Vorticity in Liquid He II Flow through Orifices*

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Recent experiments in wide channels have shown a marked difference between the critical velocity observed in pure superfluid flow and thermal counterflow; because of the lack of a satisfactory hydrodynamical theory, the reason for this difference remains obscure. In the present work we have investigated the onset of superfluid vorticity for these two types of flow through small orifices of some tenths of a millimeter i.d., by means of the technique of trapping of negative ions. Our results show that the same difference is also observed in our much simpler geometry; namely, we find the same paradox already found in channels, that is the law $v_{s,c}d=\text{const}$, expected to hold in pure superfluid flow, is instead followed only in the thermal counterflow, and the effect is geometry-independent.

1. INTRODUCTION

Experimental data on critical velocities in liquid He II have led to contradictory conclusions. Recent experiments[1] show that critical velocities in pure superfluid flow through channels depend on the lateral dimension d of the channel, following the empirical law $v_{s,c}d^{1/4}=1$ (cgs units), and are temperature-independent. On this basis it was argued[2] that other observed critical velocities, the values of which disagree with the formula quoted above, are caused by classical turbulence of the normal fluid.

On the other hand, recent measurements by Ricci and Vicentini-Missoni[3] show that, in counterflow through wide channels, critical superfluid velocities vary according to the relation $v_{s,c}d=3.5\times10^{-3}$ cm^2/sec. This is the well-known[4] formula of Feynman for the creation of vortex lines. Moreover, in these measurements normal-fluid turbulence is detected as a second threshold at higher velocities.

Thus, considering only these most recent measurements, the experimental situation presents a contradictory and perhaps paradoxical aspect, namely, that the $v_{s,c}d=\text{const}$ relation, expected to hold in pure superfluid flow, instead is obeyed only in thermal counterflow.

Craig[5] has tried to give an explanation of the $v_sd^{1/4}=1$ law for superfluid flow, but the very reason for the different dependence of the critical velocity on diameter in the two different kinds of flow remains obscure.

In the present experiment we have studied these two kinds of flow, investigating the behavior of orifices, the simpler geometry of which permitted[6] a theoretical prediction of superfluid critical velocities.

Our measurements confirm for this geometry the same behavior as observed in channels, and exclude the possibility that classical turbulence might be responsible for the low critical velocities observed.

2. EXPERIMENTAL TECHNIQUES AND APPARATUSES

The creation of vorticity is detected by the technique of trapping negative ions.[7] With this technique we have studied both the superfluid-flow and the counterflow regime in orifices of 0.030 and 0.070 cm i.d.

A "diode" is placed on each side of the orifice, so that the ionic beams cross the axis of the orifice at right angles at a distance of 5–15 mm. In one modified version of the superfluid-flow apparatus, the orifice itself is the receiving electrode, so that the ionic beam is parallel to the axis of the orifice. The vorticity created at the orifice, when carried by the flow, interacts with the beam of negative ions, traps some of them, and so alters the total current received on the plate of the diode as well as the current density distribution in the beam.

In both cases, the flow is created by means of the thermomechanical effect. Thus, one run consists in measuring the current I carried in the diode as a function of heat input $\dot{Q}$ in the heater which serves to create the flow. We find that, upon heating, a critical heat input $\dot{Q}_c$ is reached, such that for $\dot{Q}>\dot{Q}_c$ a marked decrease $-\Delta I$ is measured in the total current. The critical superfluid velocity is calculated from this $\dot{Q}_c$ in both types of experiment, namely, superfluid flow and counterflow. For the superfluid flow it was possible to measure the velocity also in an independent manner by means of visual observation and cathetometer readings.

A. Superfluid-Flow Apparatus

The isothermal flow of pure superfluid is created by means of the thermomechanical effect. The normal-

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[1] W. Vermeer, W. M. Van Alphen, J. F. Olijhoek, K. W. Taconis, and R. De Bruyn Ouboter, Phys. Letters **18**, 265 (1965); W. M. Van Alphen, J. F. Olijhoek, R. De Bruyn Ouboter, and K. W. Taconis, Physica **32**, 1901 (1966).

