Three-Body Associated Production in Proton-Proton Collisions and the One-Boson-Exchange Model*

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The reactions $pp \to YKN$ are studied under the hypothesis of a dominant contribution from the exchange of a pseudoscalar meson (π or K) between the initial protons. The difficulty of a rigorous theoretical treatment of a true three-body final state is circumvented by assuming that the various damping effects at large momentum transfers (form factors, absorption) can be described in terms of a simple multiplicative cutoff function. Also, the assumptions and approximations which usually come into play in the theoretical analysis of these reactions are discussed critically. It is found that the agreement between theory and experiment is fairly good, within the limits of the model.

1. INTRODUCTION

ASSOCIATED production in proton-proton collisions has been studied in detail only in the recent years, some time after a first pioneer work with rather poor statistics.[1] Low-energy data have been obtained in fixed-angle counter experiments.[2,3] At higher energies there exist bubble-chamber data up to 8 GeV/c.[4–7] The interpretation of the data obtained has been mainly concerned with the reactions leading to three-body final states, although four- and five-body final states have a larger cross section at the higher energies. In particular, the reactions we are interested in are the following:

$$pp \to p\Lambda K^+, \quad pp \to p\Sigma^+K^0,$$
$$pp \to p\Sigma^0K^+, \quad pp \to n\Sigma^+K^+. \qquad (1)$$

In such reactions, the observed strong peaking of the final hyperons and nucleons in the center-of-mass system (c. m. s.) has suggested the dominance of a peripheral mechanism, favoring low-momentum transfers between final and initial baryons. The mechanism which has been studied the most extensively is the exchange of a pseudoscalar boson (π or K meson) between the initial protons, as illustrated by the diagrams of Fig. 1.

A remarkable feature of all the reactions (1) is that none of them exhibits an appreciable formation of resonant states, contrary to what happens in non-strange inelastic final states, where the N_{33}^* is copiously produced. Therefore, the diagrams of Fig. 1 cannot be replaced by Born terms describing a quasi-two-body reaction mediated by the exchange of a virtual boson. This fact is of crucial importance in the theoretical study of these processes. Indeed, most of the theoretical knowledge how to treat peripheral collisions can be used only for quasi-two-body reactions, and cannot be easily generalized to effective three-particle final states. In this paper, we shall discuss the difficulties connected with this situation, and the possibility of circumventing them in the attempt to find a reasonable way of comparing theory and experiment.

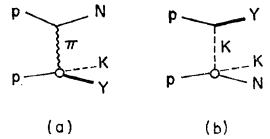

FIG. 1. Typical diagrams representing any of the reactions (1) through: (a) pion exchange; (b) kaon exchange. p = incoming protons, N = outgoing nucleon, Y = outgoing hyperon.

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[1] R. I. Louttit, T. W. Morris, D. C. Rahm, R. R. Rau, A. M. Thorndike, and W. J. Willis, Phys. Rev. **123**, 1465 (1961).

[2] A. C. Melissinos, N. W. Reay, J. T. Reed, T. Yamanouchi, E. Sacharidis, S. Lindenbaum, S. Ozaki, and L. C. L. Yuan, Phys. Rev. Letters **14**, 604 (1965); J. T. Reed, thesis, University of Rochester, 1965 (unpublished).

[3] J. T. Reed, A. C. Melissinos, N. W. Reay, T. Yamanouchi, E. J. Sacharidis, S. J. Lindenbaum, S. Ozaki, and L. C. L. Yuan, Phys. Rev. (to be published).

[4] E. Bierman, A. P. Colleraine, and U. Nauenberg, Phys. Rev. **147**, 922 (1966).

[5] G. Alexander, O. Benary, G. Czapek, B. Haber, N. Kidron, B. Reuter, A. Shapira, E. Simopoulou, and G.Yekutieli, Phys. Rev. **154**, 1284 (1967).

[6] W. Dunwoodie, W. E. Slater, H. K. Ticho, G. A. Smith, A. B. Wicklund, and S. G. Wojcicki, communication at the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, Calif., 1966. We are aware that the experimental analysis has progressed much since the presentation of these preliminary data. However, we were unable to obtain the most recent data. The data presented in the above paper are 75% at 6.6 GeV/c and 25% at 5.4 GeV/c; we compare them with the theoretical calculations at 6.6 GeV/c, and assume a total cross section of 50 ± 5 μb at this energy. G. A. Smith (private communication).

[7] G. Ascoli, M. Firebaugh, E. L. Goldwasser, U. E. Kruse, and R. D. Sard, communication at the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, Calif., 1966. M. Firebaugh, thesis, University of Illinois, 1967 (unpublished). M. Firebaugh, G. Ascoli, E. L. Goldwasser, R. D. Sard, and J. Wray, University of Illinois Report C00 1195-109 (1967).

