# Born-Approximation Formulation of Neutron-Deuteron Scattering with Tensor Interaction*

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The Born approximation for *n-d* scattering is calculated for a nucleon-nucleon interaction which includes a tensor term. The $l=0$ parts of the resulting scattering amplitude are adjusted to obtain a fit to the *n-d* elastic angular distributions. Spin-polarization phenomena are then calculated, with good agreement with experiment.

## I. INTRODUCTION

THE theory of elastic nucleon-deuteron scattering with central potentials is well known.[1] Recent polarization measurements,[2] however, require a description of *n-d* scattering which takes into account noncentral forces. This problem has been discussed briefly by Wu and Ashkin[3] and Verde.[4] Horie *et al.*[5] considered the question in some detail but ignored the deuteron *D* state. Bransden, Smith, and Tate[6] gave a thorough dynamical treatment of elastic *n-d* scattering with tensor forces, but polarization phenomena were not discussed and numerical results have not been forthcoming. Budianskii[7] and Goldberg[8] have described phase-shift analyses of nucleon-deuteron scattering and have discussed certain polarization measurements in detail. Goldberg, in fact, deduced a set of phase shifts which gave an excellent fit to his 40-MeV *p-d* scattering data. This has been to date the only significant numerical work on the problem, and its usefulness is limited by the fact that it ignores doublet-quartet transitions and treats only the polarization of the scattered nucleon. The present work is a Born-approximation description of all neutron-deuteron polarization phenomena, in the presence of a tensor force. The deuteron *D* state is included.

## II. THEORY

As is well known,[3,9] the application of the Born approximation to *n-d* scattering leads to a scattering matrix of the form

$$M(\theta,\varphi)=\frac{1}{4\pi}\frac{4M}{3\hbar^2}\int e^{-i\mathbf{k}'\cdot\mathbf{q}}\varphi^*(\mathbf{r})[V_n(\mathbf{q}-\tfrac{1}{2}\mathbf{r})+V_p(\mathbf{q}+\tfrac{1}{2}\mathbf{r})]$$
$$\times[\varphi(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{q}}-\varphi(\mathbf{q}+\tfrac{1}{2}\mathbf{r})e^{-i\mathbf{k}\cdot(\frac{1}{2}\mathbf{q}-\frac{3}{4}\mathbf{r})}P_{13}{}^{\sigma}]d\mathbf{r}d\mathbf{q}. \quad (1)$$

The notation is that of Fig. 1. $P_{13}{}^{\sigma}$ is the spin-exchange operator for the two neutrons and appears as a result of antisymmetrization.

$\varphi(\mathbf{r})$ is the deuteron wave function and will be written[9]

$$\varphi(\mathbf{r})=v(r)+[w(r)/\sqrt{8}]s_{12},\ \text{etc.}, \quad (2)$$

where

$$S_{12}=3\boldsymbol{\sigma}_1\cdot\hat{r}\boldsymbol{\sigma}_2\cdot\hat{r}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2.$$

If we assume the same radial form for the *S*- and *D*-state contributions to the deuteron ground-state wave function, then we may write

$$\varphi(r)=v(r)[1+(P_D{}^{1/2}/\sqrt{8})S_{12}],\quad \int v(r)^2r^2\,dr=P_S,$$

$$\int w(r^2)r^2\,dr=P_D,$$

in which $P_S$ and $P_D$ are the fractional contributions of the $L=0$ and $L=2$ parts of the wave function. Then, writing $\eta^2=P_D$, we have

$$\varphi(\mathbf{r})=v(r)[1+(\eta/\sqrt{8})S_{12}],\quad \text{etc.} \quad (3)$$

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The radial form $v(r)$ is taken to be the Hulthén wave function[10]:

$$v(r)=\left[\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}\right]^{1/2}\frac{e^{-\alpha r}-e^{-\beta r}}{r}. \tag{4}$$

The potentials $V_n$ and $V_p$ are, if we write $\mathbf{w}=\mathbf{q}+\frac{1}{2}\mathbf{r}$ and $\mathbf{z}=\mathbf{q}-\frac{1}{2}\mathbf{r}$,

$$\begin{aligned} V_n(\mathbf{z})={}&{}^3V_n{}^-(\mathbf{z})\tfrac{1}{4}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)[\tfrac{1}{2}(1-P_{13}{}^x)]+{}^1V_n{}^+(\mathbf{z})\\ &\times\tfrac{1}{4}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)[\tfrac{1}{2}(1+P_{13}{}^x)],\\ V_p(\mathbf{w})={}&{}^3V_p{}^+(\mathbf{w})\tfrac{1}{4}(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)[\tfrac{1}{2}(1+P_{23}{}^x)]\\ &+{}^3V_p{}^-(\mathbf{w})\tfrac{1}{4}(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)[\tfrac{1}{2}(1-P_{23}{}^x)]+{}^1V_p{}^+(\mathbf{w})\\ &\times\tfrac{1}{4}(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)[\tfrac{1}{2}(1+P_{23}{}^x)]+{}^1V_p{}^-(\mathbf{w})\\ &\times\tfrac{1}{4}(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)[\tfrac{1}{2}(1-P_{23}{}^x)]. \end{aligned} \tag{5}$$

Here ${}^3V_p{}^+(\mathbf{w})={}^3V_{pC}{}^+(|\mathbf{w}|)+{}^3V_{pT}{}^+(|\mathbf{w}|)S_{23}$, etc., with $S_{23}=3\boldsymbol{\sigma}_2\cdot\hat{w}\boldsymbol{\sigma}_3\cdot\hat{w}-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3$. We use for the radial part of the interactions a Yukawa well

$$\frac{V_0e^{-r/\lambda}}{r/\lambda}.$$

The integrand is then of the form

$$\begin{aligned} J={}&\varphi^*(\mathbf{r})V_p(\mathbf{w})\varphi(\mathbf{r})f(3)+\varphi^*(\mathbf{r})V_n(\mathbf{z})\varphi(\mathbf{r})f(3)\\ &+[\varphi^*(\mathbf{r})V_p(\mathbf{w})\varphi(\mathbf{w})f(1)+\varphi^*(\mathbf{r})V_n(\mathbf{z})\varphi(\mathbf{w})f(1)]P_{13}{}^\sigma, \end{aligned} \tag{6}$$

where

$$f(3)=e^{i\mathbf{k}\cdot\mathbf{q}},$$
$$f(1)=P_{13}{}^xf(3)=e^{-i\mathbf{k}\cdot(\frac{1}{2}\mathbf{q}-\frac{3}{4}\mathbf{r})}.$$

