Periodic Critical Velocities of Ions in Liquid Helium II. Temperature Dependence

L. Bruschi, P. Mazzoldi, and M. Santini
Istituto di Fisica, Università di Padova, Padova, Italia
and
Gruppo Nazionale Struttura della Materia, del Consiglio Nazionale delle Ricerche, Roma, Italia
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The behavior of positive- and negative-ion mobility in liquid helium as a function of the electric-field intensity has been investigated from 1.0 up to 2.3°K. Periodic discontinuities have been observed up to the λ point, while within the limits of the present technique no discontinuity has been observed above T_λ. It has been confirmed that the velocity at which the discontinuity occurs is temperature-independent below 1.1°K, but above this point a striking temperature dependence is reported. Various theoretical models proposed to account for this phenomenon around 1°K are discussed, and are found to be unsatisfactory at higher temperatures.

INTRODUCTION

A. Phenomenon around 1°K

EXPERIMENTAL investigations of the mobility of ions in liquid helium, performed by Careri, Cunsolo, and Mazzoldi, have revealed a very interesting effect.[1] These authors measured the mobility as a function of the electric field, in the temperature range between 0.9 and 1°K, using a fine time-of-flight method developed by Cunsolo.[2] They found that the mobility decreases sharply to a lower level when the ion reaches a characteristic drift velocity $\langle v_c \rangle$. The results of the experiments may be summarized as follows:

(1) The positive and negative ions display different critical velocities: 5.2 m/sec for the positive ion, and 2.4 m/sec for the negative one.

(2) The two critical velocities are temperature-independent.

(3) When the ions reach a drift velocity equal to $2\langle v_c \rangle$, the mobility falls again sharply to another lower level. The same occurs again at $3\langle v_c \rangle$, $4\langle v_c \rangle$, and $5\langle v_c \rangle$, i.e., the phenomenon is periodic in the drift velocity.

(4) The fractional charge $\Delta\mu/\mu$ is around 6%.

(5) Changing the ionic beam density and the geometry of the apparatus has no detectable effect on the phenomenon investigated.

The first discontinuity was investigated also with a completely different technique by Careri, Cunsolo, and Vicentini.[3] They used a heat-flush method[4] for the measurement of the drift velocity, and confirmed the time-of-flight results. This experiment is very important because the authors were able to learn an essential fact, namely:

(6) The new dissipation that sets in at $\langle v_c \rangle$ is "mainly" due to an extra interaction with the normal fluid.

Cope and Gribbon[5] have repeated the experiment of Careri *et al.*, in the same temperature range, again with the method of Cunsolo. They also confirmed the early experimental results, and performed systematic investigations on the influence of the extracting field. The result was the following:

(7) The size of the discontinuity, $\Delta\mu/\mu$, depends on the magnitude of the extracting field, in the sense that when it exceeds the critical field, $E_c = \langle v_c \rangle/\mu_0$ (where μ_0 is the zero-field mobility), the size becomes smaller, as if some kind of memory of the past were attached to the ions.

B. Theoretical Models

Careri and co-workers[1] tried to connect the periodicity with the multiple quantized circulation of the superfluid velocity. They advanced the hypothesis that the ion at the multiple critical drift velocities creates a vortex ring of the same multiple of quantum circulation. This was in fact the basic idea of the experiment, namely to test the prediction of "the formation of quantized vortex rings behind the ion in motion."[6] In order to explain the heat-flush experiment, the vortex ring is pictured to be in some way closely bound to the ion, thus providing an extra interaction with the normal fluid. A difficulty arose for these ideas when clear experimental evidence was found that for drift velocities around 30 m/sec the ion is really trapped by a vortex ring of one quantum of circulation.[7] These experiments were made in the same temperature range where the periodic steps were first discovered. Therefore, it was difficult to understand why

[1] G. Careri, S. Cunsolo, and P. Mazzoldi, Phys. Rev. Letters **7**, 151 (1961); Phys. Rev. **136**, A303 (1964).

[2] S. Cunsolo, Nuovo Cimento **21**, 76 (1961).

[3] G. Careri, S. Cunsolo, and M. Vincentini Missoni, Phys. Rev. **137**, A311 (1964).

[4] G. Careri, J. Reuss, F. Scaramuzzi, and J. Thomson, in *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, Wisc., 1958).

[5] J. A. Cope and P. W. Gribbon, *Proceedings of the Ninth International Conference on Low-Temperature Physics, Columbus, Ohio, 1964*, edited by J. A. Daunt *et al.* (Plenum Press Inc., New York, 1965), p. 153.

[6] G. Careri, in *Progress in Low-Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1961), p. 75, Vol. 3.

[7] G. Careri, S. Cunsolo, P. Mazzoldi, and M. Santini, in *Proceedings of the Ninth International Conference on Low-Temperature Physics, Columbus, Ohio, 1964*, edited by J. A. Daunt *et al.* (Plenum Press Inc., New York, 1965), p. 335.

the ion should trap itself in a vortex ring on one quantum after the creation of vortex rings up to five quanta of circulation.

Huang and Olinto[8] tried to overcome this difficulty in a phenomenological theory connecting the first region (periodic steps) with the second one (giant discontinuity). The theory is valid only for temperatures around 1°K, and the fundamental points are the model for the creation process, and the stability criterion for the vortex-ion complex. The critical velocities $\langle v_n \rangle$ are determined by the condition

$$6\pi R\langle v_n\rangle = nk\,, \quad n=1,2,3,\cdots.$$

Here, R is the effective radius of the ion, and k is the quantum of circulation ($k=0.998\times10^{-3}$ cm^2 sec^{-1}). When the ion in its flight reaches the nth critical velocity, it experiences an additional drag due to the creation of "turbulence" in the superfluid, and remains at this drift velocity, converting electrical energy in turbulence energy. When the total energy imparted to the superfluid equals the vortex-ring energy, "the turbulence in some way becomes an n ring." The stability criterion states that the vortex ring cannot be coupled to the ion, if the applied electric field is not higher than the critical field of the giant discontinuity. Without entering into greater detail, we want to stress here that the sharp decrease of the mobility at the critical velocity is explained and calculated in this theory by an extra interaction with the superfluid component alone.

The basic ideas of the model advanced by Di Castro[9] are similar to those of the Huang-Olinto model. However, many rings are assumed to be created in the flight, instead of only one, the electric field providing the additional momentum. As in the Huang-Olinto model there is no additional dissipation in the normal fluid.

Cope and Gribbon[10] interpret the steps with a mechanism like that of the Franck-Hertz experiment. The ion collides with thermal rotons and is excited into an elastic oscillation which rapidly decays with emission of a phonon. The critical velocities turn out to be integral multiples of the first one, and the steps should be observable only in the temperature range 0.8–1.0°K.[11]

C. Open Questions

It is very difficult to decide which of the models proposed is right, or closest to the real nature of the phenomenon. There are essentially two models: vortex-ring creation, and excitation of ion vibrational states. For the vortex rings the main difficulties are the following:

(i) It is hard to explain the heat-flush experiment in the Huang-Olinto and in the Di Castro pictures, because they do not provide the needed extra interaction with the normal fluid, required by the experimental results. The interpretation of the heat-flush experiment, as given later by Huang,[12] does not seem convincing to us.

