correct for the $\frac{9}{2}-$ member of the $\frac{1}{2}-[521]$ band. The next band observed is the $\frac{5}{2}-[512]$ rotational band with members at 88, 179, $\approx 284$, and 420 keV, with spins and parities $\frac{5}{2}-$, $\frac{7}{2}-$, $\frac{9}{2}-$, and $\frac{11}{2}-$, respectively. The state at $\approx 284$ keV may well be an $Er^{169}$ level. It must be considered tentative and its energy considered inaccurate because it is largely masked by group 5 and because groups 13 and 14 in $Er^{167}$ make a small contribution in this region. For these reasons it is shown as dashed. The next intrinsic Nilsson band expected has the configuration $\frac{7}{2}+[633]$. Two members of this band should be populated with sufficient intensity to be observed in this experiment, namely the $\frac{9}{2}+$ and $\frac{13}{2}+$ states. States with approximately the right intensity to be the $\frac{9}{2}+$ and $\frac{13}{2}+$ states are observed at 319 and 504 keV. The $\frac{7}{2}+$ and $\frac{11}{2}+$ states are then calculated with Eq. (2) to be at approximately 241 and approximately 407 keV. Since the $\frac{7}{2}+$ and $\frac{11}{2}+$ states have not been clearly observed in this experiment, they are shown as hashed lines in Fig. 6. Their lack of observation is entirely consistent with calculated cross sections. The assignment for the $\frac{7}{2}+[633]$ band must be considered to be tentative because the cross sections are so small and only two states were ob-

served. The assignment is shown with a question mark in Fig. 6. It is of interest that a similar assignment has very recently been suggested.[12]

States are observed at 395, 472, and 485 keV, which might correspond to one or more members of the Nilsson orbitals $\frac{7}{2}-[514]$ and/or $\frac{9}{2}+[624]$. Although it seems worthwhile to mention these possibilities, these assignments must be considered speculative at this time, and they are unassigned in Fig. 6. There is also a possibility that some of these states might belong to the $\frac{3}{2}-[521]$ band.

In $Er^{169}$ we see clear evidence for the $\frac{1}{2}-[510]$ band. States at 600, 655, 743, and 824 keV have a relative intensity pattern which suggests they are the $\frac{3}{2}-$, $\frac{5}{2}-$, $\frac{7}{2}-$, and $\frac{9}{2}-$ members of the $\frac{1}{2}-[510]$ band. Using these states we estimate the $\frac{1}{2}-$ band-head position at $\approx 555$ keV, where isotopic-impurity groups prevent an accurate energy determination. This band-head position is shown as a dashed line in Fig. 6. Energy-level systematics indicate this band would have been expected[5] to occur at $\approx 1.3$ MeV. This energy discrepancy, coupled with a relatively low cross section for the band at the observed position, leads to the proposal that the $(d,p)$ reaction has selected the $\frac{1}{2}-[510]$ component of this band, which may also contain a large component of the $(K-2)$ $\gamma$ vibration associated with the $\frac{5}{2}-[512]$ band.[31,32]

Finally, the states at 850 and 927 keV are assigned as the $\frac{5}{2}-$ and $\frac{7}{2}-$ members of the Nilsson band, $\frac{5}{2}-[523]$. The validity of this $\frac{5}{2}-[523]$ assignment is supported by the very recent measurement[12] of an allowed, unhindered $\beta$ decay from the $\frac{7}{2}-[523]$ ground state of $Ho^{169}$. It is interesting to note that one might expect[31] the $\frac{5}{2}-[523]$ band to mix with the $(K+2)$ $\gamma$ band built on the $\frac{1}{2}-[521]$ ground state. This $(K+2)$ vibration might be expected in this energy region, but our $(d,p)$ data can neither confirm nor dispute this possibility.

With the exception of the states at 395, 472, 485, and 715 keV, all states are assigned up to the 1082-keV state.

In Fig. 7 the experimental and calculated differential cross sections are shown for the reaction, $Er^{168}(d,p)Er^{169}$. Cross sections for the band beginning at approximately 555 keV, when compared with the cross section predicted for a pure $\frac{1}{2}-[510]$ intrinsic Nilsson orbital, seem to be systematically lower than cross sections for other strongly populated bands. Except in this case, where band mixing may furnish a natural explanation for lower cross sections, reasonable agreement was obtained between experimental and calculated intensities.

