Unusual Possibilities for the Pomeranchon*

JOHN H. SCHWARZ
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
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Two classes of unusual possibilities for the leading angular momentum singularities in vacuum quantum numbers are explored. The first class attempts to eliminate the need for an essential singularity at $J=1$ by requiring the Pomeranchon to be a "self-reproducing" singularity. The second class attempts to implement a maximum-strength principle through a "colliding poles" mechanism. None of the models studied is believed likely to correspond to the actual situation.

In this paper we examine various possibilities for the structure of angular momentum plane singularities in amplitudes having the quantum numbers of the vacuum. There are a number of indications, both experimental and theoretical, that these quantum numbers have some unique features. One obvious way in which vacuum channels are singled out is that the forces in them are the most attractive ones so that the leading vacuum Regge pole, called the Pomeranchon, is the highest lying one of all, at least in the vicinity of $t=0$. Indeed, it appears to be at, or at least very close to, the maximum value allowed by unitarity, i.e., the Froissart limit. Another unique experimental feature is the apparent lack of slope of the Pomeranchuk trajectory, as manifested by the absence of shrinkage of diffraction peaks in elastic scattering.[1] On the theoretical side there is some consternation over the possibility that angular momentum branch points may pile up in such a way as to produce an essential singularity at $J=1$. It is ironic that the existence of these branch points is deduced from the requirement that another class of essential singularities not be present.[2]

In the following we consider a mechanism to eliminate the need for an essential singularity, namely that the Pomeranchon is a "self-reproducing" singularity. We also comment upon another mechanism which has been recently suggested. Next, we explore the consequences of a conjecture that a principle of maximum strength is implemented by the existence of "colliding poles." The significance of the phase at high energy is discussed, leading to the conclusion that the colliding poles mechanism probably does not occur, but that some effects attributable to angular momentum branch points may have been seen already.

Suppose that the Pomeranchon is a simple pole with position $J=\alpha(t)$. Then arguments given by Mandelstam[3] imply that there should also be branch points, which may be regarded as arising from exchange of n Pomeranchons, with positions given by

$$J=\alpha_n(t)=n\alpha(t/n^2)-(n-1). \tag{1}$$

One notices, in particular, that if $\alpha(0)=1$, then $\alpha_n(0)=1$. Also, the slopes are related by

$$\alpha_n'(0)=\frac{1}{n}\alpha'(0).$$

Thus we see that an infinite number of singularities intersect at the point $J=1$, $t=0$, whereas for $t\neq 0$ they accumulate at the value $J=1$, making this point an essential singularity.

It is not surprising that no serious attempts have been made to utilize the complete structure of an essential singularity. The phenomenologists understandably prefer to assume that branch points give negligible contributions compared to poles, even though it is clear that in the scattering region ($t<0$) the branch points are the higher lying singularities. Another suggestion made recently[4] is that $\alpha(0)=1-\epsilon$ (with $\epsilon\approx 0.07$), so that $\alpha_n(0)=1-n\epsilon$, and the accumulation of singularities is averted. Several authors[4,5] have tried to utilize such a suggestion to account for certain experimental facts. It seems, however, that the arguments they give do not provide compelling evidence for such an interpretation. On the esthetic side, we tend to feel that the elimination of the essential singularity in this fashion is no great victory. There is no reduction in the number of singularities present, just a slight displacement of their positions. Also, the abandoning of the simple principle $\alpha(0)=1$ seems to be a high price to pay, especially as the most recent experiments[6] appear to allow very little latitude to the range of acceptable nonzero values for ϵ.

A somewhat different way to avoid the essential singularity would be for the singularities arising from many-Pomeranchon exchange to coincide with the Pomeranchon itself, i.e.,

$$\alpha(t)=2\alpha(\tfrac{1}{4}t)-1. \tag{2}$$

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[1] In practice, the situation is complicated by competition from other quantum numbers, so that some diffraction peaks are observed to shrink while others remain unchanged or even grow.