So far, theoretical analyses of these reactions can be found mainly in the experimental papers, where rather different conclusions are reached. Bierman *et al.*[4] and Dunwoodie *et al.*[6] claim evidence for pion exchange, with some reservations in the latter paper.[8] Ascoli *et al.*[7] consider also the possibility of K exchange with diffraction scattering in the $K^+p \to K^+p$ vertex, and conclude that neither this mechanism nor simple pion exchange are satisfactory. Early theoretical works[9,10] are of more limited interest, and are mainly devoted to the comparison of total cross sections.

In this paper, a comprehensive study of the available data at all energies will be made. The conclusions reached do not look, after all, so different from those already obtained, at least from a qualitative point of view. However, we try to discuss critically the assumptions and approximations which come into play. We hope that our analysis clarifies what one can expect to learn from the comparison between theory and experiment.

2. CHARACTERISTIC FEATURES OF THE ONE-BOSON-EXCHANGE MODEL

It is well known that the one-pion-exchange (OPE) model, modified by the introduction of form factors,[11] or by absorption,[12] or both,[13] or by other types of cutoff on the high-momentum transfers[14] gives a satisfactory account of single and double pion production in pp and $\bar{p}p$ collisions, in the incident momentum range from 2 to 6 GeV/c,[15–16] and, for the reaction $pp \to nN_{33}^*$, up to 10 GeV/c.[17] It is reasonable to assume that the same mechanism governs also those inelastic pp collisions which lead to the production of strange particles. This is perhaps the main reason (apart from criteria of simplicity) why we have not considered vector exchange in addition to pseudoscalar exchange. The conclusion that vector exchange should not be important is also suggested by the analysis by Sweig,[16] who shows that the inclusion of vector exchange in $\bar{p}p$ collisions (both for strangeness-transferring and for strangeness-nontransferring reactions) always spoils the agreement with the experiment.

However, if we look closely at the analogy with inelastic pp collision producing nonstrange final states (such as single pion production, described by the diagram of Fig. 2, where the $N\pi$ system in the lower vertex is usually in a resonant state), we notice that in the latter diagram the exchange of the final nucleons always produces another one-pion-exchange diagram, which *must* be taken into account in the description of the process (unless one restricts oneself to the production of a particular isobar). In the case of strange particle production, if we exchange the final baryons in the same way, we transform the diagram of Fig. 1(a) (with an intermediate π) into the one of Fig. 1(b) (with an intermediate K) and vice versa. Therefore one is consistently led to retain both mechanisms, especially in the absence of isobar production. Thus the old question: "π or K?", to which the experimentalists have tried to give an answer, does not seem to be very sound. A more logical answer is that, if one assumes π exchange, he should *a priori* expect K exchange, too.

The major objection against the introduction of K exchange is that, if one calculates its contribution without any modification for being the K meson off its mass shell, it comes out about one order of magnitude larger than the experimental values, whereas π-exchange diagrams, if evaluated in the same way, are still too large, but only by a factor of two. However, this is a difficulty that can be easily overcome. In all the existing applications of the one-particle-exchange models, the contribution of the high-momentum transfers between final and initial particles has always proved to be too large, so that the agreement with experiment can be obtained only by cutting it off in some way. The cutoff mechanisms have been justified in terms of physical effects (form factors, absorption) and have proven to be very strong in a variety of cases. Therefore the K-exchange mechanism can be brought consistently into the picture by assuming that it must be cut off much more strongly than the pion exchange mechanism. At present it seems to be difficult to find arguments which may prove or disprove this statement; however, from a purely empirical point of view, it is more sound to postulate that the enhancement or suppression of a mechanism is to be found within the mechanism itself, rather than ignoring such a mechanism *a priori*, or using percentage combinations (like, e.g., 80% pion exchange, 20% kaon exchange), which

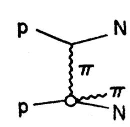

FIG. 2. Typical diagram representing single pion production in proton-proton collisions through pion exchange. p = incoming protons, N = outgoing nucleons.

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[8] It should be pointed out that the agreement claimed in Ref. 4 with the predictions of the one-pion-exchange model is only apparent, because all the theoretical values should be multiplied by a factor of 2. Such a wrong multiplication by $\frac{1}{2}$ is also present in one of the early theoretical works [Ref. 9, Eq. (15)].

[9] E. Ferrari, Phys. Rev. **120**, 988 (1960).

[10] T. Yao, Phys. Rev. **125**, 1048 (1962).

[11] E. Ferrari and F. Selleri, Nuovo Cimento Suppl. **24**, 453 (1962).

[12] J. D. Jackson, Rev. Nod. Phys. **37**, 484 (1965).

[13] P. C. M. Yock and D. Gordon, Phys. Rev. **157**, 1362 (1967).

[14] B. Haber and G. Yekutieli, Phys. Rev. **160**, 1410 (1967).

[15] For applications of the one-pion-exchange (OPE) model with form factors to pp and $\bar{p}p$ collisions, see, e.g., E. Ferrari and F. Selleri, Phys. Rev. Letters **7**, 387 (1961); Nuovo Cimento **27**, 1450 (1962); E. Ferrari, *ibid.* **30**, 240 (1963); T. Ferbel, A. Firestone, J. Johnson, J. Sandweiss, and H. D. Taft, *ibid.* **38**, 19 (1965).

