Theory of Avalanche Breakdown in InSb and InAs

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We have constructed a theory of avalanche breakdown for polar semiconductors such as InSb and InAs in which the primary electron-scattering mechanism is polar-optical-mode scattering. Because of the anisotropy of polar scattering, previous theories of avalanche breakdown involving the assumptions of an isotropic scattering probability and/or a nearly isotropic electron distribution should not be appropriate for polar semiconductors. We have assumed a very anisotropic electron distribution which is narrowly drawn out in the direction of the electric field. We also distinguish between small- and large-angle scattering, since for polar scattering, the probability of scattering to a state close to the initial state is much greater than the probability for scattering through a large angle, although the latter process results in a greater loss of energy, since it places the electron in a state in which it is decelerated by the field. The electron distribution function is obtained analytically, and from it the pair-generation rate and electron drift velocity are calculated for InSb and InAs. The pair-generation rate for InSb is obtained using the ionicity which best fits the low-field-mobility data and a hyperbolic conduction band, and it agrees very well with experimental data on the generation rate in InSb. The calculated drift velocity at fields above 200 V/cm is also in good agreement with the drift-velocity measurements by Glicksman and Hicenbothem, showing that the electron distribution is, indeed, highly anisotropic.

INTRODUCTION

IT has been known for some time[1,2] that avalanche breakdown occurs in InSb at rather low electric fields. The breakdown field of 200–300 V/cm is approximately three orders of magnitude smaller than values of the breakdown field encountered in Si and Ge; however, because of its low effective mass and small energy gap a low breakdown field is not entirely unexpected.

There are several theories which could be applied to the problem of breakdown in InSb. Among these is the theory of Stratton,[3] who calculates the electron-distribution function of electric field in a polar semiconductor with a simple constant electron effective mass. Stratton assumes that the density of electrons is sufficient to insure that electron-electron scattering will maintain the electrons in equilibrium with each other in a quasi-Maxwellian distribution which is only slightly anisotropic. This assumption both facilitates the solution of the Boltzmann equation and produces a curious type of cooperative breakdown. With increasing field and electron temperature, the electron distribution picks up energy from the field at an increasing rate but is not able to dissipate it as rapidly, resulting in a sudden breakdown at a critical electric field.

Objections to the use of Stratton's theory to describe the electron distribution in InSb include (1) the rather strong dependence of effective mass on electron energy in the conduction band which is much more nearly hyperbolic than parabolic, (2) the rather high electron concentration required (10^{16}–10^{17} cm^{-3}) to obtain dominant electron-electron scattering, and (3) the high degree of anisotropy of the electron distribution as is indicated by the electron drift velocity near breakdown.

Other treatments of electron generation such as those of Baraff[4] and of Keldysh,[5] although they no longer use the assumption of a largely isotropic electron distribution, find that the solution of the electron distribution is feasible only for isotropic scattering mechanisms. For polar scattering, the scattering probability between any two states[6] can be accurately given in terms of the ionicity of the lattice, and this scattering probability is highly anisotropic. There is no unique way in which to relate such an anisotropic scattering mechanism to an isotropic mechanism and Baraff is forced to introduce, as an additional parameter, the scattering mean free path, which he assumes to be independent of energy. Like Stratton, Baraff assumes parabolic energy bands.

We have constructed a theory of avalanche breakdown which should be appropriate for n-type InSb and InAs. We assume that there is in the conduction band a hyperbolic dependence electron energy on momentum as is suggested by the calculations of Kane[7] for InSb. We also have taken into account the anisotropic nature of polar scattering, for which small angle scattering is dominant. Although we have no undetermined parameters to adjust, as in any theory of avalanche breakdown, there are several assumptions which are necessary in order to simplify our calculations, and these will be discussed shortly.

ELECTRON-PHONON SCATTERING

In InSb of reasonable purity, the only important scattering mechanism for energetic carriers is polar scattering. Unless the impurity or carrier concentrations are rather high ($>10^{16}$ cm^3), impurity scattering and electron-electron scattering are relatively infrequent.