(ii) The creation process of the vortex ring requires too high an ion mass if it is to satisfy the Landau criterion.[1,9] Therefore the energy and momentum must be supplied by the electric field, in a rather long creation time.[8,9]

But it is difficult to visualize the creation of a so highly organized a motion, as a 100 Å vortex ring from turbulence, by an ion of 10 Å diameter, over a long creation path.[13]

Among the objections which can be made to the vibrating ion model, the strongest is probably the one advanced by Reif,[13] i.e., the surprising fact that positive and negative ions, so different in structure, can display vibrational frequencies differing only by a factor of 2.

The conclusion at the beginning of the present experiment was that in order to understand more about the phenomenon, further experimental work was required, not only because serious objections can be made to all the models, as we have seen above, but also because there were other important open questions:

(a) The first thing to be decided was whether the steps in mobility are peculiar to liquid helium II only, or if they persist above the λ point also.

(b) The second aspect to be investigated was the role played in this effect by the normal fluid, and its influence on the critical velocity, on the periodic feature, and on the size of the discontinuity.

(c) Another point, is still to be clarified at the present time: Are the steps connected with the giant discontinuity, or are they completely independent thereof?

In the general hope of gaining a deeper insight into the nature of this effect in liquid helium, and particularly with the aim of giving an answer to the questions (a) and (b), we undertook experiments at temperatures higher than the previously investigated ones.

Preliminary results on positive ions have been reported in a paper by the present authors,[14] and will be discussed in the following sections.

EXPERIMENTAL APPARATUS

A. Experimental Cell

The cells used in this experiment are essentially the same as those described in Ref. 1. Slight changes of relative dimensions, of the spacings, and of the diameter

[8] K. Huang and A. C. Olinto, Phys. Rev. **139**, A1441 (1965).
[9] C. Di Castro, Nuovo Cimento **42**, 251 (1966).
[10] J. A. Cope and P. W. F. Gribbon, Phys. Letters **16**, 128 (1965).
[11] J. F. Allen, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), p. 333.
[12] K. Huang, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), p. 332.
[13] D. Pines, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland Publishing Co., Amsterdam, 1966), p. 328; F. Reif, *ibid.*, p. 345.
[14] L. Bruschi, P. Mazzoldi, and M. Santini, Phys. Rev. Letters **17**, 292 (1966).

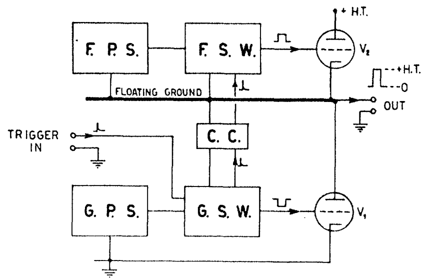

FIG. 1. Block diagram of the high-voltage square-wave generator used at high temperatures. GPS: grounded power supply. GSW: square-wave generator. FPS: floating power supply. FSW: floating square-wave generator. CC: coupling circuits between the grounded and the floating channels.

of the grid's wires have been introduced, without systematic effect on the physical results. Cells directly connected to the helium bath, as well as sealed in a stainless steel container, have been used.

Radioactive sources of various intensities, emitting α particles of 0.5 MeV from Po^{210} and β particles of 18 keV from tritium, have been employed. The resulting ionic density of the beam in the drift space ranged between 10^5–5×10^7 ions/cm^3.

The grid-to-collector distance was ranged between 3 and 6 mm.

B. Electronics and Measurements

We used always the method of measurements developed by Cunsolo,[2] in which a symmetrical square-wave electric field, with zero mean value, is applied in the drift space. The critical electric field E_c at which the first discontinuity occurs is defined by

$$\langle v_c\rangle = E_c\mu_0. \quad (1)$$

The zero-field mobility μ_0 is a rapidly decreasing function of the temperature going as $(e^{B/T})$.[15] Both from previous experimental results and from theoretical models it was expected that the critical velocity would not change with increasing temperature. The electric field necessary to detect the discontinuities was then expected, from Eq. (1), to increase rapidly with temperature. Using a reasonable drift distance one would therefore need square-wave voltages as large as ≈ 5 kV peak to peak. For this purpose we made up a square-wave generator, the block diagram of which is shown in Fig. 1. The power tubes (RCA 3E29) $V1$ and $V2$ are connected in series between voltage supply and ground. They are used as switches driven by two square-wave voltages—180° out of phase—acting directly between the grids and the cathodes. Thus when $V1$ is open, $V2$ is closed and vice versa. If V_0 is the dc high voltage, a square-wave form going from zero to V_0 appears at the output. The dc component $\frac{1}{2}V_0$ is then removed with a CR filter, thus giving the desired waveform, $\pm\frac{1}{2}V_0$. The high-voltage power supply must provide only the small current to charge the capacity of the coaxial cables connecting the output to the drift chamber ($\sim$100 pF), and the one flowing in the 1 MΩ filter resistor. Moreover, because $V1$ and $V2$, when closed, act as low resistances in series, and not in parallel, with the load, the charging and discharging of the load capacity is fast. Thus with modest power drain from the high-voltage supply, good square waves are obtained. In order to drive $V1$ directly between the grid and the cathode, a floating square-wave generator is needed, as one can see from the block diagram of Fig. 1.

Because the change of the mobility to be measured is small, careful measurements are needed to avoid spurious results. The mobility is defined by

$$\mu = \langle v\rangle/E = d^2/V\tau, \quad (2)$$

where d is the grid-to-collector distance, V the value of the voltage applied between these electrodes, and τ is the time of flight. In the method of Cunsolo the time of flight is measured by the zeroing condition of the mean current at the collector according to the relation

$$\bar{i} = \tfrac{1}{2}I(1-\tau/T) = \tfrac{1}{2}I(1-\tau f), \quad (3)$$

where T is the half-period of the applied square-wave voltage, which in turn is determined by the frequency of the pulser triggering the flip-flop unit. An important feature of the method is that the null frequency can be determined with good accuracy by linear extrapolation in a current-frequency plot. The quantities to be measured are therefore the current at the collector, the frequency of the pulser, and the applied voltage. In measuring the mobility as a function of the electric field the distance d is a fixed quantity; therefore, it cannot introduce spurious steps on the results.

The current has been measured with a Keithley 610B picoammeter driving a Philips 2212 A/00 recorder. No major trouble is encountered in measuring the current. If the cell, and particularly the grid, are clean, the relation (3) between current and frequency is indeed fulfilled, and the "zero level" of the current does not introduce any ambiguity in the measurement.

If, on the contrary, the electrodes are not clean, the zero at high frequency cannot be properly determined, and near the flight frequency the current is no longer a

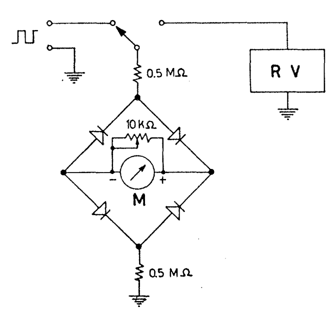

FIG. 2. Four-diode bridge comparator for the measurement of the square-wave voltage. RV: dc reference voltage, Keithley 241.

[15] F. Reif and L. Meyer, Phys. Rev. 119, 1164 (1960).

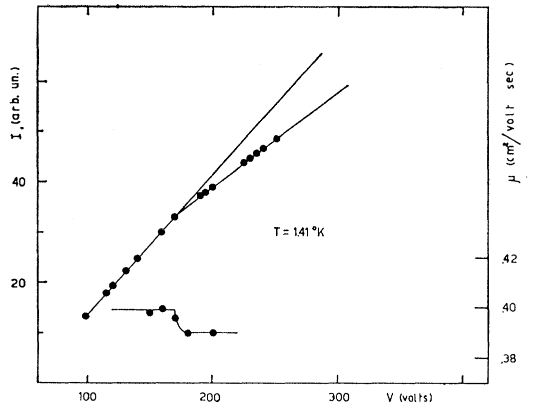

FIG. 3. The mean current collected at the receiving electrode at various square-wave voltages. The frequency of the SW is fixed at 50 cps. A change in the slope occurs at the same value of the voltage at which the mobility falls to the second level. In the lower part of the figure are reported five values of the mobility measured around the discontinuity.

linear function of frequency. A smoothed tail appears and the zeroing is not sharp. In this situation the results must be disregarded.

