hyperfragment decay channel ${}_{\Lambda}H^4 \to \pi^- + He^4$. We note that the imaginary portion of H contributes a negligible contribution. From (4a) and (4b), we calculate the ratio of the partial rates $\Gamma({}_{\Lambda}H^4 \to \pi^- + He^4)$, with and without final-state interactions, to be

$$\frac{\Gamma_{\text{plane}}({}_{\Lambda}H^4 \to \pi^- + He^4)}{\Gamma_{\text{inter}}({}_{\Lambda}H^4 \to \pi^- + He^4)} = 1.21 \pm 0.04. \quad (5)$$

We note a substantial lowering of the partial decay rate. The error above reflects the uncertainty in the experimental value of δ_0. The calculation is quite insensitive to the nuclear size parameter. Further, since in (5), the same hyperfragment wave function is used in both the numerator and denominator, the result is also insensitive to the details of the hyperfragment wave function used.

It has been pointed out[7] that a measurement of the branching ratio $({}_{\Lambda}H^4 \to e^- + \bar{\nu} + He^4)/({}_{\Lambda}H^4 \to \pi^- + He^4)$ allows one to measure the pure Fermi coupling for lambda β-decay, since the β decay is a $0 \to 0$ transition. Of course, it is necessary to have a reliable theoretical estimate of the partial rate ${}_{\Lambda}H^4 \to \pi^- + He^4$; thus, the 21% lowering due to final-state effects plays a critical role in the interpretation of the above experiment.

In conclusion, we modify the estimate of the ${}_{\Lambda}H^4$ lifetime $\tau({}_{\Lambda}H^4) = 0.65\ \tau(\Lambda)$ calculated by Dalitz and Rajasekaran,[8] to be

$$\tau({}_{\Lambda}H^4) = 0.73\tau(\Lambda) = 1.75 \times 10^{-10}\ \text{sec}, \quad (6)$$

where the lifetime in (6) reflects the effects of final-state interactions. Unfortunately, no comparison with experiment is possible, since no accurate experimental measurement of $\tau({}_{\Lambda}H^4)$ has as yet been performed.

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[7] M. M. Block (to be published).

[8] R. H. Dalitz and G. Rajasekaran, Phys. Letters 1, 58 (1962).

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$QQ\bar{Q}$ Model of Meson-Baryon Processes

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A $QQ\bar{Q}$ model of beson-baryon scattering and production of both positive- and negative-parity mesons is developed as a generalization of the "elementary meson" model proposed earlier by the authors. The model is based on the assumption that mesons are tighter structures than baryons, which allows the former to "see through" the latter, but not vice versa. The formalism allows the full inclusion of multiple-scattering effects within the meson-quark system via the $QQ\bar{Q}$ structure, and leaves scope for the inclusion of similar effects between the meson and the quarks in the baryon, as a second-stage process. The model is more restricted than the usual quark models (based on additivity), which in general enable correlations of meson-baryon, baryon-baryon, and baryon-antibaryon amplitudes. The model makes little use of dynamics, except for the method of "spectator functions" to calculate the amplitudes for the various processes in terms of residues at the appropriate poles of the relevant spectator functions. However, the possible uses of simple dynamical considerations are indicated, by which the larger amplitudes could be distinguished from the smaller ones. The amplitudes are evaluated in a two-stage process, the first step indicating the "$SU(3)$ level" of predictions, involving only one spin-parity type at a time, while the second step gives $SU(6)$-type predictions which in principle connect mesons of different spin-parity assignments. For the negative-parity mesons (0^- and 1^-), the results are similar to those obtained by previous authors. However, the model provides, with no extra physical assumptions but considerably more algebraic manipulations, amplitude relations for the production of positive-parity mesons (0^+, 1^+, and 2^+) which, according to the quark picture, are structures of the form ${}^1P_1$ and ${}^3P_{0,1,2}$, and are expected to simulate mesons of the type B, A_1, and A_2 (and perhaps also the scalar mesons). No attempt is made in this paper to confront the predictions with experiment, which will be the subject of a subsequent communication.

I. INTRODUCTION

SINCE the additive quark (Q) model for hadron scattering was first proposed by Levin and Frankfurt,[1] as well as by Lipkin and Scheck,[2] there have been a number of investigations on its more detailed effects which included inelastic processes.[3,4] Thus the

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[1] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaksiyu 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)].

[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966); H. J. Lipkin, *ibid.* 16, 71 (1966).

[3] J. L. Friar and J. S. Trefil, Nuovo Cimento 49A, 642 (1967).

[4] M. Jacob and C. Itzykson, Nuovo Cimento 48A, 909 (1967).

"antisymmetric sum rule" in meson-baryon scattering, which follows from mere additivity (and no further symmetry principle) is found to be well satisfied, but that the "symmetric sum rule" or the Johnson-Treiman relations[5] which require the additional assumption of $SU(3)$ symmetry, are not so good.[6] Further, the predictions of the additivity model concerning relations between meson(M)-baryon(B) and BB scattering are good only to about 10%, and this indicates either a breakdown of additivity or a nontrivial effect of binding on the effective QQ amplitudes.[7] Another effect which cannot possibly be included under the additivity assumption is the $B\bar{B}$ annihilation contribution to the high-energy total cross sections,[8] and this has been shown to improve greatly the agreement of the symmetric sum rule over its original form.[6] Finally, the effects of double scattering in QQ amplitudes have also been proposed for an understanding of the sharp bend in high-energy pp scattering.[9,10]

Since all these results seem to indicate a general recognition of the limitations of the simplest additivity assumption, it should be of interest to have a wider formulation of the problem which would allow the inclusion of nonadditive contributions within its framework. In this regard it is helpful to keep in mind the dynamical aspects of the problem which would distinguish between systems for which additivity is bad, and those for which it is at least tolerable. For example, it is reasonable to assume *a priori* that additivity would be less justified for systems which have tighter quark structures than for others whose quark structures are not so tight. Thus if we regard two-body QQ or $Q\bar{Q}$ forces as responsible for hadron binding, then an examination of the relative masses of baryons as $3Q$ composites, and mesons as $Q\bar{Q}$ composites reveals that $Q\bar{Q}$ forces are appreciably stronger than QQ forces. Therefore, in a first approximation one may even regard mesons as "elementary particles" and only baryons as quark composites. Such a point of view was recently advocated in a series of papers on meson-baryon scattering,[11] photoproduction,[12] and strong decays of baryons,[13] as an alternative to the pure additivity model. While the predictive capacity of this elementary-meson model is much more limited than that of additivity, the limitation is confined only to those areas where the predictions of the additivity model are not so good (e.g., relations between MB, BB, and $B\bar{B}$ processes).

Now it is possible to carry the elementary-meson model a step further within a well-defined dynamical discipline so as to make it capable of predicting meson inelastic processes, e.g., vector meson (V) production by pseudoscalar (P) mesons. Thus while in Ref. 11 the assumption of the basic meson-quark amplitude $P+Q\to P+Q$, with a postulated elementarity of the pseudoscalar (P) meson, prevented the prediction of V-production amplitudes within its framework, a logical extension of this simple model which would easily remedy this shortcoming would now be to regard P as a $Q\bar{Q}(^1S_0)$ composite. A $Q\bar{Q}$ structure of P would in turn reduce the study of the PQ amplitude to that of a (three-body) $QQ\bar{Q}$ amplitude. Within such a three-body model, which can readily be made to include spin and $SU(3)$ effects, it is an easy matter to express the amplitudes for $P+Q\to P+Q$ as well as $P+Q\to V+Q$ processes within a common framework which automatically takes account of all multiple-scattering effects within the $QQ\bar{Q}$ system. If these basic amplitudes which clearly include elastic and inelastic components are now folded into the initial- and final-baryon systems, it should be possible to correlate meson-production amplitudes with those of meson scattering, in association with baryons of different types. Of course, the predictive power of this extended model still falls short of pure additivity in that the former is still incapable of connecting processes like $MB\to MB$ with those involving BB or $B\bar{B}$. However, even these restricted predictions should be sufficiently wide ranged to facilitate a meaningful comparison with a wide class of experimental results. More important, these predictions would clearly be based on a certainly well-defined dynamical discipline relating to the degree of compositeness of hadrons, in contrast to those of the additivity model whose dynamical basis is much less clear and which derives almost its entire strength from its experimental success.

The dynamical basis of the $QQ\bar{Q}$ model consists essentially in classifying the hadrons according to the tightness of their structures. Thus the assumption that mesons as $Q\bar{Q}$ composites are more tight than baryons as $3Q$ systems should enable the mesons to see through the structure of baryons without a reciprocal advantage of the latter. On the other hand, when one considers a quark-meson system, the assumed elementarity of the quarks should enable them to see through the $Q\bar{Q}$ structures of the mesons. In this sense, the $QQ\bar{Q}$ model does not conflict with the elementary-meson model proposed earlier,[11,12] but merely makes use of a certain hierarchy of compositeness to give a wider class of predictions. Another advantage of the $QQ\bar{Q}$ model is that it takes account of all multiple-scattering corrections at least within the meson-quark subsystem of the full meson-baryon state. While it does not yet include

[5] See, also, V. Barger and H. M. Rubin, Phys. Rev. **140**, B1366 (1965).
[6] H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966).
[7] J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42**, 711 (1966).
[8] J. J. Kokkedee and L. Van Hove, Nucl. Phys. **B1**, 169 (1967).
[9] D. R. Harrington and A. Pagnamenta, Phys. Rev. Letters **18**, 1147 (1967).
[10] V. Franco, Phys. Rev. Letters **18**, 1159 (1967).
[11] G. C. Joshi, V. S. Bhasin, and A. N. Mitra, Phys. Rev. **156**, 1572 (1967).
[12] S. Das Gupta and A. N. Mitra, Phys. Rev. **156**, 1581 (1967).
[13] A. N. Mitra and M. H. Ross, Phys. Rev. **158**, 1630 (1967).

multiple-scattering effects outside the $QQ\bar{Q}$ subsystem, it at least separates such effects into two distinct types, one due to the $QQ\bar{Q}$ structure of meson-quark amplitudes, and the other due to the effect of binding of quarks within a baryon. Such a division provides a basis for inclusion of multiple-scattering effects as a two-step process. We are at present interested only in the first step of this process which we believe concerns (dynamically) the tighter part of the meson-baryon system, but we hope that the formulation to be presented here is broad enough to warrant an extension to the second step in the near future, with the help of available multiple-scattering techniques.[14]

The emphasis in this paper is on formulation rather than on a detailed comparison with experiment. We shall be interested first in obtaining the relations between various meson-quark amplitudes which will include both positive- and negative-parity mesons, and then folding these relations into the quark structures of the initial and final baryons. The initial meson will always be chosen as a pseudoscalar (P), but the final meson will have any one of the following spin-parity states:

$$0^-,1^-;0^+,1^+,2^+.$$

According to our usual ideas of $Q\bar{Q}$ structures of mesons, we should, in the exact symmetry limit, expect geometrical relations between various 0^- and 1^- amplitudes, to the extent that these mesons have the same radial structures (${}^1S_0$ and ${}^3S_1$, respectively). Likewise, we should expect geometrical relations between amplitudes involving positive-parity meson production to the extent that the 0^+, 1^+, and 2^+ mesons have the respective $Q\bar{Q}$ structures ${}^3P_{0,1,2}$ and ${}^1P_1$ (all having the same radial wave function). Dynamically, the former would be generated by s-wave $Q\bar{Q}$ forces and the latter by p-wave $Q\bar{Q}$ forces. Any relation between these two sets of amplitudes is clearly dynamical. While we shall present a semidynamical formulation of the $QQ\bar{Q}$ system with a view to calculating the above amplitudes, we shall try as far as possible to keep distinct the essentially geometrical features of the results from the characteristic dynamical aspects. Even within the latter, our aim will be to distinguish the qualitative dynamical predictions from the more model-dependent relations between amplitudes.

