or an 8.1% decrease in the gap at H_{cb}. If, however, Eqs. (12) are averaged over the superconducting half-space, weighted with a factor $\exp(-2x/\lambda_L)$ to represent the penetration of an electromagnetic field, the decrease at H_{cb} is 6.6%. Our value of 7% is thus in fair agreement with these predictions. It would perhaps be unwise to expect more exact agreement on this point; the applied field cannot be made exactly parallel to the sample film at all points, and the perpendicular component will result in flux penetration in the form of quantized vortices.[60] Such vortices are characterized by position-dependent order parameters, so that it can become quite difficult to characterize the entire sample by a single gap value; we then no longer have the clean situation to which the theory pertains.

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Boundary Condition on the Order Parameter at a Tunneling Barrier in a Pure Superconductor ($T=T_c$)

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We consider the nucleation of superconductivity at a δ-function plane barrier in a free-electron superconductor. We calculate the exact kernel of Gorkov's linear integral equation for the order parameter Δ, and compare it with the kernel derived by de Gennes for a similar model on the basis of a correlation-function argument which has not been rigorously justified as yet. The two kernels are not the same, particularly near the barrier, but the difference oscillates with wavelength π/k_F and is negligible for the purpose of calculating asymptotic boundary conditions on the order parameter. We thus confirm for our model the boundary condition predicted by de Gennes, $\psi_+=\psi_-+(\langle 1-t\rangle/\langle t\rangle)\xi_0 d\psi/dx|_0$.

I. INTRODUCTION

In this paper, we study the functional form of the order parameter $\Delta(\mathbf{r})$ near a plane tunneling barrier in a pure superconductor.

We restrict our attention to the neighborhood of a second-order phase transition, where the order parameter $\Delta(r)$ obeys Gorkov's equation[1,2]

$$\Delta(\mathbf{r})=\int K(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r}')d^3r'. \tag{1}$$

In particular, we assume $T=T_c$ and $H=0$. Gorkov's equation then has one-dimensional solutions $\Delta(x)$.

We represent the superconductor with the BCS free-electron model with pairing interaction of strength V. We represent the barrier with the repulsive δ-function potential $(\kappa/m)\delta(x)$. The electron wave functions ψ_n satisfy the Schrödinger equation[3]

$$[-(\nabla^2/2m)+(\kappa/m)\delta(x)]\psi_n=E_n\psi_n, \tag{2}$$

which can be solved exactly.

In Sec. II, we calculate the thermal Green's function[4]

$$G_\omega(\mathbf{r},\mathbf{r}')=\sum_n \psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')/(i\omega-\epsilon_n) \tag{3}$$

from the exact wave functions. We then calculate the kernel of Gorkov's equation[5]

$$K(\mathbf{r},\mathbf{r}')=VT_c\sum_\omega G_\omega(\mathbf{r},\mathbf{r}')G_\omega^*(\mathbf{r}',\mathbf{r}). \tag{4}$$

We have not found an exact solution to Gorkov's equation. Instead, we proceed approximately by substituting zero-order solutions $\Delta_0(x)$ in Gorkov's

[1] L. P. Gorkov, Zh. Eksperim. i Teor. Fiz. **36**, 1918 (1959) [English transl.: Soviet Phys.—JETP **9**, 1364 (1959)].

[2] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, N.J., 1963), Chap. 7.

[3] Natural units $\hbar=c=k_B=1$ are used throughout this paper.

[4] See Ref. 2, Chap. 3. Here $\epsilon_n=E_n-\mu$.

[5] See Ref. 2. The sum is over $\omega=2\pi T_c(n+\frac{1}{2})$ from $n=-\infty$ to $+\infty$.

equation

$$\Delta_1(x) = \int K(\mathbf{r}, \mathbf{r}')\Delta_0(x')d^3r' \quad (5)$$

to obtain first-order solutions $\Delta_1(x)$ in Sec. III.