The frequency can be measured with high accuracy. We used the Hewlett-Packard 3734-A digital frequency meter.

Great care is required for the measurement of the square-wave voltage. A meter with high input impedance should be used, otherwise the *RC* time constant would not be high enough with the use of a reasonable high-voltage capacitor, and the waveform would not be satisfactory at low frequencies. Commercial meters involving more or less compensated attenuators were found to be unsatisfactory for a measurement with 1% accuracy. Major troubles arose in the change of scale. In the high-voltage range, say above 400 V peak to peak, the generator used was that of Fig. 1, described above, which resolves in this range, quite simply, the problem of voltage measurement. Indeed it essentially interrupts periodically the dc high voltage applied at the plane of $V1$. If the two switches $V1$ and $V2$ open and close accurately and if the load resistance is high with respect to the resistance of the closed $V1$, then the peak-to-peak output voltage is simply the dc applied voltage. This can easily be measured within 1%, or even better if one uses a digital voltmeter. The problems of compensated attenuators, linearity of amplifiers and diodes, etc., are then dispensed with. In the low-voltage range, say below 400 V peak to peak, we have adopted the circuit of Fig. 2. The square wave to be measured, applied to a four-diode bridge, gives some indication on the microammeter. This indication can now be calibrated switching the bridge from the square wave to the dc supply. The dc voltage that gives the same deviation as the square wave is the required value. As a dc reference voltage a Keithley dc power supply model 241 has been used. In this way measurements of the square-wave voltage well within 1% are easily obtained.

The intensity of the extracting electric field applied between the radioactive source and the grid was kept below the critical field E_c, unless there resulted too low a current. However, the intensity of the electric field affects only the size $\Delta\mu/\mu$ of the discontinuity, not the value of the critical velocity. The effect is the same as that observed by Cope and Gribbon[5] and quoted above, in the Introduction, and has been found by us rather troubling when one tries to detect the first step for negative ions.

We want to report here another effect found during the course of these experiments. We fixed the frequency of the square-wave (sw) electric field applied—as usual—between the grid and the collector, at a very low value with respect to the flight frequency, say 20–50 cps. In these experimental conditions, we measured the mean current at the collector as a function of the SW electric-field intensity, and we obtained diagrams like those of Fig. 3, where one can see that the slope is clearly discontinuous at the critical field E_c. Sometimes the change in slope is quite sharp and large, as, for example, that of Fig. 4 where a 30% change is displayed. This effect has not been investigated in a systematic way, and therefore has not been completely understood. It is probably connected to a time rearrangement of the charge density near the grid wires, which occurs at the transition. In some instances we have made use of this effect, as discussed in the next section.

The temperature has been measured with an accuracy of 10^{-2} °K and was kept constant within 10^{-3} °K during the measurements.

EXPERIMENTAL RESULTS

The first measurements were made with positive ions, and the central result is that the phenomenon also occurs up to temperatures near the λ point.[14] The

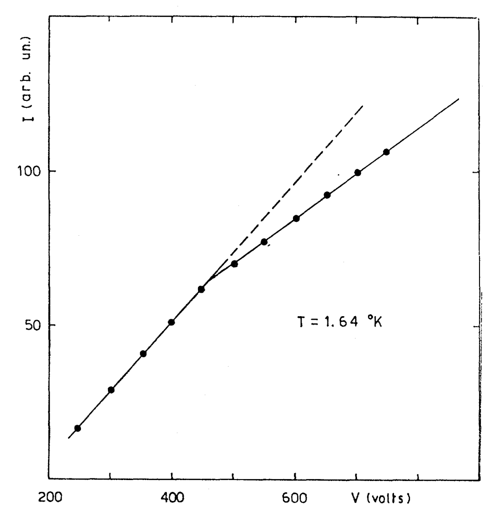

FIG. 4. Another example of the "square-wave current effect" which occurs at the critical voltage.

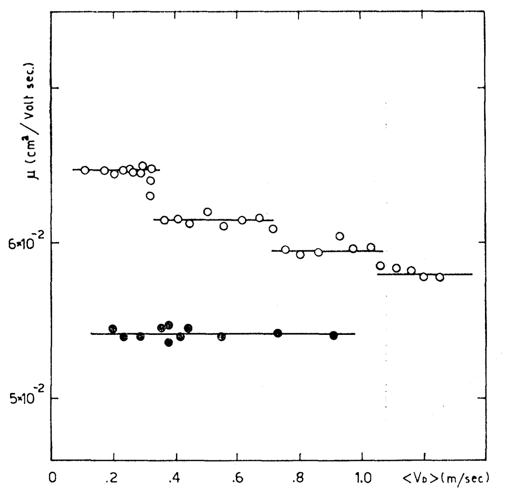

FIG. 5. Mobility versus drift velocity for positive ions at two temperatures below and above the λ point. Upper curve $T=2.06$°K, vapor pressure. Lower curve $T=2.29$°K, pressure $P=1$ atm.

feature of periodicity is also preserved at high temperatures, and the relative size of the step always has roughly the value found by previous workers at low temperatures.[1] An example of experimental data (positive ions $T=2.06$°K) from which we obtain the critical velocity is shown in Fig. 5. As one can see, the steps are well defined, and the periodic feature appears clear, also. The critical velocities can be estimated from this plot to be

$$\langle v_c\rangle_1=(0.35\pm0.02)\text{ m/sec},$$
$$\langle v_c\rangle_2=(0.70\pm0.04)\text{ m/sec},$$
$$\langle v_c\rangle_3=(1.05\pm0.05)\text{ m/sec}.$$

Figure 6 represents an analogous example for negative ions ($T=2.00$°K). The first discontinuity was not displayed by this particular run, which we wish to report here as an example of the periodicity up to the seventh step. We have found the same difficulties quoted by previous authors in detecting the first discontinuity for the negative ions, probably due to the influence of the extracting field, as discussed above. Nevertheless the first step has been clearly detected many times for negative ions. We did not extend the investigation up to so high a multiple in every run, but in most of them at least the first and second steps were detected. It is necessary to detect at least two steps in order to decide whether the first step encountered in the measurement is indeed the first multiple, or if it is the second multiple. A few critical velocities near 1.3°K reported by Bruschi *et al.*[14] were indeed affected by this indeterminacy, i.e., they were reported as values of the first step, and have been found later to be those of the second one.[16]

[16] L. Bruschi, P. Mazzoldi, and M. Santini, in *Proceedings of the Tenth International Conference on Low-Temperature Physics* (Publishing House VINITI, Moscow, 1967), Vol. I.