[2] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters **2**, 239 (1962); C. E. Jones and V. L. Teplitz, Phys. Rev. **159**, 1271 (1967); S. Mandelstam and L.-L. Wang, *ibid.* **160**, 1490 (1967); A. H. Mueller and T. L. Trueman, *ibid.* **160**, 1296 (1967); **160**, 1306 (1967); J. H. Schwarz, *ibid.* **162**, 1671 (1967).

[3] S. Mandelstam, Nuovo Cimento **30**, 1148 (1963).

[4] N. Cabibbo, J. J. J. Kokkedee, L. Horwitz, and Y. Ne'eman, Nuovo Cimento **45A**, 275 (1966); N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters **22**, 336 (1966).

[5] L. P. Horwitz and H. Neumann, Phys. Rev. Letters **19**, 765 (1967); Y. Ne'eman and J. D. Reichert, *ibid.* **18**, 1226 (1967); Y. Srivastava, *ibid.* **19**, 47 (1967).

[6] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters **19**, 193, (1967); **19**, 330 (1967); **19**, 857 (1967).

A solution of this equation is given by

$$\alpha(t)=1+\gamma t^{1/2}. \qquad (3)$$

This is not the only solution of Eq. (2), but we shall argue later that the others are too pathological to be seriously considered. It is necessary that an amplitude with vacuum quantum numbers be regular at $t=0$, and it therefore follows that an additional singularity is required at

$$\tilde{\alpha}(t)=1-\gamma t^{1/2}. \qquad (4)$$

$\alpha(t)$ and $\tilde{\alpha}(t)$ have the property of being self-reproducing in the sense that exchange of several of either one of these singularities gives rise to a new singularity at the same position. It is trivial to verify, but nevertheless remarkable, that no singularities arise from simultaneous exchange of singularities at $\alpha(t)$ and $\tilde{\alpha}(t)$.

It should be remarked that even though poles with trajectories given by Eqs. (3) and (4) give rise to branch points having the same positions, the analytic structure still remains rather complicated since a frightening superposition of singularities results. Sawada[7] has made the interesting observation that if the Pomeranchon is a double pole, then it is possible for the singularities arising from multi-Pomeranchon exchange to be double poles also, at least in a model of the type considered by Amati, Fubini, and Stanghellini.[8] Thus in this case one can conceive of the entire singularity structure in the neighborhood of $J=1$ and $t=0$ being described by a pair of double poles with positions given by Eqs. (3) and (4). The close relationship between the principles of maximal analyticity and maximum strength in this model is quite striking.

Analytically continued elastic unitarity

$$b(J,t)-b(J,t_{\mathrm{II}})=2i\rho_J(t)b(J,t)b(J,t_{\mathrm{II}}) \qquad (5)$$

must not be forgotten. Since the singularities described by Eqs. (3) and (4) do not contain the normal threshold branch points in their trajectory functions, they have the same position in $b(J,t)$ and $b(J,t_{\mathrm{II}})$, i.e., on the two sheets of the normal threshold. Thus an infinite singularity (double pole, single pole, infinite logarithmic branch point, etc.) is inconsistent with Eq. (5), unless there is a suitable set of moving branch points that collide with all the normal thresholds when $J=\alpha(t)$ or $J=\tilde{\alpha}(t)$. Such a possibility has been discussed in connection with the suggestion that the Pomeranchon is a simple pole with $\alpha(t)=1$.[9] Other authors[10] have pointed out that there is no known mechanism that would produce the required branch points, and indeed it is quite unlikely that there is an unknown one either. The same remarks apply to the more general case ($\gamma\neq 0$), which we have been discussing.

As another possibility, suppose the Pomeranchon itself were not a pole but a branch point of finite type, then there would be no obvious inconsistency between elastic unitarity and a trajectory function not containing normal thresholds. There would still be the possibility of just one or two (depending on whether or not $\gamma=0$) isolated singularities. This suggestion seems quite unreasonable as it is hard to understand how the dynamics would generate a branch point that could not be interpreted as arising from the exchange of several poles. It has been previously observed, however, that square-root-type branch points are present in potential theory with a singular potential[11] and, also in certain model field theories.[12] Their possible relevance in the present context has also been discussed.[13] Even though it is unclear how such branch points could be understood in S-matrix theory, the possibility of their occurrencc is of interest because this is the only suggestion we can offer [other than $\alpha(0)<1$] that eliminates the need for an infinite accumulation of singularities. It is probably consistent with the experimental situation as it gives no shrinkage, total cross sections which vanish logarithmically, and reasonable phases (a point to be discussed some more later). One difficulty with such a model would be a breakdown of factorization, but there probably is sufficient flexibility in any phenomenological application to avoid trouble on this score.

