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$Al^{27}(p,p\pi^+)Mg^{27}$ Reaction from 0.6 to 28 GeV*

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Measurements have been made of the excitation function and integral recoil properties of the $Al^{27}(p,p\pi^+)$-Mg^{27} reaction at incident proton energies from 0.6 to 28 GeV. The integral recoil properties yield values of the average forward momentum transfer. Calculations based on the model of Ericson *et al.* as modified by Selleri are in good agreement with the excitation function but predict somewhat smaller values for the average forward momentum transfer.

I. INTRODUCTION

INTEREST in the $(p,p\pi^+)$ reaction has been greatly stimulated by the theoretical calculation of Ericson, Selleri, and Van de Walle.[1] They calculated the excitation function of the reaction in the impulse approximation, using one-pion-exchange theory for the elementary-particle interaction and a Fermi gas model for a description of the nucleus. Remsberg[2] presented an excitation function and some integral recoil properties for the $Cu^{65}(p,p\pi^+)Ni^{65}$ reaction, and the relatively good agreement with the calculation indicated that this description of the reaction is basically valid. The predicted importance of a pion-nucleon resonance [the isobar called $\Delta(1236)$] was clearly demonstrated by Remsberg's experimental data. Remsberg[2] also improved upon the calculation by taking into account the Fermi momentum of the struck particle in the kinematics of the elementary-particle interaction. Recently, Selleri[3] has further improved upon the calculation by allowing a kinematic factor in the one-pion-exchange theory to be off the mass shell, and by including a form factor for the exchanged pion.

In the present work on the $Al^{27}(p,p\pi^+)Mg^{27}$ reaction, we present the excitation function together with more extensive integral recoil measurements, which yield, in this case, the average forward momentum transfer. These measurements are interpreted using only two-body kinematics to yield the "missing mass" of the outgoing particles in the reaction. Also, the data are compared with the improved calculation to further test the model of Ericson *et al.* in a lighter target nucleus.

The earliest measurement of the $Al^{27}(p,p\pi^+)Mg^{27}$ reaction was by Benioff,[4] who obtained a cross section of 0.1±0.07 mb at 5.7 GeV. The excitation function from threshold to 0.6 GeV has been determined by Kuznetsova, Pokrovskii, and Rybakov,[5] and their data will be presented later. An early value of our cross section at 28 GeV has been published,[6] and a preliminary report of this work, emphasizing the recoil properties, was presented[7] in 1963.

II. EXPERIMENTAL

The cross-section and recoil properties were studied in the same target stack, which consisted of an Al target and plastic catcher foils, while the beam was monitored by the production of Na^{24} in the target itself. A target stack consisted of a 0.001-in. Al target foil (99.99% pure) and three 0.00025-in. Mylar foils which acted as forward recoil catcher, backward recoil catcher, and activation blank. The Mylar foils were trimmed with a scalpel so that they protruded slightly beyond the target. In addition, another Mylar foil, twice as large, was folded around the stack covering the leading edge. This served to guard the catcher and blank foils and hold the stack together. In the case of the irradiations to measure the recoil properties perpendicular to the beam, the target stack was held an an angle of 10° to the beam, and a second activation blank foil was added to the stack, so that there was one blank on each side of the target.

The irradiations were performed at the Brookhaven Cosmotron and AGS, and at the latter machine it was necessary to reduce the beam intensity to prevent the burning of the plastic foils. The irradiations sometimes lasted as long as 20 min, but corrections were made for the variation of the beam intensity during the bombardments.

The Mg^{27} was separated chemically from the target

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[1] T. Ericson, F. Selleri, and R. T. Van de Walle, Nucl. Phys. **36**, 353 (1962).

[2] L. P. Remsberg, Phys. Rev. **138**, B572 (1965).

[3] F. Selleri, Phys. Rev. **164**, B1475 (1968).

[4] P. A. Benioff, Phys. Rev. **119**, 316 (1960).

