Effects of Final-State Interactions on the Decay Rate $_\Lambda H^4 \to \pi^- + He^4$

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The effects of final-state interactions on the hyperfragment decay rate $_\Lambda H^4 \to \pi^- + He^4$ have been calculated, using a recent measurement of the s-wave phase shift from elastic π-He^4 scattering at 139 MeV/c. The repulsive nature of the s-wave interaction leads to a 21% lowering of the partial decay rate calculated by Dalitz and Liu using noninteracting (plane-wave) particles in the final state. A new total lifetime estimate $\tau(_\Lambda H^4) = 1.75\times 10^{-10}$ sec is made.

THE partial decay rate $_\Lambda H^4 \to \pi^- + He^4$ has been calculated by Dalitz and Liu,[1] who assume a plane wave for the final pion–α-particle system. This is, of course, the same as the assumption that there is no interaction between the pion and the α particle. The point of this note is to modify the Dalitz-Liu calculation in order to estimate the effects of the strong final-state interaction on the decay channel $_\Lambda H^4 \to \pi^- + He^4$.

Dalitz and Liu start with the premise that the hyperfragment $_\Lambda H^4$ is describable as a composite of a bound Λ and a triton, where the bound Λ decays essentially as a free particle (the impulse model). The α-particle wave function is taken as a Gaussian, whose size parameter is matched to the electron-He^4 scattering experiments. They then calculate a "sticking factor," the overlap integral[2] $G(q)$, where

$$G(q) = \left(\frac{3\alpha_4}{\pi}\right)^{3/4} \int \exp(-\tfrac{3}{2}\alpha_4 r^2) \frac{\sin\frac{3}{4}qr}{\frac{3}{4}qr} u_\Lambda(r) d_3 r. \quad (1)$$

In (1), the α-particle wave function is taken to be proportional to $\exp(-\frac{1}{2}\alpha_4 \sum r_{ij}^2)$, where $\alpha_4 = \frac{9}{32}R_4^2$, with R_4 the He^4 nuclear radius, q the momentum transfer, and $u_\Lambda(r)$ the hyperfragment wave function. The wave function $u_\Lambda(r)$ was numerically evaluated[2] as a function of the binding energy, and $G(q)$ computed.[2] Since $_\Lambda H^4$ has been shown to be spin zero,[3] the only partial wave entering in the decay $_\Lambda H^4 \to \pi^- + He^4$ is the s wave. We note that in (1), the factor $(\sin\frac{3}{4}qr)/\frac{3}{4}qr = j_0(\frac{3}{4}qr)$ represents the s-wave portion of the plane wave $\exp(i\frac{3}{4}\mathbf{q}\cdot\mathbf{r})$. Therefore, in order to take final-state interactions into account, we should replace $j_0(\frac{3}{4}qr)$ by the actual s-wave radial wave function $R_0(\frac{3}{4}qr)$, and hence define a new overlap integral

$$H(q) = \left(\frac{3\alpha_4}{\pi}\right)^{3/4} \int \exp(-\tfrac{3}{2}\alpha_4 r^2) R_0(\tfrac{3}{4}qr) u_\Lambda(r) d_3 r. \quad (2)$$

We calculate (2) at $q = 136$ MeV/c, in order to compute the effects of final-state interaction on the partial rate $_\Lambda H^4 \to \pi^- + He^4$.

In order to estimate R_0 in (2), we appeal to experiment. The Northwestern University helium bubble-chamber group has recently completed measurements on elastic pion-α scattering, i.e., $\pi^\pm + He^4 \to \pi^\pm + He^4$, at a laboratory momentum of 139 MeV/c, which corresponds to the same momentum transfer as the hyperfragment decay. A preliminary phase-shift analysis[4] indicates that the s-wave π-He^4 phase shift corresponds to a repulsive potential, and that it is given by $\delta_0 = (-8.7\pm 0.7 + i0.60 \pm i0.06)$ deg. The imaginary part of the phase shift reflects the fact that both α-particle breakup and absorption processes (inelastic channels) occur in π-He^4 scattering. Using the electron scattering results for the He^4 system,[5,6] and suitably modifying it to take into account the finite proton size, we choose $R_4 = 1.41$ F. With this Gaussian size, we assume the complex nuclear potential to be given by $U(r) = (U_0 - iW_0)e^{-\beta r^2}$, with $\beta = 3/2R_4^2$. We then solve the Schrödinger scattering equation for $l=0$,

$$\frac{d^2\chi_0}{d\rho^2} + \left(1 - \frac{V_0(\rho)}{q^2}\right)\chi_0 = 0, \quad (3)$$

where $\rho = qr$, $\chi_0 = \rho R_0$, and V_0, the reduced potential, is given by $V_0(r) = 2\mu U(r)$, with μ the reduced π-He^4 mass. We integrate (3) numerically on the computer to find those values of U_0 and W_0 that give rise to the observed s-wave scattering phase shifts (real and imaginary parts, respectively). This in turn leads to a choice of a radial wave function R_0, which is then inserted in (2) to calculate $H(q)$. We have also evaluated the bound-state radial wave function $u_\Lambda(r)$ numerically, and hence the plane-wave integral $G(q)$. We obtain, at $q = 136$ MeV/c,

$$G = 0.735, \quad (4a)$$

$$H = 0.650 + i0.0051. \quad (4b)$$

It is clear on physical grounds that the smaller magnitude of H reflects the repulsive nature of the final-state interaction. We are finally interested in the ratio $G^2/|H|^2$, which gives the change in the rate of the

