Three-Pion Decay Modes of the η Meson: Is $T=3$ Necessary?*

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Predicted values of the branching ratio $R=\Gamma(\eta\to 3\pi^0)/\Gamma(\eta\to\pi^+\pi^-\pi^0)$ anywhere between 1.6 and 0.7 are perfectly consistent with the Dalitz plot of 640 background-free η decays $\eta\to\pi^+\pi^-\pi^0$. These events were analyzed under the assumption of pure $T=1$ final states with a complex matrix element that includes terms cubic in pion energies. To obtain low values of R, final-state interactions are needed. $T=3$ is not needed.

In the decays

$$\eta\to\pi^+\pi^-\pi^0 \quad (1)$$

and

$$\eta\to 3\pi^0 \quad (2)$$

the three pions must have isotopic spin $T=1$ or 3. (We assume C invariance.) If the decay is electromagnetic and if the photon can carry only isospin 0 or 1, then the $T=3$ amplitude carries one more power of α than the $T=1$ amplitude. In that case $T=1$ may dominate.[1]

On the basis of the similarity between the τ, τ', and η Dalitz plots,[2] it has long been conjectured[3] that the three pions are mostly in the totally symmetric $T=1$ state, with a small admixture of the other two (nonsymmetric) $T=1$ states. For the totally symmetric $T=1$ state the branching ratio

$$R\equiv\Gamma(\eta\to 3\pi^0)/\Gamma(\eta\to\pi^+\pi^-\pi^0)$$

is given by $R=(1.15)(3/2)=1.7$.

The experimental Dalitz plot for reaction (1) is not totally symmetric. However, it is well fitted by a real linear matrix element[4]

$$M(+-0)=1+by_3, \quad (3)$$

where $y_3=(3T_0/Q)-1$, and b is real. The fit gives typically $b=-0.45\pm0.05$, which predicts $R=1.63\pm0.03$.[5,6]

Two published direct measurements of R give the results $R=0.83\pm0.32$ (Ref. 7) and $R=0.90\pm0.24$ (Ref. 6), under the assumption that there are no decays $\eta\to\pi^0\gamma\gamma$. The existence of $\eta\to\pi^0\gamma\gamma$ (Ref. 8) would give corrections which would reduce these two results to values as low as 0.5.[9] Other experiments have been combined[10] to give the indirect result $R=0.93\pm0.16$. Thus several experiments are in poor agreement with the value of R predicted by the real linear matrix-element model with $T=1$.[11] If one fits the Dalitz plot to other models having $T=1$, such as that involving the hypothetical σ meson,[12] or involving ρ dominance,[13] one can obtain predicted values of R as low as about 1.2.[5,6]

By including contributions from both ρ and σ one can reach predicted values of R as low as unity.[14] However, the existence of the σ mesons is not experimentally established.

The poor agreement has led Veltman and Yellin[15] and Woo[16] to conclude that $T=3$ must be present, since, as is well known,[1] the (totally symmetric) $T=3$ amplitude can interfere with the totally symmetric part of the $T=1$ amplitude to give R as small as one pleases, for example between 0.5 and 1.0. The possible existence of a large $T=3$ amplitude has led Adler[17] to suggest that there may be an isotensor part to the electromagnetic interaction.

What we wish to demonstrate is that, *assuming only $T=1$*, we can find excellent fits to the Dalitz plot of a sample of 640 η decays of type (1), giving *predicted values of R anywhere between 1.6 and 0.7*. We emphasize that those fits that correspond to predicted low values of

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[10] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Lawrence Radiation Laboratory Report No. UCRL-8030, revised 1967 (unpublished).

[11] More recently, unpublished experimental results presented by Baltay *et al.* at the Heidelberg Conference give $R=1.55\pm0.25$ and show no evidence for the existence of $\eta\to\pi^0\gamma\gamma$. For more recent results, see the January 1968 revision of Ref. 10 [Rev. Mod. Phys. **40**, 1 (1968)]. If R is eventually found to be closer to 1.5 than to 1.0, it will not affect the conclusions of this paper.

