quantum numbers. While a number of exotic suggestions—such as a self-reproducing branch point or colliding poles could not be rigorously excluded, in our opinion the orthodox picture of a single simple Regge pole with intercept unity (plus all the associated branch points) is preferred. We have suggested that important information can be derived from the phase at high energy. Perhaps, branch points are required to fully account for it.

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CP Violation in Current-Current Models*

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The usual current-algebra techniques are used in two separate models for relating *CP*-violating effects in $K \to 2\pi$ decays with possible *CP* violations in ordinary and strange β decays. In the first model, one meson is removed from the $K \to 2\pi$ matrix element, and the resulting current-current spurion is saturated with a limited number of intermediate states. The second model is essentially that of Glashow, Schnitzer, and Weinberg. In this model, it is found that the *CP*-violating rate can be calculated in terms of the chiral-symmetry spectral-function sum rules. The *CP*-conserving rate, on the other hand, required the use of $SU(3)$ spectral-function sum rules. It is found that (a) the leptonic *CP* violations appear to be quite small, (b) the $\Delta I=\frac{3}{2}$ *CP*-violating transitions do not seem to be suppressed by the usual $\Delta I=\frac{1}{2}$ dynamical mechanism, and (c) there is a (model-dependent) tendency for the nonleptonic *CP* violation to vanish in the limit of chiral symmetry.

I. INTRODUCTION

In this paper, we use the hypothesis of partially conserved axial-vector current[1] (PCAC) and the chiral $SU(3)\otimes SU(3)$ current algebra[2] to get an estimate of the relation between the nonleptonic and possible leptonic *CP*-violating effects[3] in the weak interactions. The assumption is made that the weak Hamiltonian can be written in current-current form.

Two separate models are considered. In the first, described in Sec. II, the effective Hamiltonian is taken to be a first-order strictly local one. Then, using the standard techniques,[4] one meson is "removed" from the $K \to 2\pi$ matrix element, and the resulting current-current spurion is "saturated" with a limited number of intermediate states.

The second model is the one which has been used by Glashow, Schnitzer, and Weinberg[5] to treat the $K \to 2\pi$, *CP*-conserving decay. The weak interaction is in this case taken to proceed by an intermediate vector boson. Furthermore, all three mesons are removed from the decay matrix element, and the resulting vacuum expectation values are calculated by assuming that the Weinberg-type sum rules[6] are strictly true. Whereas this approach involves the $SU(3)$ spectral-function sum rules for the *CP*-conserving case, it turns out to involve the chiral-symmetry sum rules for the *CP*-violating case. The second model is described in Sec. III.

It is clear that each model involves a number of somewhat drastic and *different* assumptions. Therefore we will not attempt to compare the two models at this time. We note, however, that the following two conclusions emerge from both:

(a) The predicted *CP* violations for ordinary and strangeness-changing β decay seem to be quite small, on the verge of or below the present experimental uncertainties.

(b) The $\Delta I=\frac{3}{2}$ *CP*-violating amplitude is not suppressed in the same way as the $\Delta I=\frac{3}{2}$ *CP*-conserving amplitude.

In the second model, and for a (plausible) special case of the first model, we have an additional conclusion:

(c) The *CP*-violating $K \to 2\pi$ decays vanish in the limit of chiral symmetry.

A discussion of the results will be given in Sec. IV.

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[3] J. Christenson, J. Cronin, V. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[4] This model is based on the one described by Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **157**, 1317 (1967).

[5] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 205 (1967). Denote this by GSW.

[6] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967); J. J. Sakurai, Phys. Letters **24B**, 619 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 205 (1967).

