CP Invariance and the Radiative Decay of the Long-Lived Neutral K Meson into Two Pions

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As a possible test to observe the effects of CP nonconservation, the radiative decay process $K_L^0 \to \pi^+ + \pi^+ + \gamma$ is discussed in detail. A method to find the effects is given by observing the dependences of the transition rate on the energy difference of the two pions and on the polarization of the γ ray. If CP nonconservation in the decay $K_L^0 \to 2\pi$ is due to the electromagnetic interaction of hadrons, the effects of the CP nonconservation on the radiative decay will be expected to be quite large. A rough estimate of the magnitude of the branching ratio of this process is also made. The arguments given here are easily adapted to the decays $K_L \to 2\pi^0+\gamma$, $K_S^0 \to 2\pi^0+\gamma$, and $K_S^0 \to \pi^+ + \pi^- + \gamma$.

1. INTRODUCTION

THE observation[1] of the interference effects between the short-lived and long-lived neutral K mesons, K_S^0 and K_L^0, establishes that CP invariance is violated in the decay $K_L^0 \to \pi^+ + \pi^-$. As the possible origins of the CP nonconservation in this decay, the semistrong interaction,[2] the electromagnetic interaction,[3] the weak interaction,[4] and a new superweak interaction[5] with $|\Delta S| = 2$ were proposed. Recent experiments performed by both the CERN group[6] and the Penn-Princeton Accelerator (PPA) group[7] in measuring the ratio of the decay rate $R(K_L^0 \to \pi^0 + \pi^0)$ to the rate $R(K_L^0 \to \pi^+ + \pi^-)$ definitely rule out the superweak interaction; also they show that the CP nonconservation in the decay $K_L^0 \to 2\pi$ is due to an amplitude with $|\Delta I| \geq \frac{3}{2}$.

We must further restrict the remaining three possible origins. Many suggestions have been made to test these possibilities; some experiments have been done, but no conclusive answer has been reached. In this paper we shall consider the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$ as a possible test from which we may get information about whether the electromagnetic interaction of hadrons violates CP invariance or not. This decay process has been discussed by Lee and Wu.[8] We shall study this process in more detail in a phenomenological manner.

Since the γ ray has two helicity states but the other particles have vanishing spin, the matrix element consists of two parts:

$$F_{\alpha\beta}(p_+)_\alpha(p_-)_\beta f_1(s,t)$$

and

$$\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}(p_+)_\gamma(p_-)_\delta f_2(s,t)\,,$$

where $F_{\alpha\beta} = i(k_\alpha \epsilon_\beta - k_\beta \epsilon_\alpha)$. The momenta of π^+, π^-, γ, and the polarization vector of γ are denoted by p_+, p_-, k, and ϵ, respectively. The structure functions f_1 and f_2 are functions of s and t, which are defined by

$$s = -(p_+ + p_-)^2\,,\quad t = -p_K(p_+ - p_-)\,, \tag{1.1}$$

where p_K is the four-momentum of K_L^0. The invariants s and t are even and odd, respectively, under charge conjugation. It is convenient to expand the structure functions f_i in terms of functions g_i and h_i which are even under charge conjugation:

$$f_i(s,t) = g_i(s,t^2) + (it/m_K^2)h_i(s,t^2)\,,\quad (i=1,2)$$

where m_K is the mass of the K_L^0. Then the general form of the matrix element is expressed as

$$\frac{1}{4(2\pi)^2}\frac{1}{m_K^6}\frac{1}{(\omega_K\omega_+\omega_-\omega)^{1/2}} \times\{F_{\alpha\beta}(p_+)_\alpha(p_-)_\beta[g_1(s,t^2)+(it/m_K^2)h_1(s,t^2)] +\tfrac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}(p_+)_\gamma(p_-)_\delta[g_2(s,t^2)+(it/m_K^2)h_2(s,t^2)]\} \times\delta^{(4)}(p_K-p_+-p_--k)\,, \tag{1.2}$$

where $\epsilon_{1230}=1$, and ω_K, ω_+, ω_-, and ω denote the energies of K_L^0, π^+, π^-, and γ, respectively.

