PHYSICAL REVIEW VOLUME 167, NUMBER 4 20 MARCH 1968

Cross-Section Measurements for the Nucleon-Transfer Reactions $^{10}B(^{19}F,^{18}F)^{11}B$ and $^{10}B(^{19}F,^{18}O)^{11}C$†

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(Received 9 November 1967)

Thick targets of ^{10}B were bombarded with ^{19}F ions accelerated in the Oak Ridge tandem Van de Graaff, and cross sections were measured for the nucleon-transfer reactions $^{10}B(^{19}F,^{18}F)^{11}B$ and $^{10}B(^{19}F,^{18}O)^{11}C$ from 10.5 to 26.5 MeV. The amount of ^{18}F and ^{11}C present in each irradiated sample was determined by the detection of the 110- and 20.5-min positron activities, respectively, characteristic of the two nuclides. It was found that the cross section for the neutron-transfer reaction was $\sim$2.5 times greater than that of the proton-transfer reaction. This is in contrast to results obtained previously for similar transfer reactions induced by ^{14}N ions on ^{10}B and ^{14}N targets. Reaction cross sections for the incident ^{14}N to transfer a nucleon to the target nuclei ^{10}B and ^{14}N have been measured, and are approximately independent of whether a neutron or a proton is transferred. Previously published data for the reactions $^{14}N(^{14}N,^{13}N)^{15}N$, $^{10}B(^{14}N,^{13}N)^{11}B$, and $^{14}N(^{19}F,^{18}F)^{15}N$, were examined together with the data for the $^{10}B(^{19}F,^{18}F)^{11}B$ reaction. By applying the tunneling theory of Breit *et al.* to these results, two values (one for each target nucleus) were extracted for the ratio of the reduced width of the transferred neutron in ^{19}F to that in ^{14}N. Both ratios were found to be $\sim$3.7. The internal consistency lends encouragement to this method of reduced-width extraction. Graphite disks were bombarded with ^{19}F ions, and the yield of ^{18}F from ^{19}F on carbon in the energy range investigated was found to be neglegible when compared with that measured for ^{10}B targets. Excitation functions were also measured for the compound-nucleus reactions $^{10}B(^{19}F,\alpha p)^{24}Na$, $^{12}C(^{19}F,2p)^{29}Al$, and $^{13}C(^{19}F,2\alpha)^{24}Na$.

I. INTRODUCTION

A PROMISING tool in the field of nuclear spectroscopy related to (d,p) and (p,d) reactions involves the transfer of neutrons between heavy ions. A quantitative description of this process for energies below the Coulomb barrier has been formulated by Breit *et al.*[1] This tunneling theory gives a good fit to angular distributions for transfer reactions that occur at low incident energies. Indeed, this success has led investigators[2,3] to derive neutron reduced widths for the reaction $^{14}N(^{14}N,^{13}N)^{15}N$ by measuring the total cross section as a function of energy, and by assuming that the reduced widths in ^{14}N and ^{15}N are equal. Good agreement was found between the reduced widths extracted in this manner and values based on shell-model calculations.

The tunneling theory has been formulated specifically for the reaction $^{14}N(^{14}N,^{13}N)^{15}N$. Surprising success was attained, however, in the extraction of the neutron reduced width in ^{11}B when the theory was applied[3] to cross-section measurements for the reaction $^{10}B(^{14}N,^{13}N)^{11}B$. The derived reduced width for ^{11}B agreed well with a shell-model calculation that assumed the ^{11}B ground state to be $^{10}B+1p_{3/2}$ neutron. This agreement may have been largely coincidental or may have been observed because the neutron states involved in the two reactions are fairly similar. Further theoretical calculations are required to resolve this point.

It has been suggested[4] that these studies could be extended to other targets and projectiles. One such projectile is ^{19}F, which, when stripped of a neutron, becomes ^{18}F, a nuclide with a convenient half-life for radioactivity studies. These $(^{19}F,^{18}F)$ reactions have Q values which are 0.1 MeV more positive than $(^{14}N,^{13}N)$ reactions induced on the same target nuclei. The direct extension of the tunneling theory to reduced-width determinations for nuclei other than $1p$ nuclei would seem to be unreasonable in the light of theory's limited applicability.[1] Systematic studies of $(^{14}N,^{13}N)$ and $(^{19}F,^{18}F)$ reactions on given targets permit, however, the extraction of ratios of the neutron reduced widths in ^{14}N and ^{19}F if, for a given target, the reduced width in the acceptor nucleus is assumed to be the same for both reactions. This ratio determination has been made[5,6] for six targets by the use of a variety of available total cross-section data. Some internal consistency in these ratios has been found[5,6]; this lends credence to the possible future use of heavy-ion reactions in reduced-width determination. The older available cross-section data have been found[2,3] to be incorrect in some instances; in other cases, measurements have not been done at low incident energies, where the tunneling theory is considered to apply.