### D. $Er^{171}$ Energy Levels

Isoya[13] suggested two intrinsic states in $Er^{171}$. The ground-state assignment is $\frac{5}{2}-[512]$ and the Nilsson orbital $\frac{1}{2}-[521]$ was assigned[13] at 100 keV.

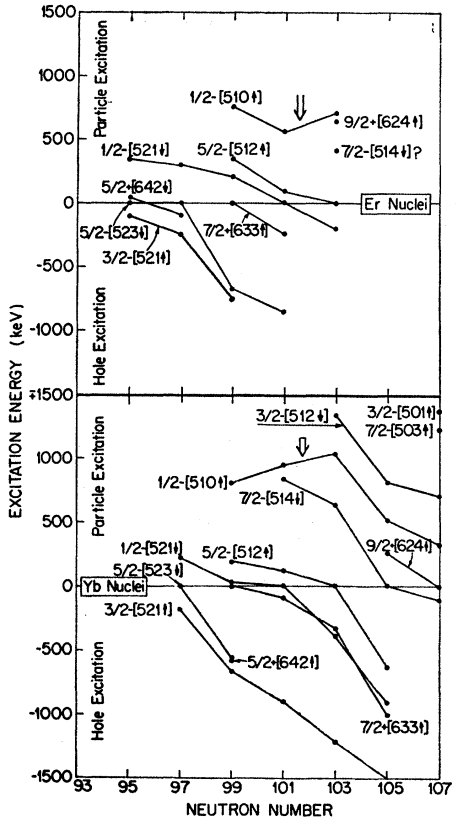


FIG. 10. Comparison of the experimental systematics of the Nilsson levels in the odd-$A$ Er and Yb isotopes. The arrows indicate where the energy systematics of the $\frac{1}{2}-[510]$ orbital begin to deviate significantly from the expected systematics. The deviations suggest mixing of this Nilsson orbital with the $(K-2)$ $\gamma$ band built on the $\frac{5}{2}-[512]$ orbital, as one proceeds toward lower neutron numbers.

Our assignments are shown in Fig. 8. Detailed discussion is similar to that given for $Er^{167}$ and $Er^{169}$. Some comment is in order on the $\frac{1}{2}-[521]$ band. Proton group 4 in Fig. 3 is somewhat broader than the usual resolution. There is a rather strong isotopic impurity arising from an $Er^{169}$ level in this position, which makes it difficult to obtain a precise measurement of the separation of the $\frac{3}{2}-$ and $\frac{5}{2}-$ rotational states. The separation of these two levels appears to be similar to that in $Er^{169}$, although the evidence is not conclusive.

We believe the present data provide some evidence for the $\frac{7}{2}-[514]$ and $\frac{9}{2}+[624]$ bands. These two bands are expected to occur in or near this region of excitation. Evidence for the $\frac{7}{2}-[514]$ band rests solely on one state at 527 keV proposed to be the $\frac{9}{2}-$ rotational state. This tentative assignment is indicated by a question mark in Fig. 8. Evidence for the $\frac{9}{2}+[624]$ band is based on rotational-energy separations. Candidates for the $\frac{9}{2}+$ and the $\frac{13}{2}+$ members of this band occur at 645 and 900 keV, respectively. The $\frac{11}{2}+$ member of this $\frac{9}{2}+$ band might be present as part of

TABLE V. Calculated structure of the rotational bands in $Er^{167}$, $Er^{169}$, and $Er^{171}$.

| Nilsson state $K\pi[Nn_z\Lambda]$ | $Er^{167}$ Soloviev and Vogel[a] | $Er^{167}$ Bes and Cho[b] | $Er^{169}$ Soloviev and Vogel | $Er^{169}$ Bes and Cho | $Er^{171}$ Soloviev and Vogel | $Er^{171}$ Bes and Cho[b] |
|---|---|---|---|---|---|---|
| $\frac{7}{2}+[633]$ | 99%[c] | | 99%[c] | | | |
| $\frac{1}{2}-[521]$ | 98% | | 96% | | 99%[d] | |
| $\frac{5}{2}-[512]$ | 91% | | | | | |
| $\frac{5}{2}-[523]$ | 97%[e], 94%[f] | | 46% +47% $(K-2)\gamma$ vib. on $\frac{1}{2}-[521]$ + 2% Octupole vib. on $\frac{5}{2}+[642]$ | | | |
| $\frac{7}{2}-[514]$ | | | | | 99%[c] | |
| $\frac{3}{2}-[521]$ | 79% +15% $(K-2)\gamma$ vib. on $\frac{1}{2}-[521]$ + 4% Octupole vib. on $\frac{3}{2}+[651]$ | 98% | | 92%[b] | | 80%[d] |
| $\frac{1}{2}-[510]$ | 32% +65% $(K-2)\gamma$ vib. on $\frac{5}{2}-[521]$ + 3% $(K-2)\gamma$ vib. on $\frac{3}{2}-[512]$ | 50% | 51%[c] + 41% $(K-2)\gamma$ vib. on $\frac{5}{2}-[512]$ +3.5% $\beta$ vib. on $\frac{1}{2}-[521]$ | 67%[c] | 48% +48% $(K-2)\gamma$ vib. on $\frac{5}{2}-[512]$ | 83%[d] |
| $\frac{3}{2}+[651]$ | 7% +88% $(K-2)\gamma$ vib. on $\frac{7}{2}+[633]$ | 1.6% +1.6% $\frac{3}{2}+[651]$ +0.6% $\frac{3}{2}+[642]$ | | | | |