[2] W. M. Van Alphen, G. J. Van Haasteren, R. De Bruyn Ouboter, and K. W. Taconis, Phys. Letters **20**, 474 (1966).

[3] M. Ricci and M. Vicentini-Missoni in *Proceedings of the Tenth International Conference on Low-Temperature Physics, Moscow, 1966* (Proizvodstvenno—Izdatelskii Kombinat, VINITI, Moscow, USSR, 1967).

[4] See, for example, K. R. Atkins, *Liquid Helium* (Cambridge University Press, London, 1959), p. 198.

[5] P. P. Craig, Phys. Letters **21**, 385 (1966).

[6] P. W. Anderson, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), p. 146.

[7] G. Careri, F. Dupré, and P. Mazzoldi in Ref. 6, p. 305.

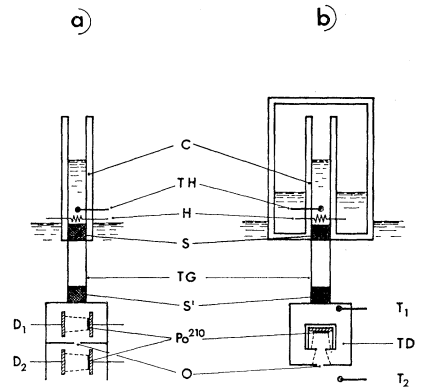


FIG. 1. Schematic diagrams of the superfluid-flow apparatuses. (a) The ionic beams (dashed) cross the axis of the orifice at right angles. (b) The ionic beam (dashed) is parallel to the axis of the orifice, and the Perspex container is shown around the hot can; H is the heater in the hot can C. TH, T_1 and T_2 are carbon thermometers. S and S′ are superleaks. TG is thermal ground. D_1 and D_2 are ionic diodes. TD is ionic triode. Po^{210} is the polonium source. O is the orfice.

fluid motion is clamped by the superleaks S and S′, made with fine pressed jeweler's rouge, and by the thin-walled copper thermal ground TG (see Fig. 1). The apparatus sketched in Fig. 1(a) was built to observe the vorticity at some distance from the orifice and to determine the direction in which it propagates with respect to the superflow (note the diodes D_1 and D_2 on both sides of the orifice). The other apparatus [Fig. 1(b)] was built to improve the sensitivity to negative-ion trapping by making the orifice itself the plate of the triode TD. The slight difference in the flow apparatus allows one to take cathetometer readings of the flowing superfluid, as clarified below; the other features of the two apparatuses are identical. When the heater H is switched on, the temperature T and the level h of the helium in the thermally insulated can C rise with respect to the outer bath, respectively, by an amount ΔT and Δh such that $\Delta h/\Delta T=S/g$, where S is the entropy per unit mass of helium and g the acceleration of gravity. With a constant heat input, the level difference Δh remains constant, and because of the temperature difference ΔT, the helium continuously evaporates to condense in the cooler outer bath; the evaporating helium is replaced by a flow through the orifice, which, because of the superleaks and thermal ground system, must be a pure superfluid flow in isothermal conditions.

Now if $\dot{Q}$ is the heat input to the heater, $\dot{Q}_l$ is the heat lost by conduction through the walls of the can; a is the area of the orifice; and ρ, S, L are, respectively, the density, entropy, and latent heat of vaporization at the temperature T; then one has the relation between $\dot{Q}$ and the superfluid velocity v_s:

$$\dot{Q}=\dot{Q}_l+a\rho v_s(TS+L). \qquad (1)$$

In the present apparatus, $\dot{Q}_l$ is less than 1% of the other term on the right in the relation (1). Moreover, at temperatures near 1°K, where our measurements are taken, one has $TS/L\simeq 10^{-2}$. Thus we have assumed

$$v_s=\dot{Q}/a\rho L, \qquad (1')$$

since we expect the measurement of v_s to be no better than 5 or 10%.