The present study was undertaken to provide cross-section measurements for the reaction $^{10}B(^{19}F,^{18}F)^{11}B$ below 18 MeV, the incident ^{19}F energy at which an

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† Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

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[1] G. Breit and M. E. Ebel, Phys. Rev. **103**, 679 (1956); M. E. Ebel, *ibid.* **103**, 958 (1956); G. Breit and M. E. Ebel, *ibid.* **104**, 1030 (1956); G. Breit, in *Handbuch der Physik*, edited by S. Flugge (Springer-Verlag, Berlin, 1959), Vol. 41, Part 1; G. Breit, Phys. Rev. **135**, B1323 (1964); G. Breit, K. W. Chun, and H. G. Wahsweiler, *ibid.* **133**, B403 (1964); G. Breit, in *Proceedings of the Second Conference on Reactions between Complex Nuclei, Gatlinburg, 1960* (John Wiley & Sons, Inc., New York, 1960), p. 1.

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[4] P. J. A. Buttle and L. J. B. Goldfarb, Nucl. Phys. **78**, 409 (1966).

[5] K. S. Toth and E. Newman, Phys. Rev. **130**, 536 (1963).

[6] R. M. Gaedke, K. S. Toth, and I. R. Williams, Phys. Rev. **140**, B296 (1965).

earlier study[7] had been terminated. These data, when combined with recently available results[3,6] for the reactions $^{14}N(^{14}N,^{13}N)^{15}N$, $^{14}N(^{19}F,^{18}F)^{15}N$, and $^{10}B(^{14}N,^{13}N)^{11}B$, permitted the determination of two reduced-width ratios. Excitation functions were also measured for the proton-transfer reaction $^{10}B(^{19}F,^{18}O)^{11}C$ and the compound-nucleus reaction $^{10}B(^{19}F,\alpha p)^{24}Na$. Since carbon is a contaminant frequently found on targets, graphite disks were bombarded with fluorine ions, and the cross section for the reaction $^{12}C(^{19}F,^{18}F)^{13}C$ was measured in the energy range of interest. Excitation functions were determined for the production of ^{29}Al and ^{24}Na from ^{19}F incident on carbon.

II. EXPERIMENTAL TECHNIQUE

The target materials used in this investigation were boron-enriched in ^{10}B (96.5%) and graphite. Boron targets were prepared by compressing the powdered material under a pressure of 5 tons/in.^{2} into brass molds $\frac{3}{4}$ in. in diam. These targets, thicker than the range of the fluorine ions, presented a hard and uniform surface to the incident particles. Carbon targets consisted of graphite discs $\frac{1}{16}$ in. thick. Bombardments were made in a Faraday-cup assembly, and beam currents up to 200 nA were recorded. The energy of the fluorine ions, accelerated in the Oak Ridge tandem Van de Graaff, was varied from 10.0 to 27.0 MeV. While the beam energy was known to ±100 keV, the beam resolution was about 10 keV.

After bombardment, the targets were counted in a fixed geometry in a low-level gas-flow β detector with a background of $\sim$0.25 counts/min. The counts were ordinarily begun 4 or 5 min after bombardment. Decay curves were resolved into their components, and the presence in each target of the various activities of interest was established by the identification of their half-lives. A computer program was used to give least-squares fits to the decay curves and to determine the magnitudes of the various decay-curve components extrapolated to time zero, i.e., to the end of bombardment. The program also supplied the standard deviations for the time-zero magnitudes.

The low-level counter was calibrated absolutely for the particular geometry used by means of a RaDEF source of known strength. The counting efficiency was 22%. Counting rates at time zero, as obtained from the computer fit, were then corrected by this efficiency to obtain the absolute yields per incident particle. The probable error in these yields results mainly from the uncertainty in the counter-efficiency determination, and is estimated to be ±15%.

Smooth curves were drawn through the thick-target yield points, and these curves were then differentiated to obtain the excitation functions. For this determination, the stopping power of the target materials for ^{19}F ions had to be known. It was calculated by using the known stopping power of aluminum for fluorine ions[8] and by assuming that the relative stopping power for protons and ^{19}F ions of the same velocity in a given material is the same. Proton stopping powers for Al, C, and ^{10}B were taken from Allison and Warshaw.[9] Probable errors in the absolute cross sections are estimated to be ±30% and are attributed to errors arising from uncertainties in the counter efficiency, the slope of the yield curves, and the stopping power.