II. GENERAL CONSIDERATIONS AND FIRST MODEL

We define l_λ,[7] $V_{b\lambda}{}^a$, and $P_{b\lambda}{}^a$ to be, respectively, the leptonic, vector-octet, and pseudovector-octet currents. Under CP, $\mathbf{V}_b{}^a(\mathbf{x},t)\to\mathbf{V}_a{}^b(-\mathbf{x},t)$ and $\mathbf{P}_b{}^a(\mathbf{x},t)\to\mathbf{P}_a{}^b(-\mathbf{x},t)$. The total weak-Hamiltonian density is written as the symmetrical product of two currents:

$$H_w=\tfrac{1}{2}(G/\sqrt{2})[J_\lambda,J_\lambda{}^\dagger]_+\,, \tag{1}$$

where

$$J_\lambda=l_\lambda+\cos\theta(V_{2\lambda}{}^1+e^{i\xi}P_{2\lambda}{}^1)+\sin\theta(V_{3\lambda}{}^1+e^{i\psi}P_{3\lambda}{}^1)\,. \tag{2}$$

In the above equations, $G\simeq 10^{-5}/M_p{}^2$, $\sin\theta\cos\theta\simeq 0.22$, and ξ and ψ are the CP-violating phases. These phases are directly measurable[8] in neutron and Λ or Σ β decay. If $\psi=\xi$, there will evidently be no CP violation in the *parity-conserving* amplitudes. Our form for H_w is similar to those of Glashow[9] and (especially) Alles.[10]

From the analysis of Wu and Yang,[11] we learn that for describing all $K\to 2\pi$ decays, it is sufficient (apart from the p and q factors) to consider the following two matrix elements:

$$\begin{aligned}M_{+0}&=[(2P_{+0})(2P_{00})(2K_0)]^{1/2}\\&\qquad\times\langle\pi^+(P_+)\pi^0(P_0)|H_w|K^+(K)\rangle\\&=(\sqrt{\tfrac{3}{2}})\tilde{A}_2e^{i\delta_2}\,,\end{aligned} \tag{3a}$$

$$\begin{aligned}M_{+-}&=[(2P_{+0})(2P_{-0})(2K_0)]^{1/2}\\&\qquad\times\langle\pi^+(P_+)\pi^-(P_-)|H_w|K^0(K)\rangle\\&=(\sqrt{\tfrac{2}{3}})\tilde{A}_0e^{i\delta_0}+\tfrac{1}{3}\sqrt{3}e^{i\delta_2}\tilde{A}_2\,,\end{aligned} \tag{3b}$$

where $\tilde{A}_0$ and $\tilde{A}_2$ are, respectively, the final $\pi\pi$ isospin-0 and isospin-2 amplitudes, while δ_0 and δ_2 are the appropriate scattering phase shifts. For simplicity, we shall neglect final-state interactions by formally setting $\delta_0=\delta_2=0$.

The usual current-algebra technique yields expressions for M_{+0} and M_{+-} in the limit where one of the meson four-momenta is extrapolated to zero. One problem is to relate these limits to the physical amplitudes. If we are only interested in the ratio $\tilde{A}_2/\tilde{A}_0$, then various methods[4] will give the same results for the CP-*conserving* case. However, this is not true when CP violation is allowed, so we shall adopt the least controversial method and take the physical amplitude to be one-half the sum of the two *pion* limits. These limits are expressed in terms of current-current spurions of the form $\langle\pi|[V_b{}^a,V_d{}^c]_+|K\rangle$ and $\langle\pi|[P_b{}^a,P_d{}^c]_+|K\rangle$. It is convenient to evaluate these spurions in the limit of $SU(3)$ symmetry.[12] The decomposition into irreducible $SU(3)$ tensors is as follows[4]:

$$\begin{aligned}V_{bd}{}^{ac}&\equiv\langle\pi|[V_b{}^a,V_d{}^c]_+|\pi\rangle\\&=\tau^VT_{bd}{}^{ac}+\delta^V[(\delta_d{}^aD_b{}^c+\delta_b{}^cD_d{}^a)\\&\qquad-\tfrac{2}{3}(\delta_b{}^aD_d{}^c+\delta_d{}^cD_b{}^a)]\\&\qquad+\sigma^V(\delta_d{}^a\delta_b{}^c-\tfrac{1}{3}\delta_b{}^a\delta_d{}^c)\langle\pi\pi\rangle\,,\end{aligned} \tag{4}$$