[1] M. C. Steele and M. Glicksman, J. Phys. Chem. Solids 8, 242 (1959).
[2] A. C. Prior, J. Electron. Control 4, 165 (1958).
[3] R. Stratton, Proc. Roy. Soc. (London) A242, 355 (1957).
[4] G. A. Baraff, Phys. Rev. 128, 2507 (1962).
[5] L. V. Keldysh, Zh. Eksperim. i Teor. Fiz. 48, 1692 (1965) [English transl.: Soviet Phys.—JETP 21, 1135 (1965)].
[6] H. Frohlich, Advan. Phys. 3, 325 (1954).
[7] E. O. Kane, J. Phys. Chem. Solids 1, 249 (1957).

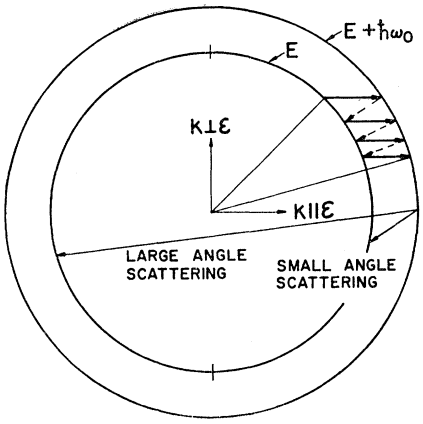

FIG. 1. Schematic representation of focusing effect on distribution of electric field. An electron is accelerated in **k** space in the direction of the field **E**, but scatters between energy surfaces with a probability which is symmetric about the radial to the initial state. For the purpose of illustration, we have assumed that the scattering takes place when the electron reaches the upper surface and that the scattering occurs to the center of the distribution of scattering probability.

Partially because of the low effective mass and, therefore, low density of conduction-band states, acoustic-mode scattering in InSb is also rather improbable. Estimated acoustic-mode scattering times are approximately two orders of magnitude longer than those for optical-mode scattering.[8]

The probability for polar scattering, with the spontaneous emission of a phonon of energy $\hbar\omega_0$, between states with wave number **k** and **k'** in a simple conduction band, is given by[6]

$$P_{\mathbf{k}\mathbf{k}'}=A\frac{\delta(E'+\hbar\omega_0-E)}{|\mathbf{k}-\mathbf{k}'|^2}. \quad (1)$$

Here, A is given in terms of the effective ionic charge for polar scattering e^*, the reduced mass M_R of the atoms in each unit cell, and the volume of a unit cell Ω by

$$A=\frac{16\pi^3 e^{*2}e^2}{M_R\omega_0\Omega}.$$

The $|\mathbf{k}-\mathbf{k}'|^2$ term in the probability causes small-angle scattering to be strongly favored over large-angle scattering, and at moderately high electron energies it is possible for the scattering probability to vary over almost three orders of magnitude. As a result, it is possible for an electron to undergo tens of scattering events without significantly changing its direction in **k** space. In a strong electric field, the energy gained from the field will at least partially be dissipated in the emission of optical-mode phonons, but after each collision the electron will continue to move in the direction of, and continue to gain energy from, the field. The electric field itself has a tendency to focus the electron distribution in **k** space along the direction of the field. This is illustrated in Fig. 1 in which an electron initially away from the direction of the field gradually approaches this direction. For the sake of this illustration it is assumed that the electron is scattered when it reaches the upper energy surfaces to the average or most probable state on the lower-energy surface. The focusing effect of the field helps to counteract the tendency of the carriers to diffuse in **k** space away from the region of highest concentration.

Although less frequent than small-angle scattering, scattering through a large angle is very effective in that it can put the electron in a state in which it is decelerated by the field. After such a loss of energy, the electron reenters that part of the electron distribution having positive velocities with respect to the field at a much lower energy than it had prior to undergoing large-angle scattering.

If an electron is to cause impact ionization, it must be able to arrive at an energy about the minimum required for impact ionization. To do this, it must not undergo large-angle scattering as it picks up an excess of energy from the field over what is lost to the optical modes in small-angle scattering. Even though the electric field may not be strong enough to offset the average rate of energy loss, fluctuations in the random-scattering rate will, nevertheless, allow a few electrons to get up to the impact ionization energy.