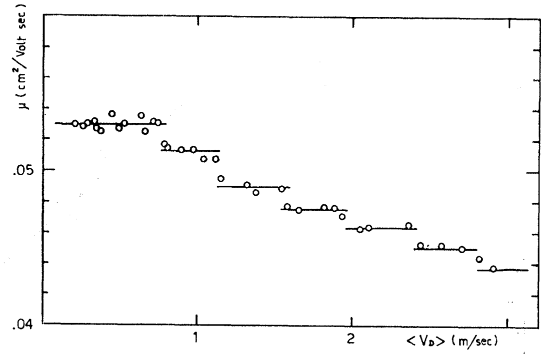

FIG. 6. An example of results with negative ions at 2.0°K, where steps up to the seventh have been detected. The first step is not present. If one takes for $\langle v_1\rangle$ the value of $\frac{1}{2}\langle v_2\rangle=0.39$ m/sec, then for the other steps one has $\langle v_n\rangle=n\langle v_1\rangle$, with $n=3.0$, 4.0, 5.0, 6.1, 7.1.

Above the λ point no mobility discontinuities were detected. The range of velocity investigated is from 20 up to 100 cm/sec. Drift velocities lower than 20 cm/sec are difficult to measure accurately with the method

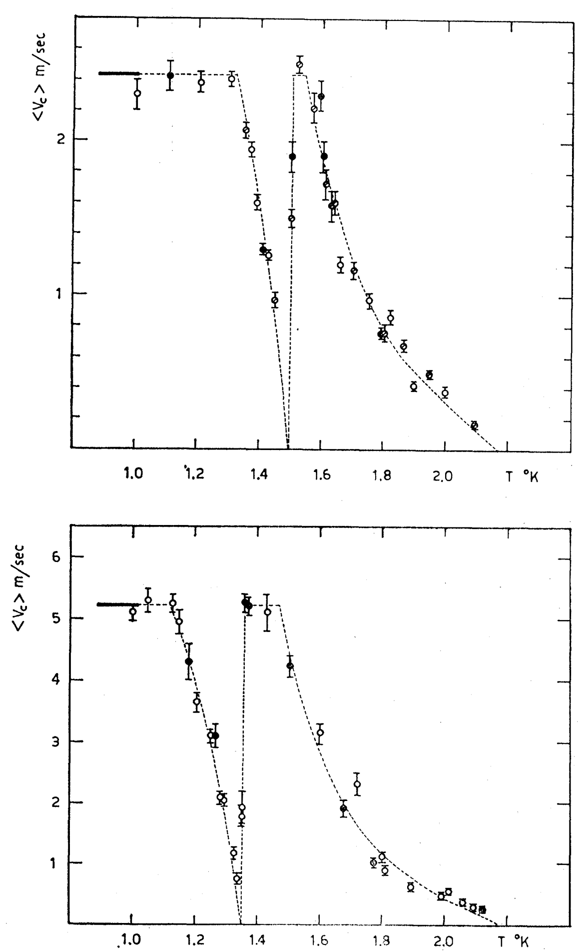

FIG. 7. Drift velocity of the first step for negative ions (upper curve) and for positive ions (lower curve) at various temperatures. The symbols are explained in the text. The dotted lines represent the functions (4)–(7) of the text.

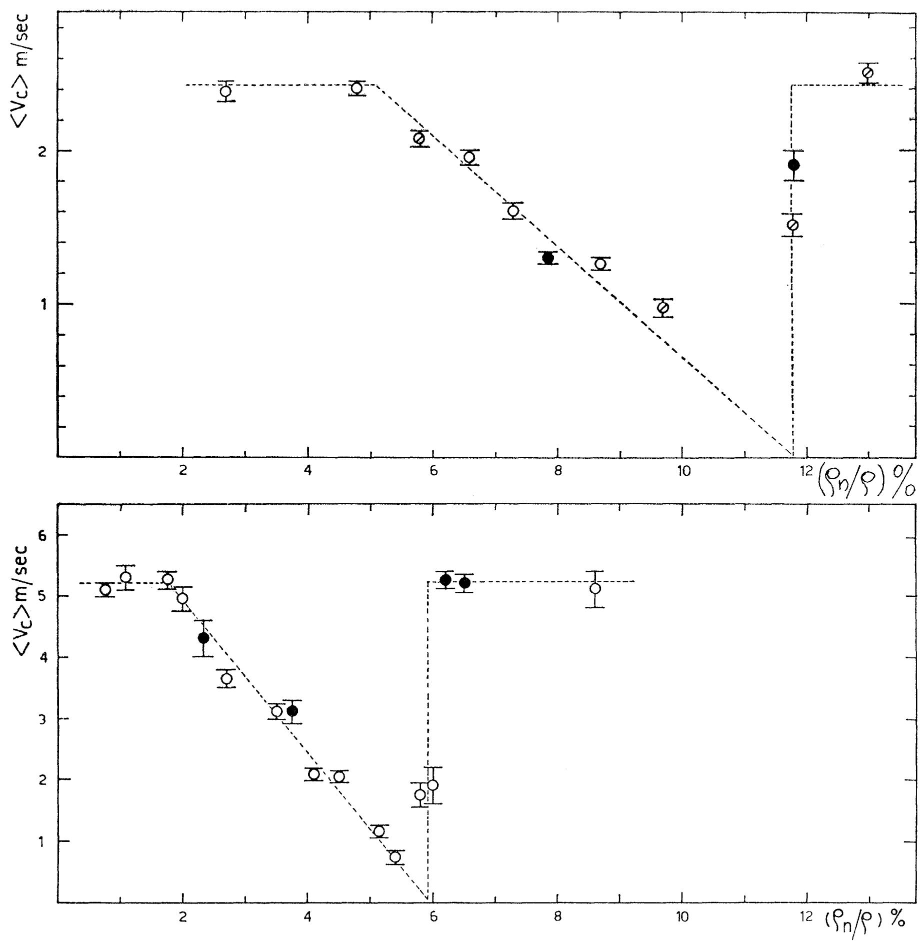

FIG. 8. $\langle v_c \rangle$ for negative ions (upper part) and for positive ions (lower part) plotted against the (ρ_n/ρ) ratio, for temperatures lower than 1.5°K.

used, because square waves of too low a frequency are needed, and moreover it is almost impossible to filter out the comparatively high currents flowing through the grid-collector capacity. Another method must be used to investigate low velocities. Above 100 cm/sec, on the other hand, owing to the low value of the mobility, square-wave voltage higher than 4 kV peak to peak are needed, and this is beyond the capability of our experimental apparatus. A comparison of the mobility's behavior below and above the λ point is displayed in Fig. 5.

The results of several runs are reported in Fig. 7, where the critical velocity of the first step is plotted as a function of temperature. The black points are results deduced from measurements with only one step investigated, whereas the open dots refer to measurements were more steps are detected. The points with a bar for negative ions are the values of the critical velocity deduced from the change in slope of the current-field plot as explained above. The results of Careri *et al.*[1] around 1°K, where $\langle v_c \rangle$ is temperature-independent, are also represented in the figure, by heavy horizontal lines. Among the errors affecting the measurement of the drift velocity, the largest is a systematic one, due to the uncertainty in the effective flight distance. Although the total error in $\langle v \rangle$ is $\pm 2\%$, the error on the critical velocity $\langle v_c \rangle$ is larger, because it is evaluated as the mean of the two measured velocities between which the break is found.

While at low temperatures the critical velocity has been confirmed to be temperature-independent, as previously observed around 1°K by several authors,[1,3,5] at a well-defined temperature an entirely different situation sets in. This temperature is 1.13°K for positive ions and 1.30°K for the negative ones. Above this temperature we can outline two different regions, where $\langle v_c \rangle$ depends in a different way on the normal and the superfluid densities. The two regions are connected by a sharp transition, at which $\langle v_c \rangle$ takes again the value it had at low temperatures.