III. RESULTS

A. ^{19}F Incident on ^{10}B

The yields per incident particle as a function of bombarding energy are shown in Fig. 1 for three reactions: $^{10}B(^{19}F,^{18}F)^{11}B$, $^{10}B(^{19}F,^{18}O)^{11}C$, and $^{10}B(^{19}F,\alpha p)^{24}Na$. The production of ^{24}Na is assumed to proceed by the evaporation of an α particle and a proton from the compound nucleus ^{29}Si, because the Q value for this reaction is 9.25 MeV. The Q values for alternate modes of production are extremely negative: -19.0 MeV for the evaporation of five nucleons and -9.12 MeV for the evaporation of a ^3He particle and a deuteron. The threshold energy corresponding to the latter

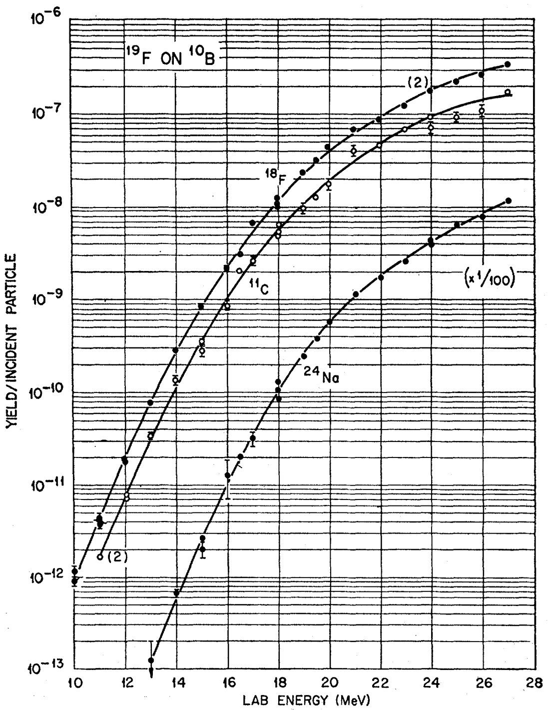

FIG. 1. Yields per incident particle as a function of bombarding energy for reactions induced by ^{19}F incident on ^{10}B. The numbers 2 (enclosed in parentheses) indicate that points next to them represent two measurements. Note that the ^{24}Na data points have been decreased by a factor of 100.

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[7] J. L. Perkin, R. F. Coleman, and D. N. Herbert, Proc. Phys. Soc. (London) **79**, 1033 (1962).

[8] L. C. Northcliffe, Phys. Rev. **120**, 1744 (1960).

[9] S. K. Allison and S. D. Warshaw, Rev. Mod. Phys. **25**, 779 (1953).

Q value is 26.4 MeV, which is near the upper limit of the energy range investigated.

Because of the half-lives involved, the three radioactive products ^{18}F (110 min), ^{11}C (20.5 min), and ^{24}Na (15.0 h) were easily identified when the experimental decay curves were resolved into their various components. Shorter-lived components were present in the decay curves—^{28}Al (2.3 min), ^{27}Mg (9.5 min), and, from carbon contamination, ^{29}Al (6.6 min). Since the products of main interest were ^{11}C and ^{18}F, and because of the difficulty of the task, no attempt was made to determine the amounts of ^{28}Al, ^{27}Mg, and ^{29}Al in each irradiated target. Instead, the decay curves were truncated at 50–60 min after the end of bombardment, and the computer program was then used to fit the remainder of the decay points with the three components ^{11}C, ^{18}F, and ^{24}Na.

Cross-section data for the three reactions are shown in Fig. 2. Our data for the neutron-transfer reaction are compared in Fig. 2 with those of Perkin *et al.*,[7] who investigated the reaction in the energy range 18–43 MeV. The two sets of data are in agreement within experimental errors, although there is substantial deviation between the two excitation functions above 22 MeV. The excitation function of Perkin *et al.*[7] reaches a value of 7 mb at ~30 MeV, however, and levels off at 7.4 mb for energies >32 MeV. As seen in Fig. 2, our excitation function appears to be leveling off at a cross section of about 7.5 mb. The apparent disagreement in the interval 22–30 MeV is probably due to the manner in which smooth curves were drawn through the yield data points measured in the two investigations. Also, it appears from the data presentation of Perkin *et al.*[7] that in the energy range 20–43 MeV their ^{10}B yields were measured at widely spaced (5- or 6-MeV) energy intervals; this would lead to additional uncertainties in their cross-section determination.