It remains to be investigated whether solutions of Eq. (2) other than Eqs. (3) and (4) could accommodate a normal-threshold branch point so as to make a pole (or other infinity) consistent with elastic unitarity. In order to explore this possibility, let us suppose that $\alpha(t)$ is a solution of Eq. (2) with a branch point at $t=t_0$. From Eq. (2) it is then apparent that there are also branch points for $t=t_0/4^n$, with $n=\pm 1, \pm 2, \cdots$. These branch points would also appear in $b(J,t)$, where they are completely unacceptable. Thus, such a solution is to be rejected.

In the preceding paragraphs, we have discussed mechanisms for eliminating the infinite accumulation of many-Pomeranchon singularities. The only ones we were unable to exclude outright were: (1) the Pomeranchon intercept is slightly below 1, and (2) the Pomeranchon itself is a finite self-reproducing branch point rather than a pole. Let us now explore a somewhat different suggestion. Start from the assumption that only simple poles need to be considered and attempt to implement a principle of maximum strength. (In this discussion we are resigned from the outset to accept the

[7] T. Sawada, Nuovo Cimento **48A**, 534 (1967); **51A**, 208 (1967).
[8] D. Amati, S. Fubini, and A. Stanghellini, Phys. Letters **1**, 29 (1962); Nuovo Cimento **26**, 896 (1962).
[9] R. Oehme, Phys. Rev. Letters **18**, 1222 (1967).
[10] J. Finkelstein and C. Tan, Phys. Rev. Letters **19**, 1061 (1967).

[11] R. Oehme, Nuovo Cimento **25**, 183 (1962).
[12] J. D. Bjorken and T. T. Wu, Phys. Rev. **130**, 2566 (1963); R. F. Sawyer, *ibid.* **131**, 1384 (1963); D. Atkinson and A. P. Contogouris, Nuovo Cimento **39**, 1082, 1102 (1965); A. P. Contogouris, *ibid.* **44A**, 927 (1966).
[13] P. G. O. Freund and R. Oehme, Phys. Rev. Letters **10**, 450 (1963). These authors consider a fixed branch point competing with a normal Pomeranchon, rather than a branch point alone such as we are discussing. Their formulas become somewhat modified when signature is taken into account.

infinite accumulation of branch points at $J=1$.) The following theorem has been proved by Martin[14]: If for some $t_1>0$, but below the leading threshold or bound state (e.g., $0<t_1<4\mu_\pi^2$ for vacuum quantum numbers), the scattering amplitude is known to be bounded by a power $\alpha(t_1)>1$, then for $0<t\leq t_1$ the amplitude has the power bound

$$\alpha(t)\leq 1+[\alpha(t_1)-1](t/t_1)^{1/2}. \qquad (6)$$

An attempt to "saturate" this bound for the Pomeranchon can be made by supposing

$$\alpha(t)=f(t)+t^{1/2}g(t)\,, \qquad (7)$$

where $f(0)=1$, f and g are analytic at $t=0$, and at least one or the other of them contains the normal thresholds in order that there be consistency with elastic unitarity. Analyticity at $t=0$ requires that there also be another pole at $J=\tilde{\alpha}(t)$, where

$$\tilde{\alpha}(t)=f(t)-t^{1/2}g(t)\,. \qquad (8)$$

Thus the partial-wave amplitude has the structure

$$a(J,t)=r(J,t)/\{[J-f(t)]^2-tg^2(t)\}\,, \qquad (9)$$

with the requirement that $r(J,t)$ is analytic at $t=0$. For $t=0$ the amplitude has a single pole or a double pole at $J=1$ depending on whether or not $r(J,0)$ contains a factor of $J-1$. We shall assume that in general $r(J,0)$ is finite at $J=1$.

