Photoproduction with Linearly Polarized Photons as a Test for Cuts in the Angular Momentum Plane

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We propose here two simple experimental tests for Regge cuts. They consist of looking at $s(d\sigma/dt)$ for the reactions $\gamma+N\to\pi^0+N$ and $\gamma+N\to N^*+\pi^0$ as a function of s and φ_γ for fixed t, where φ_γ is the angle the polarization vector makes with the scattering plane. The behavior of $s(d\sigma/dt)$ at $\varphi_\gamma=0$ determines whether cuts are present or not.

INTRODUCTION

RECENTLY there has been renewed interest in the effects of Regge cuts on scattering processes.[1] Most of these papers have been concerned with trying to predict polarization phenomena.[2] In this paper we propose two simple experiments which will reveal the existence of cuts. The experiments consist of looking at the differential cross section for single-pion and N^*-pion photoproduction for different photon polarization angles at fixed t and various energies. The use of polarized photons enables one to select to order $1/s$ eigenstates of parity in the t channel, so that trajectories of opposite parity are suppressed. By eliminating the leading trajectories by this parity filter, we can look for whatever else is present in the asymptotic energy region. The theorem proving this, as well as some of its possible ramifications, is left to the Appendix.

The first experiment consists of doing photoproduction of π^0's off protons (or neutrons). In neutral-pion photoproduction we can have contributions from the ω, ρ, φ, and B mesons. From $SU(3)$ arguments we expect the φ contribution to be small, if not zero, and the ρ contribution to be much smaller than that of the ω. Regardless of this, the contributions of ρ, ω, and φ form an effective Regge vector-meson exchange as far as the parametrization is concerned. The contribution of the B is expected to be small because the trajectory has a low intercept and thus is relatively unimportant at high energies. If we consider neutral-pion photoproduction off deuterons [or the isospin combination $T^+=T(\pi^0)+T(N\pi^0)$],[3] then only ω and φ can contribute, and we do not have to worry at all about an opposite-parity trajectory. However, in that case we have to worry about the validity of the impulse approximation.

The helicity decomposition of photoproduction in terms of the "parity conserving" helicity amplitudes in the t channel, $f^\pm$ and $f^{J\pm}$, is given by[4]

$$f_{\frac12\frac12,10}{}^{\pm}=(1/\sin\theta_t)[f_{\frac12\frac12,10}\pm f_{-\frac12-\frac12,10}],$$

$$f_{\frac12-\frac12,10}{}^{\pm}=\frac{1}{1+\cos\theta_t}f_{\frac12-\frac12,10}\pm\frac{1}{1-\cos\theta_t}f_{-\frac12\frac12,10}.$$

We formally introduce linearly polarized γ rays by means of the following density matrix:

$$\rho_{\lambda\lambda'}=\frac14\begin{pmatrix}1 & -e^{-2i\varphi_\gamma}\\ -e^{2i\varphi_\gamma} & 1\end{pmatrix}.$$

Here φ_γ is the angle that the polarization vector makes with the plane of scattering. We extend the work of Gottfried and Jackson[5] of determining final-state density matrices in terms of crossed-channel helicity amplitudes to the case of an initially polarized beam. This straightforward extension leads to the following expression for $d\sigma/dt$ in terms of t-channel helicity amplitudes[6]:

$$d\sigma/dt=\pi(s-M_N{}^2)^{-2}\times[|f_{\frac12\frac12}|^2+|f_{\frac12-\frac12}|^2+|f_{-\frac12-\frac12}|^2+|f_{-\frac12\frac12}|^2 -2\cos2\varphi_\gamma\,\mathrm{Re}(f_{\frac12\frac12}f_{-\frac12-\frac12}{}^*-f_{\frac12-\frac12}f_{-\frac12\frac12}{}^*)],$$

where, in f_{ab}, a and b refer to the $\bar N$ and N helicities, respectively; $\lambda_\gamma=1$, $\lambda_\pi=0$. But