For purposes of clarity, the ordinate scale for the ^{24}Na reaction was decreased by a factor of 100 in Fig. 1 and 10 in Fig. 2. This reaction, which presumably proceeds via a compound-nucleus mechanism, has a measurable cross section even at energies far below the Coulomb barrier, which for an r_0 value of 1.5 F is 25.9 MeV. The results do indicate, however, that, as expected, the compound-nucleus excitation function drops off more steeply with decreasing energy than the two transfer excitation functions.

B. ^{19}F Incident on C

Two main activities were observed in the ^{19}F bombardments of carbon: ^{29}Al and ^{24}Na. These nuclides are assumed to be produced from the following evaporation reactions: ^{12}C(^{19}F,2p)^{29}Al and ^{13}C(^{19}F,2α)^{24}Na. (The threshold energy for the production of ^{24}Na from ^{12}C is 27.0 MeV, if it is assumed that an α particle and a ^{3}He particle are evaporated from the compound nucleus.) Above 22 MeV, a small amount of ^{18}F was observed; the threshold energy for the transfer reaction ^{12}C(^{19}F,^{18}F)^{13}C is 14.2 MeV. The Q values for the production of other radioactive nuclides with reasonable half-lives, e.g., ^{13}N (10 min), ^{28}Al, ^{27}Mg, and ^{28}Mg (21.3 h), are quite negative; the lowest threshold energy (for ^{13}N) is 15.7 MeV, so that the yields for these nuclides would be greatly reduced in comparison to those of ^{29}Al and ^{24}Na. The threshold energy for the neutron-transfer reaction ^{12}C(^{19}F,^{20}F)^{11}C is 31.3 MeV.

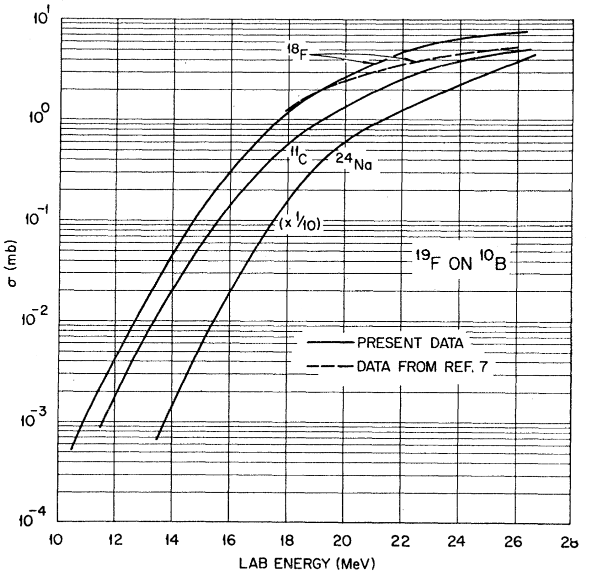

FIG. 2. Excitation functions for reactions induced by ^{19}F incident on ^{10}B. Results from Ref. 7 are indicated by the dashed curve. Note that the ^{24}Na curve has been lowered by a factor of 10.

The yields and cross sections for reactions resulting from ^{19}F on carbon are shown in Figs. 3 and 4, respectively. The point of interest is that the ^{18}F yield from carbon is low in the investigated energy range, and therefore the production of ^{18}F from carbon contamination on the ^{10}B targets need not be considered.

IV. DISCUSSION

In the investigated energy range, the cross section for the neutron-transfer reaction ^{10}B(^{19}F,^{18}F)^{11}B is about 2.5 times greater than that for the proton-transfer reaction ^{10}B(^{19}F,^{18}O)^{11}C. Such is not the case for the corresponding transfer reactions induced by ^{14}N ions incident on ^{10}B. Cross sections for the reactions ^{10}B(^{14}N,^{13}N)^{11}B and ^{10}B(^{14}N,^{13}C)^{11}C have been measured[3] to be roughly equal in the laboratory energy range 8.8–17.0 MeV. The same result has been noted for the reactions ^{14}N(^{14}N,^{13}N)^{15}N and ^{14}N(^{14}N,^{13}C)^{15}O. There again, for ^{14}N incident energies 12.4–20.0 MeV, cross sections for the two transfer reactions have been measured[3] to be about the same. These results are illustrated in Fig. 5, where we have plotted, for the three pairs of reactions under consideration, the ratio (neutron-to-proton transfer) of cross sections as a