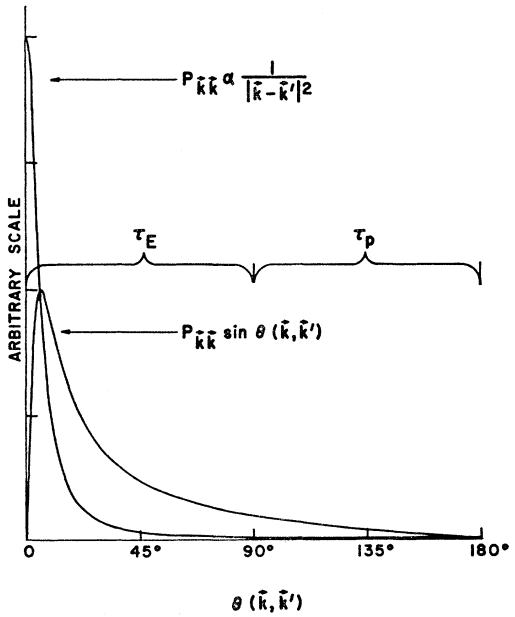

FIG. 2. Anisotropy of polar scattering for a typical case corresponding to scattering between two states **k** and **k** midway up to the ionization energy in InSb. The angle $\theta(k,k')$ is the angle between **k** and **k'**. The scattering probability $P_{\mathbf{k}\mathbf{k}'}$ is sufficiently peaked around $\theta=0$ for small-angle scattering to predominate even when this probability is weighted by the angular density of states $\sin\theta$. Small-angle scattering $\theta<90°$, is characterized by the time τ_E and large-angle scattering $\theta>90°$ by τ_p.

[8] Estimated from a spherical-band deformation-potential model. See J. Bardeen and W. Shockley, Phys. Rev. **80**, 72 (1950).

MATHEMATICAL TREATMENT

The mathematical treatment of the avalanche-breakdown problem can most conveniently be formulated in a transport theory in energy space. We will assume that the electron distribution in $\mathbf{k}$ space is asymmetrically drawn out in the direction of the field, and that if θ is the angle between the field and the electron velocity, then $\langle\cos\theta\rangle_{\text{av}}\cong 1$. The accuracy of this assumption can be determined by a comparison of the calculated values of the electron drift velocity versus electric field with the available drift velocity data for InSb. It will be seen that this is a far better assumption than the frequently made one of a nearly isotropic distribution ($\langle\cos\theta\rangle_{\text{av}}\ll 1$).

In order to take into account the anisotropy of polar scattering, we have lumped together in small-angle scattering all those events in which the electron scatters through an angle less than $\frac{1}{2}\pi$ (see Fig. 2). Electrons traveling initially in the direction of the field will continue to move at least partially in this direction after such scattering. The small-angle scattering rate $1/\tau_E$ is given by

$$\frac{1}{\tau_E}=A\int_0^{\pi/2}\frac{2\pi k'^2\sin\theta d\theta}{|\mathbf{k}-\mathbf{k}'|^2(\partial E/\partial k')\times(2\pi)^3}$$

$$=B\ln\frac{k^2+k'^2}{(k-k')^2}, \tag{2}$$

where

$$B=\frac{2\pi e^{*2}e^2k'}{M_R\omega_0\Omega(\partial E/\partial k')k}.$$

All scattering through an angle greater than $\frac{1}{2}\pi$ will result in a negative-velocity component with respect to the field. The large-angle scattering rate is given by

$$\frac{1}{\tau_p}=A\int_{\pi/2}^{\pi}\frac{2\pi k'^2\sin\theta d\theta}{|\mathbf{k}-\mathbf{k}'|^2(\partial E/\partial k')\times(2\pi)^3}$$

$$=B\ln\frac{(k+k')^2}{k^2+k'^2}\cong B\ln 2. \tag{3}$$

Consider now the motion of electrons in a one-dimensional energy space. The electrons gain energy from the electric field $\mathcal{E}$ at a rate given by $e\boldsymbol{\mathcal{E}}\cdot\mathbf{v}$ and lose energy by small-angle scattering events at a rate given by $\hbar\omega_0/\tau_E$. Although this rate is exactly defined, in any interval of time, Δt, there will be probabilities that various numbers of phonons will be emitted and these probabilities will form a distribution around the average number emitted, given by $\Delta t/\tau_E$. If we started a distribution of electrons sharply peaked around a given energy, we would find that under the combined effects of the field and small-angle scattering this distribution would have not only drifted in energy but also spread because of the statistical nature of the phonon-emission processes. This spreading of the distribution corresponds to a diffusion process in one-dimensional energy space. The "diffusion constant" appropriate to this process is given in analogy with the problem of one-dimensional diffusion by $D_E=(\hbar\omega_0)^2/2\tau_E$, where $\hbar\omega_0$ is the mean free path and τ_E is the average time between collisions. The flux of electrons past a point E in energy is given by

$$\Phi=\left(e\boldsymbol{\mathcal{E}}\cdot\mathbf{v}-\frac{\hbar\omega_0}{\tau_E}\right)n-D_E\frac{\partial n}{\partial E}, \tag{4}$$

where $n(E)$ is the concentration of electrons per unit energy.