$$f_{\frac12\frac12}=\tfrac12\sin\theta_t(f_{\frac12\frac12}{}^{+}+f_{\frac12\frac12}{}^{-}),$$
$$f_{-\frac12-\frac12}=\tfrac12\sin\theta_t(f_{\frac12\frac12}{}^{+}-f_{\frac12\frac12}{}^{-}),$$
$$f_{\frac12-\frac12}=\tfrac12(1+\cos\theta_t)(f_{\frac12-\frac12}{}^{+}+f_{\frac12-\frac12}{}^{-}),$$
$$f_{-\frac12\frac12}=\tfrac12(1-\cos\theta_t)(f_{\frac12-\frac12}{}^{+}-f_{\frac12-\frac12}{}^{-}).$$

Therefore, in terms of f^+ and f^- we obtain

$$d\sigma/dt=[2/\pi(s-M_N{}^2)^2][|f_{\frac12\frac12}{}^{+}|^2\sin^2\theta_t\sin^2\varphi_\gamma +|f_{\frac12\frac12}{}^{-}|^2\sin^2\theta_t\cos^2\varphi_\gamma+|f_{\frac12-\frac12}{}^{+}|^2(1-\sin^2\theta_t\sin^2\varphi_\gamma) +|f_{\frac12-\frac12}{}^{-}|^2(1-\cos^2\varphi_\gamma\sin^2\theta_t)+2\cos\theta_t\,\mathrm{Re}f_{\frac12-\frac12}{}^{+}f_{\frac12-\frac12}{}^{-}].$$

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[1] S. Mandelstam, Nuovo Cimento **30**, 1148 (1963).

[2] V. de Lany, D. Gross, I. Muzinich, and V. Teplitz, Phys. Rev. Letters **18**, 149 (1967).

[3] G. Chew, M. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[4] M. Gell-Mann, M. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[5] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

[6] A complete density matrix analysis of one- and two-pion photoproduction processes in terms of t-channel Regge trajectory exchange will be published elsewhere. Here we are concerned primarily with the s dependence of the differential cross section.

Now, f^+ and f^- have partial-wave expansions in terms of the parity conserving partial-wave amplitudes $F^{J\pm}$ which are eigenstates of parity $\pm(-1)^J$:

$$f_{\lambda_c\lambda_d,\lambda_a\lambda_b}{}^{\pm}(s,t)=\sum\,(2J+1)E_{\lambda\mu}{}^{J+}F_{\lambda_c\lambda_d,\lambda_a\lambda_b}{}^{J\pm} +E_{\lambda\mu}{}^{J-}F_{\lambda_c\lambda_d,\lambda_a\lambda_b}{}^{J\mp},\quad \lambda=|\lambda_a-\lambda_b|,\quad \mu=|\lambda_c-\lambda_d|\,.$$

Using the asymptotic properties of the E functions,

$$E_{\lambda\mu}{}^{\alpha+}\sim S^{\alpha-a},\quad E_{\lambda\mu}{}^{\alpha-}\sim S^{\alpha-a-1},\quad E_{0\mu}{}^{\alpha-}=0\,,$$

where

$$a=\max(|\lambda|,|\mu|)\,,$$

we recognize that $f^{\pm}$ are eigenstates of parity to order $1/s$, for single trajectory exchange.

Let us now look at how the various trajectories contribute to the parity-conserving t-channel helicity amplitudes.

VECTOR MESONS

The ω, ρ, and φ mesons contribute in a similar fashion to $F_{\frac{1}{2}\frac{1}{2},10}{}^{J+}$ and $F_{\frac{1}{2}-\frac{1}{2},10}{}^{J+}$ only because of parity. Therefore, the various t-channel helicity amplitudes have the following s dependence:

$$f_{\frac{1}{2}\frac{1}{2}}{}^{+}\sim a(t)s^{\alpha_\omega-1}+b(t)s^{\alpha_\rho-1}+c(t)s^{\alpha_\varphi-1}\,,$$
$$f_{\frac{1}{2}-\frac{1}{2}}{}^{+}\sim d(t)s^{\alpha_\omega-1}+e(t)s^{\alpha_\rho-1}+f(t)s^{\alpha_\varphi-1}\,,$$
$$f_{\frac{1}{2}-\frac{1}{2}}{}^{-}\sim d(t)s^{\alpha_\omega-2}+e(t)s^{\alpha_\rho-2}+f(t)s^{\alpha_\varphi-2}\,,$$

where $SU(3)$ implies that c and f are almost zero, and that b and e are small. The trajectories are almost degenerate and can be lumped into one "effective vector-meson Regge exchange" irrespective of $SU(3)$ arguments.

B MESON

Parity and G-parity arguments limit the B-meson contribution to $F_{\frac{1}{2}\frac{1}{2},10}{}^{J-}$ only. Therefore, we find

$$f_{\frac{1}{2}\frac{1}{2}}{}^{-}\sim g(t)s^{\alpha_B-1}\,.$$

Thus we find that if there are no cuts in the complex j plane and no as yet undiscovered trajectories with the right quantum numbers, then the differential cross section has the following s dependence:

$$d\sigma/dt\sim a(t)s^{2(\alpha_V-1)}\sin^2\varphi_\gamma +\cos^2\varphi_\gamma[b(t)s^{2(\alpha_B-1)}+c(t)s^{2(\alpha_V-2)}]\cdots +d(t)s^{2(\alpha_V-2)}\,,$$

where we have lumped the contribution from the various vector mesons into an effective vector-meson trajectory function $\alpha_V(t)$. We next assume that all trajectories lie on straight lines which are approximately parallel to the known ρ trajectory with a mass displacement given by

$$\alpha_x-\alpha_\rho=m_\rho{}^2-m_x{}^2\,.$$

Thus we have the following parametrization of the trajectory functions:

$$\alpha_V\approx 0.5+t\,,$$
$$\alpha_B\approx -0.3+t\,.$$

We thus find that[7]

$$s d\sigma/dt\approx s^{2t}\{a(t)\sin^2\varphi_\gamma+\cos^2\varphi_\gamma \times[(1/s^{1.6})b_B(t)+(1/s^2)c(t)]+(1/s^2)d(t)\}\,.$$

Therefore, if we keep t and s fixed, we see that we get a dip at $\varphi_\gamma=0$ and the value of $s d\sigma/dt$ at the dip should approach zero as $1/s^{1.6}$. We note also that if our approximate parametrization is correct at $\varphi_\gamma=0$, then

$$s^{2.6}d\sigma/dt+e(t)|\gamma_{\frac{1}{2}\frac{1}{2}}{}^B|^2+O(1/s^{0.4})\,,$$

where $e(t)$ contains known kinematic factors. Therefore, we can isolate the explicit t dependence of the B-meson coupling constant.

Suppose, however, that there is a cut in the complex angular momentum plane due to exchange, say, of "ω" and the Pomeranchon. This cut can contribute to $f_{\mu,\lambda}{}^-$. The contribution will have the following s dependence:

$$f_{\mu,\lambda}{}^-\sim s^{\alpha_c(t)-a}D(t,1/\ln s)\,.$$

If we assume that all the trajectories lie on straight lines, we find[8]

$$\alpha_c(t)=\alpha_\omega\left[\left(\frac{\alpha_P'}{\alpha_\omega'+\alpha_P'}\right)^2 t\right]+\alpha_p\left[\left(\frac{\alpha_\omega'}{\alpha_P'+\alpha_\omega'}\right)^2 t\right]-1\,,$$

where $\alpha_\omega(t)$ and $\alpha_P(t)$ are the ω and Pomeranchon trajectory functions, and α_ω' and α_P' are their slopes, which are assumed to be constants for the values of t to be considered. Thus if we have contributions from the leading cut and from the "ω" pole, the asymptotic behavior of $d\sigma/dt$ has the following s behavior. (The B contribution is unimportant compared to that of the cut.)