One motivation for studying the colliding poles mechanism for the Pomeranchon is a desire to understand why there is little shrinkage when vacuum quantum numbers are exchanged and appreciable shrinkage when other quantum numbers are exchanged. The principal observation to be made in this regard is that if $f(t)\approx 1$ for $t<0$, then we have $\mathrm{Re}\alpha(t)\approx 1$ for $t<0$. Thus, as $\mathrm{Re}\alpha(t)$ determines the power behavior of the amplitude, little shrinkage would be expected to result from Pomeranchon exchange. For other quantum numbers the shrinkage depends on the position and strength of the leading pole and perhaps the competing branch points. As a simple example take the ρ trajectory to be described approximately by

$$\alpha_\rho(t)=\alpha_0+\alpha' t\,, \qquad (10)$$

and the Pomeranchuk trajectory by

$$\alpha_P(t)=1+\gamma t^{1/2}. \qquad (11)$$

The branch point which arises from simultaneous exchange of these two poles then has the trajectory

$$\alpha_{P\rho}(t)=\alpha_0-(\gamma^2/4\alpha')+\gamma t^{1/2}. \qquad (12)$$

An interesting feature of this result is that the $t=0$ intercept of the branch point is displaced downwards from the intercept of the ρ trajectory. This fact suggests that the pole may control the shrinkage for some range of values of t, even if the branch points have appreciable strength.

The maximal strength principle, as implemented above, does not give completely unique predictions, but it does suggest that there may be a pair of poles which collide for $t=0$ so as to effectively produce a double pole for $t=0$. As a consequence one obtains the prediction that total cross sections grow proportionally to $\ln s$. Present experimental data cannot, of course, distinguish such a behavior from constancy, especially if there is also competition from inverse logarithms (branch points) and decreasing powers (other poles). Another experimental prediction of this double pole behavior is apparently useful. One finds from the Sommerfeld-Watson formula the asymptotic behavior at $t=0$

$$A(s,0)\underset{s\to\infty}{\sim}(\mathrm{const})s[\ln^2 s-\ln^2(-s)]$$

$$=(\mathrm{const})s[i\ln s+\pi]. \qquad (13)$$

This formula implies that at high energy the ratio $\mathrm{Re}A/\mathrm{Im}A$ in the forward direction is positive[15] and approaches zero logarithmically. At $s=100$, for example, the ratio is still about 0.7.

Experimentally, the vacuum quantum numbers can be isolated by combining amplitudes for π^+p and π^-p scattering. The ratio $\mathrm{Re}A/\mathrm{Im}A$ is found[6] (both experimentally and by dispersion relations) to be about -0.13 at 20 GeV/c. This fact rules out the colliding poles mechanism we have described for all practical purposes. The only way to reconcile it with the experimental phase information would be for the competing branch points (which give rise to logarithmically vanishing negative values for $\mathrm{Re}A/\mathrm{Im}A$) to account for the major portion of the total cross section at energies in the tens of GeV. Such a model seems too artificial to pursue. It is also of some interest that a knowledge of the phase at high energy is useful information in attempting to account for the data from a more conventional point of view. From crude calculations we have concluded that a simple Pomeranchon pole with $\alpha(0)=1$ together with a P' pole, constrained to fit the total cross-section data, may give a real part that is too large, although of proper sign. Agreement probably can be restored by allowing an admixture of branch points, also having intercept unity. The situation here is rather similar to the problem of polarization in charge-exchange scattering.[16] In the present case, one has data at higher energies, however, and it therefore appears safer to neglect effects due to the direct channel.

In conclusion, we have examined a number of possibilities for the leading J-plane singularities in vacuum

[14] A. Martin in *Strong Interactions and High Energy Physics* edited by R. G. Moorhouse, (Oliver and Boyd, Edinburgh, 1964).

[15] In obtaining the relation $\ln(-s)=\ln s-i\pi$ one uses the same arguments that give rise to the signature factor $e^{-i\pi\alpha}\pm 1$, *not* $e^{i\pi\alpha}\pm 1$.