Our fundamental assumption is a nonrelativistic $QQ\bar{Q}$ system interacting through $Q\bar{Q}$ and QQ pairs, of which the former are governed by much stronger forces than the latter. The $Q\bar{Q}$ forces are taken to operate in s and p waves only. These and other assumptions on the symmetry of the $QQ\bar{Q}$ system are spelled out in detail in Sec. II, which discusses the construction of the $QQ\bar{Q}$ wave function taking account of the spin and $SU(3)$ degrees of freedom, in a representation that is particularly suited to the boundary condition of an initial octet of pseudoscalar mesons scattering (elastically or inelastically) against a quark. Section III is concerned with certain general dynamical considerations for the $QP\rightarrow QP$ and $QP\rightarrow QV$ processes which facilitate a distinction between the small and the large parts of the spatial amplitudes. In Sec. IV we explain the construction of the $QP\rightarrow QP$ and $QP\rightarrow QV$ amplitudes from the $QQ\bar{Q}$ wave function, and also obtain a catalog of the actual meson-baryon amplitudes. Section V is devoted to a method of construction of the amplitudes for the production of positive-parity mesons on lines similar to the earlier sections for the negative-parity cases. Section VI includes a short summary of results obtained together with a comparison with contemporary approaches.

II. $QQ\bar{Q}$ WAVE FUNCTION: GENERAL CONSIDERATIONS

Our $QQ\bar{Q}$ model of meson-quark amplitudes consists of three nonrelativistic particles (two quarks and an antiquark) interacting in pairs. Since it has already been said that $Q\bar{Q}$ interactions are stronger than QQ forces, we may ignore the latter to simplify the formalism. This is really not a limitation for a class of results in the form of relations between various amplitudes, as long as we are not interested in their actual numerical values. In any case, it will always be possible to point out the formal modifications implied by a subsequent imposition of QQ forces. As for $Q\bar{Q}$ forces, we shall only take them to be in s and p waves which, respectively, generate negative- (0^- and 1^-) and positive-parity ($0^+,1^+,2^+$) mesons. With little loss of generality these forces will be subsequently assumed factorable, ostensibly to simplify the formal analysis, but without any particular risk of obtaining model-dependent relations between amplitudes of physical interest.[15]

Once we have a complete $QQ\bar{Q}$ wave function, it is essentially a matter of quantum-mechanical discipline to obtain the various quark-meson amplitudes through suitable interpretation of the different components of the $QQ\bar{Q}$ wave function in conjunction with appropriate boundary conditions.[16] For example, in the limit of exact $SU(6)$, the $QP\rightarrow QP$ and $QP\rightarrow QV$ amplitudes are expressible in terms of the same orbital function obtained by evaluating the residue of the $QQ\bar{Q}$ wave function at the appropriate pole corresponding to the (degenerate) mass of the meson. Similar considerations apply to the production of positive-parity mesons. $SU(6)$ breaking effects due to various mass differences

[14] See, e.g., R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Britten and L. G. Dunham (Interscience Publishers, Inc., New York, 1959), Vol. I.

[15] A fuller discussion of the *raison d'être* for the assumption of such forces within a nonrelativistic framework has been given in a recent paper by the author [A. N. Mitra, Ann. Phys. (N. Y.) 43, 126 (1967)].

[16] Such boundary value manipulations within a nuclear three-body framework dealing with scattering and stripping reactions are discussed more fully in A. N. Mitra, Phys. Rev. 139, B1472 (1965).

should manifest through the shifts of the pole position corresponding to the actual masses of the mesons in relation to the available separation energy between the initial pseudoscalar meson (P) and the quark. Such results can be visualized very rapidly with the help of the so-called spectator functions characteristic of three-body wave functions in the context of separable potentials.[17] We wish to emphasize, however, that even with separable potentials, the algebraic structures of the quark-meson amplitudes are general enough to make them independent of this particular assumption, and that these could well have been written down without introducing such potentials. On the other hand, relations between the amplitudes for negative- and positive-parity meson production are dynamical in origin, and the separable model provides a possible means to calculate such relations.

After these physical preliminaries, we proceed to spell out the further assumptions needed for a concrete construction of the $QQ\bar{Q}$ wave function. The most important question to be settled is of course the type of statistics to be assumed for quarks. In conformity with our earlier approach in a series of recent papers,[11–15] we shall assume parastatistics for the like particles (quarks), so as to make the QQ part of the $QQ\bar{Q}$ state a para-Bose system.[18] This requires the QQ part of the $QQ\bar{Q}$ wave function to be symmetric with respect to an interchange of all the coordinates (spin, $SU(3)$, and orbital). Since $\bar{Q}$ is a different particle in the nonrelativistic limit, one need not impose any symmetry requirement on this particle *vis-a-vis* the two quarks. Finally we note that this para-Bose assumption is a mere formality so long as results in the form of sum rules between various amplitudes are desired, as in the present investigation. Indeed, the same relations would be formally obtained with the opposite assumption of Fermi statistics for the two quarks, just like the earlier results on the meson-baryon processes,[11] photoproduction,[12] and strong decays of hadrons.[13] However, there exists a different (and much more restricted) set of physical quantities like baryon form factors[19] and perhaps also the **56** baryon mass formulas[20] which could discriminate between the two forms of statistics, within the Gell-Mann–Zweig model.

Our next task is to express the $QQ\bar{Q}$ wave function in terms of all the available degrees of freedom. We denote the indices by 1 and 2 and the antiquark index by 3. We shall now write down the various functions using such a basis function as to bring out the boundary condition of a pseudoscalar octet of mesons, made up of Q_1 and $\bar{Q}_3$, scattering on Q_2. The effect of rearrangement as between Q_1 and Q_2 would of course be incorporated through the symmetry requirement on these two quarks. This would ensure that a good part of the internal polarization of the meson as a $Q\bar{Q}$ composite is automatically incorporated in the $QQ\bar{Q}$ wave function, via the symmetry requirement.

A. Spin Functions

Let α_i and β_i represent the spin-up and spin-down states, respectively, for quark number i. For positive-parity wave functions it is most convenient to use the basis function

$$\chi'(2;13)=2^{-1/2}\alpha_2(\alpha_1\beta_3-\alpha_3\beta_1)\equiv\chi\,, \qquad (2.1)$$

which is appropriate to the boundary condition of a pseudoscalar meson (P) made up of Q_1 and $\bar{Q}_3$, scattering on Q_2 (elastically or inelastically). Now to maintain the over-all symmetry of the $QQ\bar{Q}$ wave function in the two quark indices, one must use spin functions which are either symmetric (s) or antisymmetric (a) in Q_1 and Q_2. The latter are obtained by using the spin-permutation operator $(12)_\sigma$ on the basis function χ, so as to yield[21]

$$\chi_{s,a}=2^{-1/2}[1\pm(12)_\sigma]\chi\,, \qquad (2.2)$$

where $(12)_\sigma$ has the representation

$$(12)_\sigma=\tfrac{1}{2}(1+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\,. \qquad (2.3)$$

For negative-parity wave functions, on the other hand, it is most convenient to use the vector basis function[15,22,23]

$$\boldsymbol{\chi}'(2;13)=i\boldsymbol{\sigma}^{(2)}\chi\equiv\boldsymbol{\chi}\,, \qquad (2.4)$$

in terms of which the s and a functions are

$$\boldsymbol{\chi}_{s,a}=2^{-1/2}[1\pm(12)_\sigma]\boldsymbol{\chi}\,. \qquad (2.5)$$

There is a second vector spin function $\boldsymbol{\chi}^s$ defined by

$$\boldsymbol{\chi}^s=2^{-1/2}(i\boldsymbol{\sigma}^{(1)}-\tfrac{1}{2}\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\chi\,, \qquad (2.6)$$

which is totally symmetric in all the particles and hence corresponds to the spin-quartet state, unlike the functions $\boldsymbol{\chi}_s$ or $\boldsymbol{\chi}_a$ which give only spin-doublet states, in the common basis χ.

Finally there is the tensor spin function $\chi_{\mu\nu}$, again corresponding to a quartet-spin state, given by

$$\chi_{\mu\nu}=2^{-1/2}(\sigma_\mu{}^{(1)}\sigma_\nu{}^{(2)}+\sigma_\nu{}^{(1)}\sigma_\mu{}^{(2)}-\tfrac{2}{3}\delta_{\mu\nu}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\chi\,, \qquad (2.7)$$

which should go with a (positive-parity) tensor function $\psi^{\mu\nu}$ of the spatial variables.

[17] The concept of spectator functions using separable potentials was first used in A. N. Mitra, Nucl. Phys. **32**, 529 (1962).
[18] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).
[19] A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966).
[20] D. L. Katyal, V. S. Bhasin, and A. N. Mitra, Phys. Rev. **161**, 1546 (1967).
[21] For a general discussion on three-body spin function see M. Verde, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 170. The symmetries in the present case are however, less complete than those used on a QQQ system (Ref. 15).
[22] See, e.g., R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., New York, 1953).
[23] A. N. Mitra, Phys. Rev. **150**, 1168 (1966).

B. $SU(3)$ Functions

In conformity with our boundary condition of a pseudoscalar octet $(Q_1\bar{Q}_3)$ of mesons, scattering on Q_2, we must now choose the basis function as

$$\phi = q_a^{(2)}(q_b^{(1)}\bar{q}_c^{(3)} - \tfrac{1}{3}\delta_{bc}q_d^{(1)}\bar{q}_d^{(3)}), \qquad (2.8)$$

where the $SU(3)$ states are indicated by the subscripts a, b, c $(=1, 2, 3)$, and the individual particles are distinguished by the superscripts (i). The corresponding functions $\phi_{s,a}$ are now given by

$$\phi_{s,a} = 2^{-1/2}[1 \pm (12)_u]\phi, \qquad (2.9)$$

where $(12)_u$, the (12) permutation operator in $SU(3)$ space, is given by[15]

$$(12)_u = \tfrac{1}{3} + \tfrac{1}{2}\boldsymbol{\lambda}^{(1)}\cdot\boldsymbol{\lambda}^{(2)}, \qquad (2.10)$$

and the scalar product on the right-hand side of (2.8) is over the eight indices of the two Gell-Mann matrices[24] $\lambda_\alpha^{(1,2)}$ $(\alpha = 1, 2, \cdots 8)$. A more convenient expression for ϕ in terms of an octet π_α of meson states[24] is found from the correspondence

$$q_a^{(1)}\bar{q}_b^{(3)} - \tfrac{1}{3}\delta_b{}^a q_c^{(1)}\bar{q}_c^{(3)} \leftrightarrow \tfrac{1}{2}\sqrt{2}[\pi_\alpha\lambda_\alpha^{(1)}]_b{}^a, \qquad (2.11)$$

where $\lambda_\alpha^{(1)}$ has the *same* representation as the Gell-Mann matrices for "quark number one." Thus one now has in matrix notation

$$\phi = 2^{-1/2}\pi_\alpha\lambda_\alpha^{(1)} \otimes q^{(2)} \qquad (2.12)$$

as a direct product of a 3×3 matrix $\lambda_\alpha^{(1)}\pi_\alpha$ in the $SU(3)$ space of Q_1 and a 3×1 matrix $q^{(2)}$ for the quark Q_2.[25]