[5] M. Ya. Kuznetsova, V. N. Pokrovskii, and V. N. Rybakov, Zh. Eksperim. i Teor. Fiz. **42**, 1451 (1962) [English transl.: Soviet Phys.—JETP **15**, 1006 (1962)].

[6] J. B. Cumming, G. Friedlander, J. Hudis, and A. M. Poskanzer, Phys. Rev. **127**, 950 (1962).

[7] A. M. Poskanzer and J. B. Cumming, Bull. Am. Phys. Soc. **8**, 325 (1963).

and catcher foils, and the main problem was eliminating the tremendous quantities of C^{11} activity induced in the plastic foils. This was accomplished by fuming off the carbon with perchloric acid. The other interfering activities, F^{18} and Na^{24}, were eliminated with holdback carriers. The chemical procedures are described in the Appendix.

The samples were mounted on Al cards of sufficient thickness to stop the β particles and assayed on 3×3-in. NaI crystals. In the usual system, four crystals were used, one for each sample, and their signals were routed to four quadrants of a 256-channel pulse-height analyzer, where successive spectra were recorded as a function of time. The counts in a fixed set of channels encompassing the two γ-ray peaks at 842 and 1013 keV were summed and resolved with a least-squares decay-curve program, assuming a half-life of 9.46 min for Mg^{27}. The contribution of small amounts of residual C^{11} activity to these channels was negligible. Several of the best spectra were analyzed to obtain the area of the 842-keV photopeak relative to the counts in the fixed set of channels. In each experiment each crystal was calibrated with the 835-keV γ ray from a Mn^{54} standard obtained from the National Bureau of Standards. The standard was mounted in such a manner to simulate the geometry of the samples, and it was assumed that the photopeak efficiency of the 842-keV peak was 1% lower than the 835-keV peak. For the abundance of the 842-keV γ ray in the decay of Mg^{27}, the value of 0.70 was used.[8] During the chemical separation, 1 ml of the solution in which the target foil had dissolved was set aside. The next day it was assayed for Na^{24} activity in a well-type NaI crystal which had been calibrated by the β-γ coincidence technique. Later, this solution was assayed for its sodium content to determine what fraction of the dissolver solution had been saved for Na^{24} assay.

The results of the measurements which are listed in Table I consist of the production cross section σ and values of *FW*, *BW*, and, in one case, *PW*. The quantities *F*, *B*, and *P* are the fractions of the activity recoiling out of the target in the forward, backward, and perpendicular directions, respectively, and the quantity *W* is the thickness of the target foil. The numbers for σ, *FW*, and *BW* represent, for the standard targets, the results for 3, 2, and 1 measurements, respectively, at each bombarding energy. The activation blanks were on the average about 20% of the backward catcher foils and 1% or less of the forward catcher foils. In addition to the error in the blank subtraction and that due to the counting statistics, it was estimated that there were other contributions to the errors of *FW* and *BW* due to the Mg chemical yields (2%), the intercalibration of the counters with the Mn^{54} source (1%), and the reproducibility of selecting the fixed set of channels (2%). It was estimated that the error in the cross-section measurements had additional contributions from the Na chemical yields (3%), the determination of the Na^{24} activity (1%), the uncertainty in the efficiency of the well counter for Na^{24} (2%), and the uncertainty in the Mg^{27} saturation factor for the irradiation (1%). The Na^{24} cross sections used were those recommended by Cumming,[9] but the 7% uncertainty of these monitor cross sections was not folded into the errors for σ listed in Table I. The cross-section values have been corrected both for recoil loss of Mg^{27} as determined by *F* and *B*, and for recoil loss of the Na^{24} monitor as determined previously.[10]

TABLE I. Uncorrected data.

