High-Energy Forward Double Exchange Production Cross Sections in the Quark Model

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Within the framework of the quark model an extended additivity assumption for products of quark amplitudes is used to obtain relations among some of the high-energy forward production cross sections for baryons belonging to the $SU(3)$ octet and decuplet and mesons belonging to the vector meson octet. In all the reactions considered, more than one unit of strangeness or charge is exchanged between incident and outgoing particles. Several relations are derived with and without assuming $SU(3)$ relations among quark scattering amplitudes. Of particular interest is that there are a few relations which do not require $SU(3)$ symmetry for quarks. They are

$$\tfrac{1}{4}\bar{\sigma}(K^-p\to K^+\Xi^-)=\bar{\sigma}(K^-p\to K^0\Xi^0)=\bar{\sigma}(K^-n\to K^0\Xi^-),$$
$$\tfrac{1}{4}\bar{\sigma}(K^-p\to K^+\Xi^{*-})=\bar{\sigma}(K^-p\to K^0\Xi^{*0})=\bar{\sigma}(K^-n\to K^0\Xi^{*-}),$$
$$\tfrac{1}{4}\sigma(K^-p\to K^{+*}\Xi^-)=\sigma(K^-p\to K^{0*}\Xi^0)=\sigma(K^-n\to K^{0*}\Xi^-),$$

where $\bar{\sigma}$ denotes the spin-averaged differential cross section in the forward direction. The last set of relations is only for the zero-helicity state of K^{+*} and K^{0*} produced in the forward direction. These are the relations which must be compared with experiment to decide whether the additivity assumption of products of quark amplitudes is valid in double exchange reactions, since $SU(3)$ seems to be badly broken on the quark level, at least for elastic scattering.

I. INTRODUCTION

OWING to the lack of a satisfactory dynamical theory to treat strong-interaction phenomena of elementary particles, it is highly desirable to have a simple model which can explain many aspects of strong interaction, and predict things starting from a set of simple and plausible assumptions. If the conclusions from the model agree with experiment, it is the task of a future complete theory to explain why it works. One such model which has consistently been successful in explaining the many facets of strong interaction is the quark model of hadrons. Recently various authors have applied a simplified quark model to high-energy elastic-scattering processes of hadrons and obtained several relations among their scattering amplitudes which agree remarkably well with experiment.[1–3] A few inelastic processes which do not involve the exchange of more than one unit of strangeness or one unit of charge[4,5] were also considered, where the agreement with available results was fair. In this article, we will study double-exchange hadron reactions. Such reactions where strange baryons belonging to the $SU(3)$ octet are produced have already been considered by Sarker,[6] who used a slightly different quark model. The comparison of our model with his and the advantages of our method will be considered in a later section.

In the original formulation of the model it is assumed that a meson is a quark-antiquark system and a baryon is a three-quark system. When a meson is scattered by a baryon, the quark and antiquark inside the meson are separately scattered by each quark of the baryon coherently, with the result that the meson-baryon forward-scattering amplitude can be written as a simple sum of the various quark-quark and quark-antiquark scattering amplitudes. Needless to say, the above model prohibits inelastic processes involving exchange of more than one unit of strangeness or charge between incident and final particles. On the other hand, some production processes for strange particles and baryon resonances belong to this type. In this paper we treat these reactions by assuming that in the forward direction they proceed as a result of the simultaneous occurrence of one quark-antiquark and one quark-quark process, if the initial state consists of a meson and a baryon. The forward reaction amplitudes for these processes are written as a sum of products of a quark-antiquark amplitude and a quark-quark amplitude. If we take the products of quark amplitudes as unknown parameters, we can find several relations among the meson-baryon forward-reaction amplitudes, since all of them are expressed in terms of a limited number of parameters. The number of parameters can be further limited by relating various quark-quark amplitudes among themselves and quark-antiquark amplitudes among themselves by assuming isospin invariance and $SU(3)$ invariance for interactions among quarks.

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[1] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz., Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)].

[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).

[3] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters 17, 412 (1966).

[4] H. J. Lipkin, F. Scheck, and H. Stern, Phys. Rev. 152, 1375 (1966).

[5] J. L. Friar and J. S. Trefil, Nuovo Cimento 49A, 642 (1967).

