Reflection and Transmission of Electromagnetic Waves by a Moving Dielectric Slab. II. Parallel Polarization*

C. YEH
Department of Engineering, University of California, Los Angeles, California 90024
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The reflection and transmission of a plane wave, with its electric vector polarized in the plane of incidence, by a moving dielectric slab are investigated theoretically. Two cases of the movement are considered: (a) the dielectric slab moves parallel to the interface, (b) the dielectric slab moves perpendicular to the interface. It is shown that, in general, the reflection and transmission coefficients for an incident plane wave with its electric vector polarized in the plane of incidence are different from those for an incident plane wave with its electric vector polarized normal to the plane of incidence, except for case (b) for normally incident waves. Detailed results on the reflection and transmission coefficients for case (a) for normally incident waves are given and discussed.

IN a previous article,[1] the problem of the reflection and transmission of a plane electromagnetic wave by a moving dielectric slab was considered. Various interesting features concerning the variation of the reflection and transmission coefficients, the angles of reflection and transmission, and the frequencies of the reflected and transmitted wave as a function of the velocity of the moving medium, were observed. However, only the case in which the electric vector of the incident wave is polarized normal to the plane of incidence (perpendicular polarization) was considered. The purpose of this work is to present the solution for the other polarization; i.e., the case in which the electric vector of the incident wave is polarized in the plane of incidence (parallel polarization) will be considered. It is found that the reflection and transmission coefficients are significantly different for the two polarizations.

A harmonic plane wave in the free-space regions with its electric vector polarized in the plane of incidence is assumed to be incident upon a moving dielectric slab of thickness d. (See Fig. 1 in I.) In the observer's system S the incident plane wave is

$$H_y^{(i)}=H_0e^{i(k_xx-k_zz)}e^{-i\omega t}, \tag{1}$$

$$D_y^{(i)}=0, \tag{2}$$

where H_0 and ω are, respectively, the amplitude and the frequency of the incident wave, $k_x=k_0\sin\theta_0$, $k_z=k_0\cos\theta_0$, and $k_0=\omega(\mu_0\epsilon_0)^{1/2}$. θ_0 is the angle between the propagation vector and the positive z axis in the x-z plane. The reflected wave and the transmitted wave, in the observer's system S, take the following forms:

For the reflected wave,

$$H_y^{(r)}=A_re^{i[k_x^{(r)}x+k_z^{(r)}z]}e^{-i\omega^{(r)}t}, \tag{3}$$

$$D_y^{(r)}=0; \tag{4}$$

for the transmitted wave

$$H_y^{(t)}=G_te^{i[k_x^{(t)}x-k_z^{(t)}z]}e^{-i\omega^{(t)}t}, \tag{5}$$

$$D_y^{(t)}=0. \tag{6}$$

The values of A_r and G_t are given later. It can be shown that $k_x^{(r)}$, $k_x^{(t)}$, $k_z^{(r)}$, $k_z^{(t)}$, $\omega^{(r)}$, and $\omega^{(t)}$ are the same as those given in I. In other words, the angle of reflection, the angle of transmission, and the frequencies of the reflected and transmitted waves are the same for both polarizations. Making use of the principle of phase invariance of plane waves, the covariance of Maxwell's equations, and the Lorentz transformation,

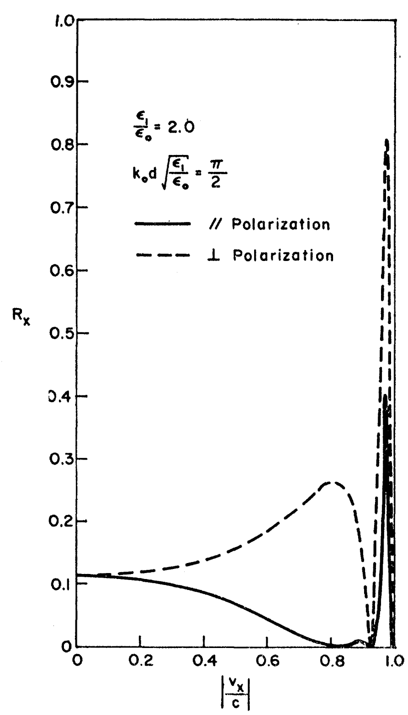

FIG. 1. The reflection coefficients R_x as functions of $|v_x/c|$ for normal incidence. (Note that $T_x=1-R_x$.)