Energy (GeV)	σ (μb)	*FW* (μg/cm^2)	*BW* (μg/cm^2)
0.6	117 ±4	495±17	5.2±0.9
1.0	152 ±5	350±11	4.2±1.4
1.7	152 ±4	265±8	5.4±0.6
1.7[a]	189 ±9	234±11	19.4±1.1
2.9	125 ±4	213±9	11.3±3.4
10	83 ±3	240±11	
28	75.8±2.0	238±10	18 ±2
28[b]	248 ±13	108±6	34 ±3
1.7		110±6[c]	

[a] In this experiment the Al and Mylar thicknesses were each increased by a factor of 5.

[b] For this experiment a normal target was sandwiched between two pieces of 0.013-in. Al.

[c] The target was oriented at 10° to the beam, and this is the value of *PW*, where *P* is half of the fraction of the activity recoiling into the two catcher foils.

To investigate the possibility of impurities in the target foil contributing to the yield of Mg^{27}, several measurements were made of the yield of Mg^{28} by assaying the target sample the next day for its 21.3-h half-life. An effective cross section of about 0.1 μb was found for Mg^{28} production at both 1.7 and 28 GeV. This could be accounted for by Cu and Fe impurities in the Al to the extent[11] of 0.02%, which is roughly consistent with the stated purity of the foil. However, the cross section for the production of Mg^{27} from the impurities should be somewhat larger than that for Mg^{28}, and the recoil ranges would be greater[12] than those for the $(p,p\pi^+)$ reaction. Assuming that the yield of Mg^{27} from Cu is four times that of Mg^{28}, and that its recoil properties are similar to those for Mg^{28} from Cu,[12] then the contributions to the values in Table I would be about 0.4 μb for σ, 2 to 4 μg/cm^2 for *FW*, and 0.8 to 1.5 μg/cm^2 for *BW*. Because of the smallness of these values and the crudeness of the guess of the yield of Mg^{27} relative to Mg^{28}, these corrections have not been made to the data.

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[8] C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (John Wiley & Sons, Inc., New York, 1967), 6th ed.

[9] J. B. Cumming, Ann. Rev. Nucl. Sci. **13**, 261 (1963).

[10] A. M. Poskanzer, J. B. Cumming, and R. Wolfgang, Phys. Rev. **129**, 374 (1963).

[11] This is calculated using the cross section of 0.5 mb, which has been measured for the production of Mg^{28} from Cu at 2 GeV. See G. Friedlander, J. M. Miller, R. Wolfgang, J. Hudis, and E. Baker, Phys. Rev. **94**, 727 (1954).

[12] The values of *FW* and *BW* for Mg^{28} from Cu are approximately 1.0 and 0.3 mg/cm^2 at both 0.7 and 3 GeV. See V. P. Crespo, J. M. Alexander, and E. K. Hyde, Phys. Rev. **131**, 1765 (1963).

TABLE II. Data corrected for the secondary reaction and scattering.

Energy (GeV)	σ (μb)	FW (μg/cm^2)	BW (μg/cm^2)	FW [a] (μg/cm^2)	$\langle q \cos\theta \rangle$ (m_0c)	$\langle w^2 \rangle^{1/2}$ (m_0)
0.6	103±5	555±25	−0.9±3.5	555±25	0.367±0.016	1.34±0.02
1.0	138±6	380±15	−0.3±2.8	380±15	0.268±0.011	1.35±0.02
1.7	138±5	287±11	1.0±2.4	287±11	0.218±0.008	1.40±0.02
2.9	111±5	234±12	6.6±4.8	227±13	0.185±0.011	1.49±0.03
10	69±4	280±19		272±20	0.211±0.016	2.36±0.08
28	$62\pm3\frac{1}{2}$	281±19	10.0±5.1	271±20	0.211±0.016	3.65±0.14
1.7		116±7[b]		370±22[c]	0.264±0.016[d]	

[a] Corrected for scattering also.
[b] This value is PW.
[c] This value is πPW corrected for the 10° angle of the target to the beam.
[d] This value is $\langle q \sin\theta \rangle$.