The g_1 and h_1 terms in (1.2) are P nonconserving, but the g_2 and h_2 terms are P conserving. As far as the final states are concerned, the g_1 and h_2 terms have even eigenvalues, but the g_2 and h_1 terms have odd eigenvalues under CP transformation. Thus the existence of the g_1 term and/or the h_2 term means that CP is not conserved in the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$.

Let us denote the relative angular momentum between π^+ and π^- by l. Then the g_1 and g_2 terms relate with $l=(2n-1)$ states, but the h_1 and h_2 terms relate with $l=2n$ states, where n is a positive integer. Thus, in the former states $I=1$ but in the latter states $I=0$ or $I=2$, where I is the total isospin of the $(\pi^+ + \pi^-)$ system. The matrix element (1.2) holds also for the

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[1] V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters 15, 73 (1965).
[2] J. Prentki and M. Veltman, Phys. Letters 15, 88 (1965).
[3] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965); S. Barshay, Phys. Letters 17, 78 (1965).
[4] See for example J. Prentki, J. S. Bell, and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles, Oxford, England 1965* (Rutherford High-Energy Laboratory, Chilton, Berkshire, England, 1966).
[5] L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964).
[6] J. M. Gaillard, F. Krienen, W. Galbraith, A. Hussri, M. R. Jane, N. H. Lipman, G. Manning, T. Ratcliffe, P. Day, A. G. Parham, B. T. Payne, A. C. Sherwood, H. Faissner, and H. Reithler, Phys. Rev. Letters 18, 20 (1967).
[7] J. W. Cronin, P. F. Kunz, W. S. Risk, and P. C. Wheeler, Phys. Rev. Letters 18, 25 (1967).
[8] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 16, 567 (1966).

decay $K_S^0 \to \pi^+ + \pi^- + \gamma$. The matrix elements of the decays $K_L^0 \to 2\pi^0 + \gamma$ and $K_S^0 \to 2\pi^0 + \gamma$ are also described by the analogous form with (1.2). But in these cases $g_1 = g_2 = 0$.

In the rest system of K_L^0, we get

$$F_{\alpha\beta}(p_+)_\alpha(p_-)_\beta = \tfrac{1}{2} i m_K \omega \boldsymbol{\varepsilon} \cdot (\mathbf{p}_+ - \mathbf{p}_-) = i m_K |\mathbf{p}_+| |\mathbf{p}_-| \sin\theta \sin\varphi ,$$

$$\epsilon_{\alpha\beta\gamma\delta} F_{\alpha\beta}(p_+)_\gamma(p_-)_\delta = i m_K \boldsymbol{\varepsilon} \cdot [\mathbf{k} \times (\mathbf{p}_+ - \mathbf{p}_-)] = 2 i m_K |\mathbf{p}_+| |\mathbf{p}_-| \sin\theta \cos\varphi , \quad (1.3)$$

where θ and φ denote, respectively, the angles between the momenta $\mathbf{p}_+$ and $\mathbf{p}_-$ and between the polarization vector $\boldsymbol{\varepsilon}$ and the normal to the decay plane. These angles may take the values $0 \leq \theta \leq \pi$ and $0 \leq \varphi \leq \frac{1}{2}\pi$. The first and the second expressions in (1.3) relate to electric and magnetic field strengths, respectively. These expressions show that the observation of the polarization of the γ ray could give information about the CP nonconservation in the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$.

In the next section, the $(\omega_+ - \omega_-)$ distribution and the s distribution will be calculated. A way to determine the existence of the CP-nonconserving terms will be discussed in Sec. 3. It is well known that if the $(\omega_+ - \omega_-)$ distribution is asymmetric with respect to the $(\omega_+ - \omega_-) = 0$ axis, then CP is not conserved in the decay. Even if it is symmetric, it will be shown that both the shape of the $(\omega_+ - \omega_-)$ distribution and the observation of the polarization of the γ ray can uniquely give information about the existence of the g_1 term and/or h_2 term. It is also shown that the s distribution will help to identify the process. The last section will be devoted to making a rough estimate of the magnitude of the branching ratio of the decay under the assumption that the CP nonconservation in the decay $K_L^0 \to 2\pi$ is due to the electromagnetic interaction of hadrons.