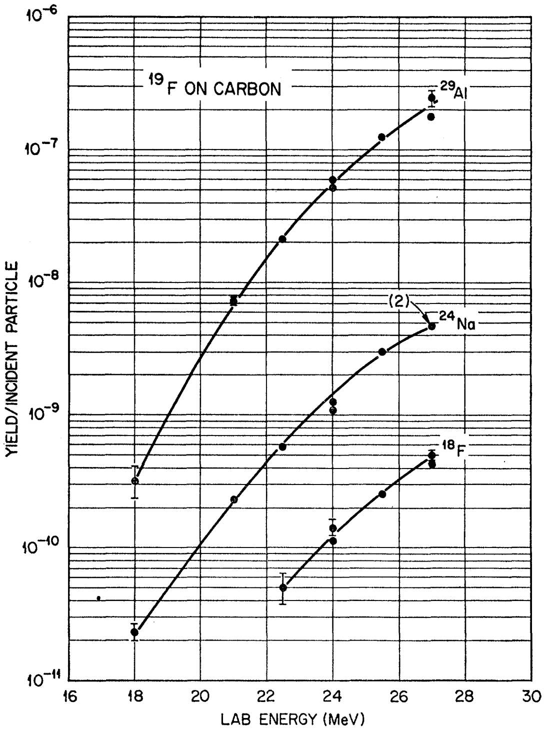

FIG. 3. Yields per incident particle as a function of bombarding energy for reactions induced by ^{19}F incident on carbon. The datum point indicated by an arrow represents two measurements.

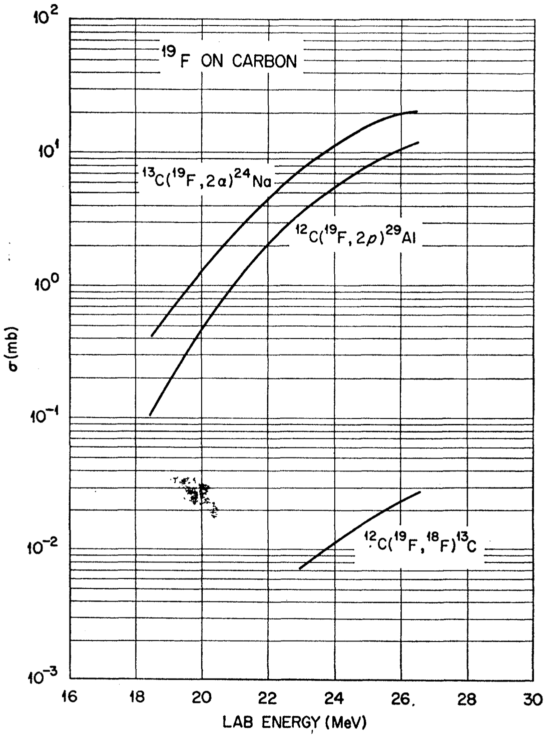

FIG. 4. Excitation functions for reactions induced by ^{19}F incident on carbon. Note that (for reasons discussed in the text) the ^{24}Na is assumed to result from reactions with ^{13}C and not ^{12}C.

function of laboratory energy. The averaged ratios, shown in Fig. 5 by the solid lines, are 2.5 for $^{19}F+^{10}B$, 1.2 for $^{14}N+^{14}N$, and 1.0 for ^{14}N on ^{10}B. The quantities plotted along the ordinate scale are actually ratios of differences between experimentally measured thick-target yields. These differences $\Delta Y/\Delta E$ are in effect cross sections, because the remaining quantities that enter into the calculation of cross sections are the same for each pair of reactions. What has been eliminated is the bias inherent in drawing a smooth curve through the yield points. The scatter in the plotted ratios is a reflection of the fact that the smoothing procedure has been dispensed with.

The above results may stem from the fact that ^{19}F has two neutrons available for transfer but only one proton in the *s-d* shell. In the nitrogen-induced reactions, ^{14}N has one $p_{1/2}$ neutron and one $p_{1/2}$ proton available for transfer. Another possible explanation is that in the nitrogen reactions we are dealing with pairs of nucleon states that are fairly similar; this is not the case for the ^{19}F reaction, where ^{18}F and ^{18}O have different values of spin and isospin.

The results also seem to be in line with the work of Volkov and Wilczynski,[10] who examined systematically the difference between neutron and proton transfers at high incident energies. For targets ranging from ^{27}Al to ^{181}Ta, they found that proton-transfer cross sections decreased rapidly with increasing atomic number of the target. They postulated[10] that this dependence (not observed for neutron-transfer cross sections) might be due to the Coulomb interaction in the proton-transfer process. The interaction could cause a large polarization of the proton wave function in the final nucleus, which in turn may result in a significant decrease of the transition matrix element. Whether the results at high energies are related to our own findings is not clear.