C. Orbital Functions

Finally, the quantities $\chi_{s,a}$ and $\phi_{s,a}$ must be associated with orbital functions of the appropriate symmetry, so as to build a totally symmetric function in Q_1 and Q_2 in all the three degrees of freedom taken together, in accordance with our assumption of the para-Bose statistics for these two particles. Thus for even-parity states, the complete $QQ\bar{Q}$ wave function is

$$\Psi^{(+)} = \psi_s\chi_s\phi_s + \psi_s'\chi_a\phi_a + \psi_a\chi_s\phi_a + \psi_a'\chi_a\phi_s + (\psi_a{}^{\mu\nu}\phi_a + \psi_s{}^{\mu\nu}\phi_s)\chi_{\mu\nu}, \qquad (2.13)$$

where $\psi_{s,a}$, $\psi_{s,a}'$ are two independent pairs of symmetric (s) and antisymmetric (a) scalar (0^+) orbital functions, and $\psi_{s,a}{}^{\mu\nu}$ represent a third pair of tensor (2^+) functions. Similarly, for odd-parity states, the corresponding $QQ\bar{Q}$ function is expressible in the vector basis as

$$\Psi^{(-)} = \boldsymbol{\psi}_s\cdot\boldsymbol{\chi}_s\phi_s + \boldsymbol{\psi}_s'\cdot\boldsymbol{\chi}_a\phi_a + \boldsymbol{\psi}_a\cdot\boldsymbol{\chi}_s\phi_a + \boldsymbol{\psi}_a'\cdot\boldsymbol{\chi}_a\phi_s + \boldsymbol{\psi}^s\cdot\boldsymbol{\chi}^s\phi_s + \boldsymbol{\psi}^a\cdot\boldsymbol{\chi}^s\phi_a, \qquad (2.14)$$

where $\boldsymbol{\psi}_{s,a}$, $\boldsymbol{\psi}_{s,a}'$, and $\boldsymbol{\psi}^{s,a}$ are three independent pairs of vector (1^-) orbital functions.

It may be noted that the expressions $\Psi^{(\pm)}$, by virtue of the symmetry requirement, contain the effect of exchange or rearrangement (as between Q_1 and Q_2) on the quark-meson amplitudes that can be constructed from them. In order words, a good part of the internal polarization of the meson as a $Q\bar{Q}$ composite is already incorporated in the $QQ\bar{Q}$ wave function because of the requirement of symmetry.

III. $QQ\bar{Q}$ WAVE FUNCTION: DYNAMICAL CONSIDERATIONS

We now try to present a semiquantitative analysis of the orbital parts of the meson-quark amplitudes within the framework of factorable two-body forces. We first consider the limit of extreme degeneracy characterized by (i) neglecting the QQ force compared with the $Q\bar{Q}$, and (ii) ignoring the spin as well as $SU(3)$ dependence of the $Q\bar{Q}$ forces. Subsequently, we shall indicate the nature of the modifications on the various amplitudes as a result of lifting these restrictions. We shall keep close to the nonrelativistic helicity formalism for the quark states (though this will not prevent us from taking account of the relativistic kinematics for the mesons where necessary). This essentially implies an analysis of the amplitudes in terms of the orbital angular momentum l rather than the total angular momentum j. Such an analysis should facilitate a clear separation of the geometrical aspects of the problem relating to the $SU(6)$-type connections between the various amplitudes, from the dynamical aspects which could, e.g., distinguish between the large and small amplitudes as seen from a partial-wave analysis.

Let the momenta of the two quarks be $\mathbf{P}_1$ and $\mathbf{P}_2$ so that the $\bar{Q}$ momentum $\mathbf{P}_3$ is $-(\mathbf{P}_1+\mathbf{P}_2)$ in the over-all c.m. frame. Using an s-wave $Q\bar{Q}$ force of the form

$$M_Q\langle\mathbf{p}_{i3}|V_{i3}^{(s)}|\mathbf{p}_{i3}'\rangle = -\lambda_0 u(p_{i3})u(p_{i3}'), \qquad (3.1)$$

where

$$2\mathbf{p}_{i3} = \mathbf{P}_i - \mathbf{P}_3 = \mathbf{P}_i + \mathbf{P}_1 + \mathbf{P}_2, \quad (i=1,2) \qquad (3.2)$$

one obtains spectator functions[15] appropriate to the description of PQ and VQ wave functions. Similarly, a p-wave $Q\bar{Q}$ interaction of the form

$$M_Q\langle\mathbf{p}_{i3}|V_{i3}^{(p)}|\mathbf{p}_{i3}'\rangle = -3\lambda_1\mathbf{p}_{i3}\cdot\mathbf{p}_{i3}v(p_{i3})v(p_{i3}') \qquad (3.3)$$

leads to spectator functions for states of the type MQ, where M is a positive-parity meson. It is convenient to consider the effect of these two interactions separately for gaining a quick insight into the structure of the respective wave functions. It is then merely a question of algebra to put these two forces together if one is interested in the more dynamical problem of connection between these two types of $QQ\bar{Q}$ wave functions, via coupled integral equations for the corresponding spectator functions. Finally, we consider the even- and odd-parity wave functions separately.

[24] M. Gell-Mann, in *Eightfold Way*, edited by M. Gell-Mann and Y. Nee'man (W. A. Benjamin, Inc., New York, 1964).

[25] In this notation, though the index 3 for the $\bar{Q}$ states does not appear explicitly, it is manifest through the various *rows* of the matrix $\lambda_\alpha^{(1)}$.

A. Even-Parity Functions

Using the interaction (3.1) in the Schrödinger equation

$$D(E)\Psi^{(+)}=-M_Q(V_{13}{}^{(s)}+V_{23}{}^{(s)})\Psi^{(+)}, \quad (3.4)$$

where

$$D(E)=\tfrac{1}{2}(P_1{}^2+P_2{}^2+P_3{}^2)-EM_Q, \quad (3.5)$$

one obtains for the orbital functions $\psi_{s,a}$

$$\psi_{s,a}=D^{-1}(E)[u(p_{13})U_{s,a}(\mathbf{P}_2)\pm u(p_{23})U_{s,a}(\mathbf{P}_1)], \quad (3.6)$$

where $U_{s,a}(\mathbf{P})$ satisfy the equations

$$[1-\lambda_0 h_0(P)]U_{s,a}(\mathbf{P})=\pm\lambda_0\int d\mathbf{q}\, K(\mathbf{P},\mathbf{q})U_{s,a}(\mathbf{q}), \quad (3.7)$$

$$K(\mathbf{P},\mathbf{q})=u(\mathbf{P}+\tfrac{1}{2}\mathbf{q})u(\mathbf{q}+\tfrac{1}{2}\mathbf{P}) \times(\mathbf{P}^2+q^2+\mathbf{P}\cdot\mathbf{q}-EM_Q)^{-1}, \quad (3.8)$$

$$h_0(P)=\int d\mathbf{q}\, u^2(q)(\tfrac{3}{4}P^2+q^2-EM_Q)^{-1}. \quad (3.9)$$

A partial-wave analysis of $U(\mathbf{P})$ according to

$$U_{s,a}(\mathbf{P})=\sum_l U_{s,a}{}^{(l)}(P)P_l(\hat{P}\cdot\hat{k})(2l+1), \quad (3.10)$$

where $\hat{k}$ is the direction of the incident meson, yields

$$[1-\lambda_0 h_0(P)]U_{s,a}{}^{(l)}(P) =\pm 4\pi\lambda_0\int_0^\infty q^2dq\, K^{(l)}(P,q)U_{s,a}{}^{(l)}(q), \quad (3.11)$$

where

$$4\pi P_l(\hat{P}\cdot\hat{k})K^{(l)}(P,q)=\int d\hat{q}\, K(\mathbf{P},\mathbf{q})P_l(\hat{q}\cdot\hat{k}). \quad (3.12)$$

In the limit of exact degeneracy (defined earlier), identical equations hold for the other pair $\bar{U}_{s,a}(P)$ associated with the functions $\psi_{s,a}{}'$.

The orbital functions $\psi_{s,a}{}^{\mu\nu}$ have the following structures:

$$\psi_{s,a}{}^{\mu\nu}=D^{-1}(E)[u(p_{13})U_{s,a}{}^{\mu\nu}(\mathbf{P}_2) \pm u(p_{23})U_{s,a}{}^{\mu\nu}(\mathbf{P}_1)], \quad (3.13)$$

where

$$U_{s,a}{}^{\mu\nu}(\mathbf{P})=(P_\mu P_\nu-\tfrac{1}{3}\delta_{\mu\nu}P^2)T_{s,a}(\mathbf{P}) \quad (3.14)$$

satisfy the equations

$$[1-\lambda_0 h_0(P)]U_{s,a}{}^{\mu\nu}(\mathbf{P}) =\pm\lambda_0\int d\mathbf{q}\, K(\mathbf{P},\mathbf{q})U_{s,a}{}^{\mu\nu}(\mathbf{q}), \quad (3.15)$$

and the functions $T_{s,a}(\mathbf{P})$ admit of the partial-wave expansion

$$T_{s,a}(\mathbf{P})=\textstyle\sum_l T_{s,a}{}^{(l)}(P)P_l(\hat{P}\cdot\hat{k})(2l+1). \quad (3.16)$$

From Eqs. (3.11) and (3.12) the following qualitative features are seen to emerge:

(i) For *even* l, the kernel $K^{(l)}$ is positive for $U_s{}^{(l)}(\mathbf{P})$ and negative for $U_a{}^{(l)}(\mathbf{P})$.

(ii) For *odd* l, the opposite is true. The only condition for such behavior is that $u(q)$ should fall off with large q, which amounts to a short-range force in the conventional nuclear sense. This qualitative feature remains valid as long as the QQ force is weaker than the $Q\bar{Q}$ force and the spin-cum-$SU(3)$-dependent forces are small. The same property also holds for the $U_{s,a}{}^{(l)}(\mathbf{P})$ functions, as well as the $T_{s,a}{}^{(l)}(\mathbf{P})$, even though the symmetry-breaking effects would make $U_{s,a}$ and $\bar{U}_{s,a}$ different. $T(P)$ is of course different from U, $\bar{U}$ as seen from (3.11) versus (3.15). For the purpose of this present investigation we shall make use of only such qualitative features that are dynamically deducible.

To calculate the meson-quark amplitudes from the spectator functions, we must use the boundary condition[26]

$$U_s(\mathbf{P})=(2\pi)^3\delta(\mathbf{P}-\mathbf{k})+D^{(+)}(\mathbf{P})(P^2-k^2-i\epsilon)^{-1}, \quad (3.17)$$

corresponding to a meson-quark momentum $\mathbf{k}$ in the c.m. system, from which the scattering amplitude on the energy shell is deduced as $D^{(+)}(p)$ with $p^2=k^2$. For the function $U_a(\mathbf{P})$, the corresponding amplitude is defined as $F^{(+)}(\mathbf{P})$. The production of a vector meson (of higher mass) is similarly expressed in terms of the boundary condition[16]

$$U_s(\mathbf{P})=(2\pi)^3\delta(\mathbf{P}-\mathbf{k})+D^{(+)}(\mathbf{P})(P^2-k_1{}^2-i\epsilon)^{-1}, \quad (3.18)$$

where k_1 ($<k$) is now the separation momentum between the final Q and V particles, so that the amplitude of $QP\to QV$ is given by $D^{(+)}(\mathbf{P})$ or $F^{(+)}(\mathbf{P})$ at $p^2=k_1{}^2$ according as we consider the U_s or U_a functions, respectively. Similarly, we use the notation $\bar{D}^{(+)}$, $\bar{F}^{(+)}$ for the corresponding amplitudes associated with $\bar{U}_s$ and $\bar{U}_a$; and $d^{(+)}$, $f^{(+)}$ for the amplitudes associated with the tensor functions $U_{s,a}{}^{\mu\nu}$. Now in the limit of $SU(6)$ degeneracy, $k_1{}^2\approx k^2$, the orbital functions for the $QP\to QP$ and $QP\to QV$ processes are described by the same quantities $D^{(+)}(P)$ or $F^{(+)}(P)$ (or their counterparts) evaluated at the common point $P^2=k^2$. While, therefore, the mass-breaking effects are easily taken into account by considering the appropriate momentum limits in the functions $D^{(+)}(P)$, $F^{(+)}(P)$, etc., the formalism provides the correct relative normalization between the scattering and production amplitudes by keeping the (geometrical) $SU(6)$ effects separate from the (dynamical) orbital effects.