In the other version of the apparatus [Fig. 1(b)], the helium evaporating from the can is condensed in a Perspex container which surrounds the can. The velocity v_s is measured by watching the velocity V of the level rise in the container and using the relation $v_s=V\cdot A\rho/a\rho_s$, where A is the container cross section and ρ_s the superfluid density. By means of the carbon resistance thermometers T_1 and T_2, it is verified that no temperature difference larger than 10^{-5}°K (the sensitivity of the thermometers) arises across the orifice during the flow, so that we are sure that any spurious normal-fluid flow must have velocities very much lower than the superfluid velocities in which we are interested.

The visual observation of the level and the continuous control of temperature in the hot can are intended to ensure that no thermal or mechanical instability arises in the flow apparatus; as the ratio of the area of the can to the area of the orifice is of the order 100, every temporal fluctuation of Δh and ΔT from the mean stationary value results in irregular uncontrolled flows through the orifice which can sometimes exceed the mean stationary value of the velocity. As we discuss below, this behavior puts a severe limitation on the maximum value of the velocity v_s which can be achieved in a stationary regime. During measurements, Δh is fixed to better than 0.01 mm and ΔT to better than 10^{-4}°K.

B. Counterflow Apparatus

Counterflow is a thermal flow with no net mass transfer; in the two-fluid model one has

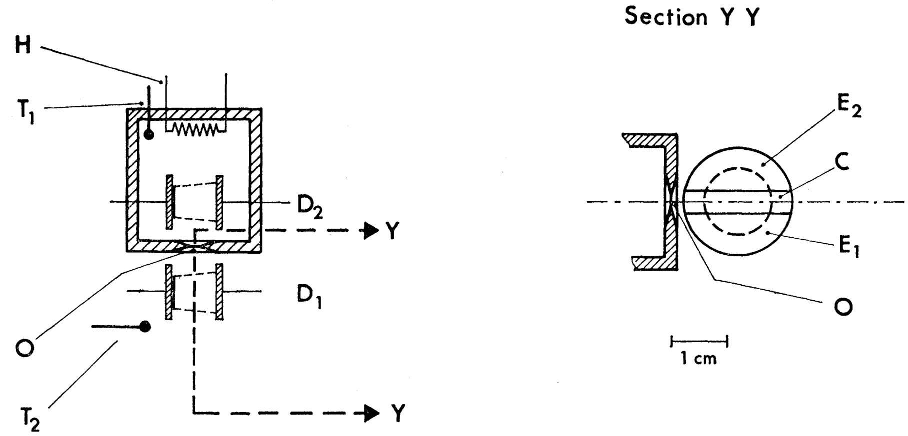
$$\rho_s v_s+\rho_n v_n=0,$$

where the indices s and n refer, respectively, to superfluid and normal fluid; the entropy continuity equation gives a relation between the velocity at the orifice and the heat input $\dot{Q}$ at the heater H (Fig. 2)

$$\dot{Q}=a\rho STv_n\equiv a(\rho\rho_s/\rho_n)STv_s. \qquad (2)$$

The hot chamber is made of thick-walled stainless steel; the heat lost by conduction through the chamber walls was found to be negligible, so we have disregarded it in Eq. (2). Various orifices were used, some of stainless and some of brass; the cutting of the rim of the orifice is shown to scale in Fig. 2; the final thickness of the rim is less than 0.02 mm. In Fig. 2 is also shown how the receiving electrode of the diode D_1 is divided in three sections to investigate the distribution of the vorticity around the axis of the orifice; the area enclosed in the dashed circumference represents to scale the cross section of the ionic beam.

FIG. 2. Diagram of the thermal counterflow apparatus. H is heater T_1 and T_2 are carbon thermometers. D_1 and D_2 are ionic diodes. O is orifice. Section YY is drawn to scale to show the collector of the diode D_1 sectioned in a central electrode and two lateral electrodes E_1 and E_2; the dashed circumference represents cross dimensions of the ionic beam as emitted by the source.