We introduce the following notation: $f=f(3)$, $f''=f(1)$, and $f'=P_{23}{}^xf=e^{i\mathbf{k}\cdot(\frac{1}{2}\mathbf{q}+\frac{3}{4}\mathbf{r})}$. Then, using the fact that $P_{23}{}^x\mathbf{r}=-\mathbf{z}$, $P_{13}{}^x\mathbf{r}=\mathbf{w}$, $P_{23}{}^x\mathbf{w}=-\mathbf{w}$, and $P_{13}{}^x\mathbf{w}=\mathbf{r}$, we obtain

$$\begin{aligned} M(\theta,\varphi)={}&\sum_\pm\frac{1}{8\pi}\frac{4M}{3\hbar^2}\int e^{-i\mathbf{k}'\cdot\mathbf{q}}\varphi(\mathbf{r})\{V_p{}^\pm(\mathbf{w})\varphi(\mathbf{r})f\\ &\pm V_p{}^\pm(\mathbf{w})\varphi(\mathbf{z})f'+V_n{}^\pm(\mathbf{z})\varphi(\mathbf{r})f\pm V_n{}^\pm(\mathbf{z})\varphi(\mathbf{w})f''\\ &+[2V_p{}^+(\mathbf{w})\varphi(\mathbf{w})f''+V_n{}^\pm(\mathbf{z})\varphi(\mathbf{w})f''\\ &\pm V_n{}^\pm(\mathbf{z})\varphi(\mathbf{r})f]P_{13}{}^\sigma\}d\mathbf{r}d\mathbf{q}. \end{aligned} \tag{7}$$

It is evident that there are three distinct contributions to (7):

$$\begin{aligned} J_1&=\int e^{-i\mathbf{k}'\cdot\mathbf{q}}\varphi(\mathbf{r})V(q\pm\tfrac{1}{2}\mathbf{r})\varphi(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{q}}d\mathbf{r}d\mathbf{q},\\ J_2&=\int e^{-i\mathbf{k}'\cdot\mathbf{q}}\varphi(\mathbf{r})V(\mathbf{w})\varphi(\mathbf{w})e^{-i\mathbf{k}\cdot(\frac{1}{2}\mathbf{q}-\frac{3}{4}\mathbf{r})}d\mathbf{r}d\mathbf{q},\\ J_3&=\int e^{-i\mathbf{k}'\cdot\mathbf{q}}\varphi(\mathbf{r})V(q\pm\tfrac{1}{2}\mathbf{r})\varphi(q\mp\tfrac{1}{2}\mathbf{r})e^{-i\mathbf{k}\cdot(\frac{1}{2}\mathbf{q}\pm\frac{3}{4}\mathbf{r})}d\mathbf{r}d\mathbf{q}. \end{aligned} \tag{8}$$

$J_1$ is the direct integral, peaked in the forward direction, and $J_2$ and $J_3$ are exchange integrals, peaked backward. At low energies $J_2$ is dominant, $J_1$ is negligible for $L>1$, and $J_3$ is unimportant.[1]

Putting in all factors, and neglecting inelastic terms, we find for the $J_1$ and $J_2$ contributions to $M$

$$\begin{aligned} M_1={}&\frac{1}{32\pi}\left(\frac{4M}{3\hbar^2}\right)\int e^{-i\mathbf{k}'\cdot\mathbf{q}}v(r)^2\{[1+(2\eta/\sqrt{8})S_{12}+\tfrac{1}{8}\eta^2S_{12}{}^2][({}^3V_{pC}{}^+(w)+{}^3V_{pC}{}^-(w))(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)\\ &+({}^1V_{pC}{}^+(w)+{}^1V_{pC}{}^-(w))(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]+[S_{23}+(\eta/\sqrt{8})S_{12}S_{23}+(\eta/\sqrt{8})S_{23}S_{12}+\tfrac{1}{8}\eta^2S_{12}S_{23}S_{12}]\\ &\times[({}^3V_{pT}{}^+(w)+{}^3V_{pT}{}^-(w))(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)+({}^1V_{pT}{}^+(w)+{}^1V_{pT}{}^-(w))(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]+[1+(2\eta/\sqrt{8})S_{12}+(\eta^2/8)S_{12}{}^2]\\ &\times[{}^1V_{nC}{}^+(z)(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)-{}^3V_{nC}{}^-(z)(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)]+[S_{13}+(\eta/\sqrt{8})S_{12}S_{13}+(\eta/\sqrt{8})S_{13}S_{12}+\tfrac{1}{8}\eta^2S_{12}S_{13}S_{12}]\\ &\times[{}^1V_{nT}{}^+(z)(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)+{}^3V_{nT}{}^-(z)(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)]+[1+(2\eta/\sqrt{8})S_{12}+\tfrac{1}{8}\eta^2S_{12}{}^2][{}^1V_{nC}{}^+(z)(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)\\ &-{}^3V_{nC}{}^-(z)(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)]P_{13}{}^\sigma+[S_{13}+(\eta/\sqrt{8})S_{12}S_{13}+(\eta/\sqrt{8})S_{13}S_{12}+\tfrac{1}{8}\eta^2S_{12}S_{13}S_{12}]\\ &\times[{}^1V_{nT}{}^+(z)(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)-{}^3V_{nT}{}^-(z)(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)]P_{13}{}^\sigma\}e^{i\mathbf{k}\cdot\mathbf{q}}d\mathbf{r}d\mathbf{q}, \end{aligned} \tag{9}$$