$$d\sigma/dt\approx a(t)s^{2(\alpha_\omega-1)}\sin^2\varphi_\gamma+D^2(t,1/\ln s)s^{2(\alpha_c-1)}\cos^2\varphi_\gamma +E(t,1/\ln s)s^{\alpha_\omega+\alpha_c-3}\cos^2\varphi_\gamma+F(t,1/\ln s)s^{\alpha_\omega+\alpha_c-3}\,.$$

At $t=0$ we have $\alpha_c\approx 0.5$. Therefore, near $t=0$ the s dependence of $s d\sigma/dt$ is given by

$$s d\sigma/dt\approx a(t)\sin^2\varphi_\gamma+D^2(t,\ln 1/s)\cos^2\varphi_\gamma +\text{terms of order } 1/s\,.$$

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[7] P. Stichel [Z. Physik **180**, 170 (1964)] proved for single-pion photoproduction, by looking at the explicit s dependence of various terms, that the asymptotic dependence of the amplitudes is as follows:

$f(s,t,\varphi)\sim\sin\varphi$ if parity of exchanged particles is $(-1)^J$,
$f(s,t,\varphi)\sim\cos\varphi$ if parity of exchanged particle is $-(-1)^J$.

This is only a special case of the more general theorem proved in the Appendix, which shows that as long as one has a photon spin-zero vertex, we are selecting almost eigenstates of parity by using linearly polarized photons.

[8] G. Bellenteni, G. Cocconi, A. N. Diddens, E. Lillenthun, J. P. Scanlon, and A. Wetherell, Phys. Letters **19**, 705 (1966).

If we plot this for fixed t as a function of φ_γ for increasing values of s, we notice that the value of $sd\sigma/dt$ at $\varphi_\gamma=0$ should approach a "constant" value. The value of this constant will give an indication as to the strength of the Regge cut. (Note that a trajectory of opposite J parity to the ω but at the same height will also lead to this behavior, but no particles with the appropriate quantum numbers are known.)

Experiment 2 involves doing a similar experiment for the reaction $\gamma+p\rightarrow N^{*+}+\pi^0$. This reaction is also relatively simple in the Regge-pole picture because the exchanged trajectory in the t channel is limited to having the quantum numbers $C=-1$ and $I=1$. Thus only the ρ or B mesons can be exchanged, with the B expected to have little importance. In this reaction, as in the last, using polarized photons again enables one to select to order $1/s$ eigenstates of parity in the t channel, so that the opposite parity trajectory will be suppressed in the differential cross section. First we write the "parity-conserving" helicity amplitudes in the t channel:

$$f_{\frac{1}{2}\frac{1}{2},-10}{}^{\pm}=(1/\sin\theta_t)(f_{\frac{1}{2}\frac{1}{2},-10}\mp f_{-\frac{1}{2}-\frac{1}{2},-10}),$$

$$f_{\frac{1}{2}\frac{3}{2},-10}{}^{\pm}=\frac{1}{1+\cos\theta_t}f_{\frac{1}{2}\frac{3}{2},-10}\mp\frac{1}{1-\cos\theta_t}f_{-\frac{1}{2}-\frac{3}{2},-10},$$

$$f_{\frac{1}{2}-\frac{1}{2},-10}{}^{\pm}=\frac{1}{1-\cos\theta_t}f_{\frac{1}{2}-\frac{1}{2},-10}\mp\frac{1}{1+\cos\theta_t}f_{-\frac{1}{2}-\frac{1}{2},-10},$$

$$f_{\frac{1}{2}-\frac{3}{2},-10}{}^{\pm}=\frac{1}{\sin\theta_t(1-\cos\theta_t)}f_{\frac{1}{2}-\frac{3}{2},-10}\pm\frac{1}{\sin\theta_t(1+\cos\theta_t)}f_{-\frac{1}{2}\frac{3}{2},-10},$$