If the electromagnetic interaction violates CP invariance so as to explain the observed magnitude of the decay amplitude of $K_L^0 \to 2\pi$, the CP-nonconserving terms in (1.2) would have about the same order of magnitude as the corresponding CP-conserving terms: $|g_1| \approx |g_2|$ and $|h_1| \approx |h_2|$. In this case, it would be relatively easy to observe the effects of the CP nonconservation in the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$. If it is not the case, $|g_1|^2 \approx \alpha |g_2|^2$ and $|h_2|^2 \approx \alpha |h_1|^2$, where α is he fine-structure constant. It is very hard in the latter case to observe the effects of the CP nonconservation in the decay. Thus, the process $K_L^0 \to \pi^+ + \pi^- + \gamma$ could give information about the possible origin of the CP nonconservation in the decay $K_L^0 \to 2\pi$.

2. DISTRIBUTIONS

In this section we shall calculate the $(\omega_+ - \omega_-)$ and the s distributions for the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$. The results will be easily translated to other cases, say, $K_L^0 \to 2\pi^0 + \gamma$, $K_S^0 \to \pi^+ + \pi^- + \gamma$, and $K_S^0 \to 2\pi^0 + \gamma$. The matrix element (1.2) leads in the rest system of K_L^0 to the transition rate,

$$d\omega_{2\pi+\gamma} = [m_K / 1024(2\pi)^3] dx dy dz \, \delta[x - \tfrac{1}{2}(1+z)] X Y , \quad (2.1)$$

where

$$x = (\omega_+ + \omega_-)/m_K, \quad y = t/m_K^2 = (\omega_+ - \omega_-)/m_K, \quad z = s/m_K^2,$$

$$X = z^3 - 2(1+2c^2)z^2 + (1+8c^2-4y^2)z - 4c^2 , \quad (2.2)$$

$$Y = a + 2by + dy^2 , \quad (2.3)$$

with $c = m_\pi / m_K$,

$$a = |g_1|^2 + |g_2|^2 + (|g_2|^2 - |g_1|^2)\cos 2\varphi + 2\,\mathrm{Re}(g_2^* g_1)\sin 2\varphi ,$$

$$b = \mathrm{Im}(h_1^* g_1 + h_2^* g_2) + \mathrm{Im}(h_2^* g_2 - h_1^* g_1)\cos 2\varphi + \mathrm{Im}(h_1^* g_2 + h_2^* g_1)\sin 2\varphi ,$$

$$d = |h_1|^2 + |h_2|^2 + (|h_2|^2 - |h_1|^2)\cos 2\varphi + 2\,\mathrm{Re}(h_2^* h_1)\sin 2\varphi . \quad (2.4)$$

Here, we denote by φ the angle between the normal to the decay plane and the polarization vector of the γ ray; it takes on values between zero and $\frac{1}{2}\pi$. The angle between the vectors $\mathbf{p}_+$ and $\mathbf{p}_-$ is denoted by θ. From the condition $\sin^2\theta \geq 0$, the inequality

$$X \geq 0 \quad (2.5)$$

is obtained. This inequality will be used later to determine the possible domains of y and z. The summation with respect to the polarization states of the γ ray gives

$$\sum_{\text{pol.}} Y = 2(|g_1|^2 + |g_2|^2) + 4\,\mathrm{Im}(h_1^* g_1 + h_2^* g_2) y + 2(|h_1|^2 + |h_2|^2) y^2 . \quad (2.6)$$

Until now, we have not used any approximation. If the small part of K_L^0 (the K_1^0 part) is neglected and the CPT theorem is assumed, as was discussed by Lee and Wu,[8] the structure functions g_1, g_2, h_1, and h_2 are relatively real except for the phase shifts which come from the final-state interactions. If the CP violation in the decay $K_L^0 \to 2\pi$ is due to the electromagnetic interaction of hadrons, the t^2 dependence of the structure functions may be neglected, because of the smallness of the effects of the inner-bremsstrahlung processes, and also because of the centrifugal-barrier effects.