where $T_{bd}{}^{ac}$, $D_b{}^a$, and $\langle\pi\pi\rangle$ are, respectively, the 27-, 8-, and 1-dimensional tensors. The pseudovector spurion has an analogous expansion with coefficients τ^P, δ^P, and σ^P. From Eqs. (1)–(4), we calculate the decay amplitudes as

$$\begin{aligned}\tilde{A}_2&\simeq\tfrac{1}{2}(\sqrt{\tfrac{2}{3}})(\lim_{P+\to 0}+\lim_{P_0\to 0})M_{+0}\\&=\tfrac{1}{2}(b/\sqrt{3})[2(\tau^V+\tau^P)+\tfrac{4}{5}i(\xi\tau^P-\psi\tau^V)\\&\qquad+(6/5)i(\xi\tau^V-\psi\tau^P)+i(\delta^P-\delta^V)(\psi+\xi)]\,,\end{aligned} \tag{5a}$$

$$\tilde{A}_0\simeq\tfrac{1}{2}(\sqrt{\tfrac{3}{2}})(\lim_{P+\to 0}+\lim_{P-\to 0})M_{+-}\simeq\tfrac{1}{2}b(\sqrt{\tfrac{3}{2}})(\delta^V+\delta^P)\,, \tag{5b}$$

where

$$b=\mu Gg_{\pi NN}\sin\theta\cos\theta/2M_Pg_A\,, \tag{6}$$

and μ is a "degenerate" pseudoscalar-meson octet mass. In deriving Eqs. (5), we have assumed that ψ and ξ are small and, for simplicity, thrown away several small terms in Eqs. (5b). The above amplitudes are related to A_0 and A_2 of Wu and Yang[11] by

$$A_0=c\tilde{A}_0\,,\quad A_2=c\tilde{A}_2\,, \tag{7a}$$

$$c=\frac{1}{4(\pi M_K)^{1/2}}\left(1-4\left[\frac{M_\pi}{M_K}\right]^2\right)^{1/4}\,. \tag{7b}$$

To go further, it is necessary to estimate τ^V, τ^P, δ^V, and δ^P in some way. From the approximate experimental validity of the $\Delta I=\frac{1}{2}$ rule for CP-conserving amplitudes, we must have $\tau^V\simeq-\tau^P$. In the special case when $\psi=-\xi$, we have

$$\mathrm{Im}A_2/\mathrm{Re}A_2=\tfrac{2}{5}\xi\quad(\psi=-\xi)\,. \tag{8}$$

For the more general case, we may attempt to "saturate" the current-current spurions with a limited set of intermediate single-particle states. If only the pseudoscalar-meson octet and a scalar singlet intermediate states are included, we found[4]

$$\begin{aligned}\tau^V&=I_\pi{}^V\,,&\tau^P&=\tfrac{1}{2}I_\sigma{}^P\,,\\\delta^V&=-(9/5)I_\pi{}^V\,,&\delta^P&=\tfrac{3}{5}I_\sigma{}^P\,,\end{aligned} \tag{9}$$

where $I_\pi{}^V$ is an integral over vector $\pi\pi$ form factors and $I_\sigma{}^P$ is an integral over pseudovector $\pi\sigma$ form factors. It turns out that $I_\pi{}^V$ and $I_\sigma{}^P$ have unambiguously

[7] We assume for the moment that l_λ has the usual $\gamma_\lambda(1+\gamma_5)$ form.