[16] V. M. de Lany, D. J. Gross, I. J. Muzinich, and V. L. Teplitz, Phys. Rev. Letters **18**, 149 (1967); C. B. Chiu and J. Finkelstein, Nuovo Cimento **48A**, 820 (1967).

quantum numbers. While a number of exotic suggestions—such as a self-reproducing branch point or colliding poles could not be rigorously excluded, in our opinion the orthodox picture of a single simple Regge pole with intercept unity (plus all the associated branch points) is preferred. We have suggested that important information can be derived from the phase at high energy. Perhaps, branch points are required to fully account for it.

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CP Violation in Current-Current Models*

Y. T. Chiu
Space Science Laboratory, Aerospace Corporation, El Segundo, California
AND
J. Schechter
Physics Department, Syracuse University, Syracuse, New York

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The usual current-algebra techniques are used in two separate models for relating *CP*-violating effects in $K\rightarrow 2\pi$ decays with possible *CP* violations in ordinary and strange β decays. In the first model, one meson is removed from the $K\rightarrow 2\pi$ matrix element, and the resulting current-current spurion is saturated with a limited number of intermediate states. The second model is essentially that of Glashow, Schnitzer, and Weinberg. In this model, it is found that the *CP*-violating rate can be calculated in terms of the chiral-symmetry spectral-function sum rules. The *CP*-conserving rate, on the other hand, required the use of $SU(3)$ spectral-function sum rules. It is found that (a) the leptonic *CP* violations appear to be quite small, (b) the $\Delta I=\frac{3}{2}$ *CP*-violating transitions do not seem to be suppressed by the usual $\Delta I=\frac{1}{2}$ dynamical mechanism, and (c) there is a (model-dependent) tendency for the nonleptonic *CP* violation to vanish in the limit of chiral symmetry.

I. INTRODUCTION

In this paper, we use the hypothesis of partially conserved axial-vector current[1] (PCAC) and the chiral $SU(3)\otimes SU(3)$ current algebra[2] to get an estimate of the relation between the nonleptonic and possible leptonic *CP*-violating effects[3] in the weak interactions. The assumption is made that the weak Hamiltonian can be written in current-current form.

Two separate models are considered. In the first, described in Sec. II, the effective Hamiltonian is taken to be a first-order strictly local one. Then, using the standard techniques,[4] one meson is "removed" from the $K\rightarrow 2\pi$ matrix element, and the resulting current-current spurion is "saturated" with a limited number of intermediate states.

The second model is the one which has been used by Glashow, Schnitzer, and Weinberg[5] to treat the $K\rightarrow 2\pi$, *CP*-conserving decay. The weak interaction is in this case taken to proceed by an intermediate vector boson. Furthermore, all three mesons are removed from the decay matrix element, and the resulting vacuum expectation values are calculated by assuming that the Weinberg-type sum rules[6] are strictly true. Whereas this approach involves the $SU(3)$ spectral-function sum rules for the *CP*-conserving case, it turns out to involve the chiral-symmetry sum rules for the *CP*-violating case. The second model is described in Sec. III.

It is clear that each model involves a number of somewhat drastic and *different* assumptions. Therefore we will not attempt to compare the two models at this time. We note, however, that the following two conclusions emerge from both:

(a) The predicted *CP* violations for ordinary and strangeness-changing β decay seem to be quite small, on the verge of or below the present experimental uncertainties.

(b) The $\Delta I=\frac{3}{2}$ *CP*-violating amplitude is not suppressed in the same way as the $\Delta I=\frac{3}{2}$ *CP*-conserving amplitude.

In the second model, and for a (plausible) special case of the first model, we have an additional conclusion:

(c) The *CP*-violating $K\rightarrow 2\pi$ decays vanish in the limit of chiral symmetry.

A discussion of the results will be given in Sec. IV.

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[3] J. Christenson, J. Cronin, V. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

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[5] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 205 (1967). Denote this by GSW.

[6] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967); J. J. Sakurai, Phys. Letters **24B**, 619 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 205 (1967).