In the absence of a barrier, Gorkov's equation has the even solution $\Delta^e(x)=1$ and the odd solution $\Delta^o(x)=x$. In the presence of a barrier, we assume a zero-order even solution $\Delta_0^e(x)=1$ and a zero-order odd solution $\Delta_0^o(x)=x+a\ \mathrm{sgn}x$. We determine the intercept a for the odd solution by requiring the average difference between $\Delta_1^o(x)$ and $\Delta_0^o(x)$ for positive x to vanish; we let

$$\int_0^\infty [\Delta_1^o(x)-\Delta_0^o(x)]dx=0. \quad (6)$$

For de Gennes's model[6] of a barrier, this requirement leads to the same intercept a as de Gennes[7] found by another method.

II. GORKOV'S KERNEL

The electron functions ψ are free-electron wave functions of form $\exp(i\mathbf{k}\cdot\mathbf{r})$ everywhere except at the barrier, where their derivatives are discontinuous. We integrate Eq. (2) with respect to x over a small interval including the barrier, obtaining the boundary condition

$$d\psi/dx\,|_{x=0+}-d\psi/dx\,|_{x=0}=2\kappa\psi\,|_{x=0}. \quad (7)$$

The Schrödinger equation has even and odd solutions which satisfy the boundary condition:

$$\psi_{ek}(\mathbf{r})=\sqrt{2}\exp[i(k_2y+k_3z)]\cos(|k_1|x-\delta\ \mathrm{sgn}x),$$
$$\psi_{ok}(\mathbf{r})=\sqrt{2}\exp[i(k_2y+k_3z)]\sin(k_1x). \quad (8)$$

The phase shift δ is given by

$$\tan\delta=\kappa/|k_1|. \quad (9)$$

The transmissivity of the barrier for Fermi electrons incident at angle ϕ is

$$t(\phi)=k_F^2\cos^2\phi/(\kappa^2+k_F^2\cos^2\phi). \quad (10)$$

Some variously weighted averages of $t(\phi)$ over angle which will be needed later are

$$\int_0^1 t(\phi)d(\cos\phi)=1-(\kappa/k_F)\tan^{-1}(k_F/\kappa), \quad (11)$$

$$\int_0^1 t(\phi)2\cos\phi d(\cos\phi)=1-(\kappa/k_F)^2\ln[1+(k_F/\kappa)^2], \quad (12)$$

$$\int_0^1 [1-t(\phi)]3\cos^2\phi d(\cos\phi) =3(\kappa/k_F)^2[1-(\kappa/k_F)\tan^{-1}(k_F/\kappa)]. \quad (13)$$

Equation (12) gives the average transmissivity seen by a current normal to the barrier (represented by a displaced Fermi sphere).

On substituting the wave functions Eq. (8) in Eq. (3), we obtain the thermal Green's function for the normal state in the form of a sum of a barrier part and a barrier-free part

$$G_\omega(\mathbf{r},\mathbf{r}')=G_\omega{}^0(R)+G_\omega{}^b(R'),$$

where

$$G_\omega{}^0(R)=\sum_k \exp(i\mathbf{k}\cdot\mathbf{R})/(i\omega-\epsilon_k), \quad (14)$$

and

$$G_\omega{}^b(R')=-i\kappa\sum_k \exp(i\mathbf{k}\cdot\mathbf{R}')/(k_1+i\kappa)(i\omega-\epsilon_k). \quad (15)$$

Here

$$\mathbf{R}=\mathbf{r}-\mathbf{r}',$$

and

$$\mathbf{R}'=(|x|+|x'|, y-y', z-z').$$

$G_\omega{}^0$ is the Green's function in the absence of a barrier. For free electrons,

$$G_\omega{}^0(R)=-(m/2\pi R)\exp[(ik_F\ \mathrm{sgn}\omega-|\omega|/v_F)R]. \quad (16)$$

We may express the barrier part of the Green's function $G_\omega{}^b$ as an integral over $G_\omega{}^0$:

$$G_\omega{}^b(R')=-\kappa\exp(\kappa u)\int_u^\infty \exp(-\kappa t)G_\omega{}^0((\rho^2+t^2)^{1/2})dt, \quad (17)$$

where

$$\rho^2=(y-y')^2+(z-z')^2,$$

and

$$u=|x|+|x'|.$$

We consider the asymptotic limit for R' large. Let $t=u+\epsilon$. We expand the radical

$$(\rho^2+t^2)^{1/2}\cong R'+u\epsilon/R'$$

and carry out the integration, getting

$$G_\omega{}^b(R')\cong-\frac{\kappa R'G_\omega{}^0(R')}{\kappa R'+[|\omega|/v_F-ik_F\ \mathrm{sgn}\omega](|x|+|x'|)}. \quad (18)$$

This expression is valid for

$$\kappa R'\gg1.$$

Since we shall be interested in values of $\kappa\gtrsim k_F$ and distances $R'\gg\lambda_F$, the approximation is a good one. Hereafter we shall neglect the term $(|\omega|/v_F)(|x|+|x'|)$ in the denominator, since it is much smaller than $\kappa R'$.