By means of the carbon thermometers T_1 and T_2, it is possible to measure the temperature difference across the orifice during the thermal flow.

In all apparatuses described in Secs. 2 A and 2 B, indium O-rings were used to seal the various parts of each apparatus in a He II-tight manner. Ionic sources are made with electrodeposited Po^{210} on gold-plated electrodes. Sources and collectors are surrounded by guard rings. The distance between source and collector in each diode was varied between 4 and 10 mm. The distance between the orifice and the nearest boundary of the ionic beam was varied between 5 and 10 mm. Ionic currents I, of the order of $\sim 10^{-12}$ A, are measured with Cary electrometers, with a minimum detectable decrease of $\Delta I \simeq 5\times 10^{-15}$ A.

3. EXPERIMENTAL RESULTS

In all measurements the electric field E applied to the diodes had values between 10 and 20 V/cm; the results are independent of the magnitude of E.

A common feature of all measurements to be presented is that no variation in the diode current was ever observed when positive ions were used; trapping was observed only for negative ions. This is in agreement with the observation that in the temperature range 0.9 to 1.35 °K, the trapping of positive ions by vortex lines is negligible.[8]

The ion velocity in every run was lower than that at the first step in the mobility[9]; in particular, for negative ions we have kept the ion velocity $v_{\text{ion}} < 2$ m/sec. No dependence at all on the ion velocity was observed in the results to be presented.

The results for superfluid flow and for thermal counterflow are widely different and will be presented separately.

A. Superfluid Flow

To discuss the measuring technique we refer to the apparatus of Fig. 1(a); the characteristics of the apparatus of Fig. 1(b) are quite similar.

As a heat input $\dot{Q}$ is supplied to the heater H, the level in the can rises rapidly to reach a value such that $\Delta h/\Delta T = S/g$.[10] This transient, if the heater is turned on rapidly, takes a time of about 10 sec for $\Delta h \simeq 1$ cm, and the superfluid velocity at the orifice is of the order of 30 cm/sec. In this case the ionic current in the diode D_2 does not change by more than 0.5%, the measuring sensitivity, while in the diode D_1, which is downstream from the orifice, the current decreases by as much as 60% of the initial value. If the heat input is quickly stopped, v_s is inverted; the diode D_1, which now is upstream, does not monitor any current change, while in the diode D_2 the current decreases strongly. Thus there is evidence that, in these transients, vorticity is created at the orifice and is carried downstream for distances of the order of 1 cm, where it is detected by the negative-ion beam.

Similarly, in the apparatus of Fig. 1(b) no current decrease is observed within 0.5% in the outflow process. In the inflow process, on the other hand, using the orifice itself as a plate for the triode TD, the current decreases, reaching 80%, were still bigger than in the apparatus of Fig. 1(a). Moreover, if we measured the current collected by the collimator of the triode, a slight current decrease of 5% was observed in the source collimator system; this result shows that the vorticity created at the orifice at temperatures between 0.9 and 1.35 °K is sufficiently energetic to be able to propagate a distance of 1.5 cm. (Note that the velocity in the chamber on which the orifice is mounted is quite negligible because the chamber has a cross section 10 000 times the area of the orifice.)

In order to create a velocity v_s stationary in time without an initial vorticity production that might

[8] G. Careri, W. D. McCormick, and F. Scaramuzzi, Phys. Letters **1**, 61 (1962); see also R. J. Donnelly, Phys. Rev. Letters **14**, 39 (1965).

[9] G. Careri, S. Cunsolo, and P. Mazzoldi, Phys. Rev. **136**, A303 (1964).

[10] In the final stationary flow vorticity can occur without being detected as a departure from London's equation because our accuracy in the determination of Δh and ΔT is not high enough.