However, as was first pointed out[13] in 1947, a significant correction to the production of Mg^{27} by the emission of a meson from Al^{27} is necessary because of the Mg^{27} produced by the (n,p) reaction from the secondary neutrons produced in the foils. In order to evaluate this correction, the two thick-target irradiations listed in Table I were performed. It has been pointed out[14] that a simple linear extrapolation with target thickness would tend to underestimate the effect of the secondary neutrons because of their oblique paths through the target. Thus equations for the average path lengths of the neutrons[2] were used in making the extrapolations.[15] It was found that at the two energies the contribution to the cross section in the standard thin targets was 14±3 μb, and that the recoil properties of the Mg^{27} produced by the secondary reaction was consistent with $FW = BW = PW = 50\pm20$ μg/cm^2. Assuming that these figures applied at all the bombarding energies, the data were corrected for the contribution of the secondary reaction and are presented in Table II. From kinematics it can be shown[2] that there cannot be any $(p,p\pi^+)$ recoils in the backward hemisphere, and thus it is encouraging to find the values of BW almost consistent with zero. However, at the highest bombarding energies the angle of the recoils can be quite close to 90°, and the positive value of BW might be due to scattering of the Mg^{27} recoils in the stopping process. It can be shown[16] that in the forward hemisphere, the excess of the scattering out of the target compared with scattering in is just given by the amount of scattering into the backward hemisphere. Thus at the three highest energies we have subtracted the values[17] of BW from FW to get the final values shown in column 5.

It should be pointed out that the contribution of impurities in the target to the Mg^{27} yield could not be much greater than that estimated above because of the smallness of the value of BW at 1.7 GeV.

The value of PW in column 5 has been divided by 0.985, the cosine of 10°, to correct for the fact that the target stack was oriented at 10° to the beam, and multiplied by the factor π to obtain the average of the projections of the recoils on an axis perpendicular to the beam.[2]

To interpret the recoil properties, a range-energy relation for Mg^{27} in Al is needed. Previous measurements of Ne^{22} recoils in Al which had been transformed to Na^{24} in Al [18] were now transformed to Mg^{27} in Al in a like manner. Using this relationship, the FW values were converted to momentum and are presented in column 6. Since range is almost proportional to velocity in this region,[18] the values in column 6 represent the average of the projections of the recoil momentum along the beam axis. Since the recoil momentum in the $(p,p\pi^+)$ reaction is equal to the momentum transfer q in the elementary-particle reaction, the values are the average forward momentum transfer $\langle q \cos\theta \rangle$.

The values of σ and $\langle q \cos\theta \rangle$ from Table II have been plotted in Figs. 1 and 2, respectively. In plotting σ, the 7% uncertainty in the cross sections of the monitor reaction has been folded into the error estimates. The data of Kuznetsova *et al.*[5] were normalized to the standard monitor cross sections[9] and corrected for the contribution of the secondary reaction in the same manner that they described. Because of an unexplained gross disagreement, it was necessary to normalize their data to ours where they overlapped at 0.6 GeV. This required multiplying all their data by 0.60 before plotting it in Fig. 1.

III. TWO-BODY KINEMATICS

It was suggested by Ericson[19] that one could apply two-body kinematics to the $(p,p\pi^+)$ reaction to obtain information directly from the recoil properties about the elementary-particle reaction at low-momentum transfer.

One considers the outgoing particles, which may consist of more than two particles as long as the residual nucleus is Mg^{27}, as a single entity with an effective mass w. The quantity w, which is the total energy of these

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[13] N. A. Bonner, G. Friedlander, L. P. Pepkowitz, and M. L. Perlman, Phys. Rev. **71**, 511 (1947).

[14] A. Turkevich (private communication).

[15] A. Stehney (private communication) has pointed out that the equation for the mean path length of the neutrons from the catchers is only approximate in Ref. 2. However, the difference in the extrapolation was not significant in the present case or in Ref. 2.

[16] W. R. Pierson and N. Sugarman, Phys. Rev. **130**, 2417 (1963).