The tunneling theory of Breit and collaborators[1] predicts the variation of neutron-transfer cross sections with energy and the shape of the angular distributions of such reactions. While the theory is specialized to the $^{14}N(^{14}N,^{13}N)^{15}N$ reaction, it has been used[3] to derive the reduced width associated with the neutron transferred to ^{11}B in the reaction $^{10}B(^{14}N,^{13}N)^{11}B$. The reduced-width value determined in this manner agreed well with a value calculated from a bound-state single-particle radial wave function which assumed that the ^{11}B ground state can be represented as $^{10}B+1p_{3/2}$ neutron. Breit and co-workers[1] have cautioned against literal acceptance of the tunneling theory in the extraction of reduced widths. The extension of the use of this theory to reduced-width determinations for nuclei other than $1p$ nuclei would seem to be even more unreasonable; this would be especially so for a nucleus such as ^{19}F, which appears to be a complicated admixture of shell-model states. The motivation for this and the similar investigation in the past[3] was to measure accurate cross

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[10] V. V. Volkov and J. Wilczynski, Nucl. Phys. **A92**, 495 (1967).

sections for single-nucleon (both proton and neutron) transfer reactions far below the Coulomb barrier, in the hope that these measurements might stimulate the formulation of theories applicable to reactions other than ^{14}N(^{14}N,^{13}N)^{15}N. Another motivation for this particular investigation was to continue systematic studies of (^{14}N,^{13}N) and (^{19}F,^{18}F) reactions. Here the examination of both reactions on given targets permits cancellation of the neutron reduced width in the nucleus to which the neutron is transferred, and thus leads to ratios of the reduced widths in ^{14}N and ^{19}F. Such determinations[5,6] may not have been completely accurate for three reasons: (a) wide energy spread[3] in the cyclotron beam in the case of ^{14}N-induced reactions, (b) low beam intensity[7] in the case of ^{19}F-induced reactions, and (c) measurements in many instances performed not far below the Coulomb barrier. With the results presented here, there are now available two pairs of (^{14}N,^{13}N) and (^{19}F,^{18}F) reactions, for which reliable data exist at energies sufficiently below the Coulomb barrier. The remainder of the discussion will be devoted to the determination of these two reduced-width ratios.

The semiclassical tunneling theory[1] predicts the variation of total cross section σ with energy to be

$$\sigma=\tfrac{1}{8}\Lambda^2\frac{1}{\alpha\bar{\alpha}\lambda\lambda'}\left(\frac{\alpha b_1}{1+\alpha b_1}\right)^2\left(\frac{\bar{\alpha}b_2}{1+\bar{\alpha}b_2}\right)^2\exp X\,, \qquad (1)$$

where

$$X=[(2M)^{1/2}/\hbar]\{E_s{}^{1/2}(b_1+b_2)[1-(E_B/E)] +\bar{E}_s{}^{1/2}(\bar{b}_1+\bar{b}_2)[1-(\bar{E}_B/\bar{E})]\}\,.$$

Here b_1 and b_2 are the radii of initial and final nuclei 1 and 2, respectively, $\Lambda=h/Mv$ is the wavelength of the transferred neutron, v is the relative velocity, E_s is the binding energy of the neutron, $\alpha=(2ME_s/\hbar^2)^{1/2}$, M is the neutron mass, $E_B=Z_1Z_2e^2/r_0(A_1{}^{1/3}+A_2{}^{1/3})$ is the Coulomb barrier, E is the c.m. energy, and unbarred and barred quantities refer to initial and final systems, respectively.