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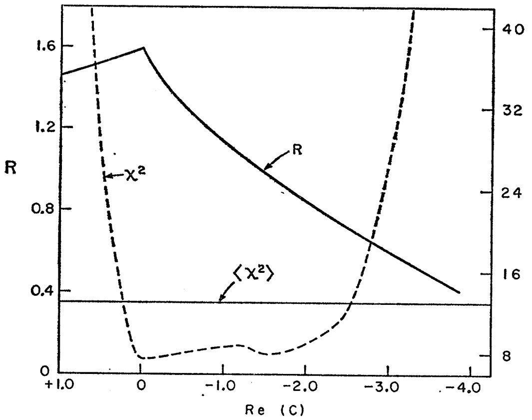

FIG. 1. Plot of the branching ratio, R (solid line), and χ^2 for the fit to the $\pi^+\pi^-\pi^0$ Dalitz plot (dashed line) versus the parameter Re(c) for the third-order fit. The horizontal line is the "expected" χ^2, $\langle\chi^2\rangle$.

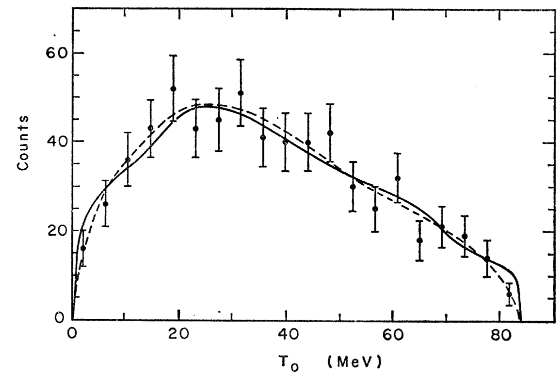

FIG. 2. Energy distribution of π^0 in $\eta\to\pi^+\pi^-\pi^0$. The 640 events are divided into 20 equal bins in T_{π^0}. Increasing bin number corresponds to increasing T_{π^0}. The dashed curve is our best fit to the third-order matrix element, which gives $R=1.6$. The solid curve is our third-order fit with $\chi^2=$"expected" χ^2 (see Table I), which gives $R=0.7$.

R are *allowed*, not demanded, by the data. (We also find good fits with predictions as high as $R=1.6$. For example, the real linear matrix element gives a good fit.)

We assume that only $T=1$ is present, and expand the matrix elements for Reactions (1) and (2) through third order in y_1, y_2, and y_3, where $y_i=(3T_i/Q)-1$. The most general C-invariant matrix elements are then[18]

$$M(+-0)=1+by_3+cy_3^2+dy_1y_2+ey_3^3+fy_1y_2y_3 \quad (4)$$

and

$$M(000)=6^{-1/2}[3+(c-\tfrac{1}{2}d)(y_1^2+y_2^2+y_3^2)+3(e+f)y_1y_2y_3]. \quad (5)$$

All parameters are complex. Thus there are ten real parameters.

We fit the matrix element $M(+-0)$ to the Dalitz plot of 640 low-background η decays of type (1), produced by pions of order 1 BeV/c in the reactions

$$\pi^{\pm}p\to\pi^{\pm}p\eta.$$

Of these events 488 were produced in the Alvarez 72-in. hydrogen bubble chamber,[5,6] and 152 were produced in the Brookhaven National Laboratory 20-in. hydrogen bubble chamber.[19] The non-η background is less than 3%. The decays $\eta\to\pi^+\pi^-\gamma$ have been removed. The Dalitz plot is given in Ref. 20.

In fitting these events to $M(+-0)$ we find that d and f are small and consistent with zero. Therefore we revise our model and set these four real parameters identically equal to zero. Then $M(+-0)$ depends only upon y_3. We therefore divide the sample into 20 equal bins in y_3.[21]

The predicted value of R turns out to be sensitive to the real part of c, Re(c), whereas it is relatively insensitive to the other parameters.[22] In Fig. 1 we plot R and χ^2 versus Re(c), where χ^2 corresponds to the best fit to the Dalitz plot that can be obtained by varying all parameters except Re(c). The horizontal line represents the "expected" χ^2 of 13. All values of Re(c) for which χ^2 is below the line correspond to excellent fits to the data. We see that *any value of R from 1.6 to 0.7 corresponds to an excellent fit.*

In Fig. 2 we show the π^0 energy spectrum and the fitted curves corresponding to predicted values $R=1.6$ and 0.7 for our complex cubic matrix-element model.