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[1] C. Yeh and K. F. Casey, Phys. Rev. 144, 665 (1966); hereafter referred to as I.

and satisfying the boundary conditions, one obtains the following relations[1,2]:

(a) If the slab is moving uniformly with a velocity v_x in the positive x direction,

$$A_r=H_0\frac{ie^{-2ik_0d\cos\theta_0}\{[\eta_x(\epsilon_0/\epsilon_1)]^2-\cos^2\theta_0\}\sin(k_0d\eta_x)}{2(\epsilon_0/\epsilon_1)\eta_x\cos\theta_0\cos(k_0d\eta_x)-i\{[(\epsilon_0/\epsilon_1)\eta_x]^2+\cos^2\theta_0\}\sin(k_0d\eta_x)},\tag{7a}$$

$$G_t=H_0\frac{2(\epsilon_0/\epsilon_1)\eta_x\cos\theta_0e^{-ik_0d\cos\theta_0}}{2(\epsilon_0/\epsilon_1)\eta_x\cos\theta_0\cos(k_0d\eta_x)-i\{[(\epsilon_0/\epsilon_1)\eta_x]^2+\cos^2\theta_0\}\sin(k_0d\eta_x)},\tag{7b}$$

with

$$\begin{aligned}\eta_x&=\gamma_x[(1-\beta_x\sin\theta_0)^2(\epsilon_1/\epsilon_0)-(\sin\theta_0-\beta_x)^2]^{1/2},\\ \gamma_x&=(1-\beta_x^2)^{-1/2},\\ \beta_x&=v_x/c,\\ c&=\text{the velocity of light in vacuum.}\end{aligned}\tag{7c}$$

(b) If the slab is moving uniformly with a velocity v_z in the positive z direction,

$$A_r=H_0\frac{i\gamma_z^2(1+2\beta_z\cos\theta_0+\beta_z^2)e^{-2ik_0d\gamma_z(\cos\theta_0+\beta_z)}\{[\eta_z(\epsilon_0/\epsilon_1)]^2-\gamma_z^2(\cos\theta_0+\beta_z)^2\}\sin(k_0d\eta_z)}{2(\epsilon_0/\epsilon_1)\eta_z\gamma_z(\cos\theta_0+\beta_z)\cos(\eta_zk_0d)-i\{[\eta_z(\epsilon_0/\epsilon_1)]^2+\gamma_z^2(\cos\theta_0+\beta_z)^2\}\sin(k_0d\eta_z)},\tag{8a}$$

$$G_t=H_0\frac{2(\epsilon_0/\epsilon_1)\eta_z\gamma_z(\cos\theta_0+\beta_z)e^{-ik_0d\gamma_z(\cos\theta_0+\beta_z)}}{2(\epsilon_0/\epsilon_1)\eta_z\gamma_z(\cos\theta_0+\beta_z)\cos(\eta_zk_0d)-i\{[\eta_z(\epsilon_0/\epsilon_1)]^2+\gamma_z^2(\cos\theta_0+\beta_z)^2\}\sin(k_0d\eta_z)},\tag{8b}$$

with

$$\eta_z=[\gamma_z^2(\epsilon_1/\epsilon_0)(1+\beta_z\cos\theta_0)^2-\sin^2\theta_0]^{1/2},\quad \gamma_z=(1-\beta_z^2)^{-1/2},\quad \beta_z=v_z/c.\tag{8c}$$

It is noted that the coefficients of the reflected and transmitted waves are significantly different for the two different polarizations of an incident plane wave.

The reflection and transmission coefficients for the parallel polarization cases are, respectively,

$$R_{x,z}=(A_rA_r^*/H_0^2)p_{xz}\tag{9}$$

and

$$T_{x,z}=G_tG_t^*/H_0^2,\tag{10}$$

where A_r and G_t are given by Eq. (7) and $p_x=1$ when the dielectric slab is moving in the x direction; and A_r and G_t are given by Eq. (8) and

$$p_z=\frac{2\beta_z+\cos\theta_0(1+\beta_z^2)}{[(1+\beta_z^2)+2\beta_z\cos\theta_0]\cos\theta_0},$$

when the dielectric slab is moving uniformly in the positive z direction. Simplifying Eqs. (9) and (10), one has for $\mathbf{v}=v_x\mathbf{e}_x$

$$R_x=\frac{\{[\eta_x(\epsilon_0/\epsilon_1)]-\cos^2\theta_0\}^2\sin^2(k_0d\eta_x)}{4[(\epsilon_0/\epsilon_1)\eta_x]^2\cos^2\theta_0\cos^2(k_0d\eta_x)+\{[(\epsilon_0/\epsilon_1)\eta_x]^2+\cos^2\theta_0\}^2\sin^2(\eta_xk_0d)},\tag{11}$$

$$T_x=1-R_x,\tag{12}$$

and for $\mathbf{v}=v_z\mathbf{e}_z$

$$R_z=\chi\frac{\gamma_z^4[1+2\beta_z\cos\theta_0+\beta_z^2]^2\{[\eta_z(\epsilon_0/\epsilon_1)]^2-\gamma_z^2(\cos\theta_0+\beta_z)^2\}^2\sin^2(k_0d\eta_z)}{4[(\epsilon_0/\epsilon_1)\eta_z]^2\gamma_z^2(\cos\theta_0+\beta_z)^2\cos^2(\eta_zk_0d)+\{[\eta_z(\epsilon_0/\epsilon_1)]^2+\gamma_z^2(\cos\theta_0+\beta_z)^2\}^2\sin^2(\beta_zk_0d)},\tag{13}$$

$$T_z=\frac{4(\epsilon_0/\epsilon_1)^2\eta_z^2\gamma_z^2(\cos\theta_0+\beta_z)^2}{4[(\epsilon_0/\epsilon_1)\eta_z]^2\gamma_z^2(\cos\theta_0+\beta_z)^2\cos^2(\eta_zk_0d)+\{[\eta_z(\epsilon_0/\epsilon_1)]^2+\gamma_z^2(\cos\theta_0+\theta_z)^2\}^2\sin^2(\eta_zk_0d)},\tag{14}$$

$$\chi=\frac{\gamma_z^2[2\beta_z+\cos\theta_0(1+\beta_z^2)]}{\cos\theta_0\{\gamma_z^4[2\beta_z+\cos\theta_0(1+\beta_z^2)]^2+\sin^2\theta_0\}^{1/2}}.\tag{15}$$

[2] C. Möller, *The Theory of Relativity* (Oxford University Press, London, 1957).