where the subscripts refer to the $\bar{p}$, N^{*+}, γ, and π^0 helicities, respectively. We extend Gottfried and Jackson's density-matrix formalism to the case of polarized γ rays and find for $(d\sigma/dt)(\gamma+p\rightarrow N^*+\pi^0)$ in terms of t-channel helicity amplitudes (suppressing γ and π indices, $\lambda\gamma=-1$)[6]

$$\begin{aligned}d\sigma/dt=&[2/\pi(s-M_N{}^2)^2][|f_{\frac{1}{2}\frac{1}{2}}{}^+|^2\sin^2\theta_t\sin^2\varphi_\gamma+|f_{\frac{1}{2}\frac{1}{2}}{}^-|^2\sin^2\theta_t\cos^2\varphi_\gamma]\\&+(|f_{\frac{1}{2}\frac{3}{2}}{}^+|^2+|f_{\frac{1}{2}-\frac{1}{2}}{}^+|^2)(1-\sin^2\theta_t\sin^2\varphi_\gamma)+(1-\sin^2\theta_t\cos^2\varphi_\gamma)(|f_{\frac{1}{2}-\frac{1}{2}}{}^-|^2+|f_{\frac{1}{2}\frac{3}{2}}{}^-|^2)\\&-2\cos\theta_t(\mathrm{Re}f_{\frac{1}{2}-\frac{1}{2}}{}^+f_{\frac{1}{2}-\frac{1}{2}}{}^{*-}-\mathrm{Re}f_{\frac{1}{2}\frac{3}{2}}{}^+f_{\frac{1}{2}\frac{3}{2}}{}^{*-})+|f_{\frac{1}{2}-\frac{3}{2}}{}^+|^2(1-\sin\theta_t\sin^2\varphi_\gamma)\sin^2\theta_t+|f_{\frac{1}{2}-\frac{3}{2}}{}^-|^2(1-\sin^2\theta_t\cos^2\varphi_\gamma)\sin^2\theta_t\\&-2\cos\theta_t\,\mathrm{Re}(f_{\frac{1}{2}-\frac{3}{2}}{}^+f_{\frac{1}{2}-\frac{3}{2}}{}^{*-})\sin^2\theta_t,\end{aligned}$$

where

$$\cos\theta_t=\frac{2st-t(m^2+m^{*2}+\mu^2)+\mu^2(m^{*2}-m^2)}{(t-\mu^2)[t-(m^*-m)^2]^{1/2}[t-(m^*+m)^2]^{1/2}}.$$

We Reggeize in the usual manner and find that in the asymptotic region the ρ-meson contribution has the following s dependence:

$$f_{\mu,\lambda}{}^+\sim s^{\alpha_\rho-a},\quad f_{\mu,\lambda}{}^-\sim s^{\alpha_\rho-a-1}\quad\text{for}\quad\mu\neq0$$
$$\sim0\qquad\text{for}\quad\mu=0,$$
$$a=\max(|\lambda|,|\mu|).$$

The B contributes in the following way to the same amplitudes:

$$f_{\mu,\lambda}{}^-\sim s^{\alpha_B-a},\quad f_{\mu,\lambda}{}^+\sim s^{\alpha_B-a-1}\quad\text{for}\quad\mu\neq0$$
$$\sim0\qquad\text{for}\quad\mu=0.$$

The major contribution of the leading cut due to the exchange of $\rho+P$ will be

$$f_{\mu,\lambda}{}^-\sim s^{\alpha_c(t)-a}D(t,1/\ln s),$$

and if we assume all trajectories lie on straight lines we then find

$$\alpha_c(t)=\alpha_\rho[t(\alpha_P'/(\alpha_P'+\alpha_\rho'))^2]+\alpha_P[t(\alpha_\rho'/(\alpha_P'+\alpha_P'))^2]-1.$$

Let us now consider three distinct possibilities.