[8] M. Burgy, V. Krohn, T. Novey, G. Ringo, and V. Telegdi, Phys. Rev. **120**, 1829 (1960); M. Clark and J. Robson, Can. J. Phys. **38**, 693 (1960); **39** 13 (1961); F. Calaprice, E. Commins, H. Gibbs, G. Wick, and D. Dobson, Phys. Rev. Letters **18**, 918 (1967).

[9] S. L. Glashow, Phys. Rev. Letters **14**, 35 (1965).

[10] W. Alles, Phys. Letters **14**, 348 (1965).

[11] T. T. Wu and C. N. Yang, Phys. Rev. Letters **14**, 35 (1965); T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

[12] This does not contradict the result that the CP-conserving $K\to 2\pi$ decays vanish in the $SU(3)$ limit because it represents a zero-energy limit rather than the physical amplitude. As shown by Y. Hara and Y. Nambu [Phys. Rev. Letters **16**, 875 (1966)], in an $SU(3)$ treatment we must consider the spurion to be multiplied by $(M_\pi{}^2-M_K{}^2)$, which does vanish in the limit.

opposite signs in the $SU(3)$ limit for the form factors and that a numerical estimate establishes the plausibility of $I_\pi^V \simeq -\frac{1}{2} I_\sigma^P$, which is required for $\tau^V \simeq -\tau^P$. Using this in Eqs. (5) and (9) then leads to

$$\mathrm{Im}A_2/A_0 = -\tfrac{1}{9}\sqrt{2}(\psi+\xi)\,. \qquad (10)$$

In this case, we have of course given up the power to predict $\mathrm{Im}A_2/\mathrm{Re}A_2$. Another possibility is to assume that only the vector-meson intermediate states contribute to the PP and VV spurions. In the same way, this would yield

$$\mathrm{Im}A_2/A_0 = \tfrac{1}{3}\sqrt{2}(\psi+\xi)\,. \qquad (10')$$

Discussion of these results is deferred to Sec. IV.

III. SECOND MODEL

In the intermediate-boson model of weak interactions, the Hamiltonian of Eq. (1) is to be replaced by

$$H_w' = gJ_\lambda W_\lambda + \mathrm{H.c.}\,, \qquad (11)$$

where J_λ is still given by Eq. (2), and $g^2 = GM_B^2/\sqrt{2}$, where M_B is the mass of the W_λ field. The effective weak-interaction Hamiltonian is then

$$H_{\mathrm{eff}} = g^2 \int d^4y\, T(J_\mu(y)J_\nu(0))\Delta_{\mu\nu}{}^B(y)\,, \qquad (12\mathrm{a})$$

$$\Delta_{\mu\nu}{}^B(y) = \frac{-i}{(2\pi)^4}\int d^4p\, e^{ip\cdot y}\frac{(\delta_{\mu\nu}+p_\mu p_\nu/M_B^2)}{p^2+M_B^2}\,. \qquad (12\mathrm{b})$$

Following GSW,[5] we may derive by current algebra and PCAC the following formulas for the matrix elements of Eqs. (3):

$$M_{+-} = (i\sqrt{2}/F_\pi^2F_K)\langle 0|([B_3{}^2,[B_1{}^2,[B_2{}^1,H_{\mathrm{eff}}]]] + [B_3{}^2,[B_2{}^1,[B_1{}^2,H_{\mathrm{eff}}]]])|0\rangle\,, \qquad (13\mathrm{a})$$

$$M_{+0} = (i/F_\pi^2F_K)\langle 0|([B_3{}^1,[(B_1{}^1-B_2{}^2),[B_1{}^2,H_{\mathrm{eff}}]]] + [B_3{}^1,[B_1{}^2,[(B_1{}^1-B_2{}^2),H_{\mathrm{eff}}]]])|0\rangle\,, \qquad (13\mathrm{b})$$

where $B_a{}^b = i\int d^3x\, P_{a4}{}^b$, and F_π, for example, is the pion-decay constant $\simeq 2M_Pg_A/g_{\pi NN}$. In the derivation of Eq. (13), several terms were neglected,[13] and the K meson was treated differently from the two π mesons.