In a steady-state distribution we may write a continuity equation of the form

$$\frac{\partial n}{\partial t}=-\frac{n}{\tau_p}-\text{div}_E\Phi=0, \tag{5}$$

where $\text{div}_E=\partial/\partial E$ and $1/\tau_p$ is the rate with which carriers are being scattered through a large angle and will, therefore, reenter the distribution after deceleration at a significantly lower energy. Similarly, there are electrons being added to the distribution at a given energy. These are electrons which were at a significantly higher energy but have undergone the process of large-angle scattering with subsequent loss of energy due to deceleration and phonon emission. The rate with which these electrons are being added is comparatively small and shall be neglected since these electrons originate from a higher energy at which the distribution is very much reduced.

It can be shown by straightforward calculation that, except very close to the bottom of a hyperbolic conduction band, τ_E and v vary relatively slowly with energy. If we neglect this variation with energy we may substitute Eq. (4) for Φ yielding

$$D_E\frac{\partial^2 n}{\partial E^2}+a\frac{\partial n}{\partial E}-\frac{n}{\tau_p}=0, \tag{6}$$

where

$$a=\hbar\omega_p/\tau_E-e\boldsymbol{\mathcal{E}}\cdot\mathbf{v}.$$

The solution of Eq. (6) for the electron-distribution function is of the form

$$n(E)\sim e^{-\int\lambda(E)dE}, \tag{7}$$

where

$$\lambda=\frac{a}{2D_E}+\left[\left(\frac{a}{2D_E}\right)^2\right]^{1/2}+\frac{1}{D_E\tau_p}.$$

Below $E=\hbar\omega_0$ there is no scattering of electrons involving the emission of optical-mode phonons. In addition, the other scattering processes are sufficiently weak such that at the fields at which avalanche breakdown is observed the electrons close to the bottom of the conduction band are accelerated to higher energies without undergoing scattering. We have therefore taken

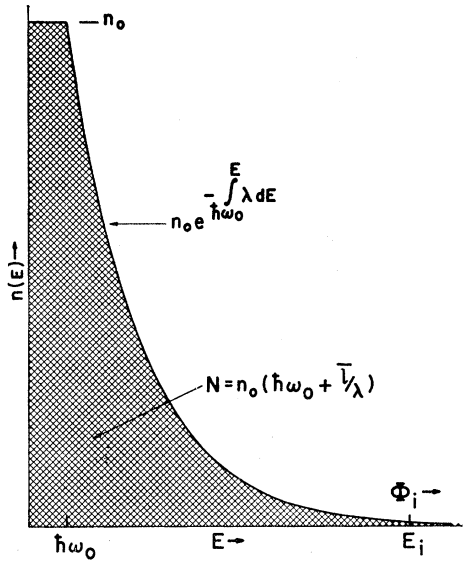

FIG. 3. Electron distribution $n(E)$ versus E. The solution has a quasi-Maxwellian decrease with increasing energy. Below the polar-scattering threshold $\hbar\omega_0$, $n(E)$ is taken to be a constant, n_0. The total area under the curve may be normalized to the number of electrons, N; and the net flux of electrons going above the ionization threshold E_i may be related to a generation rate.

the electron distribution between $E=0$ and $E=\hbar\omega_0$ to be a constant, n_0. Normalizing to the total number of electrons N,

$$N=n_0\left[\hbar\omega_0+\int_{\hbar\omega_0}^{\infty}\exp\left(-\int_{\hbar\omega_0}^{E}\lambda dE'\right)dE\right]$$
$$=n_0(\hbar\omega_0+\bar{1}/\lambda), \tag{8}$$

the distribution becomes

$$n(E)=N\exp\left(-\int_{\hbar\omega_0}^{E}\lambda dE'\right)\Big/(\hbar\omega_0+1/\lambda). \tag{9}$$