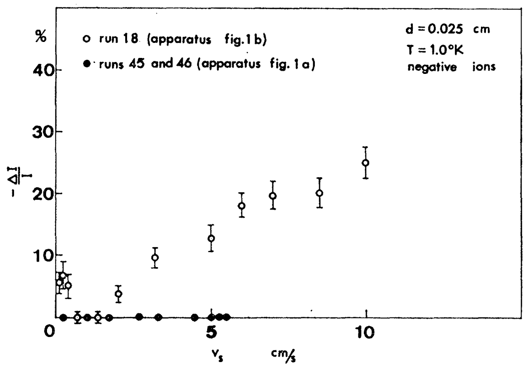


FIG. 3. Typical runs for superfluid-flow experiment. The percentage decrease in ionic current is plotted as a function of superfluid velocity. For the different features of the runs see discussion in the text.

create hysteresis, we have adopted the following procedure: The heat input $\dot{Q}$ is slowly increased to fixed values; in this manner the transients are such that the transient velocity never exceeds the final stationary velocity; after each $\dot{Q}$ is reached, the current is measured in the diodes. Following this procedure, we have reached superfluid velocities v_s of the order of some cm/sec with no current change in the diode (to within 0.5%). This is the main result; in Table I we show the largest stationary velocity v_s at which the current change in each diode is zero within the precision indicated, for two orifices and at different temperatures. The stability of the level h and of the temperature T in the can in the limits, respectively, $\delta h=0.01$ mm and $\delta T=5\times10^{-5}$°K ensures that during the measurements, uncontrolled variations of v_s around its mean value $\bar{v}_s$ were less than 10% of the superfluid velocities quoted.

Nevertheless, these uncontrolled flows limit the maximum stationary value of v_s attainable, and on the other hand can create considerable vorticity at mean velocities apparently lower than those quoted in Table I. Let us consider the orifice of diam $d=0.025$ cm at the temperature $T=1$°K. To create $\bar{v}_s\simeq5$ cm/sec, a heat input of $\dot{Q}\simeq10$ mW is required. With so high a heat input, the temperature in the can may easily become noisy in spite of every attempt to stabilize it; the limits quoted above for δh and δT are not satisfied, and the random noise induced on v_s can exceed the mean value measured. If this is the case, there is a stable decrease in the current in the diode D_1, which is a function of $\bar{v}_s$, but at the same time the current becomes 10–20 times more noisy than in the case of stationary flow to which Table I refers. In Fig. 3 is shown a typical example of this behavior: the relative current decrease $-\Delta I/I$ is plotted as a function of $\bar{v}_s$ for two typical runs; the vertical bars are not standard errors, but represent the peak-to-peak amplitude of the noise. Runs 45 and 46 represent typical results of the selected runs to which Table I refers; as can be seen, the current in run 18 is much more noisy. Striking phenomena have been observed in this run in connection with flow instabilities: A curious "resonance" pattern was observed at velocities $\bar{v}_s\simeq0.2$ cm/sec. Correspondingly, the temperature and the level in the can were found to oscillate regularly at several cps, and in this case both diodes D_1 and D_2 monitored trapping of negative ions. It was calculated that during these oscillations the instantaneous velocity exceeded 10 cm/sec. Another example of the behavior common to all these "noisy" runs is present as well in run 18: A "threshold" for creation of vorticity appears at about 2 cm/sec. Note that at the same velocities no vorticity was detected in runs 45 and 46. The behavior

TABLE I. Maximum values of superfluid velocity v_s with zero decreases $-\Delta I$ in the current of negative ions—at various temperatures and for two orifices of different diameter d.

d (cm)	v_s cm/sec	$-\Delta I$ (A)	T (°K)
0.025	5.5±0.5	$(0\pm5)\times10^{-15}$	0.9, 1.0, 1.3
0.070	3.0±0.5	$(0\pm1)\times10^{-14}$	1.0, 1.35

TABLE II. Measured values of $\dot{Q}_c$ at various temperatures for the two orifices of diameter d_1 and d_2.