[16] For applications of the OPE model with absorption to pp and $\bar{p}p$ collisions, see, e.g., G. Alexander, B. Haber, A. Shapira, G. Yekutieli, and E. Gotsman, Phys. Rev. **144**, 1122 (1966); B. E. Y. Svensson, Nuovo Cimento **39**, 667 (1965); M. J. Sweig, Phys. Rev. **157**, 1420 (1967).

[17] H. C. Dehne, J. Diaz, K. Strömer, A. Schmitt, W. P. Swanson, I. Borecka, G. Knies, and G. Wolf, Nuovo Cimento **53**, A232 (1968).

are meaningful in a statistical picture, but are devoid of significance for any approach, like the OPE model, which predicts absolute cross sections.

3. THEORETICAL TREATMENT OF THE ONE-BOSON-EXCHANGE DIAGRAMS

The simplest way of calculating the contributions of the diagrams of Fig. 1 is to use the standard Chew-Low formula,[18] which contains: the pion-nucleon coupling constant and the experimental cross section for $\pi p \to YK$, in the case of pion exchange; the $p\Lambda K$ (or $p\Sigma K$) coupling constant and the experimental cross section for $KN \to KN$, in the case of kaon exchange. The relevant formulas to be used are substantially those contained in Ref. 11; the coupling constants and cross sections to be inserted in the calculation can be found in Ref. 9. However, we are going to disregard consistently all the interference terms between different diagrams, both because they cannot be correctly calculated at present, and because a rough evaluation of them always gives a small contribution for incident momenta $\gtrsim 5$ GeV/c.

As is well known, the unmodified peripheral graphs always give too large a contribution for large values of the momentum transfer Δ^2 (We call Δ^2 the square of the four-momentum of the exchanged particle, and choose a metric such that $\Delta^2>0$). The various off-shell corrections induced by the fact that the exchanged particle is virtual (absorption, form factors) have generally been found to produce a strong damping effect on Δ^2 and to depend little on the other kinematical variables. In many instances such an effective cutoff has been shown to be well represented by a function $F(\Delta^2)$ depending on Δ^2 only and multiplying the unmodified peripheral matrix element (see, e.g., the first paper quoted in Ref. 15). In the case of low-energy inelastic pp collisions, such a function can be parametrized in the following simple way:

$$F(\Delta^2)=(1+(\Delta^2+\mu^2)/\alpha)^{-1}\,, \qquad (2)$$

where μ is the physical mass of the exchanged particle and α is an adjustable parameter [which, for the reactions $NN \to NN\pi$, has been found to lie around $60m_\pi^2$ (1.18 GeV^2)].

For the reactions discussed in this paper, since the peripheral graphs cannot be represented as Born terms leading to quasi-two-body final states, we are not able to generalize the proper treatment of those off-shell corrections currently applied to peripheral resonance production. However, we can assume that the cutoff effect on Δ^2 can still be obtained by the use of a multiplicative function of the type (2). The use of such a function reminds one of the form-factor type of correction (Ref. 15); however, it must be clear that $F(\Delta^2)$ has not the meaning of a form factor, but it is intended to lump *all* the effects which suppress the high-momentum transfers, in an approximate, but consistent way. Therefore, α plays the role of a purely empirical parameter, and we do not have arguments which may allow us to guess its value. However, in order to have not too many parameters to deal with, we will assume that α is the same for all the diagrams of Fig. 1(a), and for all the diagrams of Fig. 1(b); however, the two α's for π and K exchange must be taken as different, if we want the K-exchange contribution to be cut more strongly than the π-exchange contribution. Furthermore, we may expect that the value of α chosen for pion exchange should not be very different from the value already obtained in nonstrange inelastic reactions.[19]

At this point one might object that the use of a multiplicative cutoff function may describe the damping for high Δ^2 adequately, but it certainly misses one of the most characteristic features of the peripheral reactions observed so far: namely, the angular correlations among the final particles, which were first extensively discussed by Gottfried and Jackson.[20] It is known that the absorption corrections cause considerable distortions to occur in the expected angular distributions for resonance decay[12,16]; this effect is lost if we lump them into the multiplicative function (2). This remark is certainly correct; however, in the present case the loss is not a serious one, because we cannot calculate the expected angular correlations anyway, even in the absence of absorption. As a matter of fact, by not having resonance production, we lose from the beginning an input information which is of invaluable help for the theoretical analysis: namely, the knowledge of the *spin and parity* of the system outgoing from the lower vertex in any of the diagrams of Fig. 1. As a matter of fact, we expect many spin-parity states to contribute with comparable magnitude, and to interfere strongly; therefore, we cannot either predict a definite pattern for the angular distribution of such a system, or determine the Jackson parameters,[20] because the off-shell corrections will be *a priori* different for each angular momentum. As a consequence, we have no reasons to believe that the angular distribution, say, in the lower vertex of the diagram in Fig. 1(a) is the same as the one experimentally observed in the reactions $\pi p \to YK$, for which the incident pion is real. In our opinion, the existing experimental comparisons of off-shell and on-shell angular distributions for the YK (or KN) system do not provide a valid test in favor or against the proposed

[18] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

[19] We can find from the existing calculations that such an "universality" of the cutoff holds approximately for different reactions involving pion exchange, like: $pp \to NN^*$ and $\pi p \to \rho p$ [see U. Amaldi and F. Selleri, Nuovo Cimento **31**, 360 (1964); J. D. Jackson, J. T. Donohue, K. Gottfried, B. Keyser, and B. E. Y. Svensson, Phys. Rev. **139**, B428 (1965)].

