transforming as I, T_x, T_y, and T_z, respectively. This correspondence holds only for $\langle N\rangle=1$ $(T=\frac{1}{2})$ because the projection operators on the right-hand side of Eq. (11) complete the group structure only for this case. Unfortunately, it is just the CP violation that destroys the $SU(2)$ symmetry of the K_S, K_L system. Even worse, the mass difference of K_S, K_L, which would correspond to the potential term of (16), is not small with respect to the K_S lifetime. These facts, and the unknown details of the strong interactions make it difficult to find new results here.

It seems then, that at present it would be more fruitful to think about low-energy systems for possible applications. Because of the number factor, the system should be small, perhaps a dilute helium gas. But in any case, it would certainly be of great interest if attempts could be made to observe quantized phase-difference phenomenon in a coherent boson-type medium.

Finally, we would like to express our gratitude to the many colleagues with whom we have had discussions on the subjects of phase and Josephson tunneling. Special thanks are due to Professor P. Carruthers for a long collaboration on the question of phase, to Professor Y. H. Kao, who has kept us informed of his experimental results and their implications, and to Professor C. N. Yang, for a number of penetrating exchanges on the Josephson effect.

PHYSICAL REVIEW VOLUME 167, NUMBER 2 10 MARCH 1968

Coherence and Quantization in Nearly Superconducting Rings

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(Received 16 October 1967)

New experiments demonstrating macroscopic coherence and quantization in partially resistive superconducting rings with point contacts are described. These include: (1) the observation of quantized magnetic transitions for frequencies $\omega>\omega_1=R/L$, where R is the resistance and L the inductance of the ring, and (2) the measurement of coherent oscillations at frequencies limited by thermal noise fluctuations, $8k_BTR/\Phi_0^2$, where k_B is Boltzmann's constant, T the absolute temperature, and Φ_0 the flux quantum. These results are interpreted by introducing into the phenomenological model of weakly connected superconducting rings a continuous time dependence of the quantum states.

WE describe new experiments which demonstrate macroscopic coherence and quantization in partially *resistive* superconducting rings. Two principal effects are displayed with the use of a superconducting point contact in series with a small inductance L and a small normal resistance R. First, for frequencies $\omega>\omega_1=R/L$ there are quantized magnetic transitions as previously reported for superconducting rings.[1–3] The classical decay $\exp(-\omega_1 t)$ in the flux-screening current can be represented as a continuous exponential shift in the quantum number of the state. Second, any constant voltage across the resistance generates an oscillating current[4] similar to the Josephson ac effect[5] and this oscillation has been detected[6] at frequencies much below ω_1; in fact, the lower frequency limit is approximately[6,7] $8k_BTR/\Phi_0^2$, where k_B is Boltzmann's constant, T the absolute temperature, and Φ_0 the flux quantum.

These properties can be described by an extension of the phenomenological model of weakly connected superconducting rings.[1–3] The primary modification for resistive rings is to permit a continuous time dependence of the macroscopic quantum states; hence, the integral quantum numbers, k, for the London fluxoid become continuous variables. However, transitions between quantum states, which occur when the current in the weak contact equals the critical current i_c, satisfy the selection rule $|\Delta k|=1$. The picture which emerges is that of a discrete set of macroscopic states all moving in unison.

The general equivalent circuit of the system is shown in Fig. 1, where Φ_x represents an applied magnetic field in terms of the intercepted magnetic flux, i is the loop current flowing in R, L, and the point contact, i_1 and i_2 are branch currents, and V_J is the voltage across the point contact. Construction of such circuits has been previously detailed.[4,8] For $R\equiv 0$ we have demonstrated[3] the complete equivalence of an input current I and applied magnetic field, Φ_x. Such an equivalence no longer applies for resistive circuits and the two external fields, Φ_x and I, must be considered separately.

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Consider first $I=0$. Since a resistive ring does not have quantized, time-independent states, we introduce this time dependence by taking the state number in the phase integral as a function of time $k(t)$. Thus,

$$\phi(t)+\Phi(t)=\Phi_0 k(t), \tag{1}$$

where $\phi(t)$ is the gauge-invariant phase difference across the contact, Φ is the magnetic flux, and $\Phi_0=h/2e$. From the time derivative of the phase integral and $\Phi=\Phi_x+Li$, we have

$$V_J+(d\Phi_x/dt)+L(di/dt)=\Phi_0(dk/dt), \tag{2}$$

where V_J is the voltage across the superconducting point contact, and from the conservation of electric potential in a closed path,

$$V_J+L(di/dt)+iR=0. \tag{3}$$

Combining Eqs. (2) and (3), we have

$$dk/dt=(\omega/2\pi)-(Li/\Phi_0)\omega_1, \tag{4}$$

where we have chosen $d\Phi_x/dt=\Phi_0\omega/2\pi$. We recognize the first term as an applied emf which drives the system and the second as a relaxation term. Recalling[3] that $|Li|_{\max}\approx\Phi_0$, we have in the low-frequency limit

$$dk/dt=-(Li/\Phi_0)\omega_1; \qquad \omega\ll\omega_1, \tag{5}$$

which when combined with the stationary-state solutions[3] for $R\equiv 0$ gives for the macroscopic current

$$\begin{aligned} Li(t)&=Li(0)\exp(-\omega_1 t)\\ &=-[\Phi_x-k(0)\Phi_0]\exp(-\omega_1 t). \end{aligned} \tag{6}$$