If one plots $\langle v_c \rangle$ in the first region as a function of the ratio ρ_n/ρ of the normal to the total density,[17] one obtains a linear dependence. This is displayed in Fig. 8 both for positive and negative ions. Instead, in the second region one finds that $\langle v_c \rangle$ is simply proportional to the ratio ρ_s/ρ_n between the superfluid and the

[17] P. J. Bendt, R. D. Cowan, and J. L. Yarnell, Phys. Rev. 113, 1386 (1959).

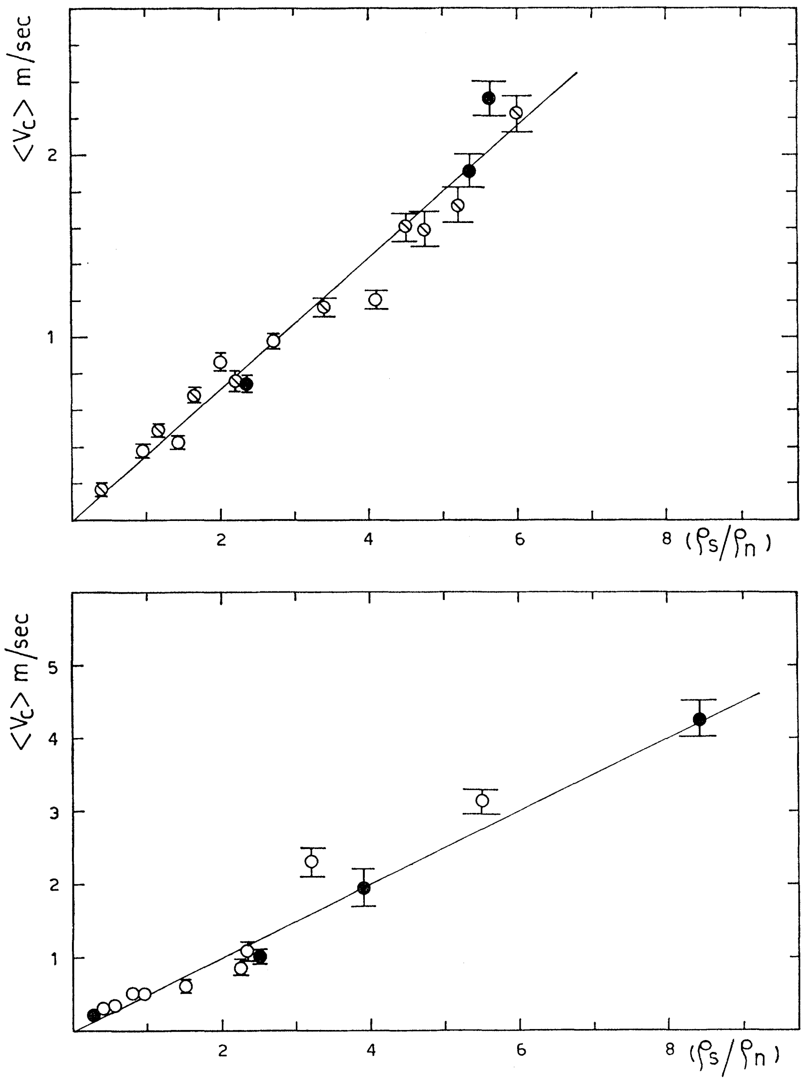

FIG. 9. $\langle v_c\rangle$ for negative ions (upper part) and for positive ions (lower part) plotted against the (ρ_s/ρ_n) ratio for temperatures above $\simeq 1.5°K$.

normal-fluid densities. Figure 9 shows the data of the second region plotted as a function of ρ_s/ρ_n.[18] The errors on the values of ρ_n/ρ and ρ_s/ρ_n in Figs. 8 and 9 are due to an uncertainty in temperature.

The equations which we have deduced from the data are the following:
positive ions:

$$\langle v_c\rangle = 5.2 - 120[(\rho_n/\rho) - (\rho_n/\rho)_0] \text{ m/sec}, \quad \text{for } 1.14°\text{K} \leq T \leq 1.35°\text{K}; \quad (4)$$

$(\rho_n/\rho)_0$ is the value of the ratio at 1.14°K, 1.7×10^{-2};

$$\langle v_c\rangle = 0.5(\rho_s/\rho_n)\ m/\text{sec}, \quad \text{for } T > 1.5°\text{K up to } \lambda; \quad (5)$$

negative ions:

$$\langle v_c\rangle = 2.4 - 36[(\rho_n/\rho) - (\rho_n/\rho)_0] \text{ m/sec}, \quad \text{for } 1.31°\text{K} \leq T \leq 1.5°\text{K}; \quad (6)$$

$(\rho_n/\rho)_0$ is the value of (ρ_n/ρ) at 1.31°K, 5.1×10^{-2};

$$\langle v_c\rangle = 0.36(\rho_s/\rho_n) \text{ m/sec}, \quad \text{for } T > 1.5°\text{K up to } \lambda. \quad (7)$$

As previously mentioned, the periodic character has has been confirmed in the whole temperature range investigated. The mean ratio $\langle v_c\rangle_2/\langle v_c\rangle_1$ between the second and the first critical velocities has the value 2.0 ± 0.2.

A plot of the relative change of the mobility $\Delta\mu/\mu$ versus temperature is shown in Fig. 10. The points appear to be statistically distributed around a mean value of 5% for the positive ions and 4% for negative ions. Each measurement of the mobility can be made with an error not larger than $\pm 1.5\%$. On the other hand, when we assign a discontinuity we look at the mean mobility levels obtained by several measurements at different electric-field values. We can therefore statistically attribute an error not greater than $\pm 1\%$ to the level of the steps. The error in $\Delta\mu/\mu$ is therefore

[18] J. G. Dash and R. D. Taylor, Phys. Rev. **105**, 7 (1957).

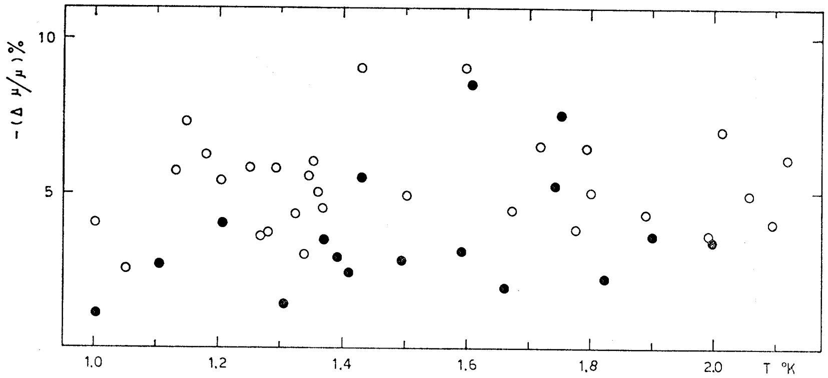

FIG. 10. The values of the relative size of the discontinuity $\Delta\mu/\mu$ for the first step at various temperatures. The errors—discussed in the text—are of 50% and are not indicated for graphical clarity. Black points refer to negative ions, white points to positive ions.

±50%. When the change $\Delta\mu/\mu$ was very small, we repeated the measurements many times at some fixed value of the electric field. In this way the error of the level can be further reduced by taking the mean value. As mentioned above, we used a low extracting field whenever possible. Nevertheless, the change $\Delta\mu/\mu$ was often rather small. It appears that the $\Delta\mu/\mu$ value is due to some effect which we cannot control from the outside while leaving the critical velocity unaffected.

DISCUSSION

(1) At this point we wish to summarize our major results, and then discuss the existing theories in terms of these.