[20] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

mechanisms of exchange. The analogy with the case of resonance production is misleading.[21]

A simple way to realize that different off-shell corrections do indeed affect the different partial waves is the following. Whenever the exchanged, virtual boson and the initial nucleon interact in a state of angular momentum l, the matrix element contains a factor $q_{\rm off}{}^l$, where $q_{\rm off}$ is the three-momentum of the incident virtual boson in the boson-nucleon c.m.s. Since $q_{\rm off}$ is a strongly increasing function of Δ^2, the very presence of such a factor will induce drastic changes in the dependence of the off-shell matrix element for different Δ^2, by ***reducing*** the cutoff effect on the higher partial waves.

In order to conclude this section, we summarize our point of view. For effective three-particle final states, one can test experimentally the predictions of the model for the total cross sections, the Δ^2 distributions,[22] and in general all those spectra which do not depend critically on the assumed behavior of the off-shell angular distributions in the four-particle vertices of the peripheral diagrams. For such spectra, the introduction of a simple cutoff function of the type (2) is hoped to be adequate in order to describe the off-shell corrections. On the other hand, one cannot expect to predict the angular correlations and the hyperon polarizations until a more detailed theory of the off-shell effects is available. Nevertheless, one can hope to study certain particular angular correlations also in the present picture, the most significant example being the Treiman-Yang angle[23] which will be discussed later (Sec. 5).

4. COMPARISON WITH EXPERIMENT

As we have already stated, we have tried to fit the experimental data by retaining both pion and kaon exchange, with different cutoff parameters.[24] For the

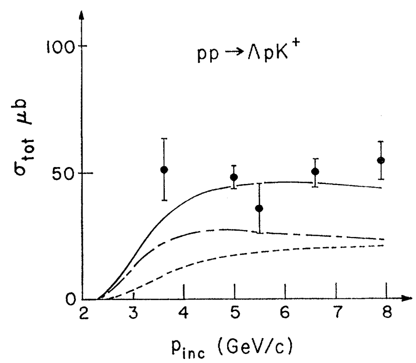

FIG. 3. Behavior of the cross section for the reaction $pp \to p\Lambda K^+$ as a function of the incident momentum. The experimental points are taken from Refs. 1, 4–7. The curves represent the predictions of the model, with the choice of parameters explained in the text. Dashed line: kaon exchange. Dot-dashed line: pion exchange. Full line: total contribution.

parameter α appearing in Eq. (1), we have chosen the values: $\alpha_\pi = 45m_\pi^2$ (0.88 GeV^2) for pion exchange; $\alpha_K = 11.2m_\pi^2$ (0.22 GeV^2) for kaon exchange. The value of α_π is not too different from the value $60m_\pi^2$ used in the reactions of pion production. The small value of α_K corresponds to a strong cutoff, as needed. Even if pion exchange were absent, a fit of the Δ^2 distributions in terms of pure K exchange would require a cutoff parameter $\alpha_K' = 19.5m_\pi^2$ (0.382 GeV^2).[25]

The cutoff functions used may not appear to be strongly decreasing with Δ^2 (even for K exchange), if we compare them with the exponential cutoffs en-

[21] We stress again that in the case of resonance production the situation is quite different. A resonance produced, say, in a pion-nucleon collision always decays the same way, according to its spin and parity, whether the incident pion be real or virtual.

[22] Unlike the case of quasi-two-body final states, here we have two characteristic momentum transfer variables (labelled $\Delta_{pp}{}^2$ and $\Delta^2{}_{\Lambda p}$ in some of the figures): namely, the momentum transfers between each of the final baryons and one of the initial protons (see Ref. 27 below for the treatment of the symmetry in the initial state). In the text, we shall use the general term Δ^2 to indicate the particular momentum transfer which is important in the study of any given spectrum (the reader can easily determine it; see also Ref. 27). We shall call Δ^2 distributions also the c.m. angular distributions of the baryons.

[23] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).