[6] A. Q. Sarker, Phys. Rev. 161, 1435 (1967).

II. OUTLINE OF THE MODEL

The forward amplitude for $A+B\rightarrow C+D$ is written $\langle CD|AB\rangle$. In order to illustrate the method of calculation and exhibit the physical assumptions in it, we give the outline of the calculation for the amplitude of the reaction $K^{-}+p\rightarrow K^{0}+\Xi^{0}$, i.e., $\langle K^{0}\Xi^{0}|K^{-}p\rangle$. It is assumed that the way in which the spins and isospins of the three quarks are coupled together to form the spin-isospin state of the baryon is represented by the totally symmetric wave function belonging to the **56** representation of $SU(6)$. The spatial state of the three quarks is taken to be an S^{3} state for all the baryons in the **56**. S-wave pseudoscalar and vector mesons are represented by $SU(6)$ wave functions belonging to the 35 representation.

So in this model, the spin-isospin wave functions of K^{0}, Ξ^{0}, K^{-}, and p are written as follows:

$$|K^{0}\rangle=(1/\sqrt{2})[\bar{\lambda}_{\uparrow}\mathcal{N}_{\downarrow}-\bar{\lambda}_{\downarrow}\mathcal{N}_{\uparrow}],$$

$$|\Xi_{+}^{0}\rangle=(1/\sqrt{18})[-2\lambda_{\uparrow}\mathcal{P}_{\downarrow}\lambda_{\uparrow}-2\lambda_{\uparrow}\lambda_{\uparrow}\mathcal{P}_{\downarrow}-2\mathcal{P}_{\downarrow}\lambda_{\uparrow}\lambda_{\uparrow}+\mathcal{P}_{\uparrow}\lambda_{\downarrow}\lambda_{\uparrow}+\mathcal{P}_{\uparrow}\lambda_{\uparrow}\lambda_{\downarrow}+\lambda_{\uparrow}\mathcal{P}_{\uparrow}\lambda_{\downarrow}+\lambda_{\uparrow}\lambda_{\downarrow}\mathcal{P}_{\uparrow}+\lambda_{\downarrow}\mathcal{P}_{\uparrow}\lambda_{\uparrow}+\lambda_{\downarrow}\lambda_{\uparrow}\mathcal{P}_{\uparrow}],$$

$$|K^{-}\rangle=(1/\sqrt{2})[\bar{\mathcal{P}}_{\uparrow}\lambda_{\downarrow}-\bar{\mathcal{P}}_{\downarrow}\lambda_{\uparrow}],$$

$$|p_{+}\rangle=(1/\sqrt{18})[2\mathcal{P}_{\uparrow}\mathcal{P}_{\uparrow}\mathcal{N}_{\downarrow}+2\mathcal{N}_{\downarrow}\mathcal{P}_{\uparrow}\mathcal{P}_{\uparrow}+2\mathcal{P}_{\uparrow}\mathcal{N}_{\downarrow}\mathcal{P}_{\uparrow}-\mathcal{P}_{\uparrow}\mathcal{P}_{\downarrow}\mathcal{N}_{\uparrow}-\mathcal{P}_{\uparrow}\mathcal{N}_{\uparrow}\mathcal{P}_{\downarrow}-\mathcal{P}_{\downarrow}\mathcal{P}_{\uparrow}\mathcal{N}_{\uparrow}-\mathcal{P}_{\downarrow}\mathcal{N}_{\uparrow}\mathcal{P}_{\uparrow}-\mathcal{N}_{\uparrow}\mathcal{P}_{\downarrow}\mathcal{P}_{\uparrow}-\mathcal{N}_{\uparrow}\mathcal{P}_{\uparrow}\mathcal{P}_{\downarrow}].$$