Performing the indicated commutations in Eqs. (13) gives the results

$$M_{+-} = \frac{i\sqrt{2}g^2}{F_\pi^2F_K}\cos\theta\sin\theta\int d^4y\,\Delta_{\mu\nu}{}^B(y) \times[\langle 0|T(V_{3\mu}{}^1(y)V_{1\nu}{}^3(0) - V_{2\mu}{}^1(y)V_{1\nu}{}^2(0))|0\rangle + (V\to P)]\,, \qquad (14\mathrm{a})$$

$$M_{+0} = \frac{2g^2(\psi+\xi)}{F_\pi^2F_K}\cos\theta\sin\theta\int d^4y\,\Delta_{\mu\nu}{}^B(y) \times[\langle 0|T(V_{2\mu}{}^1(y)V_{1\nu}{}^2(0) + V_{3\mu}{}^1(y)V_{1\nu}{}^3(0))|0\rangle - (V\to P)]\,. \qquad (14\mathrm{b})$$

As in Sec. II, we have made the assumption that ψ and ξ are small. In deriving Eqs. (14), isotopic-spin invariance was assumed. Equation (14a), which represents essentially the CP-conserving decay, evidently vanishes in the limit of exact $SU(3)$ invariance, where

$$\langle 0|T(V_{3\mu}{}^1(y)V_{1\nu}{}^3(0))|0\rangle = \langle 0|T(V_{2\mu}{}^1(y)V_{1\nu}{}^2(0))|0\rangle\,.$$

On the other hand, Eq. (14b), which represents the CP-violating decay, remains finite in the limit of exact $SU(3)$ but vanishes in the exact chiral-symmetry limit, where, for example,

$$\langle 0|T(P_{2\mu}{}^1(y)P_{1\nu}{}^2(0))|0\rangle = \langle 0|T(V_{2\mu}{}^1(y)V_{1\nu}{}^2(0))|0\rangle\,.$$

In this model, the real part of M_{+0} is zero, so that the $K^+\to\pi^+\pi^0$ decay is suppressed. This decay must therefore arise from some correction to this formula. One interesting point of view is that the correction is due to electromagnetic mass splittings.[14]

A trivial modification of Eqs. (13), which we consider for the sake of completeness, is to symmetrize the K meson along with the two π mesons.[15] Instead of Eqs. (14), we would have in this case

$$M_{+-}' = \frac{1}{3}\frac{i\sqrt{2}g^2}{F_\pi^2F_K}\cos\theta\sin\theta\int d^4y\,\Delta_{\mu\nu}{}^B(y) \times[\langle 0|T(V_{3\mu}{}^1(y)V_{1\nu}{}^3(0) - V_{2\mu}{}^1(y)V_{1\nu}{}^2(0))|0\rangle + (V\to P)]\,, \qquad (14\mathrm{a}')$$

$$M_{+0}' = \frac{g^2}{F_\pi^2F_K}(\psi+\xi)\cos\theta\sin\theta\int d^4y\,\Delta_{\mu\nu}{}^B(y) \times[\langle 0|\{3T(V_{2\mu}{}^1(y)V_{1\nu}{}^2(0)) + T(V_{3\mu}{}^1(y)V_{1\nu}{}^3(0))\}|0\rangle - (V\to P)\}\,. \qquad (14\mathrm{b}')$$

We note that (14b) and (14b′) become equal in the $SU(3)$ limit.

To evaluate Eqs. (14), we follow GSW[5] and assume

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[13] See, for example, S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

[14] L. J. Clavelli, Phys. Rev. **160**, 1384 (1967); Y. Hara, Progr. Theoret. Phys. (Kyoto) **37**, 470 (1967); S. Okubo, R. E. Marshak, and V. S. Mathur, Phys. Rev. Letters **19**, 407 (1967); J. Schechter, Phys. Rev. **161**, 1660 (1967).