[17] At 10 GeV we interpolated a value of 8±5 μg/cm^2 for BW.

[18] A. M. Poskanzer, Phys. Rev. **129**, 385 (1963).

[19] T. Ericson (private communication). See also Ref. 7.

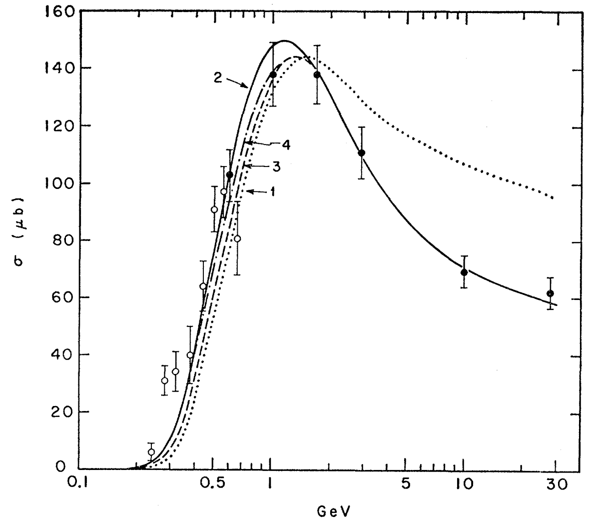

FIG. 1. Excitation function for the $Al^{27}(p,p\pi^+)Mg^{27}$ reaction. The solid points are from this work. The open points are from Ref. 5 and have been multiplied by the factor 0.60. Curve 1 (dotted) is calculated as described in Ref. 2. Curve 2 (solid) includes the off-mass-shell factor. Curve 3 (dashed) also includes the pion form factor. Curve 4 (dot-dashed) is the same as curve 3 with the Fermi energy raised to 75 MeV. Where the curve is not shown, it lies underneath the solid curve.

outgoing particles in their own center-of-mass system, is calculated as a "missing mass" based on the observation of the recoil. In the $(p,p\pi^+)$ reaction this mass has particular significance[20] since the outgoing particles may indeed be a single isobar when they leave the nucleus. The relevant kinematic equation is

$$q\cos\theta=(w^2-1+q^2+2E\Delta E-\Delta E^2)/2p\,, \qquad (1)$$

where q is the momentum transfer, and E and p are the total energy and momentum of the incident proton, all in units of the proton rest mass. The quantity ΔE is the energy transferred to the recoil nucleus and consists of the excitation energy of the residual nucleus, the difference in rest masses of Mg^{27} and Al^{27}, and the kinetic energy of the residual nucleus, all small quantities. Since w^2 is the dominant term on the right-hand side of Eq. (1), it follows that the forward momentum transfer in such a reaction is directly related to w. Since our measurements of FW yield $\langle q\cos\theta\rangle$, we can determine the root-mean-square value of w.

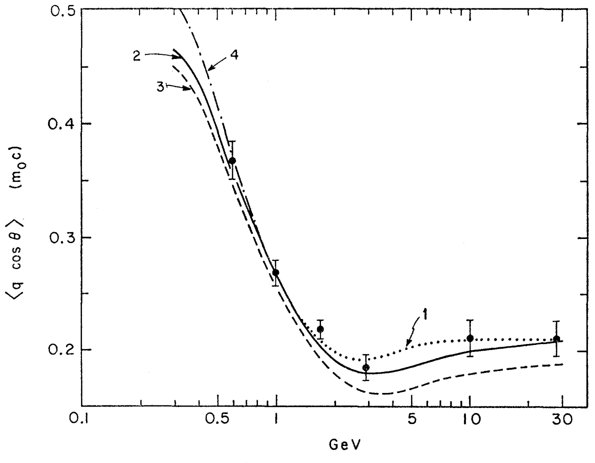

FIG. 2. Plotted against incident proton energy is the average forward momentum transfer as derived from the forward recoil measurements FW. For an explanation of the curves, see the caption of Fig. 1.