(a) Only the ρ contributes. We then find

$$d\sigma/dt\approx a(t)s^{2[\alpha_\rho(t)-1]}\sin^2\varphi_\gamma+s^{2[\alpha_\rho(t)-2]}[b(t)+c(t)\cos^2\varphi_\gamma].$$

This leads to a dip at $\varphi_\gamma=0$, and the value of $sd\sigma/dt$ at this point goes to zero at $1/s^2$ if we set $\alpha_\rho(t)=0.5+t$.

(b) Both the ρ and B mesons contribute. In the following we assume that the B trajectory is parallel to the ρ trajectory with a mass displacement given by $\alpha_\rho-\alpha_B=m_B{}^2-m_\rho{}^2$. Then

$$\alpha_B(t)=-0.30+t,$$

so that

$$sd\sigma/dt=s^{2t}[a(t)\sin^2\varphi_\gamma+d(t)(\cos^2\varphi_\gamma)/s^{1.4}+e(t)/s^{1.8}].$$

With our approximate parametrization of the trajectory function, we see that the following plot might allow us to distinguish the B meson from the secondary effects of the ρ mesons:

$$s^{2.4}d\sigma/dt=s^{2t}[s^{1.4}a(t)\sin^2\varphi_\gamma+d^B(t)\cos^2\varphi_\gamma+O(1/s^{0.4})].$$

However, the general behavior of the dip is the same as in the previous case.

(c) The ρ, the B, and the leading cut due to $\rho+P$ all contribute. Then the leading terms in the asymptotic expansion are

$$d\sigma/dt=a(t)s^{2(\alpha_\rho-1)}\sin^2\varphi_\gamma+\cos^2\varphi_\gamma(s^{2(\alpha_c-1)}F^2(t,1/\ln s)+b(t)s^{2(\alpha_B-1)})+G(t)s^{\alpha_c+\alpha_\rho-3}+\cdots.$$

If we are near $t=0$, then $\alpha_c=\alpha_\rho=0.5$ and $\alpha_B=-0.3$, so that

$$sd\sigma/dt\approx a(t)\sin^2\varphi_\gamma+\cos^2\varphi_\gamma(F^2(t,1/\ln s)+b(t)/s^{1.6})+G(t,1/\ln s)/s+\cdots.$$

Thus if there is a cut (or a trajectory with the same quantum numbers of the B but at the height of the ρ trajectory) there should be a dip in the plot of $sd\sigma/dt$ versus φ_γ for fixed t and s, and the dip should stay relatively constant as s increases.

SUMMARY OF RESULTS

In this paper we have made the following assumptions:

(1) The high-energy behavior of the photoproduction amplitudes is governed solely by Regge poles, with the possibility of Regge cuts also considered.

(2) The only trajectories contributing are those associated with the particles found in the latest table of meson resonances.

(3) The relative importance of trajectories is determined by the intercept of the trajectory functions at $t=0$ and experimental evidence suggests the following approximate form for the trajectory functions:

$$\alpha_\rho\approx 0.5+t\approx\alpha_\omega\approx\alpha_\varphi,$$
$$\alpha_B\approx -0.3+t.$$

Making these assumptions, we find that if we plot $sd\sigma/dt$ versus that angle which the photon polarization vector makes with the production plane for the two reactions $\gamma+N\to\pi^0+N$ and $\gamma+N\to\pi^0+N^*$, then there should be a dip at $\varphi_\gamma=0$. The energy dependence of the value of $sd\sigma/dt$ at this point should be $\sim 1/s^2$ in the absence of cuts or some unknown high-intercept trajectory of parity $-(-1)^J$. On the other hand, the presence of cuts or another trajectory would imply that the value of $sd\sigma/dt$ at $\varphi_\gamma=0$ should remain relatively constant with increasing energy.

APPENDIX

We shall show how using polarized photons is equivalent for certain reactions to choosing combinations of helicity amplitudes which are asymptotically (or actually) eigenstates of parity, and how these amplitudes can also be used to get superconvergence relations.