The rate of generation of pairs is given by the product of the flux of electrons entering the energy range in which ionization is possible and the function which actually ionize rather than undergo large-angle scattering. This generation rate is

$$g=\frac{1}{N}\frac{dN}{dt}=\frac{\tau_p(E_i)}{\tau_p(E_i)+\tau_i}\frac{\Phi(E_i)}{N}, \tag{10}$$

where τ_i and E_i are the time for ionization and the minimum energy for ionization. Evaluating Φ at E_i we obtain

$$g=\frac{\tau_p(E_i)}{\tau_i+\tau_p(E_i)}\frac{[\lambda(E_i)D_E(E_i)-a]}{\hbar\omega_0+\bar{1}/\lambda}$$
$$\times\exp\left(-\int_{\hbar\omega_0}^{E_i}\lambda(E)dE\right). \tag{11}$$

If we use average values of v, τ_p, and τ_E in Eq. (11), since these quantities do not vary greatly, we obtain

$$g=\frac{\tau_p}{\tau_p+\tau_i}\frac{(\lambda D_E-a)}{\hbar\omega_0+\bar{1}/\lambda}e^{-\lambda(E_i-\hbar\omega_0)}. \tag{12}$$

DRIFT VELOCITY

In addition to predicting the pair-generation rate, we may also determine the electron-drift velocity from $n(E)$. The only uncertainty is in the contribution to the current made by those electrons having an energy less than $\hbar\omega_0$. At high electric fields, when the average electron energy is fairly large, this contribution will, in any case, not be large and a calculation of the drift velocity will furnish a test of our assumptions concerning the electron distribution.

We can estimate the average drift velocity of those electrons with energy $<\hbar\omega_0$ by assuming that they are scattered into states of wave number between $-k_0$ and k_0, where k_0 is the wave number corresponding to the electron energy $\hbar\omega_0$, and are then accelerated to k_0 before they can scatter. The number of electrons which pass through a state k is then proportional to $k-(-k_0)$. If we assume a reasonably parabolic E-versus-k dependence up to $E=\hbar\omega_0$, we obtain the result that the average drift velocity of electrons of energy below $E=\hbar\omega_0$ is $\frac{1}{3}v_0$, the velocity at $E=\hbar\omega_0$.

The drift velocity will be given by

$$v_d=\frac{1}{N}\int_0^{\infty}vn(E)dE$$
$$=\left[\hbar\omega_0\tfrac{1}{3}v_0+\int_{\hbar\omega_0}^{\infty}v\exp\left(-\int_{\hbar\omega_0}^{E}\lambda dE'\right)dE\right]\Big/$$
$$\left[\hbar\omega_0+\int_{\hbar\omega_0}^{\infty}\exp\left(-\int_{\hbar\omega_0}^{E}\lambda dE'\right)dE\right]. \tag{13}$$

EXTENSION TO HIGHER TEMPERATURES

At somewhat higher temperatures the optical modes become partially excited, thereby increasing the rate of optical-mode scattering. The band structure is also slightly modified. In InSb[9] and InAs[10] there are known changes in the band gap. A corresponding change in the electron effective mass has not been experimentally or theoretically determined.

With the optical modes partially excited, the electrons can be scattered with phonon absorption and also with stimulated phonon emission. The scattering due to phonon absorption is proportional to the average

[9] V. Roberts and J. E. Quarrington, J. Electron. **1**, 152 (1955). The most accurate values of E_g for 298°K are to be found in S. Zwerdling, B. Lax, and L. M. Roth, Phys. Rev. **108**, 1402 (1957).
[10] J. R. Dixon and J. M. Ellis, Phys. Rev. **123**, 1560 (1961).

number of phonons per mode,

$$\eta=\frac{1}{e^{\hbar\omega_0/kT}-1}.$$

If we include the spontaneous processes, the rate of scattering with phonon emission is proportional to $\eta+1$.

It is convenient, with only a small loss of accuracy, to discuss the modification of the theory which results from the additional scattering using the simplification that v, τ_E, and τ_p are independent of energy. For small-angle scattering the rate of loss of energy $1/\tau_E$ remains constant since the additional loss due to events involving stimulated phonon emission are just balanced by the energy gained due to those involving phonon absorption.