(a) No steps have been observed above T_λ within the observable range of velocities, 20 to 100 cm/sec. It is possible that steps exist outside of this range. However, the phenomenon does not lose its periodic feature over the entire temperature range in which it was observed. Therefore, if above T_λ a critical velocity below 20 cm/sec exists, one should find its multiples above 20 cm/sec. Thus, if the phenomenon does not change its nature in crossing the λ point, only a critical velocity larger than 1 m/sec is possible. On the other hand we have seen that the critical velocity depends in a very simple way on the normal and superfluid densities, namely through linear functions. This fact, together with the probable lack of discontinuity above T_λ, leads us to believe that the steps are connected with the properties of liquid helium II.

(b) These experiments investigate a range in which the normal-fluid density changes by a factor of 300. In this range the phenomenon preserves its fundamental properties: It remains periodic, and the ratio $\Delta\mu/\mu$ appears to be constant, within experimental errors. The effect of the normal fluid is found only in the critical velocity. Approaching the λ point it is the critical velocity $\langle v_c\rangle$ which approaches zero, and not the relative discontinuity $\Delta\mu/\mu$.

(c) The ion's diameter governs not only the absolute value of $\langle v_c\rangle$ but also its temperature dependence. The temperature dependence of $\langle v_c\rangle$ starts at different values for positive and negative ions.

(2) We shall now reanalyze the models so far proposed in light of these new experimental results. We want to emphasize that the phenomenon is the same at 1°K as near the λ point. Therefore a satisfactory explanation cannot be limited to a narrow temperature range, but must be applicable over the whole range investigated. We have two experimental quantities to consider: the size of the steps $\Delta\mu/\mu$, and the critical velocities $\langle v_c\rangle$. We consider first the theoretical model proposed by Cope and Gribbon.[5] We think that this model is unsatisfactory at high temperatures, for the following reasons: First, the mechanism considered by the authors is the ion-roton collision. This is meaningful, in our opinion, only in the regime of kinetic theory. On the other hand, we have found steps up to 2.1°K, where the zero-field mobility follows Stokes's law, rather than kinetic theory.[19] Second, we think it would be difficult to explain the temperature dependence of $\langle v_c\rangle$, either treating the ion as a spherical harmonic oscillator, or as an acoustic oscillator. Indeed, both the effective mass of the ion and the velocity of the first sound change too little to account for the approach of $\langle v_c\rangle$ to zero in the first and in the second region. Third, one can show (see Appendix) that $\Delta\mu/\mu$, if calculated from the Cope-Gribbon (CG) model, is proportional to $\langle v_c\rangle^{-1}$. Taking for $\langle v_c\rangle$ the experimental values for positive ions, for example, one finds that $\Delta\mu/\mu$ should increase from approximately 5% to 35% in going from 1 to 1.33°K, in disagreement with the experiments. An analogous wrong increase in the $\Delta\mu/\mu$ is predicted also above 1.5°K for temperatures where kinetic theory is applicable.

(3) Next, we consider the models which try to explain the steps in terms of quantized vortex ring.[1,8,9] In these models one supposes that the ion, when it reaches the critical velocity $\langle v_c\rangle$, creates a vortex ring which must have the same drift velocity as the ion. The radius of the vortex ring is therefore imposed by the value of the critical velocity $\langle v_c\rangle$. Now at temperatures around the minimum of the first region, and near the λ point, where $\langle v_c\rangle$ reaches very low values, vortex rings as large as 3000 Å should be created. One could overcome this difficulty if one supposes the existence of an

internal velocity field in the superfluid, in such a way that the conditions for the creation process are not fixed only by the critical drift velocity of the ion. In this way the radius of the ring could be the same over the whole temperature range. We shall discuss this point later.

We want to show, for the moment, that even if we were able to explain the temperature dependence of $\langle v_c \rangle$ we would still be in serious trouble with $\Delta\mu/\mu$. If we calculate $\Delta\mu/\mu$ with the Huang-Olinto (HO) model, we have (see Appendix)

$$\Delta\mu/\mu \simeq 5.10^{-1}(\rho_s/\rho)(\mu_0/\langle v_c \rangle)\,,$$

where μ_0 is the zero-field mobility in cm^2/V sec and $\langle v_c \rangle$ is in cm/sec. The calculated $\Delta\mu/\mu$ has the right order of magnitude around 0.9°K, but decreases rapidly with increasing temperature, reaching a nearly constant value of 5×10^{-4}, 100 times smaller than the observed values, between 1.4°K and λ.

The ratio $\Delta\mu/\mu$ could be put in quantitative agreement with experiment if one assumes that the ion creates many vortices in its drift motion, as in the model proposed by Di Castro. With this assumption we have the following difficulty. In the models dealing with vortex rings, the radius of the created vortex rings is fixed by the ion diameter. Therefore the ion can create vortex rings only when it has a well-defined drift velocity, the critical drift velocity $\langle v_c \rangle$. It cannot create at drift velocities greater than $\langle v_c \rangle$. But if multiple creation is postulated, then the drift velocity $\langle v \rangle$ can never get past the threshold value $\langle v_c \rangle$, at least until the whole set of vortex rings are created. This model gives no mechanism by which one can justify the hypothesis of a multiple creation at a constant drift velocity $\langle v_c \rangle$, and at the same time account for the existence of drift velocities greater than $\langle v_c \rangle$. The thermal fluctuation of the ion velocity could provide the necessary mechanism, but detailed calculations are still lacking.

In the model of Careri and co-workers[1] the vortex was pictured to be bound in some way to the ion. As already indicated by the authors,[1] the main difficulty was to justify the stability of the complex. However, the stability condition developed by Huang and Olinto makes it clear that the ion-ring complex cannot be stable at low electric fields.

All the models analyzed above were advanced when the available results were those in a narrow temperature range around 1°K, with a constant critical velocity. From the above considerations it appears now that none of the models can be simply extrapolated to high temperatures. We think that completely new ideas are needed.

(4) The present results have led us to a new approach, and we wish to outline it here. The observation that $\Delta\mu/\mu$ is constant, within experimental error, over the whole temperature range, suggests that, for drift velocities greater than $\langle v_c \rangle$, the ion suffers an extra interaction with the normal fluid, proportional to the dissipative force which determines the zero-field mobility. This is in agreement with the conclusion drawn from the heat-flush experiment at low temperatures.[3] On the other hand, we think that an explanation of the critical velocities should be found in the study of the properties of the superfluid velocity field created by the ion motion. If this is true, one should find a temperature-independent critical velocity, a function of the ion diameter only. This is in fact the case at low temperature, below 1.1°K. Furthermore, after the "transition" from the first to the second region, $\langle v_c \rangle$ again takes on the same value it had at low temperatures. This lead us to believe that a new dissipative process against the normal fluid occurs when the superfluid velocity field reaches a characteristic flow pattern, i.e., when it reaches particular values of velocity point by point. This flow pattern is reached when the ion reaches a well-defined drift velocity v', where $v' = \langle v_c \rangle$ at low temperatures. But $\langle v_c \rangle$ does not remain temperature-independent. Let us look at temperatures above 1.5°K. From the experiment we have

$$\langle v_c \rangle = v^* \rho_s/\rho_n\,, \tag{8}$$

where v^* is a temperature-independent velocity. Equation (8) is analogous to the familiar relation between the normal and superfluid velocity in a thermal counterflow process,

$$v_n = v_s \rho_s/\rho_n\,.$$

This analogy suggests to us that the superfluid velocity field, in this temperature range, is determined by a counterflow process. We suppose that the ion drags the normal fluid around it, with a density $\beta\rho_n$, and because the ion is thermalized, there should be a superfluid field v_s given by

$$\beta\langle v \rangle \rho_n = v_s \rho_s\,.$$

When v_s reaches the temperature-independent critical value v', we observe a critical drift velocity $\langle v_c \rangle$ for the ion,

$$\langle v_c \rangle = (v'/\beta)\rho_s/\rho_n\,.$$

The value of β for the positive ions is about 10. The normal density around the ion should in fact be greater than ρ_n because of electrostriction. It seems a reasonable hypothesis that the ion in motion drags along some normal fluid. Indeed the roton-roton mean free path becomes shorter than the ion diam above 1.5°K, This hypothesis is also supported by the results of effective-mass measurements made by Dahm and Sanders.[19] If we plot their values of effective mass as a function of ρ_n (Fig. 11), we obtain a linear dependence, within experimental errors, as if a constant volume of normal fluid were dragged by the ion. We want to remark that present results above 1.5°K and our interpretation are not in disagreement with those obtained with experi-