The subscript arrows on the quarks show their helicity states, arrow being up denoting positive helicity state and arrow being down indicating negative helicity states. The "+" subscript on Ξ^{0} and p shows that they are taken at their positive helicity state. We are able to change the above nonrelativistic spin decomposition into a helicity representation because of the simplifying assumption that the internal motion of the quarks within a meson or a baryon is negligible compared to the over-all motion of the quarks, as a result of the high velocity imparted to the composite particles taking part in the reaction.[5] The axis of quantization is taken to be the direction of motion of the composite particle. Helicity amplitudes must be considered since we are considering processes at very high energies so that relativistic speeds are involved. It is clear that in order that the reaction occur, the state consisting of $\bar{\mathcal{P}}$ in K^{-} and $\mathcal{P}$ in p_{+} (briefly $\bar{\mathcal{P}}\mathcal{P}$) must go over into $\bar{\lambda}\lambda$, with $\bar{\lambda}$ in K^{0} and λ in Ξ_{+}^{0}, and the state consisting of λ in K^{-} and $\mathcal{N}$ in p_{+} (briefly $\lambda\mathcal{N}$) must go over into $\mathcal{N}\lambda$, with $\mathcal{N}$ in K^{0} and λ in Ξ_{+}^{0}, simultaneously. If ξ_{i} for $i=1, 2, \cdots 6$ denote the six quark states, then

$$\langle K^{0}\Xi_{+}^{0}|T|K^{-}p_{+}\rangle=\langle K^{0}\Xi_{+}^{0}|K^{-}p_{+}\rangle=\langle\sum_{ij}a_{ij}'\xi_{i}\bar{\xi}_{j}\sum_{lmn}b_{lmn}'\xi_{l}\xi_{m}\xi_{n}|T\times|\sum_{qr}a_{qr}\xi_{q}\bar{\xi}_{r}\sum_{stu}b_{stu}\,\xi_{s}\xi_{t}\xi_{u}\rangle,$$

where a_{ij} and b_{ijk} are coefficients occurring in the $SU(6)$ wave functions of mesons and baryons, respectively. For example, $|a_{ij}|^{2}$ is the probability of finding the meson in the quark state $\xi_{i}\bar{\xi}_{j}$.

According to the extended additivity assumption, this can be written

$$\langle K^{0}\Xi_{+}^{0}|K^{-}p_{+}\rangle=36\sum_{ij,lmn}\sum_{qr,stu}a_{ij}'b_{lmn}'a_{qr}b_{stu}\times\langle il|qs\rangle\langle\bar{j}m|\bar{r}t\rangle\delta_{nu}.$$

Needless to say, $\langle il|qs\rangle$ and $\langle\bar{j}m|\bar{r}t\rangle$ are nonzero only when the quantum numbers corresponding to conserved quantities in strong interactions are the same in the initial and final states. In writing down the above equation we have used the fact that b_{lmn}' and b_{stu} are symmetric with respect to the interchange of their indices, the $SU(6)$ wave functions of the baryons being totally symmetric.

In the forward direction, helicities cannot flip and so the only nonzero amplitudes are $\langle K^{0}\Xi_{+}^{0}|K^{-}p_{+}\rangle$ and $\langle K^{0}\Xi_{-}^{0}|K^{-}p_{-}\rangle$. But by the space-reflection invariance of the strong interaction,

$$\langle K^{0}\Xi_{+}^{0}|K^{-}p_{+}\rangle=\langle K^{0}\Xi_{-}^{0}|K^{-}p_{-}\rangle.$$

Therefore, the spin-averaged differential cross section in the forward direction can be written

$$\bar{\sigma}(K^{-}p\rightarrow K^{0}\Xi^{0})=|\langle K^{0}\Xi_{+}^{0}|K^{-}p_{+}\rangle|^{2}\times F,$$

where F is the phase-space contribution to the differential cross section. $F=P_{\mathrm{c.m.}}^{\mathrm{out}}/SP_{\mathrm{c.m.}}^{\mathrm{in}}$, where S is the square of the c.m. energy; $P_{\mathrm{c.m.}}^{\mathrm{out}}$ and $P_{\mathrm{c.m.}}^{\mathrm{in}}$ are the c.m. momenta of the outgoing and ingoing particles, respectively. In deriving the above expression for F we have to assume $\langle K^{0}\Xi_{+}^{0}|K^{-}p_{+}\rangle$ is the relativistically invariant scattering amplitude.

