S-Wave $\pi^- + p$ Scattering and a Possible η-n Bound State

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The $\pi^- + p$ $T=\frac{1}{2}$, $J=\frac{1}{2}^-$ scattering length and phase shift are calculated in a coupled-channel model of π-N and η-N scattering. The η-n system is treated as a Regge pole associated with quasibound state. The Born terms associated with ρ and nucleon exchange are included in the calculation. The results are compared with experimental data.

An extension of Regge[1] representation for studying the low-energy behavior of scattering amplitudes has been applied to $T=\frac{1}{2}$, $J=\frac{1}{2}^-$ state of π-N scattering. The Khuri representation[2] was modified by Bohm and Rashid[3] for $T=\frac{1}{2}$, $J=\frac{1}{2}$ π-N scattering, and by Pal[4] for $\bar{K}$-N scattering, by introducing Born terms where possible. Bulos *et al.*[5] and Peterson *et al.*[6] found a rapid rise in the inelasticity in $\pi^- p$ scattering associated with the threshold for η production in the reaction $\pi^- + p \rightarrow \eta + n$. The elegant phase-shift analysis of Auvil *et al.*[7] suggests that a discrete baryon state may exist in the neighborhood of $\eta + n$ threshold. Further evidence for such a state is obtained from the recent experimental data,[8] which show that the total cross section for the reaction $\pi^- + p \rightarrow \eta + n$ rises steeply from threshold to a value of 1 mb at an incident pion kinetic energy (T_{π^-}) between 655 and 704 MeV, and then falls gradually to 0.25 mb at 1300 MeV. The quantum numbers of this baryon state are of course $T=\frac{1}{2}$, $J=\frac{1}{2}^-$.

In the present paper, we use the modified Khuri representation for π-N scattering and the foregoing data on the η-n system to calculate the $T=\frac{1}{2}$, $J=\frac{1}{2}^-$ scattering length and phase shift. The η-n system will be treated as a Regge pole associated with a quasibound state in all the channels $\pi^- + p \rightarrow \pi^- + p$, $\eta + n \rightarrow \eta + n$ and $\pi^- + p \rightarrow \eta + n$ in S-wave $T=\frac{1}{2}$ scattering.

The $S_{1/2}$ partial wave for these reactions can be written as a matrix in the following way:

$$T=\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}. \tag{1}$$

Using the Khuri representation, the contribution of the η-n Regge pole to the elements of the T matrix can be calculated in a straightforward way. The result for T, including the Born terms associated with nucleon and ρ-meson exchange in $\pi^- + p \rightarrow \pi^- + p$ elastic scattering, is

$$\begin{aligned} T_{11}=&\frac{\beta}{\alpha(W)-\frac{1}{2}}[e^{(\alpha-1/2)\xi_1}+e^{(\alpha-1/2)\xi_2}]-\frac{g_{NN\pi}{}^2}{4\pi W^2}\bigg\{[(W+m)^2-m_\pi{}^2]\left(\frac{W-m}{2k^2}\right)Q_0\left(\frac{2E^2+2m_\pi{}^2-W^2-m^2}{2k^2}\right) \\ &+[(W-m)^2-m_\pi{}^2]\left(\frac{W+m}{2k^2}\right)Q_1\left(\frac{2E^2+2m_\pi{}^2-W^2-m^2}{2k^2}\right)\bigg\}-\frac{2g_{\rho\pi\pi}g_{\rho NN}}{4\pi W^2}\bigg\{[(W+m)^2-m_\pi{}^2] \\ &\times\left(\frac{W-m}{k^2}\right)Q_0\left(\frac{2E^2-2m^2+m_\rho{}^2}{2k^2}\right)+[(W-m)^2-m_\pi{}^2]\left(\frac{W+m}{k^2}\right)Q_1\left(\frac{2E^2-2m^2+m_\rho{}^2}{2k^2}\right)\bigg\}, \end{aligned} \tag{2}$$

$$T_{12}=\frac{\beta_{12}}{\alpha(W)-\frac{1}{2}}[e^{(\alpha-1/2)\xi_1''}+e^{(\alpha-1/2)\xi_2''}], \tag{3}$$

$$T_{22}=\frac{\beta_2}{\alpha(W)-\frac{1}{2}}[e^{(\alpha-1/2)\xi_1'}+e^{(\alpha-1/2)\xi_2'}]. \tag{4}$$

The coupling constants $g_{NN\pi}$, $g_{\rho\pi\pi}$, and $g_{\rho NN}$ are taken from experiment. In (2)–(4), $\alpha(W)$ is the trajectory function, and the β's are the corresponding residue functions. The ξ are given by

$$\cosh\xi_1=1+8m_\pi{}^2/k^2, \tag{5}$$

$$\cosh\xi_2=[W^2-(m-m_\pi)^2]/2k^2-1, \tag{6}$$

$$\cosh\xi_1'=1+2m_\pi{}^2/q^2, \tag{7}$$

$$\cosh\xi_2'=(W^2-m^2-2m_\pi{}^2)/2q^2-1, \tag{8}$$

$$\cosh\xi_1''=[(m_\pi+m_\eta)^2-2m^2+2(k^2+m^2)^{1/2}(q^2+m^2)^{1/2}]/2kq, \tag{9}$$

$$\cosh\xi_2''=[W^2+m^2-m_\pi{}^2-m_\eta{}^2-2(k^2+m^2)^{1/2}(q^2+m^2)^{1/2}]/2kq. \tag{10}$$