$$\begin{aligned} M_2={}&\frac{1}{16\pi}\frac{4M}{3\hbar^2}\int e^{-i\mathbf{k}'\cdot\mathbf{q}}v(r)v(w)\{[1+(\eta/\sqrt{8})S_{12}+(\eta/\sqrt{8})S_{23}+\tfrac{1}{8}\eta^2S_{12}S_{23}]\\ &+[{}^3V_{pC}{}^+(w)(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)+{}^1V_{pC}{}^+(w)(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]+[S_{23}+(\eta/\sqrt{8})S_{12}S_{23}+(\eta/\sqrt{8})S_{23}S_{12}+\tfrac{1}{8}\eta^2S_{12}S_{23}S_{12}]\\ &\times[{}^3V_{pT}{}^+(w)(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)+{}^1V_{pT}{}^+(w)(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]P_{13}{}^\sigma\}e^{-i\mathbf{k}\cdot(\frac{1}{2}\mathbf{q}-\frac{3}{4}\mathbf{r})}d\mathbf{r}d\mathbf{q}. \end{aligned} \tag{10}$$

Then, using the fact that

$$\int e^{-i\mathbf{k}\cdot\mathbf{r}}f(r)[S_{12}(\mathbf{r})]^nd\mathbf{r}=-5\left(\frac{4\pi}{5}\right)^{(n+1)/2}[S_{12}(\mathbf{k})]^n\int j_2(kr)f(r)r^2dr, \tag{11}$$

[10] L. Hulthen and M. Sugawara, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39.

where $S_{12}(\mathbf{k})$ is the Fourier transform of $S_{12}(\mathbf{r})$,

$$S_{12}(\mathbf{k})=3\boldsymbol{\sigma}_1\cdot\hat{k}\boldsymbol{\sigma}_2\cdot\hat{k}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2,$$

and defining

$$\begin{aligned}\alpha_L(k)&=\int j_L(k\xi)v(\xi)^2\xi^2\,d\xi, &\quad \beta_L(k)&=\int j_L(k\xi)V(\xi)\xi^2\,d\xi,\\ \gamma_L(k)&=\int j_L(k\xi)v(\xi)\xi^2\,d\xi, &\quad \zeta_L(k)&=\int j_L(k\xi)v(\xi)V(\xi)\xi^2\,d\xi,\end{aligned}\tag{12}$$

where $\xi$ may be $r$, $w$, or $z$, we obtain for $M_1$ and $M_2$

$$\begin{aligned}M_1=\frac{(4\pi)^2}{32\pi}\left(\frac{4M}{3\hbar^2}\right)\{&[\alpha_0(k_1)\beta_0(k_2)-(2\eta/\sqrt{8})\alpha_2(k_1)\beta_0(k_2)S_{12}-\tfrac{1}{8}\eta^2(\tfrac{4}{5}\pi)^{1/2}\alpha_2(k_1)\beta_0(k_2)S_{12}{}^2][({}^3V_{pC}{}^{+}+{}^3V_{pC}{}^{-})(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)\\&+({}^1V_{pC}{}^{+}+{}^1V_{pC}{}^{-})(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]+[-\alpha_0(k_1)\beta_2(k_2)S_{23}+(\eta/\sqrt{8})\alpha_2(k_1)\beta_2(k_2)(S_{12}S_{23}+S_{23}S_{12})+(\tfrac{4}{5}\pi)^{1/2}\tfrac{1}{8}\eta^2\alpha^2(k_1)\\&\times\beta_2(k_2)S_{12}S_{23}S_{12}][({}^3V_{pT}{}^{+}+{}^3V_{pT}{}^{-})(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)+({}^1V_{pT}{}^{+}+{}^1V_{pT}{}^{-})(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]+[\alpha_0(k_1)\beta_0(k_2)-(2\eta/\sqrt{8})\\&\times\alpha_2(k_1)\beta_0(k_2)S_{12}-\tfrac{1}{8}\eta^2(\tfrac{4}{5}\pi)^{1/2}\alpha_2(k_1)\beta_0(k_2)S_{12}{}^2][{}^1V_{nC}{}^{+}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)+{}^3V_{nC}{}^{-}(3+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_3)]+[-\alpha_0(k_1)\beta_2(k_2)S_{13}\\&+(\eta/\sqrt{8})\alpha_2(k_1)\beta_2(k)(S_{12}S_{13}+S_{13}S_{12})+\tfrac{1}{8}\eta^2(\tfrac{4}{5}\pi)^{1/2}\alpha_2(k_1)\beta_2(k_2)S_{12}S_{13}S_{12}][{}^1V_{nT}{}^{+}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)+{}^3V_{nT}{}^{-}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)]\\&+[\alpha_0(k_1)\beta_0(k_2)-(2\eta/\sqrt{8})\alpha_2(k_1)\beta_0(k_2)S_{12}-\tfrac{1}{8}\eta^2(\tfrac{4}{5}\pi)^{1/2}\alpha_2(k_1)\beta_0(k_2)S_{12}{}^2][{}^1V_{nC}{}^{+}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)\\&-{}^3V_{nC}{}^{-}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)]P_{13}{}^{\sigma}+[-\alpha_0(k_1)\beta_2(k_2)S_{13}+(\eta/\sqrt{8})\alpha_2(k_1)\beta_2(k_2)(S_{12}S_{13}+S_{13}S_{12})\\&+\tfrac{1}{8}\eta^2(\tfrac{4}{5}\pi)^{1/2}\alpha_2(k_1)\beta_2(k_2)S_{12}S_{13}S_{12}][{}^1V_{nT}{}^{+}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)-{}^3V_{nT}{}^{-}(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]P_{13}{}^{\sigma}\},\end{aligned}\tag{13}$$