B. Odd-Parity Functions

For the odd-parity case, the orbital wave function which is now a *vector*, is expressible in terms of a

[26] A. N. Mitra and V. S. Bhasin, Phys. Rev. **131**, 1265 (1963).

spectator function $V(P)$ according to

$$\psi_{s,a}=D^{-1}(E)[u(p_{13})\mathbf{P}_2V_{s,a}(\mathbf{P}_2) \pm u(p_{23})\mathbf{P}_1V_{s,a}(\mathbf{P}_1)], \quad (3.19)$$

where

$$[1-\lambda_0 h_0(P)]\mathbf{P}V_{s,a}(\mathbf{P}) = \pm\lambda_0\int d\mathbf{q}\, K(\mathbf{P},\mathbf{q})\mathbf{q}V_{s,a}(\mathbf{q}). \quad (3.20)$$

Similar equations hold for the scalar functions $\bar{V}_{s,a}(\mathbf{P})$ associated with $\psi_{s,a}$, $\psi_{s,a}'$ as well as $V^{s,a}(\mathbf{P})$ asscoiated with $\psi^{s,a}$. One can now make a partial-wave analysis of (3.2) analogous to (3.0), but there would in general be two coupled equations connecting $V^{(l\pm1)}(P)$, except for the (physically important) case of $V^{(0)}(P)$ which satisfies an uncoupled equation. From this last case it is clear that since $\int d\mathbf{q}\,\mathbf{q}K(\mathbf{P},\mathbf{q})$ has a *negative sign* for a short-range force $[u(P)\rightarrow 0$ as $p\rightarrow\infty]$, the kernel of (3.2) is attractive for $V_a(\mathbf{P})$ and repulsive for $V_s(\mathbf{P})$.

The orbital parts of the meson-quark amplitudes contained in these vector functions are now obtained by calculating the residues $D^{(-)}$ and $F^{(-)}$, respectively, of the spectator functions $V_{s,a}(\mathbf{P})$ at the appropriate poles, in exact correspondence to the positive-parity case.

To summarize the results of this section, we have shown that the orbital parts of the $PQ\rightarrow PQ$ and $PQ\rightarrow VQ$ amplitudes are expressible in terms of (i) the residues $D^{(\pm)}(\mathbf{P})$ and $F^{(\pm)}(\mathbf{P})$ of the respective spectator functions U_s, V_s, U_a, and V_a; (ii) the corresponding quantities $\bar{D}^{(\pm)}(\mathbf{P})$ and $\bar{F}^{(\pm)}(\mathbf{P})$ associated with $\bar{U}_s$, $\bar{V}_s$, $\bar{U}_a$, and $\bar{V}_a$; and (iii) the quantities $d^{(\pm)}$, $f^{(\pm)}$ going with T_s, V^s, T_a, V^a. In general these functions are different for $PQ\rightarrow PQ$ and $PQ\rightarrow VQ$ since they must be evaluated at different momenta corresponding to the mass differences between vector and pseudoscalar mesons. Where necessary, these differences would be indicated by a prime attached to the $PQ\rightarrow VQ$ amplitudes, but in the limit of exact degeneracy, the primes would be dropped. Finally, from the general dynamical considerations of this section, the following inequalities are indicated:

$$(D^{(+)},\bar{D}^{(+)},d^{(+)})\gg(F^{(+)},\bar{F}^{(+)},f^{(+)}); \quad (3.21)$$

$$(F^{(-)},\bar{F}^{(-)},f^{(-)})\gg(D^{(-)},\bar{D}^{(-)},d^{(-)}). \quad (3.22)$$

These follow directly from the remarks made in the paragraph immediately preceding Eq. (3.17), concerning the signs of the various kernels, and also the fact that in the partial-wave expansion of an amplitude, the lowest ones play the dominant role.

We close this section with a few additional remarks on the rather passive role of dynamics in the model developed so far. So long as the parameters D, F, etc. are used as mere symbols, the dynamics is completely hidden, and the various amplitude relations to be developed in the next section are entirely kinematical, apart from the standard quark structures assumed for the hadrons. Any limitations on the dynamics behind the model would therefore not show up, as long as inequalities like (3.21) and (3.22) are not explicitly used. We shall therefore keep the formulation flexible enough so as not to make use of these inequalities right from the start. Rather we shall, in a subsequent paper, try to test these inequalities in relation to experiment via the density-matrix formalism applied to observable reactions. Such a point of view would still maintain the kinematical framework of the paper, irrespective of whether relations like (3.21) or (3.22) may or may not be vitiated by more adequate dynamical considerations.

The dynamical limitation of this model is roughly twofold. First, as we have considered the nonrelativistic model of quarks, it might be thought at first sight that such an assumption is good only near threshold, while the impulse approximation is valid at very high energies. However, to the extent that the *kinematical* framework of this paper does allow the *mesons* to have relativistic energies, it should be possible to go appreciably above threshold energies for the purpose of confronting the model with experiment. These energies ($\sim$2–3 GeV) could still be well below the limit which ($\gtrsim$10 GeV) (we believe) is adequate for the validity of the additivity model, and yet be high enough for multiple-scattering effects (between the meson and the baryon quarks) not to be the dominant phenomenon.

The second limitation concerns the detailed nature of the QQ force. For the sake of illustration of the techniques we have here considered only the simplest attractive rank-one potentials. Such potentials would in general lead to $QQ\bar{Q}$ bound states appreciably below the quark mass and might well compete with low-lying hadronic masses. This is a general feature of strong QQ or $Q\bar{Q}$ potentials like, e.g., the prediction of diquarks of about one-half the quark mass, because of the QQ force.[15] It is not clear at this stage how such states can be prevented within the present day quark models characterized by nonrelativistic dynamics and strong forces. Nor is it clear that the predictions of such states is necessarily harmful unless observation techniques are able to pronounce definite verdicts on their existence or otherwise. In other words, it is not at all obvious that the very prediction of low masses, can by itself bring such states within reach of easy observability which depends on several factors, especially their production mechanism. From the theoretical point of view, such $QQ\bar{Q}$ bound states would correspond to low-lying poles in the direct channel for quark-meson scattering, but their specific effects on the quark-meson amplitudes $D^{(\pm)}$, $F^{(\pm)}$, etc., are probably not of immediate interest before the *general framework* of this model which does not depend on these detailed dynamical features, is subjected to experimental test.

Within a dynamical framework it is of course possible to prevent the formation of low-lying $QQ\bar{Q}$ bound states through a suitable modification of the $Q\bar{Q}$ potential. Thus, as had been assumed for the QQ force in connection with baryon-baryon processes,[12] if the $Q\bar{Q}$ potential is assumed to be an attractive well surrounded by a suitable repulsive barrier, it can prevent the external quark (in quark-meson scattering) from automatically entering the meson's $Q\bar{Q}$ well, and hence effectively bar the formation of a low-lying $QQ\bar{Q}$ bound state by meson-quark collision. In the present spectator model, a $Q\bar{Q}$ force with the above features can be represented by a rank-two potential to a good approximation, the repulsive term having the longer range. Such a modification in the potential would of course lead to (more complicated) coupled equations for the $QQ\bar{Q}$ spectator functions whose kernels would not lend themselves to such easy interpretations (in terms of signs) as were employed in this section to obtain the in equalities like (3.21) and (3.22). But again this is a question of detailed dynamics which is not the subject of the present investigation.

IV. CALCULATION OF PQ AND VQ AMPLITUDES

The evaluation of the meson-quark amplitudes can be regarded as a three-step process:

(i) The orbital part obtained from the residues of the various spectator functions at the poles corresponding to the meson-quark separation momenta;
(ii) overlap of spin functions between the initial and final states; and
(iii) overlap of the initial and final $SU(3)$ functions.

For the PQ and VQ cases, we first ignore the mass difference between the P and V mesons, though we have seen in Sec. III the modification needed to incorporate this mass difference. In the first step, the residues of the spectator functions for the even- and odd-parity cases yield the respective expressions

$$R^{(+)}=D^{(+)}\chi_s\phi_s+\bar{D}^{(+)}\chi_a\phi_a+F^{(+)}\chi_s\phi_a+\bar{F}^{(+)}\chi_a\phi_s +k_\mu'k_\nu'\chi^{\mu\nu}(f^{(+)}\phi_a+d^{(+)}\phi_s)\,, \quad (4.1)$$

$$R^{(-)}=D^{(-)}\mathbf{k}'\cdot\boldsymbol{\chi}_s\phi_s+\bar{D}^{(-)}\mathbf{k}'\cdot\boldsymbol{\chi}_a\phi_a+F^{(-)}\mathbf{k}'\cdot\boldsymbol{\chi}_s\phi_a +\bar{F}^{(-)}\mathbf{k}'\cdot\boldsymbol{\chi}_a\phi_s+f^{(-)}\mathbf{k}'\cdot\boldsymbol{\chi}^s\phi_a+d^{(-)}\mathbf{k}'\cdot\boldsymbol{\chi}^s\phi_s\,. \quad (4.2)$$

Here k' denotes the momentum P_2 at the separation energy of the quark and the final-state meson; the functions $D^{(\pm)}$, $F^{(\pm)}$, $\bar{D}^{(\pm)}$, $\bar{F}^{(\pm)}$, in the notation of the last section, are defined according to the convention that D ($\bar{D}$) and F ($\bar{F}$) represent the symmetric (s) and antisymmetric (a) functions, respectively, and the superscripts ($\pm$) distinguish between the amplitudes associated with even- and odd-parity states. The magnitudes of these D and F functions may be expected to satisfy the rough inequalities (3.21) and (3.22).