For the diffusion process, however, both the scattering processes with stimulated phonon emission and those with phonon absorption are random processes which both contribute to the diffusion of electrons in energy space. The diffusion constant at finite temperatures will be given by

$$D_E(T)=(1+\eta)D_E{}^{(0)}+\eta D_E{}^{(0)}=(1+2\eta)D_E(0). \quad (14)$$

The modification of the large-angle scattering time is straightforward. Since the most important effect of large angle scattering is the deceleration of the electron after scattering, it is not important whether it gains or loses energy in the scattering process. The large-angle scattering frequency thus becomes

$$\frac{1}{\tau_p(T)}=\frac{(1+2\eta)}{\tau_p(0)}. \quad (15)$$

CALCULATIONS

For calculations of the generation rate and the drift velocity, we must obtain information concerning the band structure, the polar coupling, and parameters involved in the actual ionization process.

The conduction-band structure of InSb to a first approximation can be thought of as resulting from the mixing with, and repulsion by, the light hole band.[7] If we ignore the free-electron contribution to the one-electron energy, such a conduction band is hyperbolic, being of the form

$$E=\tfrac{1}{2}E_g\left[1-\left(1+\frac{2\hbar^2\mathbf{k}^2}{m^*E_g}\right)^{1/2}\right], \quad (16)$$

where m^* is the effective mass at the bottom of the band. There are also partially canceling contributions to the conduction-band shape from the split-off valence band and from higher conduction bands. Because of its small energy gap, however, these other contributions can be taken to be relatively small.

TABLE I. Values of the constants used in calculations of the generation rate and drift velocity in n-type InSb and InAs.

	e^*	$\hbar\omega_0$ (eV)	m^*/m	$E_g(0)$ (eV)	T (°K)	$E_g(T)$ (eV)	τ_i (sec)
InSb	0.16	0.025	0.14	0.225	198	0.205	2×10^{-11}
InAs	0.29	0.029	0.23[a]	0.41	229	0.390	2×10^{-11}

[a] E. D. Polik and J. R. Stevenson, Phys. Rev. 130, 1344 (1963).

We have assumed a hyperbolic conduction band for both InSb and InAs. This may be somewhat less justified for InAs than for InSb since the energy gap and also, therefore, the ionization energy of InAs are larger than they are in InSb. Also, the contribution of the split-off valence band should be more important in InAs.

The electron velocity versus energy is given after appropriate substitutions by

$$v=\frac{\partial E}{\partial\hbar k}=\left(\frac{E_g}{2m^*}\right)^{1/2}\frac{[(2E/E_g+1)^2-1]^{1/2}}{2E/E_g+1}. \quad (17)$$

The velocity quickly approaches, with increasing energy, the asymptotic velocity

$$v_\infty=(E_g/2m^*)^{1/2}.$$

Even at, say, $E/E_g=0.2$, $v=0.7\ v_\infty$, whereas for $E/E_g=1$, $v=0.94\ v_\infty$. Table I lists the values of E_g and m^* used in our calculations.

The scattering probability is quantitatively given in terms of e^*, M_R, and Ω. Of these quantities, only e^* is not known to a high degree of accuracy. It can be shown that

$$e^{*2}=(M_R\omega_0\Omega/4\pi)(1/\epsilon_\infty-1/\epsilon_0), \quad (18)$$

where ϵ_0 and ϵ_∞ are the static and infinite frequency-limit dielectric constants. Unfortunately, expression (18) involves a small difference between two nearly equal quantities neither of which is known precisely. Using published values of $\epsilon_\infty=16$ and $\epsilon_0=18.7$ we obtain $e^*=0.16$ for InSb. This also corresponds to the value which best fits the observed electron mobility. Ehrenreich's calculations[11] indicate that the best fit of the mobility data for InSb would lie roughly midway between the values $e^*=0.13$ and $e^*=0.20$. For InAs, the value used was that calculated[12] from Eq. (18), $e^*=0.22$.

The threshold electron energy for pair production can be calculated using energy and momentum conservation and knowing the conduction- and valence-band structures. Because the heavy-hole mass is relatively large, the ionization energies are only slightly greater than the band gap, as shown in Table I.

The time for pair ionization by an energetic electron may be estimated from the quantum-efficiency data of

[11] H. Ehrenreich, J. Phys. Chem. Solids 8, 130 (1959).
[12] H. Ehrenreich, J. Phys. Chem. Solids 12, 97 (1959).