where $\mathbf{k}_1=\frac{1}{2}(\mathbf{k}'-\mathbf{k})$, and $\mathbf{k}_2=\mathbf{k}'-\mathbf{k}$;

$$\begin{aligned}M_2=\frac{(4\pi)^2}{32\pi}\left(\frac{4M}{3\hbar^2}\right)\{&[\gamma_0(k_1)\zeta_0(k_2)-(\eta/\sqrt{8})\gamma_2(k_1)\zeta_0(k_2)S_{12}-(\eta/\sqrt{8})\gamma_0(k_1)\zeta_2(k_2)S_{23}+\tfrac{1}{8}\eta^2\gamma_2(k_1)\zeta_2(k_2)S_{12}S_{23}]\\&\times[{}^3V_{pC}{}^{+}(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]+[-(\eta/\sqrt{8})\gamma_0(k_1)\zeta_2(k_2)S_{23}+(\eta/\sqrt{8})\gamma_2(k_1)\zeta_2(k_2)(S_{12}S_{23}+S_{23}S_{12})\\&-\tfrac{1}{8}\eta^2(\tfrac{4}{5}\pi)^{1/2}\gamma_2(k_1)\zeta_2(k_2)S_{12}S_{23}S_{12}]\times[{}^3V_{pT}{}^{+}(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)]P_{13}{}^{\sigma}\},\end{aligned}\tag{14}$$

in which $\mathbf{k}_1=\frac{1}{2}\mathbf{k}'+\mathbf{k}$ and $\mathbf{k}_2=\mathbf{k}'+\frac{1}{2}\mathbf{k}$. The indicated integrations can be performed in a straightforward way. The task of evaluating the matrix elements of $M=M_1+M_2$ (neglecting the $J_3$ contributions) is a much more difficult one.

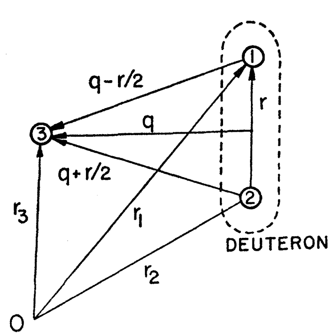


FIG. 1. Coordinates used in the text.

Some typical terms which appear are as follows:

$$\chi^*(12,3)S_{12}\chi(23,1),$$

$$\chi^*(12,3)S_{23}\chi(23,1),$$

$$\chi^*(12,3)S_{12}S_{23}S_{12}\chi(23,1),$$

$$\chi^*(12,3)\begin{pmatrix}3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3\\1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3\end{pmatrix}\chi(12,3),$$

$$\chi^*(12,3)S_{12}S_{13}S_{12}\begin{pmatrix}3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3\\1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3\end{pmatrix}\chi(23,1),$$

etc.

Using the fact that

$$S_{ij}\chi_{m_s}{}^{s}(ij,k)=(32\pi)^{1/2}\sum_{m}\sum_{m'}\sum_{s''}C(Sm_s\,1m\tfrac{1}{2}m_s-m)$$
$$\times C(1m2m'\,1m-g)C(s''m_s''\,1\,m-m'\,\tfrac{1}{2}m_s-m)$$
$$\times Y_{m'}{}^{2}(\hat{k})\chi_{m_s''}{}^{s''}(ij,k)\,,\quad\text{with}\quad m'=m_s-m_s''\,,\qquad(15)$$

we obtain, for example,

$$\chi_{m_s'}{}^{s'*}(ij,k)S_{ij}\chi_{m_s}{}^{s}(ij,k)$$
$$=(32\pi)^{1/2}\sum_{m}C(Sm_s\,1m\tfrac{1}{2}m_s-m)C(1m2g\,1\,m-g)$$
$$\times C(s'm_s'\,1\,m-g\tfrac{1}{2}m_s-m)Y_g{}^{2}\,,$$

with $g=m_s-m_s'$. More complicated terms may be handled in the same manner, upon introducing appropriate complete sets of states, e.g.,

$$\chi^*(12,3)S_{12}S_{23}S_{12}\chi(23,1)$$
$$=\sum\sum\sum\sum\sum\chi^*(12,3)S_{12}\chi(12,3)\chi^*(12,3)\chi(23,1)$$
$$\times\chi^*(23,1)S_{23}\chi(23,1)\chi^*(23,1)\chi(12,3)\chi^*(12,3)$$
$$\times S_{12}\chi(12,3)\chi^*(12,3)\chi(23,1)\,.\qquad(16)$$

Then we require terms like

$$\chi^*(12,3)\chi(23,1)\,.$$

Explicitly,

$$\chi_{m_s'}{}^{*s'}(12,3)\chi_{m_s}{}^{s}(23,1)$$
$$=\sum_{m''}\sum_{m'''}C(s'''m''';\tfrac{1}{2}g\tfrac{1}{2}m_s-m_s'')C(s''m'';\tfrac{1}{2}g\tfrac{1}{2}m''-g)$$
$$\times C(sm_s;s''m''\tfrac{1}{2}m_s-m'')$$
$$\times C(s'm_s';s'''m'''\tfrac{1}{2}m''-g)\,,\qquad(17)$$

where $m_s=m_s'$, $s'''=s''=1$, and $m_s=m'''+m''-g$.