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hyperfragment decay channel ${}_\Lambda H^4 \to \pi^- + He^4$. We note that the imaginary portion of H contributes a negligible contribution. From (4a) and (4b), we calculate the ratio of the partial rates $\Gamma({}_\Lambda H^4 \to \pi^- + He^4)$, with and without final-state interactions, to be

$$\frac{\Gamma_{\text{plane}}({}_\Lambda H^4 \to \pi^- + He^4)}{\Gamma_{\text{inter}}({}_\Lambda H^4 \to \pi^- + He^4)} = 1.21 \pm 0.04. \quad (5)$$

We note a substantial lowering of the partial decay rate. The error above reflects the uncertainty in the experimental value of δ_0. The calculation is quite insensitive to the nuclear size parameter. Further, since in (5), the same hyperfragment wave function is used in both the numerator and denominator, the result is also insensitive to the details of the hyperfragment wave function used.

It has been pointed out[7] that a measurement of the branching ratio $({}_\Lambda H^4 \to e^- + \bar{\nu} + He^4)/({}_\Lambda H^4 \to \pi^- + He^4)$ allows one to measure the pure Fermi coupling for lambda β-decay, since the β decay is a $0 \to 0$ transition. Of course, it is necessary to have a reliable theoretical estimate of the partial rate ${}_\Lambda H^4 \to \pi^- + He^4$; thus, the 21% lowering due to final-state effects plays a critical role in the interpretation of the above experiment.

In conclusion, we modify the estimate of the ${}_\Lambda H^4$ lifetime $\tau({}_\Lambda H^4) = 0.65\ \tau(\Lambda)$ calculated by Dalitz and Rajasekaran,[8] to be

$$\tau({}_\Lambda H^4) = 0.73\tau(\Lambda) = 1.75 \times 10^{-10} \text{ sec}, \quad (6)$$

where the lifetime in (6) reflects the effects of final-state interactions. Unfortunately, no comparison with experiment is possible, since no accurate experimental measurement of $\tau({}_\Lambda H^4)$ has as yet been performed.

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$QQ\bar{Q}$ Model of Meson-Baryon Processes

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A $QQ\bar{Q}$ model of beson-baryon scattering and production of both positive- and negative-parity mesons is developed as a generalization of the "elementary meson" model proposed earlier by the authors. The model is based on the assumption that mesons are tighter structures than baryons, which allows the former to "see through" the latter, but not vice versa. The formalism allows the full inclusion of multiple-scattering effects within the meson-quark system via the $QQ\bar{Q}$ structure, and leaves scope for the inclusion of similar effects between the meson and the quarks in the baryon, as a second-stage process. The model is more restricted than the usual quark models (based on additivity), which in general enable correlations of meson-baryon, baryon-baryon, and baryon-antibaryon amplitudes. The model makes little use of dynamics, except for the method of "spectator functions" to calculate the amplitudes for the various processes in terms of residues at the appropriate poles of the relevant spectator functions. However, the possible uses of simple dynamical considerations are indicated, by which the larger amplitudes could be distinguished from the smaller ones. The amplitudes are evaluated in a two-stage process, the first step indicating the "$SU(3)$ level" of predictions, involving only one spin-parity type at a time, while the second step gives $SU(6)$-type predictions which in principle connect mesons of different spin-parity assignments. For the negative-parity mesons (0^- and 1^-), the results are similar to those obtained by previous authors. However, the model provides, with no extra physical assumptions but considerably more algebraic manipulations, amplitude relations for the production of positive-parity mesons (0^+, 1^+, and 2^+) which, according to the quark picture, are structures of the form ${}^1P_1$ and ${}^3P_{0,1,2}$, and are expected to simulate mesons of the type B, A_1, and A_2 (and perhaps also the scalar mesons). No attempt is made in this paper to confront the predictions with experiment, which will be the subject of a subsequent communication.

I. INTRODUCTION

SINCE the additive quark (Q) model for hadron scattering was first proposed by Levin and Frankfurt,[1] as well as by Lipkin and Scheck,[2] there have been a number of investigations on its more detailed effects which included inelastic processes.[3,4] Thus the

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