[a] Reference 31.
[b] The values shown in these columns were obtained from amplitudes given in Table V of Ref. 32. In general, most of the remaining calculated strength resides in $\gamma$ vibrations associated with appropriate Nilsson bands, but other admixtures may be present.
[c] Calculated for $Yb^{171}$.
[d] Calculated for $Yb^{173}$.
[e] Calculated for $Yb^{169}$.
[f] Calculated for $Dy^{165}$.

one of the strong proton groups occurring in this excitation-energy region. These assignments must be considered tentative.

Evidence from cross sections indicates that the band beginning at 701 keV is largely the $\frac{1}{2}-[510]$ Nilsson state. Calculations[31,32] suggest that a component of the $(K-2)$ $\gamma$ band built on $\frac{5}{2}-[512]$ state should be present, but in smaller amounts than was the case in $Er^{167}$ and $Er^{169}$. In Fig. 9 experimental and theoretical differential cross sections for the reaction $Er^{170}(d,p)Er^{171}$ are compared. Agreement between experiment and theory is generally satisfactory.

### E. Level Systematics and Structure in Odd-Mass Er Nuclei

It has been recognized that many bands in nuclei in this mass region can exhibit a very complicated structure.[31,32] Such bands may have varying amounts of several one-quasiparticle and three-quasiparticle states mixed with collective states. Of these, the $(d,p)$ reaction is sensitive to the one-quasiparticle states. In those complex states which probably have considerable amounts of $(K-2)$ $\gamma$-vibrational admixtures, the present data furnish three possible kinds of evidence indicating complex structure. This evidence consists of deviations in (1) energy-level systematics, (2) decoupling parameters, and (3) $(d,p)$ cross sections from values expected by systematics and/or theory.

A comparison of energy-level systematics of Nilsson states in odd-mass Er and Yb nuclei is presented in Fig. 10. In Fig. 10 hole states are indicated below 0 keV and particle states are indicated by positive values. Data on Yb nuclei used in Fig. 10 for those of Burke *et al.*[16] and of Vergnes and Sheline.[17] The energies of the $\frac{1}{2}-[510]$ band seem to deviate for neutron Nos. 99 and 101, but begin to approach expected behavior after neutron No. 103. We have marked the change in energy systematics with broad arrows in Fig. 10. A more complete survey[5] of states in odd-$A$ nuclei in this deformed region indicates energy deviations of the $\frac{1}{2}-[510]$ band are approximately $\approx 0.6$, $\approx 0.7$, and $\approx 0.3$ MeV below the positions expected for the $\frac{1}{2}-[510]$ bands in $Er^{167}$, $Er^{169}$, and $Er^{171}$, respectively. Except for a minor crossover, the other energy-level systematics presented in Fig. 10 qualitatively follow Nilsson-model[6] predictions.

A second indication of mixing may be given by decoupling parameters. In this work the experimental decoupling parameters $a$ were obtained solely from energy levels deduced from the emitted-proton energy spectra, using Eq. (2). Decoupling parameters obtained from data of the present work may be subject to relatively large errors, because of the uncertainties inherent in the method together with errors occasioned by the presence of isotopic impurity peaks. Values determined for the decoupling parameters of these $K\pi=\frac{1}{2}-$ bands are $+0.13$ in $Er^{167}$, $+0.067$ in $Er^{169}$,

and +0.11 in $Er^{171}$. An approximate "typical" experimental value[17] for this band is +0.2. One might expect that decreasing amounts of the $\frac{1}{2}-[510]$ orbital would lead to a decrease in the decoupling parameter. Although values of decoupling parameters obtained with the energy resolution of these $(d,p)$ experiments do not furnish an extremely sensitive test of the decoupling-parameter criterion, the decoupling parameters so determined for the complex $\frac{1}{2}-[510]$ band are lower than the "expected" value of +0.2. This may indicate considerable mixing of the $\frac{1}{2}-[510]$ band with other bands.