III. RESULTS

The following relations are obtained for the forward reaction amplitudes:

$$\tfrac{1}{2}\langle K^{+}\Xi_{+}^{-}|K^{-}p_{+}\rangle=\langle K^{0}\Xi_{+}^{0}|K^{-}p_{+}\rangle=\langle K^{0}\Xi_{+}^{-}|K^{-}n_{+}\rangle, \quad (1)$$

$$\tfrac{1}{2}\langle K^{+}\Xi_{1/2}^{*-}|K^{-}p_{+}\rangle=\langle K^{0}\Xi_{1/2}^{*0}|K^{-}p_{+}\rangle=\langle K^{0}\Xi_{1/2}^{*-}|K^{-}n_{+}\rangle, \quad (2)$$

$$\tfrac{1}{2}\langle K_{0}^{+*}\Xi_{+}^{-}|K^{-}p_{+}\rangle=\langle K_{0}^{0*}\Xi_{+}^{0}|K^{-}p_{+}\rangle=\langle K_{0}^{0*}\Xi_{+}^{-}|K^{-}n_{+}\rangle. \quad (3)$$

Here $\Xi_{1/2}^{*-}$, for example, means that the spin component along the direction of Ξ^{*-} momentum is $\frac{1}{2}\hbar$. Similarly K_{0}^{+*} denotes that state of K^{+*} for which the component of K^{+*} spin along its direction of motion is zero.

Because of the invariance of the S operator with respect to parity transformation, all the above relations also hold true when the initial- and final-state helicities are both simultaneously reversed.

Relation (1) was already derived by Sarker.[6] It is important to note that our relation does not depend on any $SU(3)$ symmetry for interaction among quarks, whereas Sarker assumed $SU(3)$ invariance for the meson-quark interaction in his treatment of these reactions. Similarly relations (2) and (3) are also not based on any $SU(3)$ relations among quark-quark and quark-antiquark reaction amplitudes. We made use of only isospin symmetry for interaction among quarks and antiquarks.

If we write relations (1) and (3) in terms of $SU(3)$-invariant amplitudes in the t channel of the reactions, we get

$$\tfrac{1}{2}T_{27}=\tfrac{1}{2}(T_{27}+T_{10})=\tfrac{1}{2}(T_{27}-T_{10}). \tag{4}$$

This implies that in order to obtain the quark-model relations (1) and (3) we must assume that the "t-channel $SU(3)$ **10** amplitude" must be zero.

One can also obtain relations among isospin t-channel amplitudes from Eqs. (1) and (3), if we make an isospin analysis of the reactions. In terms of the isospin t-channel amplitudes, Eqs. (1) and (3) become

$$\tfrac{1}{2}I_1=\tfrac{1}{2}(I_1+I_0)=\tfrac{1}{2}(I_1-I_0). \tag{5}$$

This shows that the zero-isospin amplitude I_0 must be zero if relations (1) and (3) are valid.

Without using any quark model, but simply by assuming complete $SU(3)$ invariance for meson-baryon interaction, one can obtain the following relations:

$$\langle K^0\Xi^0|K^-p\rangle=\langle\pi^+\Sigma^-|K^-p\rangle, \tag{6}$$

$$\langle K^0\Xi^-|K^-n\rangle=\langle K^+\Sigma^-|\pi^-p\rangle. \tag{7}$$

One can check that our model also gives the same relations when we require $SU(3)$ symmetry relations among quark-quark and quark-antiquark amplitudes. With this requirement the model gives stronger equalities, equating each of the above reaction amplitudes in the forward direction, as it should be, if Eqs. (6) and (7) have to be consistent with relation (1). So in our model, the following set of relations holds true:

$$\begin{aligned}\tfrac{1}{2}\langle K^+\Xi_+{}^-|K^-p_+\rangle&=\langle K^0\Xi_+{}^0|K^-p_+\rangle\\&=\langle K^0\Xi_+{}^-|K^-n_+\rangle\\&=\langle\pi^+\Sigma_+{}^-|K^-p_+\rangle\\&=\langle K^+\Sigma_+{}^-|\pi^-p_+\rangle.\end{aligned} \tag{8}$$

Note that all of these are forward amplitudes.