A. Spin Overlap Functions

The next step which concerns the spin evaluations is best shown separately for the positive- and negative-parity cases. For the positive-parity functions (4.1) we must take the overlap with the basis function χ of (2.1) which corresponds to an initial PQ state. This gives

$$\chi^\dagger\chi_a=\tfrac{1}{2}(\surd\tfrac{1}{2})\chi^\dagger(1-\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\chi\,, \quad (4.3)$$

$$\chi^\dagger\chi_s=\tfrac{1}{2}(\surd\tfrac{1}{2})\chi^\dagger(3+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\chi\,. \quad (4.4)$$

Now the second terms on the right-hand sides of (4.3) and (4.4) are indeed zero, as seen from the fact that $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\chi$ is orthogonal to χ. Such terms can however be interpreted as spin-inelastic effects, i.e., V production by P mesons. Indeed, the operator $\boldsymbol{\sigma}^{(1)}$ now plays the role of the polarization vector $\hat{V}$ of the V meson, the exact normalization being provided by

$$\boldsymbol{\sigma}^{(1)}=\sqrt{3}\hat{V}\,. \quad (4.5)$$

In this way, the spin overlaps give rise to the quantities

$$\chi^\dagger\chi_a\Rightarrow\tfrac{1}{2}\times(\surd\tfrac{1}{2})(1-\sqrt{3}V_0)\,, \quad (4.6)$$

$$\chi^\dagger\chi_a\Rightarrow\tfrac{1}{2}\times(\surd\tfrac{1}{2})(3+\sqrt{3}V_0)\,, \quad (4.7)$$

$$V_0=\boldsymbol{\sigma}^{(2)}\cdot\hat{V}\,, \quad (4.8)$$

and

$$\chi^\dagger\chi_{\mu\nu}k_\mu'k_\nu'\Rightarrow(\surd\tfrac{3}{2})V_3\equiv(\surd\tfrac{3}{2})k_\mu'k_\nu'\hat{V}_\lambda \times(\sigma_\mu^{(2)}\delta_{\nu\lambda}+\sigma_\nu^{(2)}\delta_{\mu\lambda}-\tfrac{2}{3}\sigma_\lambda^{(2)}\delta_{\mu\nu})\,, \quad (4.9)$$

where the basis states χ and $\chi^\dagger$ have been suppressed. It is seen from Eqs. (4.6)–(4.8) that the spin-flip terms V_0, which are associated with V-meson production, bear geometrical relations to the non-spin-flip terms corresponding to PQ scattering. The term V_3, on the other hand, has no counterpart in P-meson scattering. Similarly, for the negative-parity functions we must consider the overlaps of $\mathbf{k}'\cdot\boldsymbol{\chi}_{s,a}$ with the initial state $\mathbf{k}\cdot\boldsymbol{\chi}$, where $\mathbf{k}$ and $\mathbf{k}'$ are the momenta of the initial and final mesons, respectively. A simple calculation yields the following expressions (in the common basis χ):

$$(\mathbf{k}\cdot\boldsymbol{\chi})^\dagger\mathbf{k}'\cdot\boldsymbol{\chi}_a=\tfrac{1}{2}\times(\surd\tfrac{1}{2})(P-\sqrt{3}V_1)\,, \quad (4.10)$$

$$(\mathbf{k}\cdot\boldsymbol{\chi})^\dagger\mathbf{k}'\cdot\boldsymbol{\chi}_s=\tfrac{1}{2}\times(\surd\tfrac{1}{2})(3P+\sqrt{3}V_1)\,, \quad (4.11)$$

$$(\mathbf{k}\cdot\boldsymbol{\chi})^\dagger\mathbf{k}'\cdot\boldsymbol{\chi}^s=(\surd\tfrac{3}{2})V_2\,, \quad (4.12)$$

$$P=k_\mu k_\nu'(\delta_{\mu\nu}+i\epsilon_{\mu\nu\lambda}\sigma_\lambda^{(2)})\,, \quad (4.13)$$

$$V_1=\hat{V}_\lambda k_\mu k_\nu'(\sigma_\mu^{(2)}\delta_{\nu\lambda}+\sigma_\nu^{(2)}\delta_{\mu\lambda}-i\epsilon_{\mu\nu\lambda} -\delta_{\mu\nu}\sigma_\lambda^{(2)})\,, \quad (4.14)$$

$$V_2=\hat{V}_\lambda k_\mu k_\nu'(\sigma_\mu^{(2)}\delta_{\nu\lambda}-\tfrac{1}{2}\sigma_\nu^{(2)}\delta_{\mu\lambda}+\tfrac{1}{2}i\epsilon_{\mu\nu\lambda} +\tfrac{1}{2}\delta_{\mu\nu}\sigma_\lambda^{(2)})\,. \quad (4.15)$$

The P and V terms are, respectively, associated with p-wave scattering and V-meson production by P mesons on quarks. We note that the $\boldsymbol{\chi}^s$ terms are associated entirely with V-meson production, and do not affect P meson scattering.

B. $SU(3)$ Overlap Functions

In the third and final step, we must evaluate the $SU(3)$ overlap of the resulting amplitude with the initial PQ state. There is indeed a close parallel between the spin and $SU(3)$ techniques in this regard, except that we now have to work in terms of the basis function ϕ of (2.10) which corresponds to the boundary condition of an initial *octet* of mesons, unlike the spin function χ which would correspond, in $SU(3)$ language, to an $SU(3)$-singlet meson. Thus, taking account of the physical conditions of the problem, the analogy in the $SU(3)$ case is more to an initial vector-meson scattering on quarks, than to a pseudoscalar meson-quark scattering. $SU(3)$-singlet production would still be obtained in this formalism as an $SU(3)$-inelastic process, just as the spin formalism provides for vector-meson production as a spin-inelastic effect. To complete the analogy with the spin case, the structure of ϕ as given by (2.12), shows the $SU(3)$ scalar product $\pi_\alpha\lambda_\alpha^{(1)}$ which looks very similar to the negative-parity spin functions, with π_α now playing the role of the momentum $\mathbf{k}$ (or $\mathbf{k}'$) and λ_α that of $\boldsymbol{\sigma}^{(2)}$. Thus we should now take the overlaps with the initial state $\pi_\alpha\lambda_\alpha^{(1)}$,[27] so that the following quantities would be evaluated as 3×3 matrices in the (suppressed) basis state $q^{(2)}$:

$$(\pi_\beta\lambda_\beta^{(1)})^\dagger(\sqrt{\tfrac{1}{2}})[1\pm(12)_u]\pi_\alpha\lambda_\alpha^{(1)}, \qquad (4.16)$$

where $(12)_u$ is given by (2.10). This requires the evaluation of products like $\lambda_\beta^{(1)}\lambda_\alpha^{(1)}$ and $\lambda_\beta^{(1)}\lambda_\gamma^{(1)}\lambda_\gamma^{(2)}\lambda_\alpha^{(1)}$. Now in the notation of Gell-Mann,[24]

$$\lambda_\beta^{(1)}\lambda_\alpha^{(1)}=\tfrac{2}{3}\delta_{\alpha\beta}+if_{\beta\alpha\gamma}\lambda_\gamma^{(1)}+d_{\beta\alpha\gamma}\lambda_\gamma^{(1)}, \quad (4.17)$$

$$\lambda_\beta^{(1)}\lambda_\gamma^{(1)}\lambda_\gamma^{(2)}\lambda_\alpha^{(1)}=\tfrac{2}{3}(if_{\alpha\beta\gamma}+d_{\alpha\beta\gamma})\lambda_\gamma^{(2)}+\tfrac{2}{3}\lambda_\alpha^{(1)}\lambda_\beta^{(2)} +\lambda_\gamma^{(1)}\lambda_\delta^{(2)}(D_{\beta\delta}^{\alpha\gamma}-\bar{D}_{\beta\delta}^{\alpha\gamma}+F_{\beta\delta}^{\alpha\gamma}), \quad (4.18)$$

where

$$D_{\beta\delta}^{\alpha\gamma}=d_{\alpha\gamma\epsilon}d_{\beta\delta\epsilon}, \quad \bar{D}_{\beta\delta}^{\alpha\gamma}=f_{\alpha\gamma\epsilon}f_{\beta\delta\epsilon}, \qquad (4.19)$$

$$F_{\beta\delta}^{\alpha\gamma}=i(d_{\alpha\gamma\epsilon}f_{\beta\delta\epsilon}+d_{\beta\delta\epsilon}f_{\alpha\gamma\epsilon}). \qquad (4.20)$$

These functions have a simple interpretation. Thus (4.17), containing the pure $\lambda^{(1)}$ terms, represents the direct $SU(3)$-elastic effects which correspond to quark-octet scattering without exchange. In (4.18), the pure $\lambda^{(2)}$ terms represent exchange $SU(3)$-elastic effects which also correspond to quark-octet-scattering, but after a rearrangement of the initial quark with the quark constituent of the meson. The term $\lambda_\alpha^{(1)}\lambda_\beta^{(2)}$ gives $SU(3)$-singlet production, while the more complicated $\lambda^{(1)}\lambda^{(2)}$ terms (involving the four-index generators D, $\bar{D}$, F) represent the production of 27-plet mesons. This last we shall not consider in this paper since it corresponds to quark-meson states of the form $Q(Q\bar{Q})(Q\bar{Q})$, though it is remarkable that even this simple formalism generates such terms. The $SU(3)$-singlet production, on the other hand, can be more easily recovered from the original form (4.16) by changing the basis function to an $SU(3)$ singlet, viz.,

$$\phi_0=(\sqrt{\tfrac{1}{3}})q_a^{(1)}\bar{q}_a^{(3)}q_i^{(2)}, \qquad (4.21)$$

[27] The corresponding terms with $\pi_0\lambda_0$, where $\lambda_0=I/\sqrt{3}$, and is an $SU(3)$ singlet, would have looked like the positive-parity terms in the earlier language of spin functions.

TABLE I. The $SU(3)$ matrix elements for the baryonic $8\to8$ transitions. The initial meson is a pion or a kaon. The symbol for the final meson is merely an $SU(3)$ symbol characterized by π, K, or η for the *octet* and X^0 for a singlet, and refers to other spin-parity states without loss of generality. For other notations, see text.

Transition	Direct	Exchange
$\langle\pi^+p\|\pi^+p\rangle$	$\frac{4}{3}A''$	$\frac{2}{3}B'$
$\langle\pi^-p\|\pi^-p\rangle$	$2A'+\frac{2}{3}A''$	$-\frac{1}{3}B'+\frac{1}{3}B''$
$\langle\pi^0n\|\pi^-p\rangle$	$-2\sqrt{2}A'+\frac{2}{3}\sqrt{2}A''$	$\sqrt{2}B'-\frac{1}{3}\sqrt{2}B''$
$\langle\eta n\|\pi^-p\rangle$	$(\sqrt{\frac{2}{3}})A'-(\frac{1}{3}\sqrt{\frac{2}{3}})A''$	$(\sqrt{\frac{1}{6}})B'-(\frac{1}{3}\sqrt{\frac{1}{6}})B''$
$\langle X^0n\|\pi^-p\rangle$	0	$(\sqrt{\frac{1}{3}})B'-(\frac{1}{3}\sqrt{\frac{1}{3}})B''$
$\langle K^+\Sigma^+\|\pi^+p\rangle$	$\frac{4}{3}A''$	0
$\langle K^0\Sigma^0\|\pi^-p\rangle$	$-\frac{2}{3}\sqrt{2}A''$	0
$\langle K^0\Lambda^0\|\pi^-p\rangle$	$2(\sqrt{\frac{2}{3}})A'$	0
$\langle K^+p\|K^+p\rangle$	0	$\frac{2}{3}B'$
$\langle K^-p\|K^-p\rangle$	$2A'+\frac{2}{3}A''$	$-\frac{1}{3}B'-\frac{1}{3}B''$
$\langle \bar{K}^0n\|K^-p\rangle$	$2A'-\frac{2}{3}A''$	0
$\langle\pi^-\Sigma^+\|K^-p\rangle$	0	$\frac{2}{3}B''$
$\langle\pi^0\Sigma^0\|K^-p\rangle$	0	$-\frac{1}{3}B''$
$\langle\pi^0\Lambda^0\|K^-p\rangle$	0	$(\sqrt{\frac{1}{3}})B'$
$\langle\eta\Sigma^0\|K^-p\rangle$	$(\frac{4}{3}\sqrt{\frac{1}{3}})A''$	$-(\frac{1}{3}\sqrt{\frac{1}{3}})B''$
$\langle\eta\Lambda^0\|K^-p\rangle$	$-\frac{4}{3}A'$	$\frac{1}{3}B'$
$\langle X^0\Sigma^0\|K^-p\rangle$	0	$-(\frac{1}{3}\sqrt{\frac{2}{3}})B''$
$\langle X^0\Lambda^0\|K^-p\rangle$	0	$\frac{1}{3}\sqrt{2}B'$

and proceeding exactly as in the spin case. This gives rise to the requisite singlet-production structure

$$\tfrac{2}{3}\lambda_\alpha^{(2)}\pi_\alpha\pi_0^\dagger \qquad (4.22)$$

which will be used in the subsequent analysis.