Similarly, if we use a complex quadratic matrix-element model we obtain excellent fits with predicted values of R anywhere between 1.6 and 1.1. For a complex linear matrix-element model we obtain excellent fits for R between 1.6 and 1.4.

In Table I we give best-fit (minimum χ^2) parameters for complex linear, quadratic, and cubic matrix-element models, and also the parameters for the complex cubic model that give $R=0.7$. Notice that in order to obtain a small predicted value of R the imaginary parts of the

[18] See, e.g., C. Zemach, Phys. Rev. **133**, B1201 (1964).

[19] H. Foelsche and H. Kraybill, Phys. Rev. **134**, B1138 (1964).

[20] These are the 640 events from the "low-momentum" πp experiments contributed to the η compilation of the Columbia-Berkeley-Purdue-Wisconsin-Yale Collaboration, Phys. Rev. **149**, 1044 (1966). In this paper the Dalitz plot has been folded over the T_{π^0} axis to yield 27 bins instead of the 54 bins given in Fig. 1b of that reference, i.e., we combine angular zones 1 and 18, 2 and 17, etc. The Dalitz plot follows: angular zone, radial zone: counts; 1, 3:8; 1, 2:11; 1, 1:21; 2, 3:12; 2, 2:12; 2, 1:22; 3, 3:12; 3, 2:14; 3, 1:20; 4, 3:13; 4, 2:16; 4, 1:29; 5, 3:22; 5, 2:18; 5, 1:26; 6, 3:17; 6, 2:21; 6, 1:28; 7, 3:23; 7, 2:24; 7, 1:38; 8, 3:32; 8, 2:43; 8, 1:30; 9, 3:42; 9, 2:40; 9, 1:46.

[21] Sample of 640 events divided into 20 equal bins in y_3. Increasing bin number corresponds to increasing y_3. We have bin: counts; 1:16; 2:26; 3:36; 4:43; 5:52; 6:43; 7:45; 8:51; 9:41; 10:40; 11:40; 12:42; 13:30; 14:25; 15:32; 16:18; 17:21; 18:19; 19:14; 20:6. This spectrum is in good agreement with the entire sample of events given in Table VIII, Ref. 20.

[22] This is understood qualitatively as follows: Square the matrix elements given by Eqs. (4) and (5). Average them over the Dalitz plot. Neglect the averages of all terms except the constant and quadratic terms. Set $\langle y_i^2\rangle=\frac{1}{4}$, its nonrelativistic value. Then $R\approx\frac{3}{2}[1+\frac{1}{2}\,\mathrm{Re}c]/[1+\frac{1}{2}\,\mathrm{Re}c+\frac{1}{4}|b|^2]$. We see that $R\approx0$ for Rec ≈-2. When the terms neglected here are included, we get the results shown in Fig. 1 for this sample of 640 decays.

TABLE I. The best-fit parameters for our first-, second-, and third-order fits and also the parameters for the complex cubic model that give $\chi^2=\langle\chi^2\rangle$. All errors correspond to increasing χ^2 by 1. We arbitrarily take $\mathrm{Im}(b)$ to be positive.

Parameter	First	Order of fit Second	Third	Third order at $\chi^2=\langle\chi^2\rangle$[a]
$\langle\chi^2\rangle$	17	15	13	13
χ^2	8.9	7.9	7.4	13.0
R	1.63	1.55	1.61	0.72
$\mathrm{Re}(b)$	$-0.43_{-0.05}^{+0.04}$	$-0.50_{-0.09}^{+0.09}$	$-0.55_{-0.13}^{+0.13}$	-0.52
$\mathrm{Im}(b)$	$0.20_{-0.27}^{+0.27}$	$0.57_{-0.60}^{+0.40}$	$0.4_{-1.2}^{+1.5}$	2.56
$\mathrm{Re}(c)$	...	$-0.20_{-0.30}^{+0.40}$	$0.03_{-0.55}^{+0.10}$	-2.53
$\mathrm{Im}(c)$	...	$0.15_{-0.75}^{+0.50}$	$-0.03_{-0.7}^{+0.7}$	-0.68
$\mathrm{Re}(e)$	...	...	$0.32_{-0.25}^{+0.25}$	0.35
$\mathrm{Im}(e)$	...	...	$-0.62_{-0.6}^{+1.3}$	-2.49