In this approximation, the relative angular momenta between π^+ and π^- are $l=1$ and $l=2$ in the g_1 and g_2 terms and the h_1 and h_2 terms, respectively. Thus the first equation of (2.4) becomes

$$a = 2[(A_1^E)^2 + (A_1^H)^2] + 2[(A_1^H)^2 - (A_1^E)^2]\cos 2\varphi + 4 A_1^E A_1^H \sin 2\varphi , \quad (2.7)$$

where the same notation as that of Lee and Wu[8] is used. The upper suffix E or H of a real structure function A denotes that the A relates with electric field or magnetic field. The second and third equations of (2.4) are also found to be consistent with Lee and Wu.

A. y Distribution

Since $(1+8c^2-4y^2)\geq 0$, the equation $X=0$ has three real and positive roots z_1, z_2, and z_3. They are

$$z_1=\tfrac{2}{3}\{(1+2c^2)+[(1-4c^2)^2+12y^2]^{1/2}\cos\tfrac{1}{3}\rho\}\,,$$
$$z_2=\tfrac{2}{3}\{(1+2c^2)+[(1-4c^2)^2+12y^2]^{1/2}\cos\tfrac{1}{3}(\rho+2\pi)\}\,,$$
$$z_3=\tfrac{2}{3}\{(1+2c^2)+[(1-4c^2)^2+12y^2]^{1/2}\cos\tfrac{1}{3}(\rho-2\pi)\}\,, \tag{2.8}$$

where the angle ρ is defined by

$$\cos\rho=\frac{36(1+2c^2)y^2-(1-4c^2)^3}{[(1-4c^2)^2+12y^2]^{3/2}}\,. \tag{2.9}$$

Call the roots with the largest, the middle, and the smallest values z_l, z_m, and z_s; then the domain of z is given by

$$z_m\geq z\geq z_s\,. \tag{2.10}$$

The domain of y is given by

$$1\geq\cos\rho\geq -1\,,$$

which gives

$$y_0\equiv\tfrac{1}{4}[(2+c^2)^{1/2}+c]^{1/2}[(2+c^2)^{1/2}-3c]^{3/2}\geq|y|\geq 0\,.$$

Thus the expression (2.1) leads to the y distribution which is proportional to

$$\omega_{2\pi+\gamma}\propto\int_{-y_0}^{y_0}dy\,[(z_m^4-z_s^4)-(8/3)(1+2c^2)(z_m^3-z_s^3)$$
$$+2(1+8c^2-4y^2)(z_m^2-z_s^2)-16c^2(z_m-z_s)]Y\,. \tag{2.11}$$

To obtain this y distribution, the s dependence of the structure functions was neglected.

B. Z Distribution

The inequality $X\geq 0$ gives the upper limit of y^2, and the lower limit of y^2 is equal to zero for any z. Thus the possible domain of y^2 is given by

$$(1/4z)[z^3-2(1+2c^2)z^2+(1+8c^2)z-4c^2]\geq y^2\geq 0\,. \tag{2.12}$$

The possible domain of z is obtained from the condition $y^2\geq 0$. It gives

$$1\geq z\geq 4c^2\,.$$

Thus the expression (2.1) leads to the z distribution.

$$\omega_{2\pi+\gamma}=\frac{m_K}{1536(2\pi)^3}\int_{4c^2}^{1}dz\frac{1}{\sqrt{z}}[z^3-2(1+2c^2)z^2$$
$$+(1+8c^2)z-4c^2]^{3/2}\{a+(1/20)(d/z)[z^3-2(1+2c^2)z^2$$
$$+(1+8c^2)z-4c^2]\}\,. \tag{2.13}$$

To obtain this expression, the t^2 dependence of the structure functions was neglected.

3. TESTS FOR THE EFFECTS OF CP NONCONSERVATION

In this section we shall discuss methods to find the effects of CP nonconservation, if they exist, in the decay $K_L^0\to\pi^++\pi^-+\gamma$. As was discussed in Sec. 1, the four terms appearing in (1.2) transform differently into each other under C (or P) and CP transformations. Phenomenologically speaking, the relative magnitudes of the structure functions g_1, g_2, h_1, and h_2 may be arbitrary; however, the t^2 dependence of the structure functions may be neglected because of the centrifugal-barrier effects. Let us first consider the y distribution and for a while forget about the polarization of the γ ray. Then the y distribution is symmetric or asymmetric with respect to the $y=0$ axis.