By combining our measurements at 1.7 GeV for $\langle q\cos\theta\rangle$ and $\langle q\sin\theta\rangle$, we obtain $\langle q\rangle^2=0.117\pm0.009$. Based on this, we assume $q^2=0.12\pm0.02$ at all energies for substituting in Eq. (1). We estimate that ΔE consists of 4 ± 2 MeV in excitation energy, 3.1 MeV for the rest-mass increase, and 2 MeV in kinetic energy, giving $\Delta E=0.010\pm0.002$. One sees that the ΔE^2 term is completely negligible, and that even the q^2 and $2E\Delta E$ terms are small. The calculated values of $\langle w^2\rangle^{1/2}$ are listed in the last column of Table II and are plotted in Fig. 3. The value of w at threshold, which consists of the rest mass of a proton and a pion, is plotted in the figure, and the value of w for the first $(\frac{3}{2},\frac{3}{2})$ isobar $\Delta(1236)$ is indicated by the dashed line. The dominance of this isobar in the region of 1-GeV bombarding energy is indicated by the appearance of a plateau near the mass of the isobar. These data could be compared directly with elementary-particle data suitably biased for the low-momentum transfers selected by the $(p,p\pi^+)$ reaction. However, because of the success of one-pion-exchange theory for this class of elementary-particle data, a comparison with the results of the calculation described in the next section is equivalent.

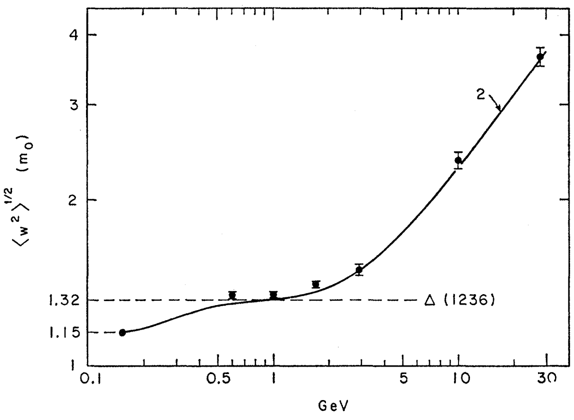

FIG. 3. The root-mean-square value of the "missing mass" as derived from $\langle q\cos\theta\rangle$ by two-body kinematics is plotted versus incident proton energy. The point at $\langle w^2\rangle^{1/2}=1.15$ indicates the threshold of the reaction, and the line labeled $\Delta(1236)$ indicates the mass of the first $(\frac{3}{2},\frac{3}{2})$ isobar. The curve is calculated, including the off-mass-shell factor with a Fermi energy of 50 MeV.

IV. CALCULATIONS

The calculation performed by Remsberg[2] for the case of Cu was repeated here for Al, and the results for a Fermi energy of 50 MeV are presented in Figs. 1 and

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[20] J. B. Cumming, Phys. Rev. **137**, B848 (1965).

2 as the dotted lines.[21] As in the previous case, the fit to the recoil properties is good, but the excitation function fits poorly. Of the two improvements suggested by Selleri,[3] the first allows a kinematic factor in the one-pion-exchange theory to be off the mass shell. This involves replacing the $R(w)$ factor used by Remsberg by

$$R(q^2,w)=\tfrac{1}{2}[w^4-2(1-q^2)w^2+(1+q^2)^2]^{1/2}.$$

The results of the calculation including this factor are shown by the solid curves in Figs. 1 through 3. There is a very dramatic improvement in the excitation function, and the fit to the recoil properties is still good. In fact, considering that there are only two adjustable parameters in this calculation—the Fermi energy and the vertical normalization of the excitation function—the over-all agreement with the experimental data is excellent. The second improvement suggested by Selleri is an empirical correction factor to one-pion-exchange theory, which is given by the term

$$9\mu^2/(10\mu^2+q^2).$$

This is described as a product of form factors associated with the vertices and the propagator of the exchanged pion. It decreases the importance of the higher-momentum transfers. The calculation including both factors is shown by the dashed curves in Figs. 1 and 2. The excitation function is not much affected, but now the recoil properties fit poorly. However, the calculation could be brought back to fit the data as well as before by raising the Fermi energy to 75 MeV as shown by the dot-dash curves. Note that a radius parameter r_0 of 1.25 F would indicate a Fermi energy of only 30 MeV.[22]