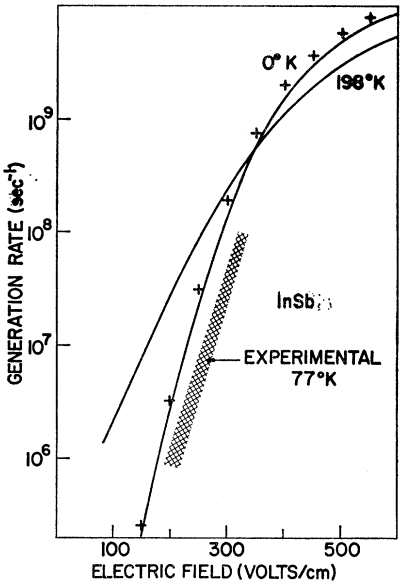

FIG. 4. Calculated generation rate in InSb versus electric field. The curve for 0°K was calculated using a value of λ dependent on energy. The indicated points were obtained evaluating λ at a mean energy (see text), as was the curve for 198°K.

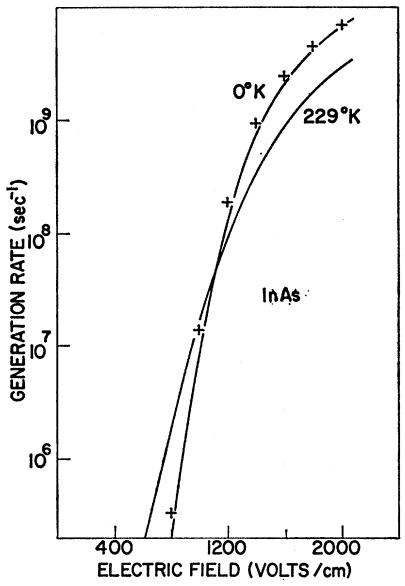

FIG. 6. Calculated generation rate in InAs versus electric field. The curve for 0°K was calculated using a value of λ dependent on energy. The indicated points were obtained evaluating λ at a mean energy, as was the curve for 229°K.

Tauc[13] for InSb. Electrons which are excited optically to energies at which they may produce hole-electron pairs can also lose energy by optical-mode emission. From Tauc's data it appears that for every 1.2 eV of excess energy another pair is produced, although only ≈ 0.25 eV is necessary for the ionization process. From this one would estimate that in addition roughly 20 phonons are emitted in the time required for impact ionization and, therefore, that the time for impact ionization τ_i is approximately 2×10^{-11} sec.

We have calculated the generation rates and drift velocities in InSb and InAs from Eqs. (11) and (13), taking into account the slow variation of v, τ_E, and τ_p. We have also calculated the generation rates using values of v, τ_E, and τ_p evaluated halfway between $E=\hbar\omega_0$ and $E=E_i$, and these values lie very close to those obtained from the more exact solution.

For both InSb and InAs we have also calculated the generation rate for temperatures at which the optical modes are moderately excited. These temperatures are 198°K for InSb and 229°K for InAs, corresponding to a phonon occupation per mode of 0.3.

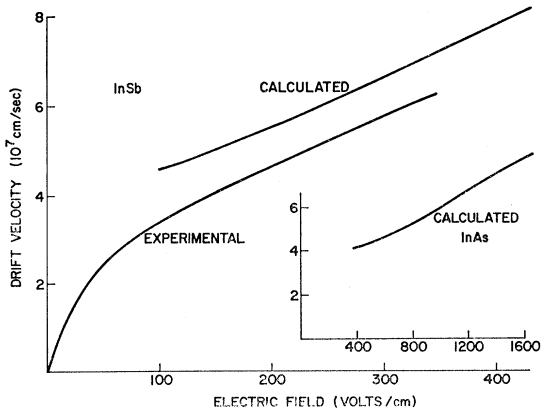

FIG. 5. Electron-drift velocity versus electric field for InSb and InAs (inset). The experimental values for InSb are from Glicksman and Hicenbothem (Ref. 15) and represent data taken at 77°K.

DISCUSSION

The pair-generation rate per electron in InSb has recently been experimentally determined by McGroddy and Nathan.[14] Values from their data are shown plotted for comparison with the theoretical results. Drift-velocity measurements have been made on n-type InSb by Glicksman and Hicenbothem,[15] and these are plotted with the calculated values of the drift velocity.