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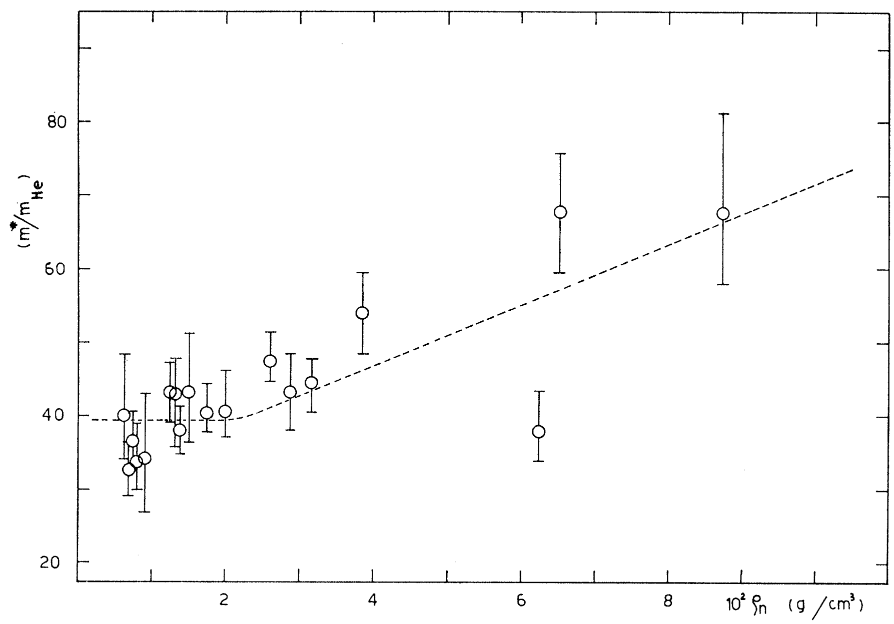

FIG. 11. The values of the effective mass of positive ions found by Dahm and Sanders (Ref. 19) plotted as a function of ρ_n.

ments with pure superfluid flow.[20,21] The critical velocities observed in these experiments are temperature-dependent only very close to the λ point. Those of Clow and Reppy,[21] in particular, are proportional to the superfluid density ρ_s. We cannot measure very low drift velocities, and so we cannot approach as close to the λ point as did Clow and Reppy[21]. We therefore do not know whether our critical velocity is still proportional to ρ_s/ρ_n in the Clow-Reppy region. Nevertheless, we want to observe that the two quoted experiments concern pure superfluid flow, while in our situation the normal fluid is also free to move. Moreover, our characteristic width is of the order of the ion diameter, much smaller than those reported in the quoted experiments.

(5) The dip of the critical velocity is the most difficult point to be explained. No properties of liquid helium are known which can account for so sharp a transition. However, in a tentative way, we would like to retain the hypothesis that the steps occur when a well-defined, temperature-independent superfluid velocity v' is reached. Then an "internal" velocity field is required in this region as well. From experimental data one can show that the "internal" velocity should be

$$v_{\rm int} \propto (\rho_n - \rho_{n0}),$$

where ρ_{n0} is the value of ρ_n at the onset of temperature dependence (i.e., 1.13°K for positive ions and 1.30°K for negative ones). A superfluid velocity field linearly dependent on ρ_n could be the one suggested by Feynman, arising from the polarization of rotons.[22] Increasing the temperature, the roton density increases "until a point is reached where spontaneous polarization appears." If there is an average background velocity $\mathbf{v}_s$, and a roton polarization $\mathbf{P}$, the actual velocity at any point is, according to Feynman,

$$\mathbf{w} = \mathbf{v}_s + \alpha \mathbf{P} \rho_0^{-1}.$$

In our process the superfluid background velocity $\mathbf{v}_s$ field is supplied by the backflow of the ion. If we assume that the polarization $\mathbf{P}$ is a function of the roton density, then the internal velocity field turns out to be a function of ρ_n. Our critical velocities start to depend on ρ_n at temperatures where the roton energy gap Δ starts to decrease, and this decrease has been explained in terms of roton-roton interaction.[17]

With an increase in temperature, the contribution of the internal field becomes greater and greater, until a temperature is reached where the critical superfluid field is produced by a very small drift velocity. Above this temperature, where the internal velocity field itself should be greater than the critical velocity v', it is not clear what should happen. However, if the critical velocity v' is fixed by the ion's dimension and cannot be increased, then the creation of the steps can not take place through the contribution of the two velocity fields. Experimentally one sees that $\langle v_c \rangle$ jumps in a very narrow temperature range to the low-temperature value.

(6) To conclude, the new data reported in this paper are in agreement with the previous experimental work on discontinuities in the mobility of ions in liquid helium.[1,3,5] However, it has become more difficult to understand these phenomena, because the existing theoretical approaches are shown to be inadequate to describe the present results at high temperatures. We

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[21] J. R. Clow and J. O. Reppy, Phys. Rev. Letters **19**, 291 (1967).
[22] R. P. Feynman, Phys. Rev. **94**, 262 (1954).

have proposed a possible explanation of the temperature dependence of the critical velocity $\langle v_c\rangle$, using the assumption of suitable internal velocity field in the superfluid. In this way, a temperature-independent critical superfluid velocity is preserved, even if the critical drift velocity of the ion depends on temperature. However, the true nature of the phenomenon and the detailed mechanism for the extra dissipation are still far from understood.