As discussed by Remsberg,[2] the necessity of fitting with a high Fermi energy may result from two approximations in the nuclear part of the calculation. First, the model of a zero-temperature Fermi gas neglects the presence of the higher-momentum components in the nucleus. Secondly, the distortion of the momentum-transfer spectrum caused by localization of the reaction site in the nucleus is not taken into account in this calculation. It does not seem likely that either of these factors would be big enough to account for the high Fermi energy.

It must be remembered that the elementary-particle interaction is calculated assuming that the only important diagram is the one in which all the pions are emitted from the vertex containing the bombarding proton. Several reasons have been given for the suppression in the nuclear reaction of the case where pions are also emitted from the other vertex.[1,2] However, it could be that the neglect of this effect is the reason the calculated values of $\langle q\cos\theta\rangle$ are too low when a reasonable Fermi energy is used.

Finally, in considering the validity of the impulse approximation, which is basic to the calculation, it must be borne in mind that the average momentum transfers are low enough to correspond to distances almost as large as nucleon-nucleon separations in the nucleus. Thus one can ask if the presence of the other nucleons in the nucleus might affect the pion form factor since it was derived from data for a free-proton target. The consideration of such a question must wait until the calculation is put on a firmer basis by the elimination of the other approximations.

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APPENDIX

The target foil was dissolved with heat in a solution consisting of 3 ml of conc. HCl, 1 mg of Mg carrier, 25 mg of Na carrier, and a few drops of $HgCl_2$ solution to hasten dissolution of the pure Al. To act as F^- holdback carrier, there was also present a few drops of 5% NH_4F and a few milliliters of saturated H_3BO_3. After the foil had dissolved, 1 ml of the solution was put aside for Na^{24} assay. To the rest of the solution NaOH was added to precipitate $Mg(OH)_2$, while keeping the Al in solution. The $Mg(OH)_2$ was dissolved in HCl, and a $Fe(OH)_3$,CuS_2 scavenge was performed. The supernate was heated, more NH_4OH was added, and the Mg was precipitated by adding dropwise a 5% oxime solution. The solution was filtered, and the sample on the filter paper was mounted for counting.

The beakers in which the Mylar foils were to be dissolved contained 1 mg of Mg carrier which had been evaporated to dryness and dissolved in seven drops of conc. HNO_3 and 14 drops of conc. $HClO_4$. After adding the Mylar foils, the beakers were covered with watch glasses and heated to dense white fumes. Then the watch glasses were removed, and the solutions were taken to dryness with the help of a hot-air blower. At this point a few drops of dilute NaF solution were added to act later as Na^+ and F^- holdback carriers. More acid was added, and the beakers were again evaporated to dryness. The residue was dissolved in HCl, and a $Fe(OH)_3$,CuS_2 scavenge was performed. The Mg oxinate was precipitated and mounted in the same manner as the target foil.

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[21] The value of B was taken to be 6.4 MeV, the separation energy of a neutron from Mg^{27}. It was shown in Ref. 1 that σ is approximately proportional to B^2.

[22] The expression for σ has a term P_F^{-3}, where P_F is the Fermi momentum. As in Ref. 2, this term, which only affects the vertical normalization, was fixed by choosing a radius parameter of 1.25 F. The values of the reduction factor needed to normalize the calculations to fit the data as in Fig. 1 are then 0.38, 0.23, 0.40, and 0.33 for curves 1 to 4, respectively.