[24] We have taken the experimental information on the cross sections needed in this paper from the following sources: (a) For all channels $\pi^- p \to YK$; O. I. Dahl, L. M. Hardy, R. I. Hess, J. Kirz, D. H. Miller, and J. A. Schwartz, Phys. Rev. **163**, 1430 (1967). This paper also contains exhaustive references to all the previous work. (b) For $\pi^+ p \to \Sigma^+ K^+$: P. Daronian, A. Daudin, M. A. Jabiol, C. Lewin, C. Kochowski, B. Ghidini, S. Mongelli, and V. Picciarelli, Nuovo Cimento **41**, A503 (1966) (which contains detailed experimental bibliography), and the more recent high-energy data by J. Bartsch, L. Bondar, R. Speth, G. Hotop, G. Knies, F. Storim, J. M. Brownlee, N. N. Biswas, D. Lüers, N. Schmitz, R. Seeliger, and G. P. Wolf, *ibid.* **43**, A1010 (1966); R. R. Kofler, R. W. Hartung, and D. D. Reeder, Phys. Rev. **163**, 1479 (1967). (c) For K^+p elastic scattering: S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, T. F. Stubbs, G. M. Djerrou, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **9**, 135 (1962); T. F. Stubbs, H. Bradner, W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Slater, D. M. Stork, and H. K. Ticho, *ibid.* **7**, 188 (1961); V. Cook, D. Keefe, L. T. Kerth, P. G. Murphy, W. A. Wenzel, and T. F. Zipf, Phys. Rev. **129**, 2743 (1963); S. Focardi, A. Minguzzi-Ranzi, L. Monari, G. Saltini, P. Serra, T. A. Filippas, and V. P. Henri, Phys. Letters **24**, B314 (1967); A. Bettini, M. Cresti, S. Limentani, L. Peruzzo, R. Santangelo, D. Locke, D. J. Crennell, W. T. Davies, and P. B. Jones, *ibid.* **16**, 83 (1965); W. Chinowsky, G. Goldhaber, S. Goldhaber, T. O'Halloran, and B. Schwarzschild, Phys.Rev. **139**, B1411 (1965); J. Debaisieux, F. Grard, J. Heugebaert, L. Pape, R. Windmolders, R. George, Y. Goldschmidt-Clermont, V. P. Henri, D. W. G. Leith, G. R. Lynch, F. Muller, J. M. Perreau, G. Otter, and P. Sällström, Nuovo Cimento **43**, A142 (1966); W. DeBaere, J. Debaisieux, P. Dufour, F. Grard, J. Heugebaert, L. Pape, P. Peeters, F. Verbeure, R. Windmolders, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, A. Moisseev, F. Muller, J. M. Perreau, and V. Yarba, *ibid.* **45**, A885 (1966); M. Ferro-Luzzi, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, Calif., 1967), p. 183. (d) For K^+n charge exchange: W. Slater, D. H. Stork, H. K. Ticho, W. Lee, W. Chinowsky, G. Goldhaber, S. Goldhaber, and T. O'Halloran, Phys. Rev. Letters **7**, 378 (1961); I. Butterworth, J. L. Brown, G. Goldhaber, S. Goldhaber, A. A. Hirata, J. A. Kadyk, B. M. Schwarzschild, and G. H. Trilling, *ibid.* **15**, 734 (1965). (e) For K^+n elastic scattering, no data are practically available. The energy behavior of the cross section can only be guessed from the total cross section, the charge-exchange data, and the inelastic channels.

[25] The values chosen for the K-exchange cutoff parameters depend on the assumed value of the $\Lambda p K^+$ coupling constant. See Ref. 33 below for the discussion of this point.

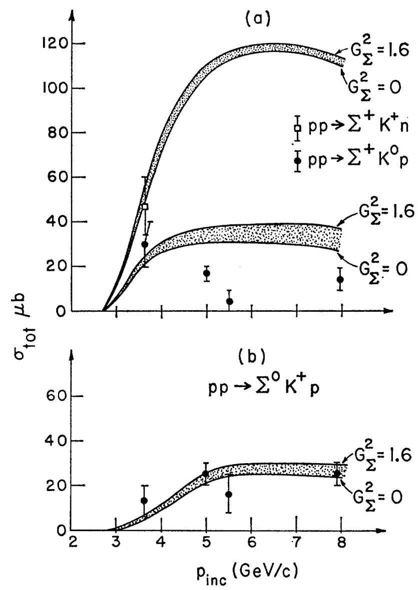

FIG. 4. Behavior of the cross section for the reactions (1) producing Σ's. The experimental points are taken from Refs. 1, 4, 5, and 7. The curves represent the predictions of the model, and are given almost completely by pion exchange. The additional contributions from kaon exchange (for the coupling constant G_Σ^2 less than 1.6) are represented by the shaded regions.