[15] Since GSW has omitted a number of terms in the identity of Weinberg (Ref. 13), we can regard their result as simply symmetrizing in an *ad hoc* manner

$$M\sim\tfrac{1}{2}[\langle\pi_1|[B_2,H]|K\rangle + \langle\pi_2|[B_1,H]|K\rangle].$$

By analogy, we can also symmetrize taking K into account, namely,

$$M\sim\tfrac{1}{3}[\langle\pi_1|[B_2,H]|K\rangle + \langle\pi_2|[B_1,H]|K\rangle + \langle\pi_1\pi_2|[B_\pi,H]|0\rangle].$$

We emphasize, however, that both are *ad hoc* assumptions pending more careful investigation of the off-mass-shell continuation of mesons.

representations like[16]

$$\langle 0|T(P_{2\mu}{}^1(y)P_{1\nu}{}^2(0))|0\rangle = \frac{-i}{(2\pi)^4}\int d^4q\, e^{iq\cdot y}\left\{\int d\mu^2 R_P(\mu^2)\frac{\delta_{\mu\nu}+q_\mu q_\nu/\mu^2}{q^2+\mu^2} + \tfrac{1}{2}F_\pi{}^2 q_\mu q_\nu/q^2\right\}. \quad (15)$$

For the currents $P_3{}^1$, $V_3{}^1$, and $V_2{}^1$, R_P in Eq. (15) is to be replaced by R_P', R_V', and R_V, respectively, while F_π is to be replaced by F_K, F_κ, and 0, respectively. Then we have

$$M_{+-}=\frac{-\sqrt{2}g^2 i}{F_\pi{}^2F_K(2\pi)^4}\cos\theta\sin\theta \times\int d^4p\left[\int d\mu^2\left\{\frac{3(M_B{}^2-\mu^2)}{(p^2+M_B{}^2)^2(p^2+\mu^2)} + \frac{3}{(p^2+M_B{}^2)^2}+\frac{1}{\mu^2 M_B{}^2}\right\}(R_V'-R_V+R_P'-R_P) + \frac{(F_\kappa{}^2+F_K{}^2-F_\pi{}^2)}{2M_B{}^2}\right], \quad (16)$$

and a similar expression for M_{+0}. To proceed we need the integral

$$\int\frac{d^4p}{(p^2+M_B{}^2-i\epsilon)^2(p^2+\mu^2-i\epsilon)}=\frac{-i\pi^2}{M_B{}^2-\mu^2} \times\left(\frac{\mu^2}{M_B{}^2-\mu^2}\ln\frac{M_B{}^2}{\mu^2}-1\right). \quad (17)$$

We must also assume the Weinberg sum rules[6] for chiral-symmetry breaking,

$$\int d\mu^2[R_V(\mu^2)-R_P(\mu^2)]=\int d\mu^2[R_V'(\mu^2) - R_P'(\mu^2)]=0, \quad (18a)$$

$$\int\frac{d\mu^2}{\mu^2}[R_V(\mu^2)-R_P(\mu^2)]=\tfrac{1}{2}F_\pi{}^2, \quad (18b)$$

$$\int\frac{d\mu^2}{\mu^2}[R_V'(\mu^2)-R_P'(\mu^2)]=\tfrac{1}{2}F_K{}^2-\tfrac{1}{2}F_\kappa{}^2, \quad (18c)$$

and for $SU(3)$-symmetry breaking,

$$\int d\mu^2[R_V(\mu^2)-R_V'(\mu^2)]=\int d\mu^2[R_P(\mu^2) - R_P'(\mu^2)]=0, \quad (19a)$$

$$\int\frac{d\mu^2}{\mu^2}[R_V(\mu^2)-R_V'(\mu^2)]=\tfrac{1}{2}F_\kappa{}^2, \quad (19b)$$

$$\int\frac{d\mu^2}{\mu^2}[R_P(\mu^2)-R_P'(\mu^2)]=\tfrac{1}{2}F_K{}^2-\tfrac{1}{2}F_\pi{}^2. \quad (19c)$$

Equations (18) are needed for the CP-violating decay, while Eqs. (19) are needed for the CP-conserving decay. There is some doubt[17] as to the validity of Eqs. (19), but we shall not consider this question here. At any rate, the $SU(3)$-breaking sum rule involving the isoscalar current—which is the most questionable one—does not appear in this calculation.