Case A

If the distribution is asymmetric with respect to the $y=0$ axis, as is shown by (2.11) with (2.6), then there exists an interference effect between the g_1 and h_1 terms and/or between the g_2 and h_2 terms; that is, it shows the existence of the CP-nonconserving g_1 term and/or h_2 term.

If the asymmetry is not observed, that may be for many reasons. The sine factors $\sin(\delta_0-\delta_1)$ and $\sin(\delta_1-\delta_2)$ included in $\mathrm{Im}(h_1{}^*g_1)$ and $\mathrm{Im}(h_2{}^*g_2)$ may be small. (Here, δ_I is the phase shift due to final-state interaction of the two pions with total isospin I in their lowest-l state; see Ref. 8.) Even if they are not so small, the centrifugal-barrier effects may suppress the effects of the h_1 term and/or the h_2 term. Another possibility is that both the sine factors and the structure functions h_1 and h_2 are large enough, but occasionally it happens that $\mathrm{Im}(h_1{}^*g_1+h_2{}^*g_2)\approx 0$. Of course there remains the possibility that the CP nonconservation in the decay $K_L^0\to 2\pi$ is due to a nonelectromagnetic interaction.

At any rate, if the y distribution is symmetric with respect to the $y=0$ axis, we shall classify the distribution in the following three categories and discuss a way to find the effects of the CP nonconservation in our process by observing the polarization of the γ ray.

Case B

The effects of the h_1 and h_2 terms in (1.2) to the y distribution are negligible compared with those of the g_1 term and/or the g_2 term. Curve B in Fig. 1 shows the y distribution when $h_1=h_2=0$ and the s and t^2 dependences of g_1 and g_2 are neglected. The t^2 dependences of g_1 and g_2 are suppressed by the centrifugal-barrier effects and the integration with respect to s was already done to get the curve, so the shape of curve B would be affected only very weakly when we neglect the s and t^2 dependences of g_1 and g_2. Since the g_1 and g_2 terms in (2.1) give the same y distribution, it is necessary to know whether the CP-nonconserving g_1 term contributes to the distribution or not. This is known by observing the polarization of the γ ray.

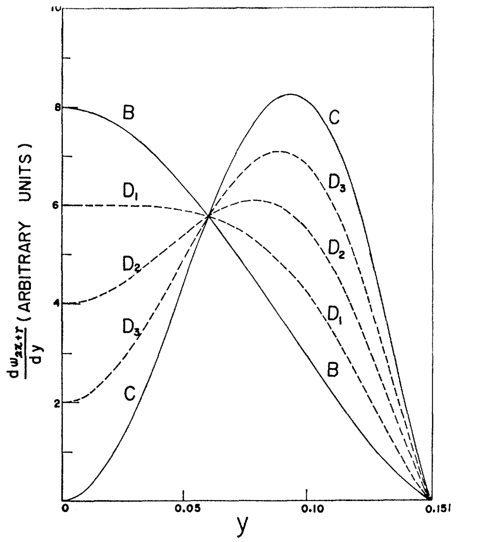

FIG. 1. Transition rates for $K_L^0 \to \pi^+ + \pi^- + \gamma$ versus y.

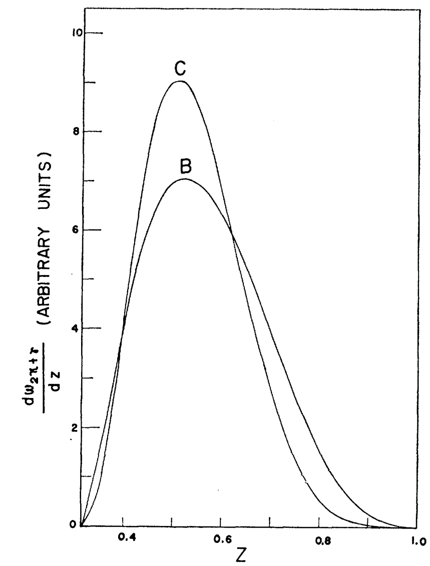

FIG. 2. Transition rates for $K_L^0 \to \pi^+ + \pi^- + \gamma$ versus z.