We now substitute the thermal Green's function from Eqs. (16) and (18) in Eq. (4) to obtain Gorkov's kernel

$$K(\mathbf{r},\mathbf{r}')=K^0(R)+K^1(R')+K^2(\mathbf{r},\mathbf{r}'),$$

[6] P. G. de Gennes, *Superconductivity of Metals and Alloys* (W. A. Benjamin, Inc., New York, 1966), p. 238.

[7] We have corrected some typographical errors in de Gennes's book, and quote his corrected result in Eq. (40).

where

$$K^0(R)=VT_c\sum_\omega |G_\omega^0(R)|^2,$$

$$K^1(R')=VT_c\sum_\omega |G_\omega^b(R')|^2,$$

$$K^2(\mathbf{r},\mathbf{r}')=VT_c\sum_\omega [G_\omega^0(R)G_\omega^{b*}(R')+\text{cc}].$$

K^0 is Gorkov's kernel in the absence of a barrier. Explicitly

$$K^0(R)=(VT_c)(m/2\pi)^2R^{-2}\sum_\omega \exp[-(2|\omega|/v_F)R].$$

The sum on ω is cut off at ω_D. It can be performed immediately:

$$\sum_\omega \exp[-(2|\omega|/v_F)R]=\frac{1-\exp[-(2\omega_D/v_F)R]}{\text{sink}[(2\pi T_c/v_F)R]}. \quad (19)$$

Hence

$$K^0(R)=(VT_c)\left(\frac{m}{2\pi}\right)^2\frac{1-\exp[-(2\omega_D/v_F)R]}{R^2\sinh[(2\pi T_c/v_F)R]}. \quad (20)$$

We may easily verify[8] that

$$\int K^0(R)\,d^3R=1.$$

The second term in K can be evaluated immediately:

$$K^1(R')=[1-t(\phi)]K^0(R'). \quad (21)$$

Here $t(\phi)$ is the transmissivity of the barrier for a Fermi electron incident at angle $\phi=\arccos[(|x|+|x'|)|R']$. The reflectivity is

$$1-t(\phi)=[1+(k_F/\kappa)^2(|x|+|x'|)^2/R'^2]^{-1}.$$

The third term in K is much messier:

$$K^2(\mathbf{r},\mathbf{r}')=-2[1-t(\phi)][\cos k_F(R-R')+(k_F/\kappa)[(|x|+|x'|)/R']\sin k_F(R-R')]\times\left\{(VT_c)\left(\frac{m}{2\pi}\right)^2\frac{1-\exp[-(\omega_D/v_F)(R+R')]}{RR'\sinh[(\pi T_c/v_F)(R+R')]}\right\}. \quad (22)$$

However, it simplifies when $\mathbf{r}$, $\mathbf{r}'$ are on opposite sides of the barrier, for then $R=R'$, and

$$K^2(R)=-2K^1(R). \quad (23)$$

Collecting these results, we get the complete kernel

$$K(\mathbf{r},\mathbf{r}')=[\theta(x)\theta(x')+\theta(-x)\theta(-x')][K^0(\mathbf{r},\mathbf{r}')+(1-t(\phi))K^0(\mathbf{r},\bar{\mathbf{r}}')+K^2(\mathbf{r},\mathbf{r}')]+[\theta(x)\theta(-x')+\theta(-x)\theta(x')]t(\phi)K^0(\mathbf{r},\mathbf{r}'). \quad (24)$$

Here $\theta(x)$ is the unit step function, and $\bar{\mathbf{r}}'$ is equal to $\mathbf{r}'$, except that x' has the opposite sign.

We note that of the two separate terms in the kernel, the first is nonzero when x, x' are on the same side of the barrier, while the second term is nonzero when x, x' are on opposite sides.