From Eqs. (16)–(19), we then have

$$M_{+-}=\frac{3\sqrt{2}g^2}{16F_\pi{}^2F_K\pi^2}\cos\theta\sin\theta\int d\mu^2\left(\frac{\mu^2}{M_B{}^2-\mu^2}\ln\frac{M_B{}^2}{\mu^2}\right) \times(R_V'-R_V+R_P'-R_P), \quad (20a)$$

$$M_{+0}=\frac{-3ig^2}{8F_\pi{}^2F_K\pi^2}\cos\theta\sin\theta(\psi+\xi) \times\int d\mu^2\left(\frac{\mu^2}{M_B{}^2-\mu^2}\ln\frac{M_B{}^2}{\mu^2}\right) \times(R_V+R_V'-R_P-R_P'). \quad (20b)$$

Equations (20) are evaluated in the vector and axial-vector pole-dominance approximation by setting

$$R_V(\mu^2)=2(M_\rho{}^2/f_\rho)^2\delta(\mu^2-M_\rho{}^2) \simeq M_\rho{}^2F_\pi{}^2\delta(\mu^2-M_\rho{}^2), \quad (21a)$$

$$R_V'(\mu^2)\simeq M_\rho{}^2F_\pi{}^2\delta(\mu^2-M_{K^*}{}^2), \quad (21b)$$

$$R_P(\mu^2)\simeq M_\rho{}^2F_\pi{}^2\delta(\mu^2-M_{A_1}{}^2), \quad (21c)$$

$$R_P'(\mu^2)\simeq M_\rho{}^2F_\pi{}^2\delta(\mu^2-M_{K_1}{}^2), \quad (21d)$$

where the relation[18] $(M_\rho/f_\rho)^2\simeq F_\pi{}^2$ was used.

Substituting Eqs. (21) in (22) and noting Eqs. (3) gives the final result

$$\frac{\mathrm{Im}A_2}{A_0}\simeq\frac{2}{3}\left(\frac{\mathrm{Im}M_{+0}}{M_{+-}}\right)=-\tfrac{2}{3}\sqrt{2}(\psi+\xi) \times\left[\frac{L(K^*)+L(\rho)-L(K_A)-L(A_1)}{L(K^*)-L(\rho)+L(K_A)-L(A_1)}\right], \quad (22a)$$

[16] For simplicity, we have omitted the Schwinger terms in Eqs. (15). These will cancel out in our calculation if we assume the validity of the Weinberg sum rules. Furthermore, the main justification for idealizing to the world of zero-pion mass and conserved pseudovector current seems to be that it gives good results in the calculation of T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters **18**, 759 (1967).

[17] See, for example, J. J. Sakurai, Phys. Rev. Letters **19**, 803 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **19**, 470 (1967).

[18] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayazuddin, Phys. Rev. **147**, 1071 (1966).

where, for example,

$$L(\rho)=M_\rho^2\ln(M_B^2/M_\rho^2). \qquad (22b)$$

If we use Eqs. (14′) instead of Eqs. (14), so as to treat the K and π's symmetrically, we would have to replace Eq. (22a) by

$$\left(\frac{\mathrm{Im}A_2}{A_0}\right)'=-\sqrt{2}(\psi+\xi)\times\left[\frac{3L(\rho)-3L(A_1)+L(K^*)-L(K_A)}{L(K^*)-L(\rho)+L(K_A)-L(A_1)}\right]. \qquad (22c)$$