An $SU(3)$ analysis in the t channel of reactions given in relation (2) also shows that T_{10} must be zero. Equations (2), when combined with pure $SU(3)$ equalities, produce another set of relations:

$$\begin{aligned}\tfrac{1}{2}\langle K^+\Xi_{1/2}{}^{*-}|K^-p_+\rangle&=\langle K^0\Xi_{1/2}{}^{*0}|K^-p_+\rangle\\&=\langle K^0\Xi_{1/2}{}^{*-}|K^-n_+\rangle\\&=\langle\pi^+Y_{1/2}{}^{*-}|K^-p_+\rangle\\&=\langle K^+Y_{1/2}{}^{*-}|\pi^-p_+\rangle.\end{aligned} \tag{9}$$

The model also directly gives all these relations. Of course, from relation (3) we can write down a further set of relations as follows:

$$\begin{aligned}\tfrac{1}{2}\langle K_0{}^{+*}\Xi_+{}^-|K^-p_+\rangle&=\langle K_0{}^{0*}\Xi_+{}^0|K^-p_+\rangle\\&=\langle K_0{}^{0*}\Xi_+{}^-|K^-n_+\rangle\\&=\langle\rho_0{}^+\Sigma_+{}^-|K^-p_+\rangle\\&=\langle K_0{}^{+*}\Sigma_+{}^-|\pi^-p_+\rangle.\end{aligned} \tag{10}$$

It is important to note that the first two relations in Eqs. (8)–(10) depend only on the particular quark model and not on any $SU(3)$ symmetry for the interaction.

Relations (8)–(10) can also be written in terms of $\bar{\sigma}$, the forward differential cross section summed over final-state helicities and averaged over initial helicities. They are

$$\tfrac{1}{4}\bar{\sigma}(K^-p\to K^+\Xi^-)=\bar{\sigma}(K^-p\to K^0\Xi^0) \tag{8$'$a}$$
$$=\bar{\sigma}(K^-n\to K^0\Xi^-) \tag{8$'$b}$$
$$=\bar{\sigma}(K^-p\to\pi^+\Sigma^-)\times F/F_{1c} \tag{8$'$c}$$
$$=\bar{\sigma}(\pi^-p\to K^+\Sigma^-)\times F/F_{1d}, \tag{8$'$d}$$
$$\tfrac{1}{4}\bar{\sigma}(K^-p\to K^+\Xi^{*-})=\bar{\sigma}(K^-p\to K^0\Xi^{*0}) \tag{9$'$a}$$
$$=\bar{\sigma}(K^-n\to K^0\Xi^{*-}) \tag{9$'$b}$$
$$=\tfrac{1}{4}\bar{\sigma}(K^-p\to\pi^+Y^{*-})\times F'/F_{2c} \tag{9$'$c}$$
$$=\tfrac{1}{4}\bar{\sigma}(\pi^-p\to K^+Y^{*-})\times F'/F_{2d}, \tag{9$'$d}$$
$$\tfrac{1}{4}\sigma(K^-p\to K^{+*}\Xi^-)=\sigma(K^-p\to K^{0*}\Xi^0) \tag{10$'$a}$$
$$=\sigma(K^-n\to K^{0*}\Xi^-) \tag{10$'$b}$$
$$=\sigma(K^-p\to\rho^+\Sigma^-)\times F''/F_{3c} \tag{10$'$c}$$
$$=\sigma(\pi^-p\to K^{+*}\Sigma^-)\times F''/F_{3d}. \tag{10$'$d}$$

In the above relations, F is the common phase factor which occurs in the reactions of relations (8′a) and (8′b). Similarly the phase factor F' is the same for all reactions occurring in relations (9′a) and (9′b). F'' is of course the phase factor common to the reactions involved in relations (10′a) and (10′b). The other phase factors, F_{1c}, F_{1d}, F_{2c}, etc., are self-explanatory.

It is to be noted that relation (10′) is only for the zero-helicity state of the final-state vector mesons.

IV. CORRECTIONS AND COMPARISON WITH EXPERIMENT

Now the question arises of how to compare relations (8′)–(10′) with experiment. When comparing them with experiment we must bear in mind two complicating factors. The first is the mass difference between the initial and final states and between the different final states in related reactions. The second is the nonzero momentum transfer of the quarks even in the forward direction.