As is shown by the second expression in (2.4), if CP is conserved, the distribution of the direction of the linear polarization should have the form $\cos^2\varphi$. If $|g_1|^2 \gg |g_2|^2$, the distribution of the polarization has the form $\sin^2\varphi$. If $|g_1|^2 \approx |g_2|^2$, the distribution will show a behavior $\approx 1 \pm \sin 2\varphi$. Thus, it may be possible to determine the existence of the CP-nonconserving g_1 term.

Case C

The effects of the g_1 and g_2 terms to the y distribution are negligible compared with those of the h_1 and h_2 terms. The curve C in Fig. 1 shows the y distribution when $g_1 = g_2 = 0$ and the s and t^2 dependences of h_1 and h_2 are neglected. Both the h_1 and h_2 terms give the same y distribution. It vanishes at $y = 0$ because of the factor y^2 in the expression of Y. If CP is conserved, as is shown by the last expression in (2.4), the polarization distribution should have the form $\sin^2\varphi$. If the distribution does not have this form, it shows the existence of the CP-nonconserving h_2 term.

Case D

At least one of the g_1 and g_2 terms and also one of the h_1 and h_2 terms contribute to the y distribution. Curves D_1, D_2, and D_3 in Fig. 1 show such cases. If CP is conserved, the function Y in (2.11) is given by

$$Y = |g_2|^2 \cos^2\varphi + 2y \,\mathrm{Im}(h_1^* g_2) \sin 2\varphi + y^2 |h_1|^2 \sin^2\varphi .$$

The second term gives the asymmetry of the polarization distribution with respect to the $y = 0$ axis. The ratio $|h_1|^2/|g_2|^2$ is given by the shape of the y distribution. The ratio is 92.7, 278, and 834 for curves D_1, D_2, and D_3, respectively.

In the cases (B) and (C), the form of the polarization distribution is independent of y but in this case it depends on y. Thus in order to know whether CP-nonconserving terms exist or not, more experimental events will be required in the last case than in the first two cases.

Figure 2 shows the z distribution. The first and the second terms in (2.13) are represented by curves B and C, respectively. Curves B and C have quite similar shapes in the z distribution, but they have different shapes in the y distribution. Thus the y distribution can be used to classify our process and the z distribution would be useful to identify our process.

The way to find the existence of CP-nonconserving terms explained here is also useful in other processes, say, $K_L^0 \to 2\pi^0 + \gamma$, $K_S^0 \to 2\pi^0 + \gamma$, and $K_S^0 \to \pi^+ + \pi^- + \gamma$. In the first two processes, $g_1 = g_2 \equiv 0$ and their y and z distributions are represented uniquely by curve C in Figs. 1 and 2, respectively.

4. RELATION BETWEEN $K_L^0 \to \pi^+ + \pi^- + \gamma$ AND $K_L^0 \to \pi^+ + \pi^-$

The branching ratio of the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$ is not known at the present moment, but its upper limit to all the decay modes is about 0.3%.[9] Under this situation, we want to estimate the rough magnitude of the branching ratio by assuming that the CP nonconservation in the decay $K_L^0 \to 2\pi$ is actually due to the electromagnetic interaction of hadrons and, by connecting the matrix element of the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$ to that of the decay $K_L^0 \to \pi^+ + \pi^-$.

As far as the lowest order of the electromagnetic radiative corrections is concerned. Figure 3 shows all possible diagrams for the decay $K_L^0 \to \pi^+ + \pi^-$. Figure

[9] B. M. K. Nefkens, A. Abashian, R. J. Abrams, D. W. Carpenter, G. P. Fisher, and J. H. Smith, Phys. Letters **19**, 706 (1965).

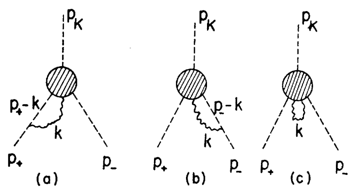

FIG. 3. Lowest-order diagrams relating the radiative decay of K_L^0 to two pions and the nonradiative decay of the same process.

3 (c) connects the matrix element of the decay $K_L^0 \to \pi^+ + \pi^- + \gamma + \gamma$ to that of $K_L^0 \to \pi^+ + \pi^-$. The contribution of this diagram to the latter matrix element would have the same order of magnitude as that of the other diagrams in Fig. 3. Since we are only making a rough order-of-magnitude estimate, we will not consider this diagram (c).