Thus we obtain the 36 matrix elements $M_{s'm_s'sm_s}$ of $M$ in the Born approximation. Only 18 of these are distinct, as may be seen by considering a partial-wave expansion of $M$:

$$M_{s'm_s'sm_s}=\sum_{J}\sum_{L}\sum_{L'}\frac{[(2L+1)4\pi]^{1/2}}{2ik}t_{L'S'LS}{}^{J}$$
$$\times C(Jm_s;L0sm_s)$$
$$\times C(Jm_s;L'm_s-m_s's'm_s')Y_{m_s-m_s'}{}^{L'}\,.\qquad(18)$$

Then, from the properties of the Clebsch-Gordan coefficients, we find that

$$M_{s'-m_s's-m_s}=M_{s'm_s'sm_s}(-1)^{s+s'+m_s+m_s'+1}\,.$$

This result may be compared with the general case, in which the form of $M$ is obtained by invoking rotational, reflection, and time-reversal invariance:

$$M=a(\theta)+b(\theta)\sigma_N+c(\theta)S_N+d(\theta)(S_PS_P+S_KS_K)$$
$$+e(\theta)(S_PS_P-S_KS_K)+f(\theta)(\sigma_NS_N)+g(\theta)$$
$$\times(\sigma_PS_P+\sigma_KS_K)+h(\theta)(\sigma_PS_P-\sigma_KS_K)+m(\theta)$$
$$\times\sigma_N(S_PS_P+S_KS_K)+n(\theta)\sigma_N(S_PS_P-S_KS_K)+p(\theta)$$
$$\times(\sigma_PS_NS_P+\sigma_KS_NS_K)+r(\theta)$$
$$\times(\sigma_PS_NS_P-\sigma_KS_NS_K)\,,\qquad(19)$$

where, as usual,

$$\hat{N}=\frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}\times\mathbf{k}'|}\,,\quad\mathbf{P}=\frac{\mathbf{k}'+\mathbf{k}}{|\mathbf{k}'+\mathbf{k}|}\,,\quad\mathbf{K}=\frac{\mathbf{k}'-\mathbf{k}}{|\mathbf{k}'-\mathbf{k}|}\,.$$

From the fact that $M$ contains 12 terms, we conclude that for $n$-$d$ scattering there are only 12 independent amplitudes. The reduction from 18 to 12 is a result of time-reversal invariance, and the Born approximation does not possess this property.[11]

Using the density-matrix formalism (see Sec. III) we write the differential cross section as

$$\sigma(\theta)=\tfrac{1}{6}\,\mathrm{Tr}MM^{\dagger}\,,\qquad(20)$$

in which $M$ is a 6×6 matrix (Hermitian in the Born approximation). We do not expect to obtain sensible angular distributions from (20) if $M$ is given by the Born approximation, since we know that the "$s$-wave" eigenphases are large at the energies under consideration and cannot be accurately treated in the Born approximation. Thus we adopt the procedure used by Christian and Gammel[1] and treat the $S$-matrix elements $S_{0S'0S}{}^{J}$ (the notation is $S_{L'S'LS}{}^{J}$) as parameters to be used to fit the experimental angular distribution and polarization measurements.[12] In practice, the following states are coupled:

$$S=\tfrac{3}{2}:\;J=L+\tfrac{3}{2},\;J=L-\tfrac{1}{2}\quad\text{and}\quad S=\tfrac{1}{2}:\;J=L-\tfrac{1}{2}$$

or

$$S=\tfrac{3}{2}:\;J=L+\tfrac{1}{2},\;J=L-\tfrac{3}{2}\quad\text{and}\quad S=\tfrac{1}{2}:\;J=L-\tfrac{1}{2}\,.$$

That is,

$$S^{\frac{1}{2}+}=\begin{pmatrix}S_{0\frac{1}{2}\,0\frac{1}{2}}{}^{\frac{1}{2}+} & S_{0\frac{1}{2}\,2\frac{3}{2}}{}^{\frac{1}{2}+}\\ S_{2\frac{3}{2}\,0\frac{1}{2}}{}^{\frac{1}{2}+} & S_{2\frac{3}{2}\,2\frac{3}{2}}{}^{\frac{1}{2}+}\end{pmatrix},$$

$$S^{\frac{3}{2}+}=\begin{bmatrix}S_{0\frac{3}{2}\,0\frac{3}{2}}{}^{\frac{3}{2}+} & S_{0\frac{3}{2}\,2\frac{3}{2}}{}^{\frac{3}{2}+} & S_{0\frac{3}{2}\,2\frac{1}{2}}{}^{\frac{3}{2}+}\\ S_{2\frac{3}{2}\,0\frac{3}{2}}{}^{\frac{3}{2}+} & S_{2\frac{3}{2}\,2\frac{3}{2}}{}^{\frac{3}{2}+} & S_{2\frac{3}{2}\,2\frac{1}{2}}{}^{\frac{3}{2}+}\\ S_{2\frac{1}{2}\,0\frac{3}{2}}{}^{\frac{3}{2}+} & S_{2\frac{1}{2}\,2\frac{3}{2}}{}^{\frac{3}{2}+} & S_{2\frac{1}{2}\,2\frac{1}{2}}{}^{\frac{3}{2}+}\end{bmatrix}.\qquad(21)$$

Thus we actually have 5 complex parameters (since $S$ is symmetric) of the form $S_{L'S'0S}{}^{J}$. If we parametrize the $S$ matrix by writing $S=U^{-1}e^{2i\delta}U$, where $\delta$ is a diagonal matrix whose elements are the eigenphases and $U$ is a unitary matrix containing the mixing parameters, then we find that the two matrices depend on a total of 9 real parameters (5 phase shifts and 4 mixing parameters) in which $L=0$ and $L=2$ are coupled. If, however, the coupling is weak, we can consider the eigenphases to belong to definite values of $L$ and $S$. Labeling the eigenphases by ${}^{2S+1}X_{2J}$, we have ${}^4S_3$, ${}^4D_3$, ${}^2D_3$, ${}^2S_1$, and ${}^4D_1$. We may then approximately consider the mixing parameters to mix the states as follows: $({}^4S_3,{}^4D_3)$,

[11] M. L. Goldberger and K. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964).

[12] The elements of the $S$ matrix and the $T$ matrix, as here defined, are related by $t_{L'S'LS}{}^{J}=S_{L'S'LS}{}^{J}-\delta_{LL'}\delta_{SS'}$.