We assume that the first difficulty can be resolved to a large extent by comparing the related cross sections with experiment at the same Q values, Q being the kinetic energy of the outgoing particles.[7]

The form-factor correction due to the nonzero momentum transfer of quarks, in the case of double collisions, is complicated[8] and is difficult to estimate. However, it should be expected that relations (8′a), (8′b), (9′a), (9′b), (10′a), and (10′b) are insensitive to form-factor corrections because of the similarity of the particles involved. (Or, in other words, these corrections would be the same for all the reactions related by these equations.) So it is of considerable interest to compare these relations with experiment. Since they do not depend on any $SU(3)$ symmetry among quark interactions, this could give a decisive check as to whether the assumption that the total reaction amplitude is a sum of products of quark amplitudes is valid in double-exchange processes.

Unfortunately, all these reactions seem to proceed via baryon exchange in the u channel, with the result that there is a backward peak (that is, when the angle between initial-state meson M^i and final-state baryon B^f is zero) in the angular distribution of reaction products. In any of the available experimental data,[9] there are not enough events in the forward direction to compare the above relations meaningfully with experiment.

V. COMPARISON WITH SARKER'S MODEL AND COMMENTS

It is instructive to compare our model with Sarker's. He used a slightly different model in which he assumed that the baryons formed an $SU(3)$ octet of three quarks while the pseudoscalar mesons constituted another $SU(3)$ octet without any quark structure. The reaction proceeds because the meson makes two encounters with the quarks within the target baryon and transforms it into the final-state baryon, while the meson itself changes into the final-state meson. In each encounter of the meson with the target baryon quark, the interaction is assumed to be completely $SU(3)$ invariant so that all quark-meson scattering amplitudes can be expressed in terms of three parameters. Several of the results which we get are identical with his. On the other hand, some of our results do not depend on $SU(3)$ invariance for interaction among quarks and antiquarks, while he used $SU(3)$ symmetry for the meson-quark interaction to derive all his results. So, in our method, we are able to see which of the results depend only on the quark model and which of them involve further assumptions.

This improvement in the model seems to be highly important in view of the fact that $SU(3)$ symmetry is badly broken on the quark level for scattering processes. Recently Barnhill[10] has evaluated the quark-quark scattering amplitudes $t(\mathcal{P}\mathcal{P})$, $t(\mathcal{N}\mathcal{N})$, $t(\mathcal{P}\mathcal{N})$, and $t(\mathcal{P}\lambda)$ from known experimental data on elastic scattering of hadrons and he found that $t(\mathcal{P}\mathcal{N})$ is much larger than $t(\mathcal{P}\lambda)$, whereas they should be the same if $SU(3)$ symmetry is valid on the quark level. Also, since we use $SU(6)$ wave functions for all hadrons taking part in the reaction, we are able to take account of the spin dependence, if any, of quark amplitudes.

We would like to remark that Carter *et al.*[11] had already derived relations (8′) and (9′) using $SU(6)_W$ symmetry. However, the predictions of $SU(6)_W$ and the quark model are not the same. $SU(6)_W$ symmetry not only gives relations (8′) and (9′), but expresses all the cross sections appearing in Eqs. (8′), (9′), and (10′) in terms of a single parameter. Thus all the reaction cross sections involved in the above relations can be related by a single equation by $SU(6)_W$. It is to be noted that the relations which Carter *et al.* give in their table for vector-meson production cross sections are for unpolarized states of initial-baryon and final-vector meson. However, it can be verified that $SU(6)_W$ relations for production cross sections of zero-helicity vector mesons are different from our relation (10′). Since data for zero-helicity vector mesons are hard to obtain, it may be difficult to verify relations (10′) experimentally. Thus a practical test of the model in the near future is to check whether the relations predicted for unpolarized cross sections by both $SU(6)_W$ and the quark model are more compatible with experiment than those which can be derived by $SU(6)_W$ only.

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[7] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

[8] A. Deloff, Nucl. Phys. **B2**, 597 (1967).

[9] J. Badier *et al.* (unpublished); J. D. Jackson (unpublished); M. L. Stevenson, University of California Radiation Laboratories Report No. UCRL-11493 (unpublished).

[10] Maurice V. Barnhill III, Phys. Rev. **163**, 1735 (1967).

[11] J. C. Carter *et al.*, Phys. Rev. Letters **15**, 373 (1965).