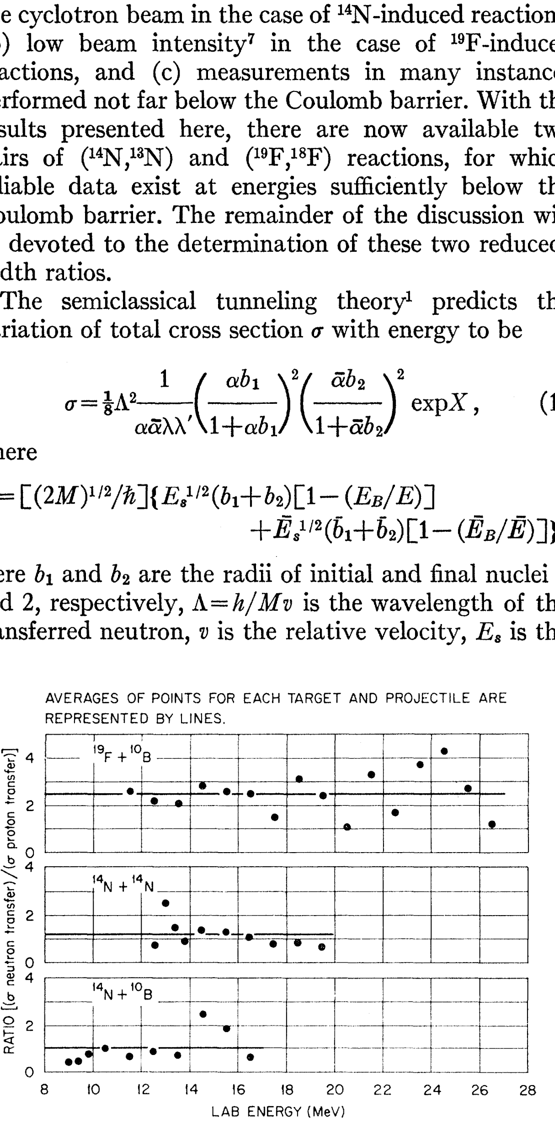

FIG. 5. Ratios (neutron transfer to proton transfer) of reaction cross sections as a function of laboratory energy. The three pairs of reactions under consideration, from top to bottom, are (1) ^{10}B(^{19}F,^{18}F)^{11}B and ^{10}B(^{19}F,^{18}O)^{11}C; (2) ^{14}N(^{14}N,^{13}N)^{15}N and ^{14}N(^{14}N,^{13}C)^{15}O; and (3) ^{10}B(^{14}N,^{13}N)^{11}B and ^{10}B(^{14}N,^{13}C)^{11}C. Quantities plotted along the ordinate scale are actually ratios of differences between experimentally measured thick-target yields.

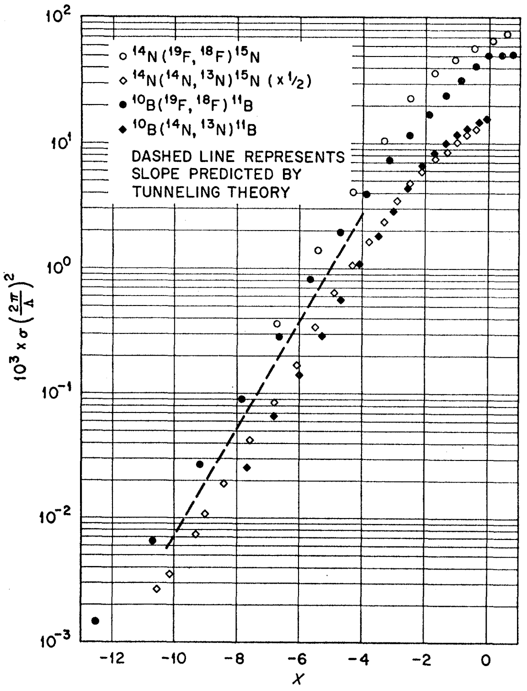

FIG. 6. Cross-section data for four single-neutron-transfer reactions replotted as $4\pi^2\sigma/\Lambda^2$ versus X; $\Lambda=h/Mv$=wavelength of the transferred neutron; X is defined in Eq. (1). For the four reactions under consideration, the tunneling theory is applicable for values of X less than about zero.

Besides the various kinematical factors, the cross-section expression [Eq. (1)] contains the product of the reduced widths in the two participating nuclei $\lambda\lambda'$, where λ refers to the nucleus donating the neutron, and λ' refers to the nucleus which has accepted the neutron. The probability of finding a neutron in a shell of unit thickness around one of these nuclei is proportional to $1/\lambda$; the quantity $1/\lambda$ is defined[1] as

$$1/\lambda=r^2R^2(r)\,. \qquad (2)$$

Here r is the nuclear radius and $R(r)$ is the radial wave function of the neutron. The kinematical factors in Eq. (1) are calculable; therefore, the reduced-width product can be determined if the cross section is known.

As first suggested by Breit,[1] the excitation functions for the (^{14}N,^{13}N) and (^{19}F,^{18}F) reactions on ^{10}B and ^{14}N targets are replotted as $\log[\sigma(2\pi/\Lambda)^2(10^3)]$ versus X in Fig. 6 to facilitate comparison with the tunneling theory. In the region where the tunneling theory is applicable ($E<E_B$ and $\bar{E}=E+Q<\bar{E}_B$), it predicts that X should change by a factor of ln10 for an order-of-magnitude change in $\sigma(2\pi/\Lambda)^2$. The dashed line in Fig. 6 indicates the predicted slope. Since an r_0 value of 1.5 F was used to calculate the Coulomb barrier, then, for the four reactions of interest, the region of applicability corresponds to X values less than about zero. As seen from Fig. 6, the agreement of the four excitation functions with the predicted slope is good for $X<-3$. Above this energy, the experimental data begin to fall below the theoretical slope. This deviation is presumably due to the effects of nuclear absorption. As the energy is increased, a greater number of incident particles are absorbed by the target nuclei; this results in a greater probability for compound-nucleus reactions and a probability for transfer which is less than that predicted by the tunneling theory, since the latter does not take nuclear absorption into account and assumes that no competing reactions are occurring. The absolute cross section for the ^{14}N on ^{14}N reaction (as displayed in Fig. 6) was decreased by a factor of 2. This is necessary because the theory was formulated to explain the identical-partical reaction ^{14}N(^{14}N,^{13}N)^{15}N; therefore, when reactions with nonidentical target and projectile nuclei are discussed, either their cross sections must be multiplied by 2 or that of the ^{14}N on ^{14}N reaction must be halved.