In the previous two reactions we noticed that there was a change in energy dependence of $d\sigma/dt$ as $\varphi_\gamma\to 0°$ or 90°. This is explained as follows: We observe that a photon polarized at an angle φ_γ with the scattering plane is represented by

$$\epsilon(\varphi_\gamma)=\frac{1}{\sqrt{2}}(|-1\rangle e^{i\varphi}-|+1\rangle e^{-i\varphi}),$$

so that the mixed amplitude

$$\langle\tfrac{1}{2}\tfrac{1}{2}|T|\varphi 0\rangle=\langle\tfrac{1}{2}\tfrac{1}{2}|T|-10\rangle e^{i\varphi}-\langle\tfrac{1}{2}\tfrac{1}{2}|T|+10\rangle e^{-i\varphi},$$

which for $\varphi=0$ is

$$f_{\frac{1}{2}\frac{1}{2},-10}-f_{\frac{1}{2}\frac{1}{2},10}.$$

But this is just the amplitude $\sin\theta_t\, f_{\frac{1}{2}\frac{1}{2}}^-$ for the process $\gamma+\pi^0\to\bar{N}+N$, and for ω and φ exchange $f_{\frac{1}{2}\frac{1}{2}}^-=0$. Similarly, at $\varphi_\gamma=0$

$$\langle\tfrac{1}{2}-\tfrac{1}{2}|T|\varphi=0,0\rangle=f_{\frac{1}{2}-\frac{1}{2}}^+ +f_{\frac{1}{2}-\frac{1}{2}}^-\cos\theta_t.$$

Thus if $f_{\frac{1}{2}-\frac{1}{2},-10}\sim s^{\alpha_\omega}$, then $f_{\frac{1}{2}-\frac{1}{2}}^+\sim s^{\alpha_\omega-1}$, $f_{\frac{1}{2}-\frac{1}{2}}^-\sim s^{\alpha_\omega-2}$, and the combination obtained by using polarized photons behaves like $s^{\alpha_\omega-1}$. This explains the dip at $\varphi_\gamma=0$. In general, if only one trajectory is exchanged, then one of the asymptotic equations

$$f_{\lambda_c\lambda_d,\lambda_a\lambda_b}\pm f_{-\lambda_c-\lambda_d,\lambda_a\lambda_b}\sim s^\alpha/s$$

holds. If $\lambda_c=\lambda_d$ or $\lambda_a=\lambda_b$, then the polarized photon combinations are exact eigenstates of parity and the kinematic singularity—free amplitudes sometimes will obey superconvergent sum rules. For example, in $\gamma+N\to N+\pi^\pm$ we know only the π, A_2 can contribute to $T^{(-)}=T(\pi^+)-T(\pi^-)$. Therefore,

$$f_{\frac{1}{2}\frac{1}{2},10}^+\sim s^{\alpha_{A2}-1}f_{\frac{1}{2}\frac{1}{2},10}^-\sim s^{\alpha_\pi-1}.$$

We notice that $f_{\frac{1}{2}\frac{1}{2}}^-$ obeys a superconvergent sum rule. This sum rule has been discussed by Halpern.[9] It corresponds to scattering of linear polarized photons off polarized targets and knowing the outgoing nucleon polarization. Similarly, we find that in the photoproduction of ρ mesons, only the ρ and B mesons contribute to $T^-=T(\rho^+)-T(\rho^-)$. Now the ρ,B can contribute only to f^+, f^-, respectively, and knowing that $\alpha_\rho\leq 0.5$, $\alpha_B\leq -0.3$ we find that we get four superconvergent sum rules, which have excellent convergence and possibly can be saturated by only a few states.

$$\int \mathrm{Im} f_{\frac{1}{2}\frac{1}{2},10}^-ds=0,\quad \int \mathrm{Im} f_{\frac{1}{2}-\frac{1}{2},11}^-ds=0,$$

$$\int \mathrm{Im} f_{\frac{1}{2}\frac{1}{2},1-1}^-ds=0,\quad \int \mathrm{Im} f_{\frac{1}{2}\frac{1}{2},1-1}^+ds=0.$$

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[9] M. B. Halpern, Phys. Rev. **160**, 1441 (1967).