T (°K)	$\dot{Q}_c$ (μW)
$d_1=0.030$ cm	
0.975	19.0±1.5
1.050	12.5±1.5
1.104	16.8±0.7
1.150	12.6±0.7
1.182	15.5±1.5
1.200	12.0±1.5
1.210	15.5±1.5
1.306	16.0±1.5
1.313	15.5±1.5
1.343	17.5±1.5
$d_2=0.070$ cm	
0.897	44.0±1.5
0.987	39.0±1.5
1.050	42 ±3
1.150	41 ±3
1.210	45 ±3
1.380	34 ±10

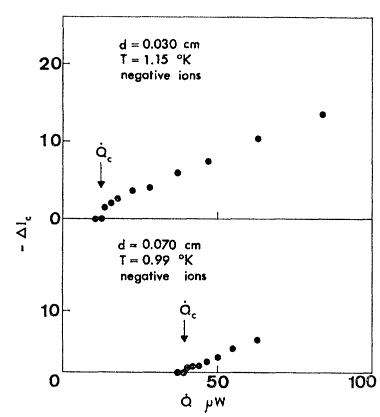


FIG. 4. Typical plot of the current decrease $-\Delta I$ (arbitrary units) in the diode D_1 as a function of the heat input $\dot{Q}$ for the counterflow experiment. The critical heat input $\dot{Q}_c$ is marked by an arrow.

just described was the same for both orifices studied and for all temperatures, and was always connected with thermal and mechanical disturbances. Insofar as these instabilities are avoided, the results of Table I are quite reproducible and no vorticity is observed. We could never distinguish whether these flow irregularities could create the detected vorticity by themselves, or whether instead the spontaneous creation of vorticity built up these instabilities.

For pure superfluid flow, we can conclude that, as long as the velocity is kept strictly stationary, there is no detectable production of vorticity up to the velocities quoted in Table I. This is an important point because both Feynman's formula[4] and Anderson's theory[6] predict critical velocities much lower, namely, of the order of $v_{s,c} \simeq 0.1$ cm/sec or less for the orifices we have studied. Moreover, the fact that it is relatively easy to have large irregularities in apparently stationary flows could be a guide to understanding the large discrepancies among existing data on critical velocities, since in other experiments no control was employed to ensure the steadiness of the flow, and large initial transients were present.

In our opinion, our results do not contradict the relation $v_s d^{1/4} = 1$ cgs. True, the values of v_s given in Table I are a factor of 2 bigger than those predicted by that relation. However, we believe there is no contradiction for the following two reasons: (i) The orifice geometry is significantly different from the channel geometry. (ii) As Keller and Hammel[11] have shown, the maximum-dissipation regime, i.e., the lowest critical velocity, is observed in gravitational flow, while in the case of mechanically controlled flow, the critical velocity can be reproducibly exceeded by a factor of 2 to 3. In our work we have employed a thermally controlled flow and so a similar behavior is to be expected.

B. Counterflow Experiments

Very different features are observed for the thermal counterflow. No transient is present on the ionic current in the diodes D_1 and D_2 (Fig. 2), when the heater H is switched on and off. Final values of the ionic current are independent of how rapidly the switching is performed, and depend only on the steady value of the heat input $\dot{Q}$ given to the heater H.

As $\dot{Q}$ is increased, a critical value $\dot{Q}_c$ is reached such that for $\dot{Q} > \dot{Q}_c$, a current decrease and a spreading is observed in the negative ion beam of diode D_1 (see Fig. 2). No alteration in the beam of diode D_2 is detected. At the same time, by means of the thermometers T_1 and T_2, the temperature difference ΔT across the orifice is measured, and we find that the ratio $\Delta T/\dot{Q}$ is constant even when $\dot{Q}$ is increased to values $\dot{Q} > 2\dot{Q}_c$. This shows the improved sensitivity to vorticity of the negative-ion trapping technique as compared to the conventional thermal technique, which defines the critical threshold as the $\dot{Q}$ value at which there is a departure from the relation $\Delta T/\dot{Q} = \text{const}$. In Fig. 4, two typical runs for two orifices of different diameter are shown; the critical heat input $\dot{Q}_c$ is marked with an arrow.

