

**Gauge Fields, Sources, and Electromagnetic Masses**, JULIAN SCHWINGER [Phys. Rev. **165**, 1714 (1968)]. The following *Note added in proof* was omitted in the printing:

(1) The discussions in the text are simplified by using the electromagnetic action term

$$w_e = \frac{1}{2}(m_\rho^2 e/g)^2 \int (dx)(dx') \rho_3^\mu(x) D_{\mu\nu}(x-x') \rho_3^\nu(x'),$$

where the property

$$\partial_\mu D^{\mu\nu}(x-x') = 0$$

guarantees the conserved nature of the photon source described by  $\rho_3^\mu$ . The momentum form of  $D_{\mu\nu}$  is

$$D_{\mu\nu}(p) = (p^2 - i\epsilon)^{-1} (g_{\mu\nu} - p_\mu p_\nu / p^2).$$

As an example, we now have

$$g^2 \delta G_{\mu\nu}(p)_\rho = e^2 [m_\rho^2 / (p^2 + m_\rho^2)]^2 D_{\mu\nu}(p),$$

while the chiral computation of  $\delta m_\pi^2$  becomes

$$\delta m_\pi^2 = 2e^2 m_\rho^2 (1/i) \int \frac{(dp)}{(2\pi)^4} D_{\mu\nu}(p) [G^{\mu\nu}(p)_\rho - G^{\mu\nu}(p)_{(A)}],$$

which no longer requires the detailed compensation of longitudinal terms in the two Green's functions. Other applications of  $D_{\mu\nu}$  occur in J. Schwinger, Phys. Rev. Letters **19**, 1154 (1967), which is later referred to as PMFF.

(2) As an alternative to introducing the empirical coupling constant  $g'$  in  $A_1\rho\pi$  coupling, one can add the coupling term

$$-\gamma(g/m_A)\rho^{\mu\nu} \cdot A_{1\mu} \times \partial_\nu \phi,$$

as mentioned in PMFF. This gives no effect in the limit  $(m_\pi/m_\rho)^2 = 0$ , and leaves the predicted mass splitting for that circumstance at 5.0 MeV.

(3) In examining finite  $(m_\pi/m_\rho)^2$  corrections we considered  $\rho + \pi$  exchange, but not  $\rho + (A_1 + \pi)$  exchange. Had we done so we would have found a logarithmically divergent result. This is discussed in PMFF from the view point of chiral transformations. The remedy proposed there is the recognition of a phenomenological form factor connecting  $\rho^0$  and the photon, which is effective also in producing finite electromagnetic modifications of nuclear  $\beta$  decay. There is a related proposal of T. D. Lee (to be published) who suggests that all photon

couplings pass through an intermediate neutral meson. This is done, presumably, since Lee's field algebra method, with its unacknowledged ambivalence between phenomenological and fundamental descriptions, has no other way to introduce an effective photon form factor. The avowedly phenomenological source theory is not so limited and allows one to consider the more plausible possibility that the  $\rho\gamma$  form factor describes the exchange of known particles, rather than a new one. A re-examination of the  $\pi$  electromagnetic mass problem suggests that the significant masses in the form factor are  $\sim 5m_\rho$  [Tung-Mow Yan (to be published)].

(4) General confirmation of the  $\rho\gamma$  coupling description comes from the clashing beam experiments of Auslander *et al.* [Phys. Letters **25B**, 433 (1967)], who clearly see the intermediary  $\rho^0$  meson in  $e^+ + e^- (\rightarrow \gamma \rightarrow \rho^0 \rightarrow) \pi^+ + \pi^-$ . They express their results through the resonance factor

$$km_\rho^4 / [(p^2 + m_\rho^2)^2 + m_\rho^2 \Gamma^2],$$

$$k = 0.59 \pm 0.15, \quad \Gamma = 93 \pm 15 \text{ MeV},$$

and also state the maximum cross section to be  $1.2 \pm 0.2 \mu\text{b}$ . The theoretical value of the latter is

$$\sigma = 4\pi (1/137)^2 (g^2/4\pi)^{-1} (m_\rho \Gamma)^{-1},$$

which assumes no relation between  $g$  and  $\Gamma$ . If the width is fixed at 93 MeV we infer

$$g^2/4\pi = 3.1 \pm 0.5,$$

to be compared with

$$g^2/4\pi = 3.4 \pm 0.2,$$

which is deduced from the strong-interaction relation  $2^{-1/2}(g/m_\rho) = f_0/m_\pi$  and the experimental  $s$ -wave  $\pi N$  scattering parameter  $f_0 = 0.84 \pm 0.03$ . The effective coupling constant of  $\rho \rightarrow \pi + \pi$  decay,

$$\hat{g} = (\frac{3}{4} + \frac{1}{4}\gamma)g$$

determines  $\Gamma$  and  $k = (\hat{g}/g)^2$ . The  $k$  measurements imply

$$\hat{g}/g = 0.77 \pm 0.10, \quad \gamma = 0.1 \pm 0.4,$$

and similar results are obtained from  $\Gamma$ . There is new experimental evidence for the  $A_1$  particle [R. Juhala *et al.*, Phys. Rev. Letters **19**, 1335 (1967)]. The measured width of  $120 \pm 15$  MeV indicates that  $\gamma = 0.5 \pm 0.1$ . A value of  $\gamma \sim 0.4$  seems compatible with the various experiments.