According to de Gennes,[9] the kernel $K(\mathbf{r},\mathbf{r}')$ is (in a sense) proportional to a correlation function

$$\langle\delta[\mathbf{r}-\mathbf{r}(0)]\delta[\mathbf{r}'-\mathbf{r}(t)]\rangle_{E_F},$$

which represents the probability of finding an electron at $\mathbf{r}'$ at time t after it was injected at $\mathbf{r}$ with the Fermi energy. Following de Gennes, we might have constructed the kernel K for our free-electron superconductor by summing K^0 over all classical electron paths connecting $\mathbf{r}$ and $\mathbf{r}'$, weighted by the probability of the path:

$$K(\mathbf{r},\mathbf{r}')\overset{?}{=}\sum_n P_nK^0(R_n),$$

where P_n is the probability of the path, and R_n is the path length. This argument[10] yields de Gennes's kernel $\bar{K}$, which is identical to the kernel given in Eq. (24,) except that it lacks the oscillatory term K^2 for $\mathbf{r}$, $\mathbf{r}'$ on the same side of the barrier. de Gennes also let the transmissivity t be independent of angle.

III. GORKOV'S EQUATION

We substitute the zero-order even solution $\Delta_0^e(x)=1$ in Eq. (5), getting (for $x>0$)

$$\Delta_1^e(x)=1+\int_{x'>0}K^2(\mathbf{r},\mathbf{r}')\,d^3r^1. \quad (25)$$

Hence for de Gennes's truncated kernel $\bar{K}$ (from which K^2 is omitted) the zero-order solution is exact.

If either x or x' is zero, then our kernel K has the simple form $t(\phi)K^0(R)$. The value of Δ_1^e on the barrier is therefore

$$\Delta_1^e(0)=\int_0^1 t(\phi)\,d(\cos\phi), \quad (26)$$

independent of the functional form of t. All angles are

[8] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals Series and Products* (Academic Press Inc., New York, 1965), Eq. 3.541.2.
[9] See Ref. 6, p. 215. See also G. Lüders, Z. Naturforsch. **A21**, 680 (1966); **A21**, 1415 (1966); **A21**, 1425 (1966); **A21**, 1842 (1966).
[10] See Ref. 6.

weighted equally in this average. If t is constant as in de Gennes's model, then the integral is simply t. The value of the integral is given in Eq. (11) for the δ-function plane barrier.

In order to evaluate the integral in Eq. (25) for x nonzero, we let t be constant. The integral can then be simplified by the substitution $p=\frac{1}{2}(R-R')$, $s=\frac{1}{2}(R+R')$, and we get

$$\Delta_1^e(x)=1-\frac{\{(1-t)\sin(2k_Fx)+[t(1-t)]^{1/2}[1-\cos(2k_Fx)]\}}{2k_Fx}\times(NV)\int_{2\pi T_cx/v_F}^{\infty}\left[1-\exp\left(-\frac{\omega_D}{\pi T_c}u\right)\right]\Big/\sinh u. \quad (27)$$

When $x=0$, the integral has the value $(NV)^{-1}$. Hence, for small $x\ll v_F/2\omega_D$,

$$\Delta_1^e(x)\cong 1-\{(1-t)\sin(2k_Fx)+[t(1-t)]^{1/2}[1-\cos(2k_Fx)]\}/2k_Fx. \quad (28)$$

For large $x\gg\xi_0$, the integral can be performed immediately:

$$\Delta_1^e(x)\cong 1-2NV\exp[-(2\pi T_c/v_F)x]\{(1-t)\sin(2k_Fx)+[t(1-t)]^{1/2}[1-\cos(2k_Fx)]\}/2k_Fx. \quad (29)$$

These asymptotic forms for zero t agree with those found by Falk[11] in a similar calculation.

Note that the oscillatory term dies off in a few Fermi wavelengths; it cannot contribute appreciably to the value of

$$\Delta_2^e(x)=\int K(\mathbf{r},\mathbf{r}')\Delta_1^e(x')d^3r'$$

for $x\gg\lambda_F$, because part of the kernel varies smoothly, and the rest oscillates out of phase in the region where the oscillatory term is large; either way, the contribution of the oscillatory term is averaged nearly to zero (to order λ_F/ξ_0).