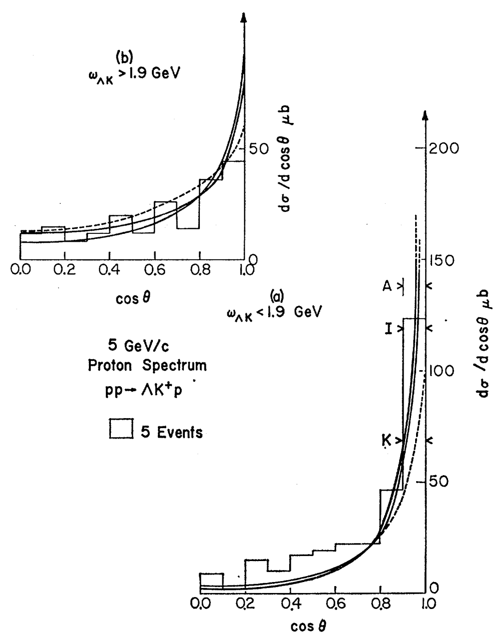

FIG. 5. Experimental c.m. angular distributions of the protons for the reaction $pp \to p\Lambda K^+$ at 5 GeV/c (from Ref. 4), with different selections of the ΛK mass $\omega_{\Lambda K}$. The solid curves represent the predictions of the model, assuming an admixture of pion and kaon exchange, with the choice of parameters explained in the text. The curve higher near $\cos\theta = 1$ and lower near $\cos\theta = 0$ is obtained by assuming the experimental (anisotropic) angular distribution for off-shell K^+p scattering; the other curve is obtained by assuming an isotropic distribution. The separate π- and K-exchange contributions are not plotted for reasons of clarity. The dashed curves correspond to pure K exchange, with anisotropic distribution and a different cutoff parameter (see end of Sec. 4). See text also for the additional indications A, I, and K on the histogram.

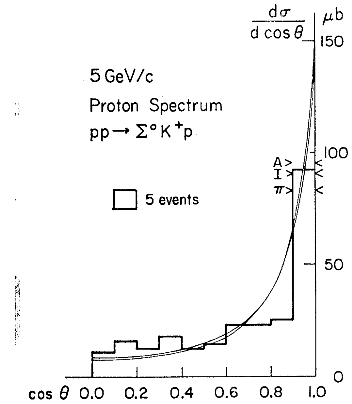

FIG. 6. The same as in Fig. 5 for the reaction $pp \to p\Sigma^0 K^+$ (all events) at 5 GeV/c (from Ref. 4). The anisotropic and isotropic curves are almost identical, due to the smallness of the K-exchange contribution. The contribution of π exchange alone in the interval $0.9 \leq \cos\theta \leq 1$ is also shown.

countered, e.g., in the applications of the absorption model. However, we remark that our function $F(\Delta^2)$ must also include the effect of the angular-momentum factors $q_{\text{off}}{}^l$, which accompany the off-shell lth partial wave for the reaction in the lower vertex. These factors do not appear in the absorptive corrections, because they are already present in the original unmodified Born-term contribution. Since such factors are strongly increasing with Δ^2, they tend to smoothen out the damping at high-momentum transfers. (It is not trivial to point out that, just because of such factors, the unmodified peripheral contributions from the Born terms are markedly different from the unmodified peripheral contributions from the Chew-Low formula; and so must be also the related cutoffs.[26])

Once we have chosen the values of the cutoff parameters, we can compare the predictions of the model with the experimental data, for a large class of distributions. We compare: the total cross sections (Figs. 3 and 4), the c.m. angular distributions of the final baryons, which are closely related to the distributions in Δ^2 (Figs. 5–8),[27] and the distributions in the in-

[26] The effect of the angular-momentum factors may be one of the causes why K exchange is cut off more strongly then π exchange. Indeed, at all energies, higher partial waves are present in the reactions $\pi p \to YK$ than in KN scattering [see Dahl *et al.*, Ref. 24 (a); Ferro-Luzzi, Ref. 24 (c)]. This implies a larger effect of the angular-momentum factors and then a slower decrease of the cutoff.

[27] More precisely: The c.m. distribution of the nucleon is related to the distribution in the momentum transfer between the final nucleon and one of the initial protons, which enters explicitly in the pion exchange formulas; the c.m. distribution of the hyperon is related to the distribution in the momentum transfer between the final hyperon and one of the initial protons, which enters explicitly in the kaon-exchange formulas. The symmetry in the c.m.s. because of the identity of the initial protons has been taken into account by "folding" the experimental and theoretical distributions around 90° in the c.m. system. Also the Δ^2 distribution of Fig. 8(a) is "folded" in an analogous way, by taking the *smaller* of the two momentum transfers between the given final particle and the two initial protons. When we speak of Δ^2 we always intend that such a selection is made.