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APPENDIX A: $\Delta\mu/\mu$ FROM THE COPE-GRIBBON MODEL

In a head-on ion-roton collision in which the roton momentum is reversed, we have a decrease in the ion velocity given by

$$2p_0/m^*, \tag{A1}$$

where p_0 is the roton momentum, and m^* is the effective mass of the ion. After the collision, the ion velocity increases toward its terminal velocity with a time constant τ, the collision time. The slower the recovery, the greater will be the effect of the exciting collision on the average drift velocity. Every exciting collision should therefore produce a mean decrease Δv in $\langle v\rangle$ proportional to τ. If n_e is the number of exciting collisions per second, one has for the mean decrease $\langle\Delta v\rangle$

$$\langle\Delta v\rangle=(\beta 2p_0/m^*)\tau n_e. \tag{A2}$$

Now n_e should be simply proportional to the total number of ion-roton collision in a second N_{ir},

$$n_e=\gamma N_{ir}, \tag{A3}$$

where γ does not depend on N_{ir}, and is therefore a temperature-independent number. We have

$$\langle\Delta v\rangle=\gamma\beta(2p_0/m^*)\tau N_{ir}. \tag{A4}$$

Now $\tau N_{ir}=1$, and therefore

$$\Delta\mu/\mu=\langle\Delta v\rangle/\langle v\rangle=\gamma\beta 2p_0/m^*\langle v\rangle.$$

Around the transition velocity $\langle v_c\rangle$,

$$\Delta\mu/\mu\propto 2p_0/m^*\langle v_c\rangle.$$

APPENDIX B: $\Delta\mu/\mu$ FROM THE HUANG-OLINTO MODEL

The drift velocity $\langle v\rangle$ which we measure is

$$\langle v\rangle=\frac{d}{\tau_t}=\frac{\tau_c\langle v_c\rangle+(\tau_t-\tau_c)\mu_0E}{\tau_t}=\mu_0E+\frac{\tau_c}{\tau_t}\times[\langle v_c\rangle-\mu_0E], \tag{B1}$$

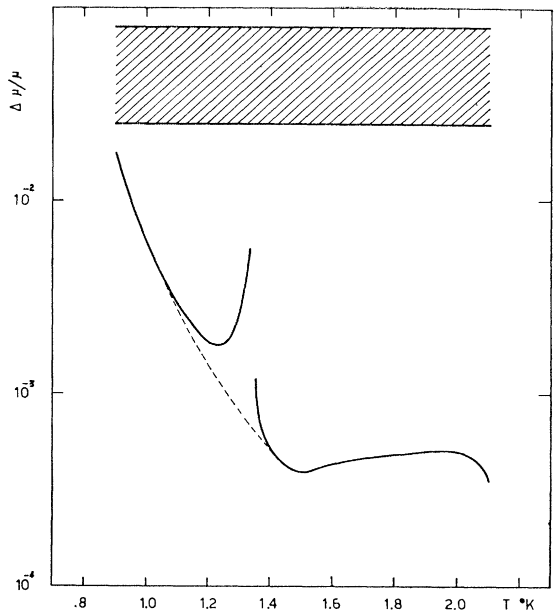

FIG. 12. The ratio $\Delta\mu/\mu$ calculated from the Huang-Olinto model using experimental data for $\langle v_c\rangle$. Our experimental values of $\Delta\mu/\mu$ fall in the shaded region.

where d is the grid-to-collector distance, τ_t is the measured time of flight, τ_c is the creation time, $\langle v_c\rangle$ is the critical velocity, and E is the applied electric field. One has

$$\bar{\mu}=\frac{\langle v\rangle}{E}=\mu_0-\frac{\tau_c}{\tau_t}\left[\mu_0-\frac{\langle v_c\rangle}{E}\right],$$

$$\frac{\Delta\mu}{\mu}=\frac{\mu_0-\bar{\mu}}{\mu_0}=1-\frac{\bar{\mu}}{\mu_0}=\frac{\tau_c}{\tau_t}\left[1-\frac{\langle v_c\rangle}{\mu_0E}\right],$$

$$\frac{\Delta\mu}{\mu}=\frac{\tau_c}{\tau_t}\left[\frac{E-E_c}{E}\right], \tag{B2}$$

where E_c is the critical electric field. The energy ϵ of the vortex ring, supplied by the electric field, is given by the HO expression:

$$\epsilon=e(E-E_c)\tau_c\langle v_c\rangle. \tag{B3}$$

The time of flight is given, as a first approximation, by

$$\tau_t=d/\mu_0E. \tag{B4}$$

From (B3) and (B4) we have

$$\tau_c/\tau_t=\epsilon\mu_0E/e(E-E_c)\langle v_c\rangle d, \tag{B5}$$

and from (B2) and (B5)

$$\Delta\mu/\mu=\epsilon\mu_0/ed\langle v_c\rangle. \tag{B6}$$

If the energy of the vortex ring is ϵ_0 at 1°K, where the density of the superfluid component ρ_s is practically the total density ρ, then at higher temperatures we have

$$\epsilon=\epsilon_0\rho_s/\rho \tag{B7}$$

to a good approximation, because of the slow change of the total density ρ from 1°K to the λ point. From (B6) and (B7) we have finally

$$\frac{\Delta\mu}{\mu}=\frac{\epsilon_0}{ed}\left(\frac{\mu_0}{\langle v_c\rangle}\right)\left(\frac{\rho_s}{\rho}\right). \quad \text{(B8)}$$

Taking $\epsilon_0=0.22$ eV, $d=0.4$ cm, one gets

$$\Delta\mu/\mu=5.5\times10^{-1}(\rho_s/\rho)(\mu_0/\langle v_c\rangle). \quad \text{(B9)}$$

The ratio $\Delta\mu/\mu$ calculated from (B9) for positive ions, using the experimental data for μ_0 and $\langle v_c\rangle$, is plotted in Fig. 12.

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Flow of Superfluid Helium in a Porous Medium

J. B. MEHL* AND W. ZIMMERMANN, JR.
Tate Laboratory of Physics, University of Minnesota, Minneapolis, Minnesota
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Flow of superfluid liquid helium has been studied experimentally in a porous medium formed by packing fine powder into a hollow glass sphere. The experiments fall into two categories, (1) the study of torsional oscillations of such a sphere filled with helium II as a function of temperature, and (2) the study of persistent circulating flow of the superfluid in such a sphere as a function of temperature and angular velocity of preparation, using a gyroscopic technique. The results at low angular velocities are shown to be in accord with an analysis of the motion of the liquid based on the two-fluid model of helium II and indicate that the superfluid can exhibit dissipationless potential flow in a porous medium with circulation constants which are remarkably stable.

I. INTRODUCTION

THE flow of liquid helium II in fine channels is of considerable interest because while the motion of the normal component of the liquid is increasingly impeded as channel size is reduced, owing to its viscosity, the ability of the superfluid component to flow with little or no dissipation is enhanced. A convenient method of forming fine channels for the study of this flow is to pack a fine powder into a container or tube. When this is done the open space remaining in the container assumes a complicated multiply connected geometry, and a network of interconnected flow channels is formed. This paper describes two related sets of experiments which we have carried out to study the flow of helium II inside a powder-filled sphere and presents an analysis of the flow taking place in these experiments based on the two-fluid model.

The first type of experiment represented a modification of the classic Andronikashvili oscillating-pile-of-disks experiment[1] in which the usual pile of disks was replaced by the powder-filled sphere. In effect, these experiments yielded measurements of the angular momentum imparted to the liquid inside the sphere as the sphere was put into rotation from rest at a temperature below T_λ, the temperature of the lambda transition.

The second set of experiments involved the use of a gyroscopic technique to study the behavior of persistent metastable circulating flow of the superfluid in the powder-filled sphere. Ever since the discovery of superfluidity in liquid helium, it has been natural to wonder whether the superfluid can flow with identically zero dissipation. A particularly sensitive test for the presence of dissipation can be made by preparing a coasting circulating current of superfluid and measuring the rate at which the flow decays. When the present work was undertaken, a number of experiments had already provided evidence that such circulating currents can have lifetimes much larger than the expected lifetimes of similar currents in helium I.[2–6] The gyroscopic technique developed in the present work,[7] and an elegant alternative gyroscopic technique developed by Reppy and co-workers,[8,9] have made it possible to establish in a particularly clear-cut way, the existence of persistent superfluid flow by providing a means of making repeated observations of the angular momentum of the flow without destroying the flow.

In our experiments of this type, long-lived circulating

* Present address: Department of Physics, University of Oregon, Eugene, Oregon 97403.

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