We now substitute the zero-order odd solution $\Delta_0^o(x)=x+a\,\mathrm{sgn}x$ in Eq. (5), getting (for $x>0$)

$$\Delta_1^o(x)=x+a+2\int_{x'>0}(1-t)K^0(\mathbf{r},\bar{\mathbf{r}}')x'd^3r' -2a\int_{x'>0}tK^0(\mathbf{r},\bar{\mathbf{r}}')d^3r'+\int_{x'>0}K^2(\mathbf{r},\mathbf{r}')(x'+a)d^3r'. \quad (30)$$

Since the kernel K is continuous at the barrier, $\Delta_1^o(0)=0$ on the barrier.

de Gennes's kernel $\bar{K}$, however, is discontinuous at the barrier:

$$\bar{K}(\mathbf{o}+,\mathbf{r}')-\bar{K}(\mathbf{o}-,\mathbf{r}') =2(1-t)[\theta(x')-\theta(-x')]K^0(\mathbf{o},\mathbf{r}'). \quad (31)$$

Consequently, the odd solution of Gorkov's equation in de Gennes's model is discontinuous at the barrier, with limiting value

$$\Delta^o(0+)=2\int_{x'>0}(1-t)K^0(\mathbf{o},\mathbf{r}')\Delta^o(x')d^3r'. \quad (32)$$

Actually, $\Delta_1^o(x)$ has the form (for t small)

$$\Delta_1^o(x)\cong a[1-\sin(2k_Fx)/2k_Fx], \quad (33)$$

within a few Fermi wavelengths of the barrier. The oscillatory term contributed by K^2 dies off in a few Fermi wavelengths, and thereafter $\Delta_1^o(x)$ changes much more slowly. de Gennes's model therefore correctly represents the behavior of $\Delta_1^o(x)$ on a scale much larger than a Fermi wavelength, since, as we noted earlier, the oscillatory part of $\Delta_1^o(x)$ does not appreciably affect the behavior of $\Delta_2^o(x)$ for $x\gg\lambda_F$.

Since the volume integral of K^0 is unity, it is consistent with Eq. (32) to assume Δ^o constant within a distance ξ_0 of the barrier with fractional error of order t. On substituting $\Delta_0^o(x)$ in Eq. (32) on both sides, we find the estimate for a

$$a=\tfrac{1}{8}\pi^2(NV)[(1-t)/t](v_F/2\pi T_c), \quad (34)$$

which is of order $\frac{1}{3}\xi_0/t$. Hence for small t, we have $a\gg\xi_0$.

We shall now look at the asymptotic form of $\Delta_1^o(x)$ for $x\gg\xi_0$. Each of the integrals in Eq. (30) is cut off exponentially as R increases more than ξ_0 beyond x. Consequently, the angular part of the integration contributes only for $\phi\sim 0$. Therefore we may set $t(\phi)=t(0)$ and remove it from under the integral signs. The relevant integrals have the asymptotic forms

$$\int_{x'>0}K^0(\mathbf{r},\bar{\mathbf{r}}')d^3r'\cong(NV)(v_F/2\pi T_c)\times\exp[-(2\pi T_c/v_F)x]/x,$$

$$\int_{x'>0}K^0(\mathbf{r},\bar{\mathbf{r}}')x'd^3r'\cong(NV)(v_F/2\pi T_c)^2\times\exp[-(2\pi T_c/v_F)x]/x,$$

$$\int_{x'>0}K^2(\mathbf{r},\mathbf{r}')d^3r'\cong-(NV)k_F^{-1}\exp[-(2\pi T_c/v_F)x]\times\sin(2k_Fx)/x,$$

$$\int_{x'>0}K^2(\mathbf{r},\mathbf{r}')x'd^3r'\cong-(NV)k_F^{-1}\exp[-(2\pi T_c)v_F)x]\times\sin(2k_Fx). \quad (35)$$

We display the K^2 and K^2x' integrals only for zero

[11] D. S. Falk, Phys. Rev. **132**, 1576 (1963). Falk calculated Δ at equilibrium, not at nucleation, so it is not *a priori* obvious that his results and ours should agree.

transmissivity. The K^2x' integral is much smaller than the K^0x' integral, except for $x>\xi_0^2/\lambda_F$, when the exponential factor is zero. Hence we discard the K^2x' integral. The K^2 integral is negligible as a coefficient of a, except for transmissivity $t<\lambda_F/\xi_0$, which is quite small.