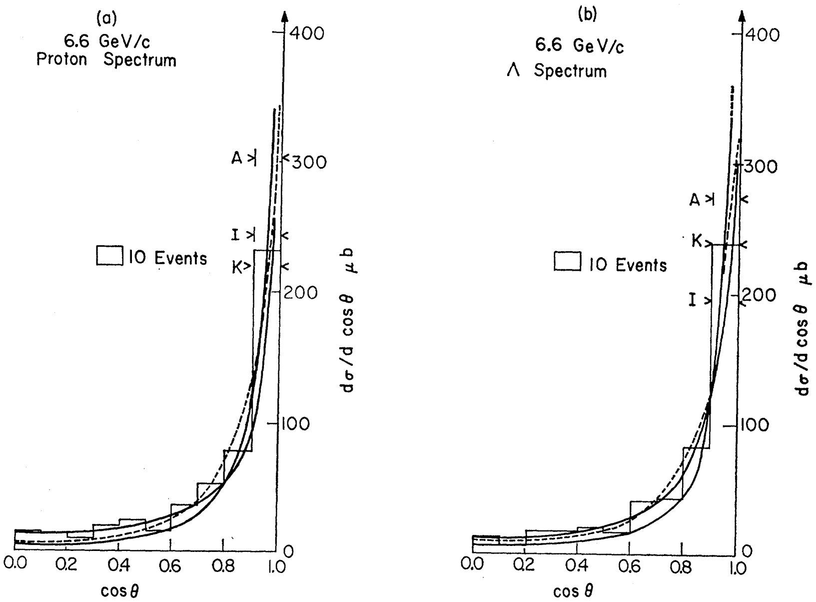

FIG. 7. The same as in Fig. 5 for the reaction $pp\rightarrow p\Lambda K^+$ (all events) at 6.6 GeV/*c*: (a) proton spectrum, (b) Λ spectrum. (Data taken from Ref. 6.)

variant masses $\omega_{\Lambda K}$, ω_{Kp} of the systems ΛK, Kp, respectively (Figs. 9–11). In the calculation of the theoretical spectra, we have retained all the experimental cuts (on Δ^2 or on the ω's) made by the experimentalists. We do not present here a fit to all the experimental spectra available, but only to what we believe to be a representative sample of them. However, the experimental information concerning the reactions of Σ production is rather scanty; we have only some total cross sections and the spectrum presented in Fig. 6.[28] For the reasons we have explained before, we have not attempted any fit to the ΛK (or Kp) angular distribution, or to the Λ polarization.

Let us now explain the meaning of the different theoretical curves presented in Figs. 5–11. Taking the final state ΛpK^+ for definiteness, and considering the pion-exchange contribution, if we want to calculate the spectrum of the final proton, or the distribution in the ΛK mass, we can apply the Chew-Low formula straightforwardly, and determine the spectrum unambiguously

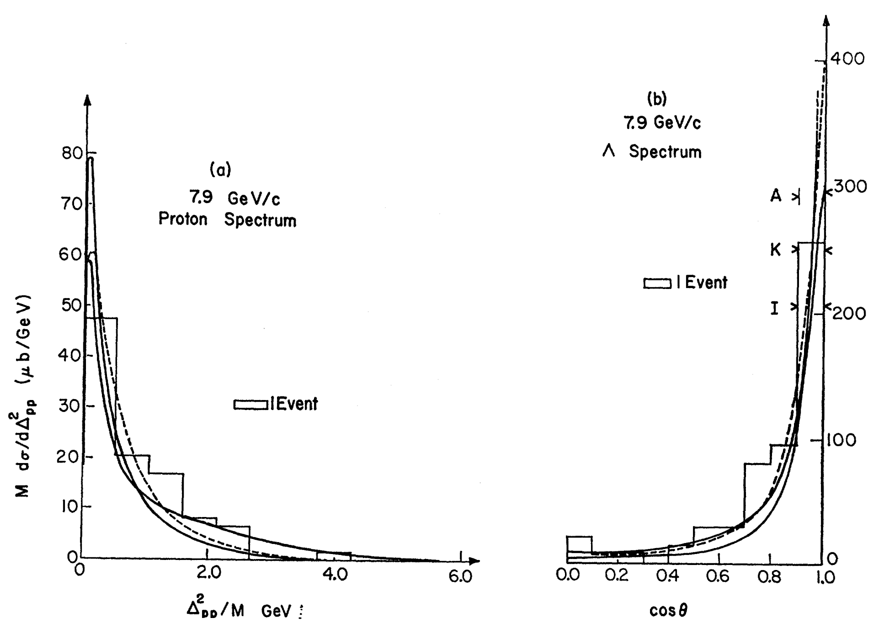

FIG. 8. (a) Experimental distribution in the momentum transfer Δ_{pp}^2 (folded as explained in Ref. 27) in the reaction $pp\rightarrow p\Lambda K^+$ (all events) at 7.9 GeV/*c* (from Ref. 7), compared with the predictions of the model (M=proton mass). The curves have the same meaning as in Fig. 5; the anisotropic case corresponds to the higher curve at low values of Δ_{pp}^2. (b) Experimental c.m. angular distribution of the Λ for the same reaction; for the meaning of the curves, see Fig. 5.

[28] This assumption might be a good one whenever diffraction scattering occurs (in our case, for K^+p scattering above 1.2 GeV/*c*). Indeed diffraction, by its very nature, can be thought to occur independently of whether the initial particle is real or virtual. However, absorptive corrections are likely to invalidate this argument.