GSW[5] evaluated M_B by equating the left-hand side of Eq. (20a) to the experimental value of M_{+-}. They found $M_B\simeq 8$ BeV. In Eq. (22a), this leads to

$$\mathrm{Im}A_2/A_0\simeq(1.43)\tfrac{2}{3}\sqrt{2}(\psi+\xi). \qquad (23)$$

If we use Eqs. (14′) instead of (14), we would find that the mass of the intermediate boson should be about 110 BeV. This difference is due to the difference in the factor $\frac{1}{3}$ between Eqs. (14a) and (14a′). Then Eq. (22c) would give

$$(\mathrm{Im}A_2/A_0)'\simeq(3.5)\sqrt{2}(\psi+\xi) \qquad (23')$$

IV. DISCUSSION

Experimentally,[19] we have $|\mathrm{Im}A_2/A_0|<3.2\times10^{-3}$. Furthermore,[19] if the $\Delta I=\frac{5}{2}$ transition in K decay is unimportant, $|\mathrm{Re}A_2/A_0|\simeq5\times10^{-2}$. For the leptonic decays, we know that ξ is about zero to within 2.8×10^{-2} rad.[20] About ψ practically nothing is known.

Depending on whether we adopt Eqs. (10), (10′), (23), or (23′), we then find that, respectively, $|\psi+\xi|<2.0\times10^{-2}$, 6.8×10^{-3}, 2.4×10^{-3}, or 6.5×10^{-4} rad. Now, since ψ is unknown, we cannot logically do more than establish an upper limit for it in each model. Nevertheless, it is interesting to note that even if ψ were zero, so that all the leptonic *CP* violation occurred in *ordinary* β decay, the value of ξ needed is in all models smaller than the present experimental uncertainty.

[19] E. Yen, Phys. Rev. Letters **18**, 513 (1967); F. Abbud, B. W. Lee, and C. N. Yang, *ibid.* **18**, 980 (1967).
[20] See F. Calaprice *et al.* (Ref. 8).

The special case $\psi=-\xi$ in the first model must be excepted from the above discussion. Here, Eq. (8) shows that we only expect $|\xi|\lesssim0.15$. The second model, on the other hand, predicts no nonleptonic *CP* violation in this limit. The first model would also give no nonleptonic *CP* violation in this limit if the intermediate "saturating" states conspired to produce $\tau^V=\tau^P=0$, instead of merely $(\tau^V+\tau^P)=0$. Then conclusion (c) of the Introduction—that the *CP*-violating decays vanish in the chiral-symmetry limit—would also hold for the first model. We may see this by substituting the chiral-limit relation $\delta^P=\delta^V$ in Eq. (5a). The following symmetrical situation seems to exist: We saw immediately from the interaction of Eqs. (1) and (2) that $\psi=+\xi$ corresponds to no *CP* violation in the parity-conserving nonleptonic amplitudes. On the other hand, our equations are telling us that when $\psi=-\xi$, the *CP* violation "wants" to vanish for the parity-violating nonleptonic amplitudes. In the case where this did not occur, the *CP*-violating phases had to be anomalously large.

A point of interest in both our models, mentioned as (b) in the Introduction, is that $\mathrm{Im}A_2$ is not suppressed by the same mechanism that suppresses $\mathrm{Re}A_2$. In the first model, this is evident on inspection of Eq. (5a), while in the second model, $\mathrm{Re}A_2=0$, but $\mathrm{Im}A_2$ is nonzero. Roughly speaking, it can be said that the approximate $\Delta I=\frac{1}{2}$ rule does not seem to "commute" with *CP*.

We will not examine here all the ambiguities and assumptions of our two models. These have been somewhat discussed elasewhere.[4,5] Our point of view is that the conclusions we have drawn may well be of a general nature and are, in any event, perhaps somewhat different from what one would expect before looking at a specific model.

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