On the other hand, for a rapidly varying external field, Eq. (4) yields

$$dk/dt=\omega/2\pi; \qquad \omega\gg\omega_1, \tag{7}$$

which is the average rate of change of k as determined by $d\Phi_x/dt$. This equation does not describe the possible quantization because Eq. (1) refers to each of the quantum states and no explicit accommodation has been made for possible discrete transitions. If the exponential decay term is negligible, then the quantization may be observed as previously described in Refs. 1–3 for various forms of $d\Phi_x/dt$. For a constant $d\Phi_x/dt$ equal to $(\Phi_0\omega/2\pi)$ we have essentially an oscillating current and voltage with a fundamental frequency $\omega/2\pi$, while for an oscillating $d\Phi_x/dt$, we should observe a periodic voltage and current of the type shown in Refs. 1 and 3.

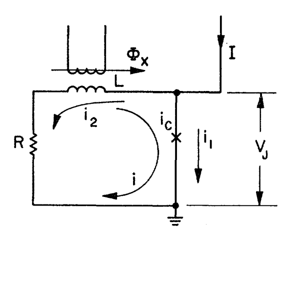

FIG. 1. Equivalent circuit of the nearly superconducting ring consisting of resistance R, inductance L, and superconducting point contact of critical current i_c. Φ_x is the externally applied magnetic field and I the external source current. Branch currents are i_1 and i_2, i is the ring current, and V_J is the voltage across the superconducting point contact.

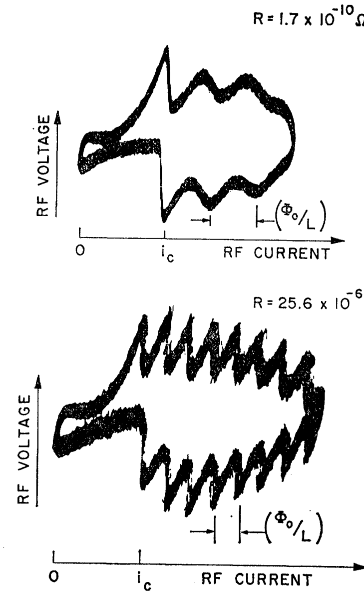

FIG. 2. Radio frequency V–I curves for the nearly superconducting ring obtained at 30 Mhz. The rf voltage amplitude across the point contact is plotted as a function of the rf current amplitude for $R=25.6\times10^{-6}$ and 1.7×10^{-10} Ω, with $L\approx10^{-10}$ H.

The effect of an external current I can be similarly demonstrated to give

$$dk/dt=(IR/\Phi_0)-(Li_1/\Phi_0)\omega_1. \tag{8}$$

Designating $IR=\Phi_0\omega/2\pi$ consider the limiting values of ω/ω_1. When $\omega\gg\omega_1$ we have the voltage-biased point contact oscillator previously described[4] where the description is now in terms of the motion of the quantum states. In the low-frequency limit, and in particular for $I<i_c$ for $t>0$, we can again use the appropriate stationary solution[3] which gives

$$k=k(0)\exp(-\omega_1 t) \tag{9}$$

and

$$Li=LI+k(0)\Phi_0\exp(-\omega_1 t). \tag{10}$$

The principal features of the electromagnetic behavior of resistive rings can be derived by superimposing these special solutions in terms of the motion of the stationary quantum states. The net effect is described by a motion of the allowed states[3] parallel to $\Phi=\Phi_x$, $i=0$ in the case of an applied field Φ_x, or parallel to $i_1=$ constant in the case of an external current I. The rate and direction of this motion is determined by ω and ω_1.