Finally, one can compare calculated cross sections shown in Figs. 5, 7, and 9 with experimental cross sections. To the extent that the bands have pure $\gamma$-vibrational components, one expects no population by the $(d,p)$ reaction. The experimentally observed fraction of the theoretical population predicted for a one-quasiparticle band observed in a complex band determines the amplitude of the one-quasiparticle state in that band. Our data are then consistent with the assumption that in $Er^{167}$ and $Er^{169}$ there is considerable dilution of the $\frac{1}{2}-[510]$ band, and that a large part of the strength of this band appears at an energy considerably lower than one might have expected if this band were unmixed. Whereas energy systematics and decoupling-parameter systematics indicate some dilution of the $\frac{1}{2}-[510]$ band in $Er^{171}$, this is not strongly indicated by cross sections for population of this band in $Er^{171}$.

Presumably the rest of the $\frac{1}{2}-[510]$ band's strength will be found in higher-lying $K=\frac{1}{2}-$ bands. Some of these bands should also contain additional strength of the $(K-2)$ $\gamma$ band built on the $\frac{5}{2}-[512]$ band. Analogous remarks might also be made about the other more tentative complex bands herein. Unfortunately, the high-level density occasioned by the use of targets with relatively poor isotopic enrichments makes it extremely difficult to identify any of these expected Nilsson level components in the higher excitation-energy regions of the $(d,p)$ spectra.

Soloviev and Vogel[31] and Bes and Cho[32] calculated the structure expected for complex states in many deformed odd-$A$ nuclei. A summary of the available results of their calculations is given in Table V. In comparing our experimental results it should be clearly noted that in some cases, when calculations were unavailable for the appropriate odd-$A$ Er nucleus, we have used the calculated structures for isotonic or other nearby nuclei. This kind of substitution is not entirely satisfactory. The fact that the two treatments[31,32] give somewhat different results in some cases is probably related to the differences in initial assumptions. Comparison with experiment is possible and is shown in Figs. 5, 7, and 9. All the bands in Table V, with the exception of the $\frac{1}{2}-[510]$ band (and possibly the $\frac{3}{2}+[651]$ band), are consistent within our experimental accuracy with our calculated cross sections for unmixed bands. Comparison between experiment and theory for the $\frac{3}{2}+$ band was not attempted because of the large number of components expected for this band. For the bands which we believe to be strongly mixed, the fractional strength of the $\frac{1}{2}-[510]$ orbital appears to be $\approx 0.6$ for $Er^{167}$ and $Er^{169}$ and $\approx 0.9$ for $Er^{171}$. The observed intensities are approximately consistent with structure calculations,[31,32] with perhaps some tendency toward less decrease in the single-particle strength than predicted.[31] It must, however, be remembered that in the $Er^{169}$ case comparison was made with an isotonic nucleus. In $Er^{171}$, the $\frac{1}{2}-[510]$ intrinsic component appears to be significantly stronger than predicted.[31]

## V. CONCLUSIONS

A large number of Nilsson orbitals have been observed together with their associated rotational bands in $Er^{167}$, $Er^{169}$, and $Er^{171}$. The energy systematics of these orbitals qualitatively follow the expectations of the Nilsson model.[6] When the Nilsson-model[6] predictions are not exactly followed, discrepancies may be understood in terms of mixing of single-particle and vibrational states. Discrepancies become appreciable at 500 to 700 keV of excitation, where vibrational states are expected to occur.

The differential cross sections experimentally measured for $(d,p)$ reactions leading to levels in $Er^{167}$, $Er^{169}$, and $Er^{171}$ agree reasonably with calculated values for those bands predicted[31,32] to have one-quasiparticle fractions $\approx 1.0$. Evidence from cross sections complements other evidence for strong mixing of the $\frac{1}{2}-[510]$ orbital with vibrational components, analogous to that observed in Yb isotopes.[16] The use of the band "fingerprints" has proved to be very useful in identifying the Nilsson bands, and Nilsson-state components in mixed bands.

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