Collecting the results of the various steps, the $SU(3)$-elastic terms of the meson-quark amplitude are contained in the expression

$$\pi_\beta^\dagger\pi_\alpha[\tilde{A}(\tfrac{2}{3}\delta_{\beta\alpha}+u_{\beta\alpha}^{(+)})+\tfrac{1}{2}\tilde{B}u_{\beta\alpha}^{(-)}], \qquad (4.23)$$

where

$$u_{\beta\alpha}^{(+)}=(if_{\beta\alpha\gamma}+d_{\beta\alpha\gamma})\lambda_\gamma^{(1)}, \qquad (4.24)$$

$$u_{\beta\alpha}^{(-)}=(-if_{\beta\alpha\gamma}+d_{\beta\alpha\gamma})\lambda_\gamma^{(2)}; \qquad (4.25)$$

$$\tilde{A}=\tfrac{1}{2}A^{(+)}(3+\sqrt{3}V_0)+\tfrac{1}{2}\bar{A}^{(+)}(1-\sqrt{3}V_0)+\sqrt{3}a^{(+)}V_3 +\tfrac{1}{2}A^{(-)}(3P+\sqrt{3}V_1)+\tfrac{1}{2}\bar{A}^{(-)}(P-\sqrt{3}V_1) +\sqrt{3}a^{(-)}V_2, \quad (4.26)$$

$$\tilde{B}=\tfrac{1}{2}B^{(+)}(3+\sqrt{3}V_0)-\tfrac{1}{2}\bar{B}^{(+)}(1-\sqrt{3}V_0)+\sqrt{3}b^{(+)}V_3 +\tfrac{1}{2}B^{(-)}(3P+\sqrt{3}V_1)-\tfrac{1}{2}\bar{B}^{(-)}(P-\sqrt{3}V_1) +\sqrt{3}b^{(-)}V_2; \quad (4.27)$$

$$A^{(\pm)}=2D^{(\pm)}+F^{(\pm)}, \quad \bar{A}^{(\pm)}=\bar{D}^{(\pm)}+2\bar{F}^{(\pm)}; \quad (4.28)$$

$$B^{(\pm)}=D^{(\pm)}-F^{(\pm)}, \quad \bar{B}^{(\pm)}=\bar{D}^{(\pm)}-\bar{F}^{(\pm)}; \quad (4.29)$$

$$a^{(\pm)}=2d^{(\pm)}+f^{(\pm)}, \quad b^{(\pm)}=d^{(\pm)}-f^{(\pm)}. \quad (4.30)$$

The first term in (4.23), proportional to $\tilde{A}$, corresponds to direct $SU(3)$ scattering, while the $\tilde{B}$ term relates to $SU(3)$-exchange effects. The $SU(3)$-singlet-production terms are similarly contained in the expression

$$(\sqrt{\tfrac{2}{3}})\pi_0^\dagger\pi_\alpha\lambda_\alpha^{(2)}\tilde{B}. \qquad (4.31)$$

Note that (4.31), which contains only $\tilde{B}$, has exactly the same spin-*cum*-orbital structure as the exchange part of (4.23).[28]

It is now a straight forward calculation to obtain the various scattering and production amplitudes for meson-baryon processes by folding expressions like (4.23) and (4.31) into the initial- and final-baryon states as $3Q$ composites. Since the formal $SU(3)$ structures of these expressions are now exactly the same as those used in Ref. 11, it should be sufficient to refer to that paper for the details of calculation. For the transitions within the **56** of baryons, we must simply use the wave functions[11]

$$\Psi_{10}=\chi^s\phi^s\psi^s, \tag{4.32}$$

$$\Psi_8=\psi^s(\chi'\phi'+\chi''\phi'')/\sqrt{2}, \tag{4.33}$$

and consider the matrix elements of the meson-quark amplitude operators (4.23) and (4.31) between the states[29] (4.32) or (4.33).

The amplitudes for transitions into various baryon-meson $SU(3)$ states are listed in Tables I and II for $\mathbf{8}\rightarrow\mathbf{8}$ and $\mathbf{8}\rightarrow\mathbf{10}$ baryonic transitions, respectively. The notation used is as follows:

$$A'=\langle\chi'|\tilde{A}|\chi'\rangle, \quad A''=\langle\chi''|\tilde{A}|\chi''\rangle; \tag{4.34}$$

$$B'=\langle\chi'|\tilde{B}|\chi'\rangle, \quad B''=\langle\chi''|\tilde{B}|\chi''\rangle; \tag{4.35}$$

$$A^s=\langle\chi^s|\tilde{A}|\chi''\rangle, \quad B^s=\langle\chi^s|\tilde{B}|\chi''\rangle. \tag{4.36}$$

Since each of the A and B coefficients contains predictions for *both* pseudoscalar (P) and vector (V) production, the meson in the final state in each process must be interpreted as a P or V, octet or singlet as the case may be, though a common symbol for both types has been used in the tables. Further simplification can be obtained by evaluating the spin-matrix elements defined by Eqs. (4.26)–(4.36). In the limit of exact mass degeneracy between the P and V mesons, both $BP\rightarrow BP$ and $BP\rightarrow BV$ amplitudes are given in terms of the *same* parameters A and B. If this restriction is dropped, one would merely obtain interrelations *within* these subsets, but not among them. We might name such relations as those at the $SU(3)$ level, though it is important to note that these relations are by themselves intermediate between the conventional $SU(3)$ type (characterized by eight parameters) and the $SU(6)$ type (characterized by four parameters). Indeed it is clear from Tables I and II, that even before the evaluation of spin-matrix elements, the entire list of amplitudes is expressed in terms of the *six* independent parameters

$$A',A''; \quad B',B''; \quad A^s,B^s; \tag{4.37}$$

eactly as in Ref. 11, so that the types of sum rules discussed in that paper would all be reproduced. In addition, one would obtain results for $SU(3)$-singlet production without introducing extra parameters.

As for relations between $BP\rightarrow BP$ and $BP\rightarrow BV$ amplitudes within this model, these would in general be obtained only if it is assumed that the variations of the A and B amplitudes with the meson-quark separation momentum $\mathbf{k}$ are small over a range which covers the gap between the P-meson and V-meson masses. This is the dynamical implication of mass degeneracy within the present framework. Under this assumption, both the $BP\rightarrow PB$ and $BP\rightarrow VB$ types of amplitudes would be expressed in terms of the twelve independent parameters

$$A^{(\pm)},\bar{A}^{(\pm)}; \quad B^{(\pm)},\bar{B}^{(\pm)}; \quad a^{(\pm)},b^{(\pm)}; \tag{4.38}$$

so that in general several $SU(6)$-type relations would be expected. These questions, together with a detailed comparison with experiment in the language of the density-matrix formation[30] will be discussed in a subsequent paper.[31]

TABLE II. $SU(3)$ matrix elements for baryonic $\mathbf{8}\rightarrow\mathbf{10}$ transitions. For explanation and notation see Table I and text.

Transition	Direct	Exchange
$\langle\pi^0\Delta^{++}|\pi^+p\rangle$	$-(2\sqrt{\frac{2}{3}})A^s$	$(\sqrt{\frac{2}{3}})B^s$
$\langle\pi^+\Delta^+|\pi^+p\rangle$	$-\frac{4}{3}A^s$	$\frac{2}{3}B^s$
$\langle\pi^0\Delta^0|\pi^-p\rangle$	$-\frac{2}{3}\sqrt{2}A^s$	$\frac{1}{3}\sqrt{2}B^s$
$\langle\pi^-\Delta^+|\pi^-p\rangle$	$\frac{4}{3}A^s$	$-\frac{2}{3}B^s$
$\langle\eta\Delta^{++}|\pi^+p\rangle$	$-\frac{2}{3}\sqrt{2}A^s$	$-\frac{1}{3}\sqrt{2}B^s$
$\langle\eta\Delta^0|\pi^-p\rangle$	$(\frac{2}{3}\sqrt{\frac{2}{3}})A^s$	$(\frac{1}{3}\sqrt{\frac{2}{3}})B^s$
$\langle\Delta^0X^0|\pi^-p\rangle$	0	$(\frac{2}{3}\sqrt{\frac{1}{3}})B^s$
$\langle\Delta^{++}X^0|\pi^+p\rangle$	0	$-\frac{2}{3}B^s$
$\langle K^+Y^+|\pi^+p\rangle$	$-\frac{4}{3}A^s$	0
$\langle K^0Y^0|\pi^-p\rangle$	$\frac{2}{3}\sqrt{2}A^s$	0
$\langle K^+\Delta^+|K^+p\rangle$	0	$\frac{2}{3}B^s$
$\langle K^0\Delta^{++}|K^+p\rangle$	0	$-(2\sqrt{\frac{1}{3}})B^s$
$\langle K^-\Delta^+|K^-p\rangle$	$\frac{4}{3}A^s$	0
$\langle\bar{K}^0\Delta^0|K^-p\rangle$	$\frac{4}{3}A^s$	0
$\langle\pi^-Y^+|K^-p\rangle$	0	$-\frac{2}{3}B^s$
$\langle\pi^0Y^0|K^-p\rangle$	0	$\frac{1}{3}B^s$
$\langle\eta Y^0|K^-p\rangle$	$-(\frac{4}{3}\sqrt{\frac{1}{3}})A^s$	$(\frac{1}{3}\sqrt{\frac{1}{3}})B^s$
$\langle X^0Y^0|K^-p\rangle$	0	$(\frac{1}{3}\sqrt{\frac{2}{3}})B^s$

V. PRODUCTION OF 0^+, 1^+, and 2^+ MESONS

We indicate, within the $QQ\bar{Q}$ model, the essential steps leading to the evaluation of quark-meson ampli-

[28] The expressions (4.23) and (4.31) have the same relative normalization to each other (though we have multipled each expression by a factor of 3 in the last steps of their respective derivations). This relative normalization has been fixed from the consideration that the $SU(3)$ **8** and **1** states are represented, respectively, by $\pi_\beta\lambda_\beta/\sqrt{2}$ and $\pi_0\lambda_0\equiv\pi_0 I/\sqrt{3}$.

[29] These notations for the orbital, spin, and $SU(3)$ wave functions of the baryons must not be confused with the earlier symbols used for similar quantities in relation to the $QQ\bar{Q}$ system. Similarly the $SU(3)$ superscripts (1) or (2) appearing in (4.19) and (4.20), or the spin superscript (2) appearing in the expressions (4.8), (4.13), and (4.14), both of which now relate to a "typical" quark in the baryon, must be distinguished from the indices appearing in (4.32) or (4.33), which refer to the three different quark constituents of a baryon.

[30] K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).