On substitution of Eqs. (35) in Eq. (30), we get the asymptotic form

$$\Delta_1^0(x)\cong x+a+(NV)(v_F/2\pi T_c x)\exp[-(2\pi T_c/v_F)x]\times\{(1-t(0))(v_F/2\pi T_c)-a[t(0)+\sin(2k_F x)(\pi T_c/v_F k_F)]\}. \quad (36)$$

The volume which dominates the contribution of the barrier to the asymptotic form is located within a distance ξ_0 of the point on the barrier closest to the point of observation $\mathbf{r}$, so it is natural that the transmissivity which enters Eq. (36) should be that for electrons travelling in a straight line between the two points (normal to the barrier). Note that if we choose $a=[(1-t)/t](v_F/2\pi T_c)$, then the coefficient of the exponential in Eq. (36) vanishes to lowest order in x^{-n} (if we neglect K^2); this affords another estimate of the order of magnitude of a. The asymptotic form of Δ_1^o is given correctly by the truncated kernel $\bar{K}$ so long as the transmissivity $t>\lambda_F/\xi_0$.

We shall now sketch the method by which de Gennes determined the intercept a. Acting on the presumption that Δ^o is nearly constant near the barrier for small t, de Gennes[12] wrote Gorkov's equation in the form

$$\Delta^+(x)-\int K^0(\mathbf{r},\mathbf{r}')\Delta^+(x')d^3r' = -\theta(-x)\int K^0(\mathbf{r},\mathbf{r}')\Delta^+(x)d^3r' + (1-2t)\theta(x)\int K^0(\mathbf{r},\bar{\mathbf{r}}')\Delta^+(x')d^3r', \quad (37)$$

where

$$\Delta^+(x)=\theta(x)\Delta(x).$$

He took the double-ended Laplace transform of Eq. (35)

$$g(p)=\int_{-\infty}^{\infty}e^{-px}g(x)dx,$$

where $p<2\pi T_c/v_F$, and let $p\to 0$, getting

$$\lim_{p\to 0}\Delta^+(p)[1-K^0(p)]=-2ta\int_{-\infty}^{0}dx\int_{x'>0}K^0(\mathbf{r},\mathbf{r}')d^3r', \quad (38)$$

where $\Delta^+(x')$ is assumed to have the constant value a within ξ_0 of the barrier. If $\Delta^+(x)_{x\to\infty}\to x+a$, then $\Delta^+(p)_{p\to 0}\to p^{-2}$. In this same limit,

$$1-K^0(p)\to -\tfrac{1}{2}p^2L,$$

where the integral

$$L=\int K^0(\mathbf{r})x^2d^3r=[7\zeta(3)/6](NV)(v_F/2\pi T_c)^2,$$

while the integral on the right-hand side of Eq. (38)

$$\int_{-\infty}^{0}dx\int_{x'>0}K^0(\mathbf{r},\mathbf{r}')d^3r'=\tfrac{1}{16}\pi^2(NV)(v_F/2\pi T_c). \quad (39)$$

Upon substituting this information in Eq. (38), we find the value of the intercept

$$a=(14\zeta(3)/3\pi^2)(1/t)(v_F/2\pi T_c)=0.501\xi_0/t. \quad (40)$$

We get this same result if we require that

$$\int_0^{\infty}[\Delta_1^o(x)-\Delta_0^o(x)]dx=0.$$

We substract $x+a$ from both sides of Eq. (30), integrate over x from 0 to ∞, and then set the result equal to zero. We solve for a, getting

$$a=\frac{\int_0^1(1-t)3\cos^2\phi\, d(\cos\phi)\int_0^{\infty}dx\int_{x'>0}K^0(\mathbf{r},\bar{\mathbf{r}}')x'd^3r'}{\int_0^1 t2\cos^2\phi\, d(\cos\phi)\int_0^{\infty}dx\int_{x'>0}K^0(\mathbf{r},\bar{\mathbf{r}}')d^3r'-\frac{1}{2}\int_0^{\infty}dx\int_{x'>0}K^2(\mathbf{r},\mathbf{r}')d^3r'}. \quad (41)$$

One of the integrals is given in Eq. (39). The other two are

$$\int_0^{\infty}dx\int_{x'>0}K^0(\mathbf{r},\bar{\mathbf{r}}')x'd^3r'=[7\zeta(3)/24](NV)(v_F/2\pi T_c)^2 \quad (42)$$

and

$$\int_0^{\infty}dx\int_{x'>0}K^2(\mathbf{r},\mathbf{r}')d^3r'=-\pi/4k_F, \quad (43)$$

where we assume $t=0$ in the K^2 integral.