[a] In the vicinity of $\mathrm{Re}(c)=2.5$ (see Fig. 1).

parameters must be comparable in magnitude to the real parts. In fact we find that, *using a real cubic matrix element, we cannot obtain any good fit that gives a predicted value of R less than 1.5*. Thus the imaginary parts are needed to obtain predicted values of R less than 1.5 That in turn implies that final-state interactions are important[23] to obtain $R<1.5$. Just how important cannot be told directly from our parameterization.

We now discuss a matrix element more general than that given by Eq. (4), namely, the matrix element obtained by multiplying $M(+-0)$ of Eq. (4) by a phase factor. Then, for example, Eq. (3) becomes

$$M'(+-0)=M(+-0)e^{i\phi_3}=(1+by_3)e^{i\phi_3}. \qquad (6)$$

This matrix element was introduced by Foster *et al.*,[24] who fitted ϕ to the branching ratio R. Notice that this phase factor can have no effect on the $\pi^+\pi^-\pi^0$ Dalitz plot, since

$$|M'(+-0)|^2=|M(+-0)|^2.$$

Thus, if such a phase exists, it is impossible to predict R from the $\pi^+\pi^-\pi^0$ Dalitz plot alone.[25]

[23] T. D. Lee, Phys. Rev. **139**, B1415 (1965).

[24] Reference 6. Fitting to their sample of 274 decays $\eta\to\pi^+\pi^-\pi^0$, they find $b=-0.4$. Their measured value $R=0.90\pm0.24$ then gives them $\gamma=1.60\pm0.40$.

[25] If ϕ were a completely symmetric function of y_1, y_2, and y_3, it would also factor out of $M'(000)$, and thus would have no observable effects.

The simplest nontrivial[25] expansion of ϕ is[24]

$$\phi_i=\gamma y_i.$$

Symmetrizing Eq. (6), we have

$$M'(000)=6^{-1/2}[(1+by_3)e^{i\phi_3}+(1+by_2)e^{i\phi_2}+(1+by_1)e^{i\phi_1}]. \qquad (7)$$

To illustrate the dependence of R upon γ, we note that if $b=0$, then as γ varies from 0 to ∞, R goes from $1.5\times1.15=1.7$ down to $0.5\times1.15=0.57$. If $b\neq0$, these limits are of course different. However, it is clear that R is strongly dependent upon γ.

As for its physical meaning, we notice that since ϕ does not affect the $\pi^+\pi^-\pi^0$ Dalitz plot, we would not expect it to be produced by final-state interactions. Thus ϕ might arise from the "intrinsic η-decay" process itself. The angle ϕ corresponds to changing the relative amounts of the symmetric and nonsymmetric states in a manner that changes the branching ratio without changing the $\pi^+\pi^-\pi^0$ Dalitz plot.

A Dalitz plot for $\eta\to3\pi^0$ will be necessary in order to answer the questions: (a) Are there quadratic and cubic terms? (b) Is there a phase factor ϕ? (c) Is there some $T=3$?

One way to answer the last question is to measure R for only those events lying very close to the center of the Dalitz plots. Call this R_0. At the very center, $R_0=\frac{3}{2}$ for the $T=1$ state *regardless* of the amount of nonsymmetric $T=1$ state present, and regardless of the energy dependence of the $T=1$ matrix element. Similarly, for $T=3$, $R_0=\frac{2}{3}$ regardless of energy dependence of the matrix element. Now, the $T=1$, and $T=3$ states must be relatively real (in the absence of final-state interactions).[23] In that case the branching ratio at the center of the Dalitz plots gives uniquely the relative amounts of $T=1$ and $T=3$. Even if there are final-state interactions, any deviation from $R_0=\frac{3}{2}$ will show that $T=3$ is present.

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