[31] S. Das Gupta, V. K. Gupta, and A. N. Mitra (to be published).

tudes where now the meson is a scalar, axial vector, or tensor. The procedure is analogous to the previous sections dealing with negative-parity mesons, except that one must now consider a p-wave $Q\bar{Q}$ force as the basic mechanism. In general, one would expect a strong coupling between the wave functions for negative- and positive-parity mesons with quarks, because of the simultaneous operation of s- and p-wave $Q\bar{Q}$ forces. If, on the other hand, we isolate the question of dynamics from the (more general) kinematical considerations, it it adequate to consider only the p-wave forces which will already indicate all the possible types of amplitudes generated, together with geometrical relations among them.

In the same notation as before, the internal variables in the $Q_1\bar{Q}_3$ pair are the relative momentum $\mathbf{p}_{13}$ and the spin vector $\boldsymbol{\sigma}^{(1)}$. All possible internal polarization states of the final (positive-parity) meson must therefore be formed out of these vectors alone. Thus the simplest quantity is the unit vector $\hat{p}_{13}$ which by itself represents the polarization of a $^1P_1$ state of $Q\bar{Q}$, corresponding to a spin-singlet structure (and hence independent of $\boldsymbol{\sigma}^{(1)}$). In the usual quark language, such a state is appropriate for a 1^+ state of even G-parity,[32] representing perhaps the B meson, when one remembers the intrinsic parity of a $Q\bar{Q}$ state as (-1). Thus we define the unit polarization vector $\hat{B}$ for a $^1P_1$ state as

$$\hat{B}=\hat{p}_{13}. \tag{5.1}$$

The ^3P states of $Q_1\bar{Q}_3$ must now be formed out of both the vectors $\hat{p}_{13}$ and $\boldsymbol{\sigma}^{(1)}$, such that the $J=0$, 1, and 2 states correspond, respectively, to the scalar, axial-vector, and tensor products of these two vectors. If we denote the unit operators corresponding to these individual states by s, $\hat{A}$, and $\hat{T}_{\mu\nu}$, respectively, we have

$$s=\hat{p}\cdot\boldsymbol{\sigma}^{(1)}, \tag{5.2}$$

$$\sqrt{2}i\hat{A}=\hat{p}\times\boldsymbol{\sigma}^{(1)}, \tag{5.3}$$

$$\hat{T}_{\mu\nu}=(\tfrac{1}{2}\sqrt{\tfrac{3}{5}})(\hat{p}_\mu\sigma_\nu^{(1)}+\hat{p}_\nu\sigma_\mu^{(1)}-\tfrac{2}{3}\delta_{\mu\nu}\hat{p}\cdot\boldsymbol{\sigma}^{(1)}), \tag{5.4}$$

where $\hat{p}$ is an abbreviation for $\hat{p}_{13}$, and each of the above operators has been normalized to a unit value for its squared modulus. Physically, one expects $\hat{A}$ and $\hat{T}_{\mu\nu}$ to represent the polarizations of the A_1 and A_2 mesons and their $SU(3)$ counterparts. The physical status of s is less sound; though one would like to associate it with δ, ϵ, or κ mesons.[33]

The construction of the wave functions goes on the same lines as before, except that one now has more freedom in this regard, because of the two available vectors $\mathbf{p}_{13}$ and $\mathbf{P}_2$ (or $\mathbf{p}_{23}$ and $\mathbf{P}_1$) in the orbital wave function. The spin functions in the scalar (χ_s,χ_a), vector $(\boldsymbol{\chi}_s,\boldsymbol{\chi}_a,\boldsymbol{\chi}^s)$, and tensor $(\chi_{\mu\nu})$ representations, that were defined in Sec. II, must be used again in association with appropriate spatial functions of even or odd parity. However, unlike the previous case, where vector spin representations went only with odd-parity spatial functions, we shall now have *both* even- and odd-parity orbitals associated with them. Indeed, the various orbital functions that can be constructed with p-wave $Q\bar{Q}$ forces of the type (3.3), are as follows (using the same general notation ψ as before).[15]

A. Even Parity

$$D(E)\psi_{s,a}=v(p_{13})\mathbf{p}_{13}\cdot\mathbf{P}_2S_{s,a}(\mathbf{P}_2)\pm v(p_{23})\mathbf{p}_{23}\cdot\mathbf{P}_1S_{s,a}(\mathbf{P}_1), \tag{5.5}$$

$$D(E)\boldsymbol{\psi}_{s,a}^{(+)}=v(p_{13})\mathbf{p}_{13}\times\mathbf{P}_2A_{s,a}(\mathbf{P}_2)\pm v(p_{23})\mathbf{p}_{23}\times\mathbf{P}_1A_{s,a}(\mathbf{P}_1), \tag{5.6}$$

$$D(E)\psi_{s,a}^{\mu\nu}=v(p_{13})T_{\mu\nu}(13;2)T_{s,a}(\mathbf{P}_2)\pm v(p_{23})T_{\mu\nu}(23;1)T_{s,a}(\mathbf{P}_1), \tag{5.7}$$

where

$$T_{\mu\nu}(13;2)=\tfrac{1}{2}(p_{13\mu}P_{2\nu}+p_{13\nu}P_{2\mu}-\tfrac{2}{3}\mathbf{p}_{13}\cdot\mathbf{P}_2\delta_{\mu\nu}), \tag{5.8}$$

with a similar expression for $T_{\mu\nu}(23;1)$.

B. Odd Parity

$$\begin{aligned}D(E)\boldsymbol{\psi}_{s,a}^{(-)}=&v(p_{13})\mathbf{p}_{13}V_{s,a}(\mathbf{P}_2)\pm v(p_{23})\mathbf{p}_{23}V_{s,a}(\mathbf{P}_1)\\&+v(p_{13})(\mathbf{p}_{13}\times\mathbf{P}_2)\times\mathbf{P}_2W_{s,a}(\mathbf{P}_2)\\&\pm v(p_{23})(\mathbf{p}_{23}\times\mathbf{P}_1)\times\mathbf{P}_1W_{s,a}(\mathbf{P}_1).\end{aligned} \tag{5.9}$$

Each of the functions S, A, T, V, and W can be expressed in a partial-wave decomposition in the standard manner. The complete $QQ\bar{Q}$ functions of even and odd parity are now as follows:

$$\begin{aligned}\Psi^{(+)}=&(\psi_s\chi_s+\psi_a'\chi_a)\phi_s+(\psi_s'\chi_a+\psi_a\chi_s)\phi_a\\&+(\boldsymbol{\psi}_s^{(+)}\cdot\boldsymbol{\chi}_s+\boldsymbol{\psi}_a^{(+)\prime}\cdot\boldsymbol{\chi}_a)\phi_s+(\boldsymbol{\psi}_s^{(+)\prime}\cdot\boldsymbol{\chi}_a+\boldsymbol{\psi}_a^{(+)}\cdot\boldsymbol{\chi}_s)\phi_a\\&+(\boldsymbol{\psi}_s^{(+)\prime\prime}\phi_s+\boldsymbol{\psi}_a^{(+)\prime\prime}\phi_a)\cdot\boldsymbol{\chi}^s\\&+(\psi_s^{\mu\nu}\phi_s+\psi_a^{\mu\nu}\phi_a)\chi_{\mu\nu},\end{aligned} \tag{5.10}$$

where $\boldsymbol{\psi}_{s,a}^{(+)\prime\prime}$ are an independent set of orbital functions like (5.6) associated with the quartet-spin function $\boldsymbol{\chi}^s$. Similarly,

$$\begin{aligned}\Psi^{(-)}=&(\boldsymbol{\psi}_s^{(-)}\cdot\boldsymbol{\chi}_s+\boldsymbol{\psi}_a^{(-)\prime}\cdot\boldsymbol{\chi}_a)\phi_s+(\boldsymbol{\psi}_s^{(-)\prime}\cdot\boldsymbol{\chi}_a+\boldsymbol{\psi}_a^{(-)}\cdot\boldsymbol{\chi}_s)\phi_a\\&+(\boldsymbol{\psi}_s^{(-)\prime\prime}\phi_s+\boldsymbol{\psi}_a^{(-)\prime\prime}\phi_a)\cdot\boldsymbol{\chi}^s,\end{aligned} \tag{5.11}$$

where $\boldsymbol{\psi}_{s,a}^{(-)\prime\prime}$ are a second set of orbital functions like (5.9) associated with the quartet-spin function $\boldsymbol{\chi}^s$.

We shall not write down the dynamical equations satisfied by the various orbital functions listed above, since these are much more involved than those with s-wave forces, and would in any case be of little immediate physical interest. Instead we shall indicate the evaluation of the $PQ\to MQ$ ($M=0^+$, 1^+, and 2^+ mesons) amplitudes, in terms of the residues of the spectator functions S, A, T, V, and W. For purposes of bookkeeping it is now useful to adopt the same notation for the residues as the spectator functions themselves,

[32] R. H. Dalitz, le Houches Lecture Notes, 1965 (unpublished).

[33] See, e.g., G. Goldhaber, in *Proceedings of the Thirteenth Annual International Conference on the High-Energy Physics, Berkeley, Calif., 1966* (Univeristy of California Press, Berkeley, Calif., 1967), p. 103.

TABLE III. The coefficients c_i, d_i (of the direct and exchange contributions) which are associated with the invariants m_i ($i=1,\cdots,12$) for positive-parity meson production.

m_i	c_i	d_i
m_1	$3S^{(+)}+\bar{S}^{(-)}+\frac{3}{2}S^{(-)}+\frac{1}{2}\bar{S}^{(+)}$	$\frac{3}{2}S^{(+)}-\frac{3}{2}S^{(-)}+\frac{1}{2}\bar{S}^{(-)}-\frac{1}{2}\bar{S}^{(+)}$
m_2	$S^{(+)}-\bar{S}^{(-)}+\frac{1}{2}S^{(-)}-\frac{1}{2}\bar{S}^{(+)}$	$\frac{1}{2}S^{(+)}-\frac{1}{2}S^{(-)}-\frac{1}{2}\bar{S}^{(-)}+\frac{1}{2}\bar{S}^{(+)}$
m_3	$3A^{(+)}+\bar{A}^{(-)}+\frac{3}{2}A^{(-)}+\frac{1}{2}\bar{A}^{(+)}$	$\frac{3}{2}A^{(+)}-\frac{3}{2}A^{(-)}+\frac{1}{2}\bar{A}^{(-)}-\frac{1}{2}\bar{A}^{(+)}$
m_4	$2A^{(+)}-2\bar{A}^{(-)}+A^{(-)}-\bar{A}^{(+)}$	$A^{(+)}-A^{(-)}-\bar{A}^{(-)}+\bar{A}^{(+)}$
m_5	$2a^{(+)}+a^{(-)}$	$a^{(+)}-a^{(-)}$
m_6	$2T^{(+)}+T^{(-)}$	$T^{(+)}-T^{(-)}$
m_7	$3V^{(+)}+\bar{V}^{(-)}+\frac{3}{2}V^{(-)}+\frac{1}{2}\bar{V}^{(+)}$	$\frac{3}{2}V^{(+)}-\frac{3}{2}V^{(-)}+\frac{1}{2}\bar{V}^{(-)}-\frac{1}{2}\bar{V}^{(+)}$
m_8	$2V^{(+)}-2\bar{V}^{(-)}+V^{(-)}-\bar{V}^{(+)}$	$V^{(+)}-V^{(-)}-\bar{V}^{(-)}+\bar{V}^{(+)}$
m_9	$2v^{(+)}+v^{(-)}$	$v^{(+)}-v^{(-)}$
m_{10}	$3W^{(+)}+\bar{W}^{(-)}+\frac{3}{2}W^{(-)}+\frac{1}{2}\bar{W}^{(+)}$	$\frac{3}{2}W^{(+)}-\frac{3}{2}W^{(-)}+\frac{1}{2}\bar{W}^{(-)}-\frac{1}{2}\bar{W}^{(+)}$
m_{11}	$W^{(+)}-\bar{W}^{(-)}+\frac{1}{2}W^{(-)}-\frac{1}{2}\bar{W}^{(+)}$	$\frac{1}{2}W^{(+)}-\frac{1}{2}W^{(-)}-\frac{1}{2}\bar{W}^{(-)}+\frac{1}{2}\bar{W}^{(+)}$
m_{12}	$2w^{(+)}+w^{(-)}$	$w^{(+)}-w^{(-)}$