Experiments were performed by the general methods of Refs. 1 and 3 by exciting the ring with an rf current $I(\omega)$ and observing the corresponding rf voltage across the contact $V_J(\omega)$. In the previous case $R\equiv 0$ we have shown[3] that a measurement of $V_J(\omega)$ as a function of $I(\omega)$ demonstrates the quantization of the ring. For $R\neq 0$, we obtain similar $V(\omega)-I(\omega)$ curves as shown in Fig. 2. The first step occurs for $|I|=i_c$ and successive

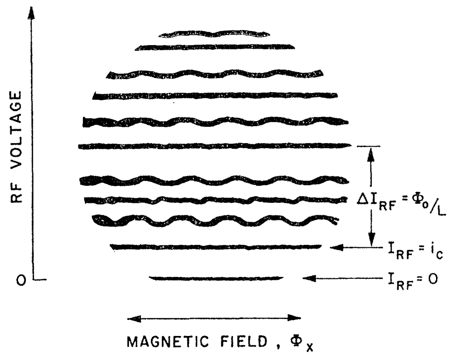

FIG. 3. Radio frequency voltage across the point contact as a function of magnetic field change for $R = 1.7\times10^{-10}$ Ω, $L\approx10^{-10}$ H for several values of rf current.

steps are periodic with interval Φ_0/L. Resistance values for the two curves are 25.6×10^{-6} and 1.7×10^{-10} Ω, giving ω_1^{-1} of 4×10^{-6} and 6 sec, respectively. Since $\omega^{-1}=3.7\times10^{-8}$ sec the requirement $\omega\gg\omega_1$ is satisfied. It should be noted that as ω_1 increases the structure of $V_J(\omega)-I(\omega)$ becomes sharper.

If we now fix the magnitude of $I(\omega)$ near any of the steps in Fig. 2 and vary Φ_x at a rate $\omega\gg\omega_1$, we observe a voltage periodic in Φ_x/Φ_0 similar to that for $R\equiv0$ as shown in Fig. 3. However, the relative phase of this signal is now arbitrary in contrast to the completely superconducting case. Upon setting $\omega=0$ we can see a shift in this periodic signal attributable to the decay term of Eqs. (4) and (6). Thus, upon excitation with an rf current and a variable magnetic field, the nearly superconducting ring exhibits quantized transitions when $\omega>\omega_1$, and classical decay of coherence when $\omega<\omega_1$.

The application of a constant external current I produces an oscillating current and voltage in the ring as previously observed and corresponding to a motion of the quantum states parallel to $i_1=$constant.[4,8] When such a constant current is added to an rf current $I(\omega)$, we observe a modulation of the $V_J(\omega)-I(\omega)$ curve at the positions of the aforementioned steps, i.e., when the amplitude of $I(\omega)$ equals $i_c+n\Phi_0/L$, for integral n. Upon biasing $I(\omega)$ to i_c, for example, we find that V_J has Fourier components at $\omega\pm\omega_0$, where $\omega_0=2\pi I_0R/\Phi_0$ and I_0 is the constant current. Detection of ω_0 from this signal is limited not to frequencies higher than ω_1, but rather by thermal noise fluctuations.[6,7] This is interpreted here as the change of the quantum number k at the rate ω_0 superimposed on the switching between states $\Delta k=\pm1$ imposed by the rf current. Since ω_0 is the Josephson frequency associated with the dc voltage I_0R, one can also look upon this as the mixing (in the point contact) of the external frequency ω and the Josephson frequency ω_0.

The frequency ω_0 can be generated by any dc voltage in the circuit and is not restricted to an external current. The first observation was, in fact, due to a thermoelectric potential resulting from a small temperature gradient across the resistance element. Frequencies as low as $\frac{1}{5}$ cycle/sec have been recorded, corresponding to a voltage of 4×10^{-16} V, when using a resistance of 1.7×10^{-10} Ω. The low-frequency limit on ω_0 and hence on the measurable voltage due to thermal noise has already been presented.[6,7] In terms of the quantum number k the thermal noise introduces an uncertainty in $k(t)$ such that $\langle k(t)k(0)\rangle\propto\exp\times(-8k_BTRt/\Phi_0^2)$.

In summary, a nearly superconducting ring incorporating a superconducting point contact (or weak link in general) exhibits quantized behavior characteristic of the entire ring and not just the contact. The chief difference between the superconducting ring and the nearly superconducting ring is the classical loss of memory in time ω_1^{-1}. Thus, while the stationary states for nearly superconducting rings have arbitrary k, hence arbitrary flux Φ, quantized transitions $\Delta k=\pm1$ are observed for $Li_c\approx\Phi_0$. Higher-order transitions, still quantized, are observed[9] for larger values of Li_c. Observation of quantized flux changes in the ring at rates lower than ω_1 is possible with the rapid cycling of the ring states at a rate much faster than ω_1. The decay of the circulating current as given in Eq. (6) is interrupted and recommences from i_c at the time of each transition. Further, the ac Josephson effect is seen to be an extensive property of superconductors and not merely associated with a given point in superconducting material. By this, we mean that although we can express the oscillatory behavior of the current density or quantum phase due to a gradient in voltage or chemical potential at some point, no macroscopic or measurable effect exists without a finite region in which the currents and fields can be defined.

ACKNOWLEDGMENT

The hospitality of the Physics Division of the Aspen Institute of Humanistic Studies is acknowledged during the preparation of this manuscript.

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