The drift velocity at fields less than 100 V/cm is considerably overestimated in our theory since at low fields the electron distribution is not strongly anisotropic. At fields of 200 V/cm and above, however, there is reasonable agreement between the theoretical and experimental values of the drift velocity. The experimental values are roughly 85% of the calculated values, indicating that the electron distribution is very nearly one-dimensional with an average value of $\cos\theta$, for the elec-

[13] J. Tauc, J. Phys. Chem. Solids 8, 219 (1959).

[14] J. C. McGroddy and M. I. Nathan, J. Phys. Soc. Japan 21, Suppl. 437 (1966).

[15] M. Glicksman and W. A. Hicenbothem, Jr., Phys. Rev. 129, 1572 (1963).

tron velocities, of 0.85. In view of the fact that there is a small density of electrons in states with a negative velocity with respect to the field, this indicates that the distribution is in fact highly anisotropic and nearly linear.

The calculated generation rate for InSb is in very good agreement with the values measured by McGroddy and Nathan.[14] For a given generation rate the electric field of the calculated generation rate seems to be roughly 15% below the experimental field although the slopes of the curves are in excellent agreement. The difference in the fields could be accounted for by saying that the effective electron velocity is not v but really $v\cos\theta$, since this would require a slight increase in the field to obtain the same theoretical generation rate. This would also be consistent with the drift-velocity results. In view of the lack of a precise value for ϵ^* and in view of the many necessary approximations which must be made in any theory of pair generation, perfect agreement should, perhaps, not be expected.

For InAs there is at present no data of the generation rate although Steele and Tosima[16] have observed a large increase in the carrier concentration at approximately 1000 V/cm. This is in excellent agreement with what we would expect from our calculations of the generation rate although the translation of a generation rate into a breakdown field is only a guess unless one has detailed information about the recombination mechanisms. The drift velocity obtained by Steele and Tosima was, however, in considerable disagreement with our predictions, having a maximum value of only 1.3×10^7 cm/sec. More recent measurements[17] of the drift velocity have yielded values of $2\text{–}3\times10^7$ cm/sec, but low-temperature measurements on suitably pure InAs have not yet been made.

[16] M. C. Steele and S. Tosima, Japan. J. Appl. Phys. 2, 381 (1963). This presents room-temperature data; however, there is no significant change in the breakdown field down to 77°K according to M. C. Steele (private communication).

[17] J. W. Allen, M. Shyam, and G. L. Pearson, Appl. Phys. Letters 9, 39 (1966); J. S. Harris (private communication).

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Bombardment-Produced Defects in p-Type Germanium at Low Temperatures*

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Lattice defects produced in nondegenerate (1–10 Ω cm) p-type germanium at 10°K by 1.0-MeV electrons cannot be detected electrically immediately following bombardment. Illumination by light having energy less than the band gap reveals the defects in the form of ionized donors. During ionization, the carrier concentration decreases as the sum of two exponentials, with time constants in the ratio ≈6 to 1. Subsequent thermal-annealing investigations show two recovery stages in the temperature range 40 to 70°K. These stages appear to be intimately related to the time constants for decay during illumination, with the low-temperature recovery associated with the fast component and the high-temperature recovery with the slow component. Both stages of recovery are independent of the type of impurity. Regardless of the extent of illumination, the electrical conductivity and the carrier concentration have essentially the same values they had prior to bombardment. When illuminated sufficiently long to ionize all the defects, the ionized donors become extremely susceptible to annihilation when the temperature is increased, and the situation after heating to 70°K represents true annealing. Illumination for shorter times results in some un-ionized defects which do not anneal upon heating to 70°K. Isochronal heating to 150°K then causes a transformation of most of the unannealed defects into a configuration which the authors call a "two-state defect," the same defect observed following irradiation at 77°K. The transformed defects break up at about 200°K. Along with the higher-temperature effects, some impurity-dependent recovery stages are noted. A model to partially account for the results is briefly discussed.

I. INTRODUCTION

ONE of the most interesting results to come from electron-irradiation experiments on germanium at low temperatures is the striking difference in behavior between n-type and p-type Ge. At 77°K, defects are introduced into both types by 1-MeV electrons, but the introduction rate for p-type material is about a fourth or a fifth of that for n-type material. For 10°K bombardments with 1.1-MeV electrons, Klontz and MacKay[1] were not able to measure any change in the electrical properties of degenerate p-type Ge due to electron bombardment, nor did they observe any changes due to annealing of defects on heating the

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[1] E. E. Klontz and J. W. MacKay, J. Appl. Phys. 30, 1269 (1959).