[12] See Ref. 6.

If t is constant and $t \gg \lambda_F/\xi_0$, we remove t from under the integral signs, neglect K^2, and find the intercept to be

$$a = 0.501[(1-t)/t]\xi_0, \tag{44}$$

which agrees with de Gennes's result Eq. (40).

If t is variable and $t \gg \lambda_F/\xi_0$, we neglect K^2 and find the intercept to be

$$a = 0.501\left[\int_0^1 (1-t)3\cos^2\phi d(\cos\phi) \Big/ \int_0^1 t2\cos\phi d(\cos\phi)\right]\xi_0. \tag{45}$$

The averages of the transmissivity and reflectivity are given in Eqs. (12) and (13), respectively, for the δ-function plane barrier. The averaged transmissivity is that seen by a current normal to the barrier.

If the transmissivity $t \ll \lambda_F/\xi_0$, we set $t=0$ in Eq. (41) and find

$$a = [7\zeta(3)/3\pi](NV)k_F(v_F/2\pi T_c)^2 \approx \xi_0^2/\lambda_F \tag{46}$$

for the intercept. This agrees with de Gennes's calculation[13] of the boundary condition on the order parameter at a plane interface between a superconductor and an insulator. For the purpose of calculating a, we may ignore K^2 if and only if $t \gg \lambda_F/\xi_0$.

IV. CONCLUSIONS

We have calculated the exact kernel K of Gorkov's equation for the model of a δ-function potential barrier in a free-electron superconductor at $T=T_c$ [Eq. (24)]. The kernel can be written as the sum of two terms, one of which is present in the absence of a barrier (call it the free kernel), and the other of which disappears when the barrier disappears (call it the barrier kernel). The barrier kernel decays exponentially at distances greater than ξ_0 from the barrier.

de Gennes's correlation-function argument[6] yields all of the free kernel and part of the barrier kernel. The missing term K^2 [Eq. (22)] is as large as the remainder of the barrier kernel, and it is necessary to include it if we wish to represent correctly the behavior of $\Delta(x)$ within a few Fermi wavelengths of the barrier. However, the behavior of Δ at greater distances from the barrier is not significantly affected by the missing term K^2. This is because K^2 oscillates with wavelength π/k_F off the barrier, while Δ is an average over $K\Delta$ [Eq. (1)], in which the oscillatory part of K is averaged out. Hence the correlation-function argument does yield $\Delta(x)$ in the region $|x| \gg \lambda_F$, even though it does not yield the entire kernel K.

We therefore confirm for our model the boundary condition on the Ginzburg-Landau order parameter ψ given by de Gennes[6,7] for a barrier in a pure superconductor:

$$\psi_+ = \psi_- + (\langle 1-t\rangle/\langle t\rangle)\xi_0(d\psi/dx)|_0, \tag{47}$$

subject to the minor restriction $\langle t\rangle \gg \lambda_F/\xi_0$. Here ψ_+, ψ_- are the intercepts on the barrier of the two linear asymptotes of Δ for positive and negative x, while $d\psi/dx|_0$ is the slope common to both asymptotes. The appropriate averages of the reflectivity and the transmissivity are written explicitly in Eq. (45). The transmissivity $\langle t\rangle$ is that seen by a current normal to the barrier (represented by a displaced Fermi sphere). We have taken the liberty of replacing the numerical coefficient 1.002 with unity.

Note added in proof. Recently an abstract appeared which reported work similar to this [B. Goodman, Bull. Am. Phys. Soc. **13**, 75 (1968)]. Professor Goodman concluded that the correlation-function argument gives the correct kernel except for interference terms, in agreement with our conclusions.

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[13] See Ref. 6, p. 229.