The reduced-width products for the four reactions were determined at $X=-6$. Since the excitation functions are essentially parallel below $X=-3$, a determination of reduced-width ratios could have been done at any value of X less than -3. The lower limit of X is set by the fact that the ^{19}F on ^{14}N reaction cross section was not measured below an energy corresponding to $X=-6.7$. For each target, e.g., ^{10}B, two reduced-width products can be calculated, $\lambda_{^{14}\mathrm{N}}\lambda_{^{11}\mathrm{B}}$ and $\lambda_{^{19}\mathrm{F}}\lambda_{^{11}\mathrm{B}}$. If it is assumed that $\lambda_{^{11}\mathrm{B}}$ is identical for the nitrogen and fluorine reactions, then the ratio $\lambda_{^{14}\mathrm{N}}/\lambda_{^{19}\mathrm{F}}$ is determined. Each target nucleus then yields an independent value of the ratio, and the similarity of the ratios serves as a consistency check of the tunneling theory. The two $(\lambda_{^{14}\mathrm{N}}/\lambda_{^{19}\mathrm{F}})$ ratios were found to be 3.67 for the pair ^{14}N on ^{14}N and ^{19}F on ^{14}N, and 3.65 for the pair ^{14}N on ^{10}B and ^{19}F on ^{10}B. The ratios are essentially equal, i.e., $\sim$3.7, and the internal consistency lends encouragement to the use of heavy-ion transfer reactions as a possible tool for the extraction of nucleon reduced widths.

The same two ratios had been determined previously[6] to be 4.6 (^{14}N target) and 1.4 (^{10}B target). Three points should be made at this time. First, the earlier determinations[6] were made at a value of $X=-1.6$, which corresponds to an energy close to the Coulomb barrier. As seen from Fig. 6, the excitation functions begin to deviate from the theoretical slope at $\sim X=-3$. A value of $X=-1.6$ had been chosen because a total of six ratios (the other target nuclei were ^{23}Na, ^{27}Al, ^{51}V, and ^{55}Mn) were evaluated, and some of the cross-section measurements had not been done at low enough bombarding energies. (The ratios must of necessity be calculated at the same value of X.) Second, the cross-section measurements[11,12] for the reactions ^{14}N(^{14}N,^{13}N)-^{15}N and ^{10}B(^{14}N,^{13}N)^{11}B used in the previous paper[6] have been shown,[3] subsequently, to be incorrect. The data utilized in the present investigation for these two particular reactions have been verified[13,14] by a thin-target technique. While we have already compared our ^{10}B(^{19}F,^{18}F)^{11}B data with those of Perkin *et al.*,[7] it would be interesting to have another check of the reaction cross section below 18 MeV. There are two sets of results available for the reaction ^{14}N(^{19}F,^{18}F)^{15}N, but it appears that the earlier[7] of the two reported cross-section measurements is in error by a factor of about 10. Our own results[6] were used in the present determination of reduced-width ratios, but here again it would be of value to have another measurement of the reaction cross section. Third, the error introduced when taking a ratio of two cross sections (good to within $\pm 30\%$) could be as high as 86%. Taken in this light, the earlier[6] and the present ratios are not in disagreement. The present results are probably more reliable because (a) the data used in the calculations were more accurate than those used previously, (b) the ratio calculations were made at energies far below the Coulomb barrier, where the tunneling theory is expected to apply, and (c) the calculations were made by using only our experimental results, so that some systematic errors in counter calibrations, stopping powers, etc., may actually have been eliminated when the cross-section ratios were determined.

These consistency checks should be made for other target nuclei, such as the four mentioned above. Earlier cross-section measurements cannot be used in these ratio determinations; they should be repeated and extended to lower bombarding energies.

ACKNOWLEDGMENTS

The cooperation of the tandem Van de Graaff accelerator crew is gratefully acknowledged. The authors would also like to thank Jerrall Glenn for his assistance in the analysis of some of the experimental data.

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