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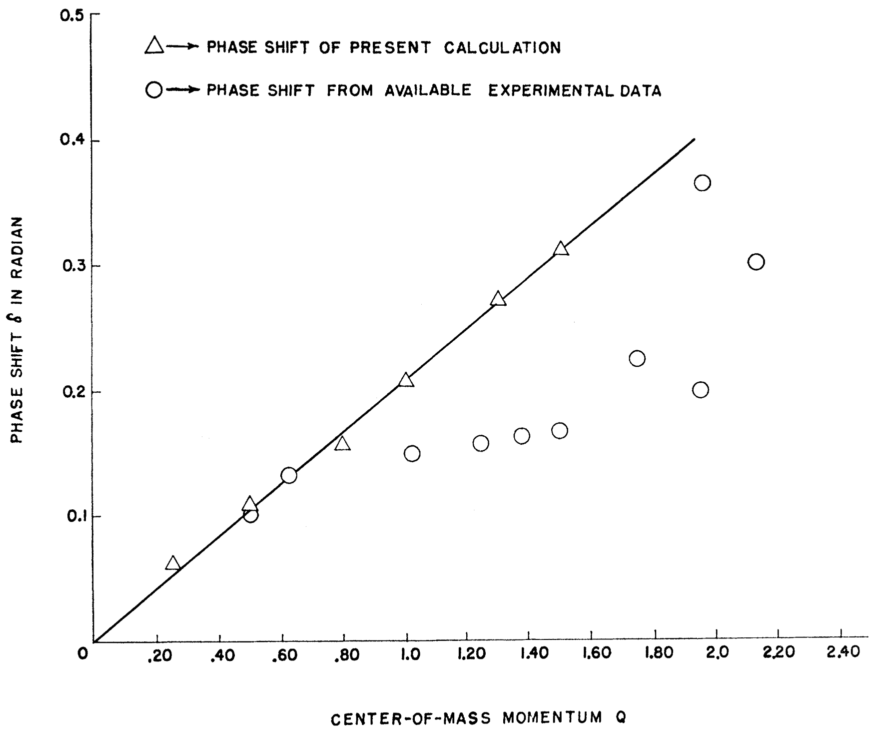

FIG. 1. Comparison of the $T=\frac{1}{2}$, $J=\frac{1}{2}^-$ phase shift calculated with the present model, and the experimental results of Miyake *et al.* (Ref. 11). Q is the pion momentum in the center-of-mass system in units of $m_\pi c$.

k and q are the center-of-mass momenta in the $\pi^-+p\rightarrow\pi^-+p$ and $\eta+n\rightarrow\eta+n$ channels, respectively. In (5)–(10), W is the total c.m. energy and m is the nucleon mass. m_π, m_η, and m_p denote the masses of the corresponding particles. The β's are given by the relations based on the threshold behavior and the factorization theorem.[9] The η-n trajectory may be written in the form

$$\mathrm{Re}\,\alpha(W)=\tfrac{1}{2}+\epsilon(W-m_{\eta n}), \quad (11)$$

$$\mathrm{Im}\,\alpha(W)=\tfrac{1}{2}\epsilon\Gamma\left(\frac{W-m-m_\pi}{m_{\eta n}-m-m_\pi}\right)^{\alpha_0}, \quad (12)$$

TABLE I. Comparison of the $T=\frac{1}{2}$, $J=\frac{1}{2}^-$ phase shift calculated with the present model, and the experimental results of Miyake *et al.* (Ref. 11).

Center-of-mass momentum of pion in units of $m_\pi c$	Phase shift δ in radians Experimental value	Present calculation
0.50	0.100	0.108
0.63	0.143	0.128
1.02	0.150	0.210
1.25	0.158	0.258
1.38	0.163	0.263
1.50	0.168	0.310

where $\alpha_0=\alpha(W)|_{k^2\rightarrow 0}$. In the above, $m_{\eta n}$ and Γ are the mass and width of the η-n state. For the slope of the η-n trajectory, we take the same value as that found for the nucleon, $\epsilon=0.4$. The complex scattering length for the reaction $\pi^-+p\rightarrow\eta+n$ is $a+ib$, where $a=0.8304$ F and $b=0.05$ F according to Peterson *et al.* The width Γ of the η-n state can be calculated very crudely in terms of a and b; we obtain $\Gamma=b/\mu_\eta a^3=83.4$ MeV, where μ_η is the reduced mass of η and n. Now the expression for T_{11} becomes

$$T_{11}=\frac{C_1}{\alpha(W)-\frac{1}{2}}\left[1+e^{(\alpha-1/2)(\xi_2-\xi_1)}\right]+\text{Born terms}. \quad (13)$$

The constant C_1 may be determined, using unitarity, in terms of ϵ and Γ. This procedure yields $C_1\simeq-\epsilon\Gamma/4k_{\eta n}\simeq-0.02$. The calculated value of the S-wave $T=\frac{1}{2}$ scattering length is $A_{11}=T_{11}(W)|_{k\rightarrow 0}\simeq 0.15$, which can be compared with the experimental data by Woolcock,[10] $A_1=0.17\pm0.005$. The $S_{1/2}$ phase shifts calculated for the reaction $\pi^-+p\rightarrow\pi^-+p$ are compared with the experimental data by Miyake *et al.*[11] in Table I and Fig. 1. The quantity Q is the momentum of meson in units of $m_\pi c$ in the center-of-mass system.

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