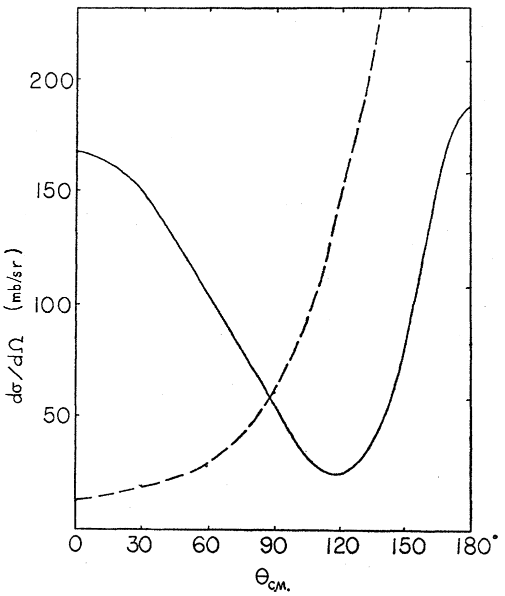


FIG. 2. Elastic scattering angular distribution for *n-d* scattering at 9.0 MeV before (dashed) and after (solid) adjusting $l=0$ parts of scattering amplitude.

$({}^4S_3,{}^2D_3)$, $({}^4D_3,{}^2D_3)$, and $({}^2S_1,{}^4D_1)$. The parameters varied in the calculations will be the ${}^4S_3$ and ${}^2S_1$ eigenphases and the three appropriate mixing parameters.

It should be pointed out that the calculation of the $J_2$ contribution to $M$ can be greatly simplified by making use of the fact that

$$[{}^3V_{pC}{}^+(\mathbf{w})+{}^3V_{pT}{}^+(\mathbf{w})S_{23}]\varphi(\mathbf{w}) = -(\hbar^2/M)(k_2{}^2+\alpha^2)\varphi(\mathbf{w}),$$

where $\alpha^2=0.05364$ F$^{-2}$. Making use of this procedure, we find

$$M_2=-\left(\frac{k_2{}^2+\alpha^2}{24\pi}\right)(4\pi)^2[\gamma_0(k_1)\gamma_0(k_2)-(\eta/\sqrt{8})\gamma_0(k_1) \times\gamma_2(k_2)S_{23}(k_2)-(\eta/\sqrt{8})\gamma_2(k_1)\gamma_0(k_2)S_{12}(k_1) +\tfrac{1}{8}\eta^2\gamma_2(k_1)\gamma_2(k_2)S_{12}(k_1)S_{23}(k_2)]P_{13}{}^\sigma.$$

This use of the deuteron equation is, in fact, an approximation since the wave function we are using is not an exact solution to the equation.

Some mention of the extremely interesting recent work on the three-body problem using separable potentials[13] may be appropriate at this point. This work, motivated or justified, as the case may be, by the contributions of Faddeev,[14] is directed toward an understanding of the purely three-body aspects of this problem. The investigations of deuteron breakup in the *n-d* problem have been especially fruitful. At the same time virtually all of this work has employed a purely central interaction, with tensor effects accomodated in only a phenomenological way. Mitra, Schrenk, and Bhasin[13] discussed the inclusion of the tensor interaction into the separable approximation, but it is not at all clear how these noncentral forces can conveniently be introduced into the formalism of Aaron *et al.*,[13] or that of Lovelace.[13] In this work we are concerned with phenomena which are sensitive to the details of the two-body interaction and are forced to forego the pleasure of a sophisticated treatment of the purely three-body aspects of the problem.

[13] R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. **140**, B1291 (1965); C. Lovelace, *ibid.* **135**, B1225 (1964); A. C. Phillips, *ibid.* **142**, 984 (1966); A. N. Mitra, G. L. Schrenk, and V. S. Bhasin, Ann. Phys. (N. Y.) **40**, 357 (1966); V. F. Kharchenko and N. M. Petrov, Nucl. Phys. **A93**, 289 (1967).

[14] L. D. Faddeev, *Mathematical Problems of the Quantum Theory of Scattering for a Three-Particle System* (Publications of the Steklov Mathematical Institute, Leningrad, 1963), No. 69 [English transl.: H. M. Stationary Office, Harwell, England, 1964].

## III. POLARIZATION

In the density-matrix formalism,[15] the (average) expectation value of the operator $A$ in the final state $|f\rangle$ is given by

$$\langle A\rangle_f=\mathrm{Tr}\rho_f A/\mathrm{Tr}\rho_f\,, \tag{22}$$

where the density matrix for the final state is, for nucleon-deuteron scattering,

$$\rho_f=\tfrac{1}{6}\,\mathrm{Tr}\rho_{\mathrm{inc}}\sum MS^\mu M^\dagger\langle S^\mu\rangle_{\mathrm{inc}}\,, \tag{23}$$

the $S^\mu$ being the 36 linearly independent 6×6 matrices $I, \sigma_i, S_i, S_{ij}, \sigma_iS_j, \sigma_iS_{jk}$, where $S_{ij}=\frac{1}{2}(S_iS_j+S_jS_i)-\frac{2}{3}\delta_{ij}$.[16] $M$ can be expressed in terms of the $S^\mu$:

$$M=A+B_i\sigma_i+C_iS_i+D_{ij}S_{ij}+F_{ij}\sigma_iS_j+G_{ijk}\sigma_iS_{jk}. \tag{24}$$

In the general case, this is equivalent to (19) so that $A$, $B_i$, etc., could be expressed in terms of $a(\theta)$, $b(\theta)$, etc., and functions of the momenta. In fact, (24) is different from (19) if we employ the Born approximation to obtain the coefficients.