Since the matrix element of the decay $K_L^0 \to \pi^+ + \pi^-$ consists of only a P-nonconserving amplitude and the final state of the decay is the eigenstate of CP with eigenvalue $(+1)$, only the g_1 term in (1.2) contributes to the matrix element. To get the matrix element from that of the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$, the term

$$[1/(2\pi)^{3/2} 1/(2\omega)^{1/2} F_{\alpha\beta}(p_+)_\alpha (p_-)_\beta g_1(s,t^2)]$$

in (2.1) should be replaced by

$$A = \frac{e}{(2\pi)^4} \int d^4k \frac{1}{[k^2 - i\epsilon][(p_+ - k)^2 + m_\pi^2 - i\epsilon]} \{[k\cdot(p_+ - k)][p_- \cdot (2p_+ - k)]$$
$$- [(2p_+ - k)\cdot(p_+ - k)][k\cdot p_-]\} g_1(s', t_1^2, k^2) + \frac{(-e)}{(2\pi)^4} \int d^4k \frac{1}{[k^2 - i\epsilon][(p_- - k)^2 + m_\pi^2 - i\epsilon]}$$
$$\times \{[(2p_- - k)\cdot(p_- - k)][k\cdot p_+] - [k\cdot(p_- - k)][p_+ \cdot (2p_- - k)]\} g_1(s', t_2^2, k^2)\,, \quad (4.1)$$

where $s' = -(p_+ + p_- - k)^2$, $t_1 = -p_K\cdot(p_+ - p_- - k)$, and $t_2 = -p_K \cdot (p_+ - p_- + k)$. This expression is invariant under the replacement $p_+ \rightleftarrows p_-$. The structure function g_1 depends now on three invariants, because the photon is a virtual one. The minus sign at the front of the charge e in the second expression of (4.1) came from the fact that π^- has the opposite charge to π^+.

To perform the k integration in (4.1), the k dependence of the structure function g_1 will be neglected. By keeping the highest divergent term alone, the expression (4.1) leads to

$$A = ie/8\pi^2(m_K^2 - 2m_\pi^2)\lambda^2 g_1\,, \quad (4.2)$$

where λ is the cutoff momentum. Now the matrix element of the decay $K_L^0 \to \pi^+ + \pi^-$, which should be compared with (1.2), is given by

$$\frac{i}{(2\pi)^{1/2}} \frac{e\lambda^2 g_1}{16\pi^2} \frac{(m_K^2 - 2m^2)}{(2\omega_K\omega_+\omega_-)^{1/2}} \delta^{(4)}(p_K - p_+ - p_-)\,. \quad (4.3)$$

This expression leads to the transition rate,

$$\omega_{2\pi} = [\alpha/(4\pi)^4](1-2c^2)^2(1-4c^2)^{1/2} m_K^3 \lambda^4 |g_1|^2\,. \quad (4.4)$$

As the second term in (2.13) shows, because of centrifugal-barrier effects, the contributions of the h_1 and h_2 terms to the transition rate of the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$ may be small compared with those of the g_1 and g_2 terms. Further, if the electromagnetic interaction of hadrons violates CP so as to explain the observed magnitude of the decay amplitude of $K_L^0 \to \pi^+ + \pi^-$, the g_1 and g_2 terms in (2.13) contribute to $\omega_{2\pi+\gamma}$ with the same order of magnitude. In order to make a rough estimate of the magnitude of $\omega_{2\pi+\gamma}$, we shall assume $|g_1|^2 = |g_2|^2$. Then a numerical calculation gives

$$\omega_{2\pi+\gamma} \approx 4.6\times 10^{-3} \times m_K |g_1|^2/384(2\pi)^3\,. \quad (4.5)$$

The expressions (4.4) and (4.5) lead to the ratio

$$\frac{\omega_{2\pi+\gamma}}{\omega_{2\pi}} \approx \frac{1}{3.5} \frac{m_K^4}{\lambda^4}\,. \quad (4.6)$$

The magnitude of λ depends on the model of the decay $K_L^0 \to \pi^+ + \pi^- + \gamma$. Assuming $\lambda \approx m_K$, the ratio (4.6) is consistent with the present upper limit of $\omega_{2\pi+\gamma}$.

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