because of the large number of items involved. The symmetric and antisymmetric functions, on the other hand, are now distinguished by the superscripts ($\pm$). The residues associated with the prime functions (ψ_s', ψ_a', etc.) are indicated by bars on the corresponding symbols, while those associated with the functions $\psi^{(\pm)''}$ are shown by the corresponding small letters. This gives the following glossary of symbols in the residue functions $R^{(\pm)}$ of $\Psi^{(\pm)}$:

$$\begin{aligned} R^{(+)}=\mathbf{p}\cdot\mathbf{k}'[&(S^{(+)}\chi_s+\bar{S}^{(-)}\chi_a)\phi_s+(\bar{S}^{(+)}\chi_a+S^{(-)}\chi_s)\phi_a] \\ &+(\mathbf{p}\times\mathbf{k}')\cdot\boldsymbol{\chi}_s[A^{(+)}\phi_s+A^{(-)}\phi_a] \\ &+(\mathbf{p}\times\mathbf{k}')\cdot\boldsymbol{\chi}_a[\bar{A}^{(-)}\phi_s+\bar{A}^{(+)}\phi_a] \\ &+(\mathbf{p}\times\mathbf{k}')\cdot\boldsymbol{\chi}^s(a^{(+)}\phi_s+a^{(-)}\phi_a) \\ &\quad+T_{\mu\nu}(13;2)\chi_{\mu\nu}[T^{(+)}\phi_s+T^{(-)}\phi_a]; \end{aligned} \tag{5.12}$$

$$\begin{aligned} R^{(-)}=\mathbf{p}\cdot\mathbf{k}'[&\mathbf{k}'\cdot\boldsymbol{\chi}_s(W^{(+)}\phi_s+W^{(-)}\phi_a) \\ &+\mathbf{k}'\cdot\boldsymbol{\chi}_a(\bar{W}^{(-)}\phi_s+\bar{W}^{(+)}\phi_a)]+(\mathbf{p}\cdot\mathbf{k}')(\mathbf{k}'\cdot\boldsymbol{\chi}^s) \\ &\times(w^{(+)}\phi_s+w^{(-)}\phi_a)+(\mathbf{p}\cdot\boldsymbol{\chi}^s)(v^{(+)}\phi_s+v^{(-)}\phi_a) \\ &+\mathbf{p}\cdot\boldsymbol{\chi}_s(V^{(+)}\phi_s+V^{(-)}\phi_a) \\ &\quad+\mathbf{p}\cdot\boldsymbol{\chi}_a(\bar{V}^{(-)}\phi_s+\bar{V}^{(+)}\phi_a). \end{aligned} \tag{5.13}$$

Here $\mathbf{k}'$ $(=\mathbf{P}_2)$ represents the separation momentum of final meson-quark system and $\mathbf{p}$ is an abbreviation for $\mathbf{p}_{13}$.

For further reduction we note that the $SU(3)$ structures of the amplitudes are formally identical with (4.23) and (4.31) for **8** meson scattering and **1** meson production, respectively, except that the quantities $\tilde{A}$ and $\tilde{B}$ have now to be replaced by alternative quantities $\tilde{C}$ and $\tilde{D}$ which depend on the symbols of (5.12) and (5.13) according to the following scheme:

$$\tilde{A}\rightarrow\tilde{C}=\sum_{i=1}^{12}m_ic_i, \tag{5.14}$$

$$\tilde{B}\rightarrow\tilde{D}=\sum_{i=1}^{12}m_id_i, \tag{5.15}$$

where m_i are a set of twelve invariants formed out of the initial and final momenta $\mathbf{k}$ and $\mathbf{k}'$, as well as the spin vectors (s, $\hat{B}$, $\hat{A}$, $\hat{T}_{\mu\nu}$) of the emitted mesons. The invariants m_i, together with their construction, are listed in the Appendix. The coefficients (c_i,d_i), expressed in terms of the various residue functions, are listed in Table III.

So far the method makes little use of explicit dynamics which is considerably more involved in this case than for the P- and V-meson production. The following features may however be noted. At the $SU(3)$ level, the predictions are formally identical to those in Ref. 11 or Sec. IV of this article. Thus for *each type* of meson emitted, all matrix elements are expressible in terms of six quantities like Eqs. (4.34)–(4.36), so that Tables I and II apply with the replacements of (5.14) and (5.15). Evaluation of the spin-matrix elements would lead to connections between amplitudes for different kinds of meson production. One must be careful in this regard, however. Thus though the 24 parameters (c_i,d_i) describe in principle, all the processes involved, this simplification is based on the assumption that all the mesons are equally massive. Since such an assumption would even *prima facie*, be absurd, the mass dependence of the coefficients should at least approximately be taken into account. While this would, in general, increase the number of parameters almost to the $SU(3)$ level, the situation can be partly redeemed by noting that the masses of at least some of the mesons with *different* spins are not very different. In this formalism, it is only for such particles, carefully chosen, that one could talk about meaningful relations among their production amplitudes. These and related questions will be discussed in a forthcoming paper.[31]

VI. SUMMARY AND CONCLUSIONS

We have tried to develop a $QQ\bar{Q}$ model of production of various types of mesons by pseudoscalar mesons on baryons. This method is somewhat different from the other contemporary quark models, in that it recognizes a hierarchy of tightness in the quark structures of hadrons, the mesons as $Q\bar{Q}$ composites being considered much tighter than baryons as QQQ composites. Under such an assumption it is meaningful to speak of an impulse approximation for individual scattering of the quarks in a baryon by the meson. The $QQ\bar{Q}$ model of the quark-meson scattering, on the other hand, takes account of all multiple-scattering effects within the meson-quark system. The corrections due to the multiple scattering of the meson by the quarks of the baryon should be possible to take into account in a second-stage process within the present formalism which appears to be general enough for such a purpose. A limitation of the model is that its predictions are restricted only to meson-baryon processes, and, unlike other quark models, it does not claim to relate processes like BB or BB with the above.

The model makes little explicit use of dynamics, except to employ the method of spectator functions to calculate the various amplitudes. The possible uses of simple dynamical considerations by which one could distinguish the larger amplitudes from the smaller ones for pseudoscalar and vector mesons, are broadly indicated. The various amplitudes are calculated in a certain hierarchy of steps which go from the $SU(3)$ level of predictions concerning final mesons of *only one* spin-parity type, at a time, to the more specific $SU(6)$-level by which the interrelations between the production amplitudes for different spin-parity types could be obtained. It is also recognized that because of the large mass-breaking effects a careful selection of the particles involved in this second step is essential, in order to obtain physically meaningful relations between the amplitudes.

The contents are devoted almost exclusively to formulation, and no attempt is made here to present any comparison with experiment, which is relegated to a further paper. Finally, while the results for negative-parity mesons are similar to those obtained in other contemporary models, the method outlined for the production of positive-parity mesons does not seem to have had a previous counterpart.

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APPENDIX

The invariants m_i ($i=1, \cdots, 12$) arise out of the evaluation of the spin-matrix elements of (5.12) and (5.13), using a common basis function χ. We use the general notations τ, α, β, s for the invariants associated with A_2-, A_1-, B-, and σ-type mesons, respectively. The following quantities appear in the derivation:

$$\beta_0 = p\mathbf{k}\cdot\hat{B}\,, \qquad \beta_0' = p\mathbf{k}'\cdot\hat{B};$$
$$\beta_1 = ip\hat{B}\cdot(\mathbf{k}\times\boldsymbol{\sigma}^{(2)})\,, \qquad \beta_1' = ip\hat{B}\cdot(\mathbf{k}'\times\boldsymbol{\sigma}^{(2)});$$
$$s_1 = ps\mathbf{k}\cdot\boldsymbol{\sigma}^{(2)}\,, \qquad s_1' = ps\mathbf{k}'\cdot\boldsymbol{\sigma}^{(2)};$$
$$\alpha_0;\, \alpha_0' = \sqrt{\tfrac{1}{2}}\sqrt{2}\,p\hat{A}\cdot(\mathbf{k};\mathbf{k}')\,,$$
$$\alpha_1;\, \alpha_1' = \sqrt{\tfrac{1}{2}}\sqrt{2}\,ip(\boldsymbol{\sigma}^{(2)}\times\hat{A})\cdot(\mathbf{k};\mathbf{k}')\,,$$
$$\alpha_2 = k'^2\alpha_0 - \mathbf{k}\cdot\mathbf{k}'(\alpha_0' + \alpha_1');$$
$$\mathbf{n} = i\mathbf{k}\times\mathbf{k}';$$
$$\tau_0';\, \tau_2 = [\sqrt{(5/3)}]p\hat{T}_{\mu\nu}(\sigma_\mu{}^{(2)}k_\nu';\, k_\mu' n_\nu)\,,$$
$$\tau_1';\, \tau_1'' = [(\sqrt{5/3})]p\hat{T}_{\mu\nu}[k_\mu k_\nu'\mathbf{k}'\cdot\boldsymbol{\sigma}^{(2)};\, k_\mu' k_\nu'\mathbf{k}\cdot\boldsymbol{\sigma}^{(2)}]\,,$$
$$\tau_3 = \tau_1' - \tau_2 - \mathbf{k}\cdot\mathbf{k}'\tau_0'.$$

In terms of these symbols the invariants m_i are as follows:

$$m_1 = \beta_0'$$
$$m_2 = \tau_0' + \alpha_1' + \tfrac{1}{3}s_1'$$
$$m_3 = \beta_1'$$
$$m_4 = \alpha_0' + \tfrac{1}{2}\alpha_1' - \tfrac{1}{2}\tau_0' + \tfrac{1}{3}s_1'$$
$$m_5 = 2\alpha_0' - \tfrac{1}{3}s_1' - \tfrac{1}{2}\alpha_1' + \tfrac{1}{2}\tau_0'$$
$$m_6 = \tfrac{1}{3}\tau_0' - (5/3)\alpha_1' + (10/9)s_1'$$
$$m_7 = \beta_0 - \beta_1$$
$$m_8 = \tfrac{1}{2}s_1 + \alpha_0 - \alpha_1$$
$$m_9 = s_1 + \alpha_1 - \alpha_0$$
$$m_{10} = \beta_0'(\mathbf{k}\cdot\mathbf{k}' + \boldsymbol{\sigma}^{(2)}\cdot\mathbf{n})$$
$$m_{11} = \tau_1'' + \tau_3 + \tfrac{1}{3}k'^2 s_1 + \alpha_2$$
$$m_{12} = \tau_1'' - \tfrac{1}{2}\tau_3 + \tfrac{1}{3}k'^2 s_1 - \tfrac{1}{2}\alpha_2.$$

The invariants m_1–m_6 are formed out of the positive-parity components of the wave function, and the remaining six quantities m_7–m_{12} out of the negative-parity components.