The differential cross section is given by

$$I_0=\mathrm{Tr}\rho_f/\mathrm{Tr}\rho_{\mathrm{inc}}. \tag{25}$$

If there is no initial polarization,

$$\rho_f=\tfrac{1}{6}\,\mathrm{Tr}\rho_{\mathrm{inc}}MM^\dagger\,, \tag{26}$$

which is just (20). Similarly, for zero initial polarization, the final nucleon polarization is

$$\langle\boldsymbol{\sigma}\rangle_f=\tfrac{1}{6}\,\mathrm{Tr}\rho_f\boldsymbol{\sigma}/\mathrm{Tr}\rho_f. \tag{27}$$

Then if

$$\hat{N}_2=\frac{\mathbf{k}_i\times\mathbf{k}_f}{|\mathbf{k}_i\times\mathbf{k}_f|}$$

is the normal to the scattering plane,

$$I_0\langle\boldsymbol{\sigma}\cdot\hat{N}_2\rangle_f=\tfrac{1}{6}\,\mathrm{Tr}M\boldsymbol{\sigma}\cdot\hat{N}_2M^\dagger \equiv I_0P_2. \tag{28}$$

[15] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).

[16] H. P. Stapp, University of California Radiation Laboratory Report No. UCRL-3098, 1955 (unpublished).

For an initially polarized beam,

$$\rho_f = \frac{1}{6}\,\mathrm{Tr}\rho_{\mathrm{inc}}[MM^\dagger + M\boldsymbol{\sigma}\cdot\hat{k}_i M^\dagger\langle\boldsymbol{\sigma}\cdot\hat{k}_i\rangle_{\mathrm{inc}} + M\boldsymbol{\sigma}\cdot\hat{N}_2 M^\dagger \times\langle\boldsymbol{\sigma}\cdot\hat{N}_2\rangle_{\mathrm{inc}} + M\boldsymbol{\sigma}\cdot\hat{P}M^\dagger\langle\boldsymbol{\sigma}\cdot\hat{P}\rangle_{\mathrm{inc}}], \quad (29)$$

in which

$$\hat{P} = \frac{\mathbf{k}_i\times\hat{N}_2}{|\mathbf{k}_i\times\hat{N}_2|}.$$

Then the differential cross section is

$$I_f = I_0(1 + P_2\langle\boldsymbol{\sigma}\cdot\hat{N}_2\rangle_{\mathrm{inc}}). \quad (30)$$

Defining $P_1$ by $\langle\boldsymbol{\sigma}\rangle_{\mathrm{inc}} \equiv P_1\hat{N}_1$, where $\hat{N}_1$ is the normal to the first scattering plane, then

$$I_f = I_0(1 + P_1P_2\hat{N}_1\cdot\hat{N}_2). \quad (31)$$

$P_2$ can clearly be measured in a left-right asymmetry scattering experiment in which the two scatterings take place in the same plane ($\hat{N}_1\cdot\hat{N}_2 = \pm 1$). Then

$$\sigma_L = I_0(1 + P_1P_2),$$
$$\sigma_R = I_0(1 - P_1P_2),$$

and $P_2 = \epsilon/P_1$, with

$$\epsilon = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}.$$

In the most general case, we must also consider initially polarized deuterons. Then

$$\rho_f = \frac{1}{6}\,\mathrm{Tr}\rho_{\mathrm{inc}}[MM^\dagger + M\sigma_i M^\dagger\langle\sigma_i\rangle_{\mathrm{inc}} + MS_iM^\dagger\langle S_i\rangle_{\mathrm{inc}} + MT_{2\mu}M^\dagger\langle T_{2\mu}\rangle_{\mathrm{inc}}], \quad (32)$$

in which we have introduced the second-rank tensor $T^{(2)}$ whose components are[17]

$$T_{22} = \tfrac{1}{2}\sqrt{3}(S_x + iS_y)^2,$$
$$T_{21} = -\tfrac{1}{2}\sqrt{3}[(S_x + iS_y)S_z + S_z(S_x + iS_y)], \quad (33)$$
$$T_{20} = \tfrac{1}{2}\sqrt{2}(3S_z^2 - 2),$$

with $T_{2-\mu} = (-1)^\mu T_{2\mu}^*$. Then

$$I_f = I_0(1 + \langle\sigma_i\rangle_{\mathrm{inc}}\langle\sigma_i\rangle_0 + \langle s_i\rangle_{\mathrm{inc}}\langle s_i\rangle_0 + \langle T_{2\mu}\rangle_{\mathrm{inc}}\langle T_{2\mu}\rangle_0), \quad (34)$$

where $\langle\sigma_i\rangle_0$ is the polarization which would result from scattering of unpolarized particles, etc. Then, if $\langle\boldsymbol{\sigma}\rangle_{\mathrm{inc}} = P_1\hat{N}_1$ and $\langle\mathbf{S}\rangle_{\mathrm{inc}} = P_1{}^D\hat{N}_1$, and we note that since $M = A + B\sigma_N + CS_N + \cdots$,

$$I_0\langle\boldsymbol{\sigma}\rangle_0 = \tfrac{1}{6}\,\mathrm{Tr}M\boldsymbol{\sigma}M^\dagger = \tfrac{1}{6}\,\mathrm{Tr}M\sigma_N M^\dagger\hat{N}_2 \equiv P_2\hat{N}_2, \quad (35)$$

$$I_0\langle\mathbf{S}\rangle_0 = \tfrac{1}{6}\,\mathrm{Tr}M\mathbf{S}M^\dagger = \tfrac{1}{6}\,\mathrm{Tr}MS_NM^\dagger \equiv P_2{}^D\hat{N}_2, \quad (36)$$

[17] W. Lakin, Phys. Rev. 98, 139 (1955).