With the collector of the diode D_1 sectioned as shown in Fig. 2, it is seen that for $\dot{Q} > \dot{Q}_c$ the current decreases on the central electrode C and correspondingly increases on the guards E_1 and E_2. This suggests that the vorticity which traps the negative ions is distributed around the

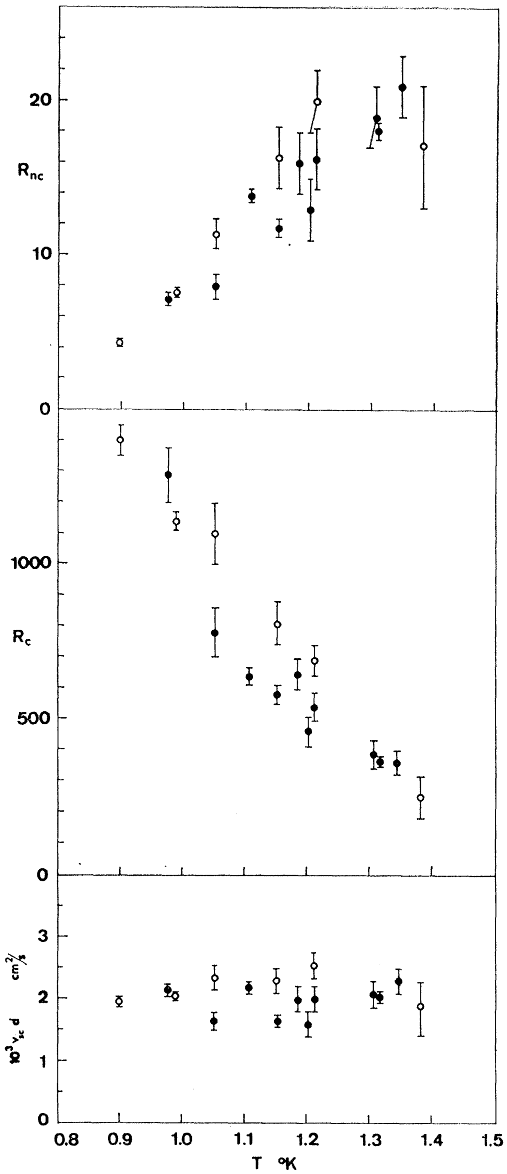


FIG. 5. Counterflow experiment. Temperature variation of the numbers $R_{n,c}$, R_c and $v_{s,c}d$ calculated from measured values of $\dot{Q}_c$. Different symbols correspond to values for the two orifices of different diameter d_1 and d_2. Filled dots: $d_1 = 0.030$ cm; unfilled dots: $d_2 = 0.070$ cm.

[11] W. E. Keller and E. F. Hammel, Physica 2, 221 (1966).

axis of the orifice, in qualitative agreement with the observation by Kapitza[12] on collimated normal-fluid jets.

To summarize: For $\dot{Q}>\dot{Q}_c$, no vorticity is detected by the diode D_2, i.e., on the superfluid downstream side with respect to the orifice, while a vorticity jet with the geometrical features of the Kapitza jet is detected on the other side.

We have measured $\dot{Q}_c$ for two orifices of diam $d_1=0.030$ and $d_2=0.070$ cm at various temperatures in the range 0.9 to 1.4 °K; results are presented in Table II.

From the measured values of $\dot{Q}_c$ we have calculated the following: the critical Reynolds number for the normal fluid, $R_{n,c}=\rho_n v_{n,c}d/\eta$; the critical Reynolds number $R_c=\rho v_{n,c}d/\eta$, where the total density is used[13]; and the product $v_{s,c}d$ of the critical superfluid velocity and the orifice diameter. The viscosity η is taken from the measurements by Woods and Hollis-Hallet[14]; for the density ρ we assume $\rho=0.145$ g/cm^3; $v_{n,c}$ and $v_{s,c}$ are calculated by means of Eq. (3).