To have a qualitative idea of how the reflection and transmission coefficients vary as a function of the velocity of the moving medium, we shall consider the limiting case of normal incidence. At normal incidence, i.e., $\theta_0=0$, Eqs. (11–14) reduce to

$$R_x=\left(\frac{\epsilon_0}{\epsilon_1}\right)^2\gamma_x^2\frac{[(\epsilon_1/\epsilon_0)-1]^2\{\beta_x^2[(\epsilon_1/\epsilon_0)+1]^2-(\epsilon_1/\epsilon_0)\}^2}{4[(\epsilon_1/\epsilon_0)-\beta_x^2]\cos^2(k_0d\eta_x^0)+(\epsilon_0/\epsilon_1)^2\gamma_x^2\{\epsilon_1/\epsilon_0[1+(\epsilon_1/\epsilon_0)]-\beta_x^2[1+(\epsilon_1/\epsilon_0)^2]\}^2}. \tag{16}$$

$$T_x=1-R_x, \tag{17}$$

$$R_z=\left(\frac{1+\beta_z}{1-\beta_z}\right)^2\frac{[(\epsilon_0/\epsilon_1)-1]^2\sin^2(k_0d\eta_z^0)}{4(\epsilon_0/\epsilon_1)\cos^2(k_0d\eta_z^0)+[(\epsilon_0/\epsilon_1)+1]^2\sin^2(k_0d\eta_z^0)}, \tag{18}$$

$$T_z=\frac{4(\epsilon_0/\epsilon_1)}{4(\epsilon_0/\epsilon_1)\cos^2(k_0d\eta_z^0)+[(\epsilon_0/\epsilon_1)+1]^2\sin^2(k_0d\eta_z^0)}, \tag{19}$$

with

$$\eta_x^0=\left[\frac{(\epsilon_1/\epsilon_0)-\beta_x^2}{1-\beta_x^2}\right]^{1/2}; \tag{20}$$

$$\eta_z^0=\left[\frac{\epsilon_1}{\epsilon_0}\left(\frac{1+\beta_z}{1-\beta_z}\right)\right]. \tag{21}$$

It is interesting to note that when the dielectric slab is moving in the z direction the reflection and transmission coefficients for a normally incident wave with parallel polarization are identical to those for a normally incident wave with perpendicular polarization [i.e., Eq. (18) is the same as Eq. (41) in I and Eq. (19) is the same as Eq. (42) in I].[3] On the other hand, when the dielectric slab is moving in the x direction the reflection and transmission coefficients for a normally incident wave are quite different for the two different polarizations.

Equation (15) is plotted in Fig. 1. The reflection coefficient is plotted as a function of the velocity of the moving slab. It is assumed that $\epsilon_1/\epsilon_0=2.0$ and $k_0d(\epsilon_1/\epsilon_0)^{1/2}=\frac{1}{2}\pi$. It can be seen from Fig. 1 that as β_x increases, the reflection coefficient for the parallel polarization case decreases monotonically until

$$\beta_x=\left[\frac{\epsilon_1/\epsilon_0}{1+(\epsilon_1/\epsilon_0)}\right]^{1/2};$$

at this velocity the reflection coefficient is zero and the transmission coefficient is unity. As β_x increases further, the oscillatory behavior of the reflection coefficient can be observed. This is because of the change of the electrical thickness of the slab as β_x varies. The reflection coefficient becomes zero at

$$\beta_x=\left[\frac{(n\pi/k_0d)^2-\epsilon_1/\epsilon_0}{(n\pi/k_0d)^2-1}\right]^{1/2}$$

for integer values of n and for $\beta_x<1$. At $\beta_x=1$, the reflection coefficient is unity, i.e., all the incident energy is reflected. For the sake of comparison, the reflection coefficient for the perpendicular polarization case is also plotted as a function of β_x in Fig. 1.

In conclusion, one observes that the characteristics of the reflection and transmission coefficients for an incident plane wave with its electric vector polarized in the plane of incidence, as a function of the velocity of the slab, are significantly different from those for an incident plane wave with its electric vector polarized normal to the plane of incidence. Even for normally incident plane waves, the reflection coefficients for the two different polarizations are different except when the slab is moving in a direction which is normal to the interface.

[3] Note that the numerator of Eqs. (39) and (40) in Ref. 1 should be multiplied by $(\epsilon_1/\epsilon_0-1)^2$, and that the right-hand side of Eq. (38) in I should be multiplied by γ_z^2.