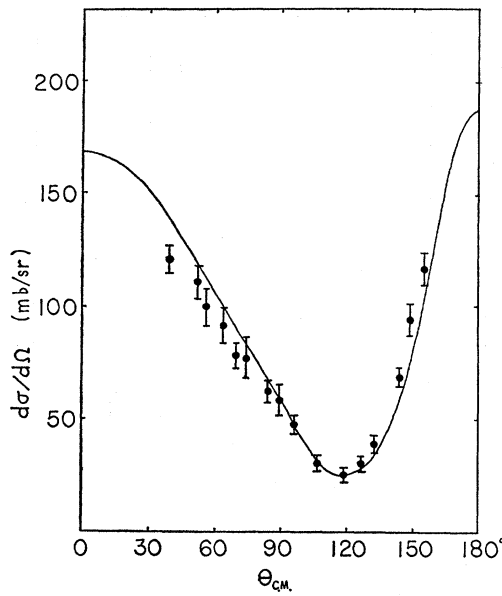


FIG. 3. Fit to the 9.0-MeV scattering data. Data from B. Bonner, Rice University thesis, 1965 (unpublished).

we find that

$$I_f = I_0'(1 + P_1P_2\hat{N}_1\cdot\hat{N}_2 + P_1{}^DP_2{}^D\hat{N}_1\cdot\hat{N}_2 + \sum_\mu \langle T_{2\mu}\rangle_{\mathrm{inc}}\langle T_{2\mu}\rangle_0). \quad (37)$$

The tensor polarization term may be reexpressed in the following manner:

$$\sum_\mu \langle T_{2\mu}\rangle_{\mathrm{inc}}\langle T_{2\mu}\rangle_0 = \sum_\mu\sum_\nu \langle T_{2\mu}\rangle_{\mathrm{inc}}\langle T_{2\nu}'\rangle_0 \mathfrak{D}_{\mu\nu}{}^2(\varphi,\theta,0), \quad (38)$$

where the $T_{2\mu}$ are referred to a coordinate system in which the polar axis is taken along $\mathbf{k}_i$ and the $y$ axis along the normal (in the conventional sense) to the *first* scattering plane and the $T_{2\nu'}$ are referred to a system in which the polar axis is along $\mathbf{k}_f$ and the $y$ axis along $\mathbf{k}_i\times\mathbf{k}_f$.

Then, if the deuterons initially have only vector polarization, and if the nucleons are unpolarized,

$$I_f = I_0(1 + P_1{}^DP_2{}^D\hat{N}_1\cdot\hat{N}_2) \quad (39)$$

and $P_2{}^D$ can be determined from a simple asymmetry experiment. If the deuterons initially have only tensor polarization, then given $\langle T_{20}\rangle_{\mathrm{inc}}$, $\langle T_{21}\rangle_{\mathrm{inc}}$, and $\langle T_{22}\rangle_{\mathrm{inc}}$, measurements at $\varphi = 0$, $\pm\pi/2$, and $\pi$ will yield $\langle T_{20}\rangle_0$, $\langle T_{21}\rangle_0$, and $\langle T_{22}\rangle_0$. The tensor polarization parameters may also be measured by scattering nucleons from polarized deuteron targets, or by scattering unpolarized particles and using the recoil deuterons to initiate an analyzing reaction. Specifically, the $He^3(d,p)He^4$ re-

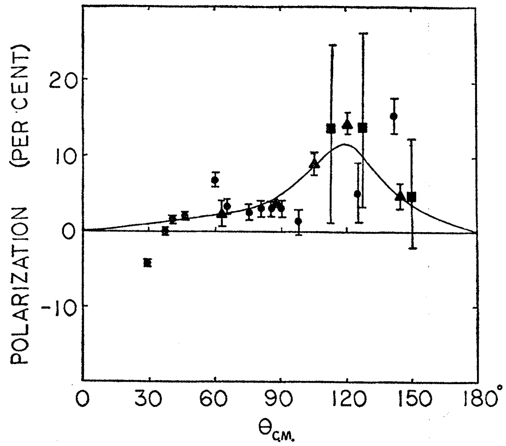


FIG. 4. Calculated neutron polarization in *n-d* scattering at 9.0 MeV. The experimental data are those of Walter and Kelsey (Ref. 2) (■) for 10.0-MeV *n-d* scattering, Grübler *et al.* (Ref. 2) (▲) for 10.0-MeV *p-d* scattering, and McKee *et al.* (Ref. 2) (●) for 11.0-MeV *p-d* scattering.

action has been used[18]; the angular distribution of the protons is then a function of $\langle T_{20}\rangle$, $\langle T_{21}\rangle$, and $\langle T_{22}\rangle$.[19]

## IV. PRELIMINARY CALCULATIONS

Preliminary calculations neglecting $J_3$ have been performed for a neutron laboratory energy of 9.0 MeV. The calculations consist of a five-parameter fit to the experimental angular distributions and polarizations. The parameters are those enumerated in Sec. II. All other eigenphases and mixing parameters are obtained from the Born approximation. The deuteron *D*-state admixture was also varied in the calculations. Some of the numerical results are shown in Figs. 2–4. The agreement between the experimental and theoretical angular distributions is good, as expected, but in addition there is substantial agreement between the measured and calculated polarizations. Both *n-d* and *p-d* polarization measurements are shown. It is to be expected that there will be little difference between the polarization distributions for the two cases except, perhaps, at small angles. The recent *p-d* asymmetry measurements by McKee *et al.* show clearly that the polarization goes negative forward of 30° in the vicinity of 10 MeV. The calculations presented here for 9.0 MeV do not show this feature although an earlier calculation at 10 MeV with $J_1$ simulated by a variation of the *P*-wave parameters does go negative at small angles. This behavior is quite sensitive to small changes in the phase shifts and mixing parameters and can be obtained with a $\chi^2$ only slightly greater than the best fit. Further calculations based on additional data will be required to clarify this point.

In view of the preliminary nature of the calculations and the need for more small-angle measurements, it does not seem appropriate to present here the phase shifts and mixing parameters derived from the best fits. These will appear in a future paper, along with deuteron-polarization predictions.

The calculations were performed on the IBM 7094 machine at the Texas A&M University Data Processing Center and on the IBM 7044 of the Tulane University Computer Center.

[18] P. Young, M. Ivanovich, and G. G. Ohlsen, Phys. Rev. Letters **14**, 831 (1965).

[19] L. G. Pondrum and J. W. Daughtry, Phys. Rev. **121**, 1192 (1961).