In Fig. 5 the temperature dependence of $R_{n,c}$, R_c, and $v_{s,c}d$ for the two orifices of diameter d_1 and d_2 is shown. As may be seen, both $R_{n,c}$ and R_c show a strong temperature dependence: The variation amounts to a factor of about 5 over the whole temperature range; on the other hand, the product $v_{s,c}d$ is temperature- and diameter-independent within the scatter, which amounts to 20%.

For these reasons we think that the vorticity detected is due to the creation of vortex lines at the orifice which, interacting with the jet of normal fluid, are observed on the v_n side of the orifice (that is, by the diode D_1); the creation of vorticity is due to a critical velocity for the superfluid, following the relation $v_{s,c}d=2\times10^{-3}$ cm^2/sec, which is a best fit to the data shown in Fig. 5. On the other hand, it is clear that classical turbulence in the normal fluid cannot play a dominant role, because in that case one would expect a temperature-independent $R_{n,c}$ or R_c in a temperature range so far from the λ point.

Moreover, our numerical value of $v_{s,c}d$ is very near the value predicted by Feynman,

$$v_{s,c}d=\frac{\hbar}{m}\ln\left(\frac{d}{a}\right)=3.5\times10^{-3}\ \text{cm}^2/\text{sec}. \qquad (3)$$

This value is about twice as large as our experimental value, but only order-of-magnitude agreement is to be expected, because the numerical value theoretically predicted is strongly dependent on a detailed vortex-line or vortex-ring model of the vorticity: Calculated values of v_sd are generally in the interval between 0.8×10^{-3} and 8×10^{-3} cm^2/sec.

[12] P. L. Kapitza, J. Phys. USSR **4**, 181 (1940).

[13] F. A. Staas, K. W. Taconis, and W. M. Van Alphen, Physica **27**, 893 (1961).

[14] A. D. B. Woods and A. C. Hollis-Hallet, Can. J. Phys. **41**, 596 (1963).

4. DISCUSSION AND CONCLUSIONS

There has been much experimental and theoretical work on critical velocities; different techniques have been used both to move the two fluids and to define the critical velocity itself, and that must be the reason for part of the discrepancies.

Tough,[15] in reviewing experimental data, has proposed that at low velocities a superfluid critical velocity is exceeded that follows a law of the type in Eq. (3), and that at higher velocities the fluid as a whole becomes turbulent. According to Tough that is the reason why ρ and not ρ_n must enter the number R_c. We refer to this work for other experimental foundations of this hypothesis; here we note that both the measurements by Ricci and Vicentini[3] in wide channels and the experimental data of the present work for thermal counterflow are in quite good agreement with Tough's hypothesis.[16]

It must be remembered at this point that a general theory of flow instabilities by Meservey[17] predicts for thermal counterflow a law $v_{s,c}d=$ const as well, and an explanation in terms of this theory would be more satisfactory than the Feynman criterion, because in the Meservey theory the presence of normal fluid is fully taken into account. However, this theory should be complemented by a consideration of the origin of the vortex lines, which are the essential ingredient giving rise to the dissipative flow and therefore to the instability.

The problem of the pure superfluid flow presents rather controversial aspects. As already pointed out, Keller and Hammel[11] have shown that the critical velocity depends on the way in which the flow is created; namely, they distinguish between gravitational flow and mechanically controlled flow. In our work we have used a thermally controlled flow, i.e., a third kind of superfluid flow. It must be observed that a common feature of all existing measurements in pure superfluid flow is that the critical velocity always exceeds the highest values predicted by Feynman-type formulas.

To conclude, the results for the orifice show the same paradox already found in wide channels, i.e., the law $v_{s,c}d=$ const, expected to hold in pure superfluid flow, is instead followed only in the thermal counterflow.

[15] J. T. Tough, Phys. Rev. **144**, 186 (1966).

[16] After this experiment was completed, we heard of an unpublished experiment on flow through orifices, performed by W. J. Trela and W. M. Fairbank [Phys. Rev. Letters **19**, 822 (1967)], where also Feynman's formula is verified.

[17] R. Meservey, Phys. Rev. **127**, 995 (1962).