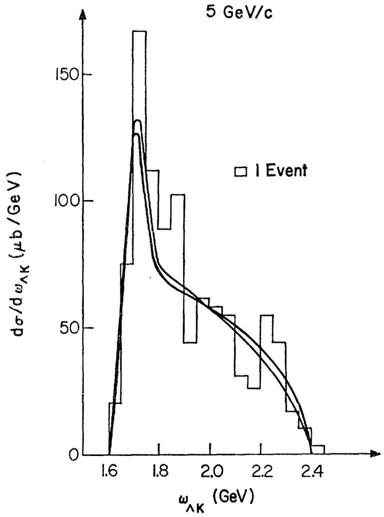

FIG. 9. Experimental distribution in the mass $\omega_{\Lambda K}$ of the ΛK system in the reaction $pp \rightarrow p\Lambda K^+$ (all events) at 5 GeV/c (from Ref. 4), compared with the predictions of the model. For the meaning of the curves, see text and Fig. 5. The anisotropic case corresponds to the higher curve at the peak.

once we have chosen the value of the cutoff. But, if we want to calculate the spectrum of the Λ, or the distribution in the Kp mass, the answer will obviously depend on the shape of the off-shell ΛK angular distribution, for which, as we pointed out, we have no sure criteria of evaluation. For K exchange the same thing happens, with the final baryons reversing their roles. So we need anyway an assumption about the off-shell angular distributions relevant to the reactions in the lower vertices of the diagrams of Fig. 1. The most straightforward thing to do is to use the experimental (on-shell) angular distributions, just because one has nothing better available.[29] However, if the results thus obtained are to make sense, we must make sure that they are rather insensitive to reasonable variations in the shape of the assumed angular distributions. We have tried to check this point by calculating all the spectra in question twice, first with the experimental angular distribution (called also *anisotropic*); and also with an isotropic angular distribution. It is likely that the correct results will lie somewhere between the two curves so obtained. The hypothesis of isotropy is certainly implausible in itself, because the forward peakings observed at high energies in the physical reactions are expected to be maintained also for virtual incident particles: but this choice represents a typical example of angular distribution drastically different from the experimental one. In all Figs. 5–11, the two solid curves refer to the two cases considered. As a rule, the anisotropic distribution favors the low-momentum transfers (cosθ close to 1) and the low invariant masses; in the diagrams of Fig. 10 the cut on Δ^2 produces a sensible difference in normalization between the two cases. However, this analysis shows the weak dependence of the spectra considered from the details of the angular distributions, and, in our opinion, confirms that the approximate treatment presented in this paper has physical meaning.

For an easier comparison of theory and experiment, we have integrated the theoretical curves over the first bin of the cosθ spectra (0.9–1.0) and shown the resulting average values, to be compared with the experimental heights of the histogram bin. The letters A and I obviously refer to the anisotropic and isotropic case, respectively.

A few words about the dashed curves (labelled K) appearing in Figs. 5–8. They have been calculated in the hypothesis of pure kaon exchange, with the experimental (diffractive) angular K^+p distribution and the

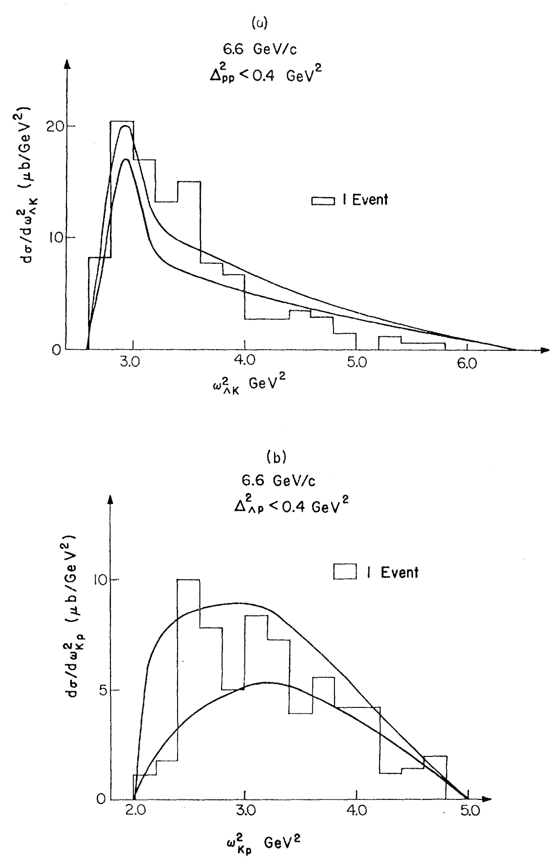

FIG. 10. Experimental distributions in the mass: (a) of the ΛK system, (b) of the Kp system, in the reaction $pp \rightarrow p\Lambda K^+$ at 6.6 GeV/c (from Ref. 6), compared with the predictions of the model. For the meaning of the curves, see text and Fig. 5. The anisotropic cases correspond to the higher curves.

[29] We have not fitted the Σ^0K^+ mass spectrum presented in Ref. 4, because its shape depends critically on the behavior of the low-energy cross section for $\pi^0p \rightarrow \Sigma^0K^+$, which is obtained as a combination of three experimental cross sections (See Ref. 9), and cannot be determined with sufficient accuracy. The same ambiguity affects the low-energy part of the total cross-section curve of Fig. 4(b) (up to about 4 GeV/c). For the high-energy part of this curve, and for the spectrum of Fig. 6, the low-energy part of the $\pi^0p \rightarrow \Sigma^0K^+$ cross section is fully integrated over energy, and the uncertainty on its shape has a much smaller effect on our results.

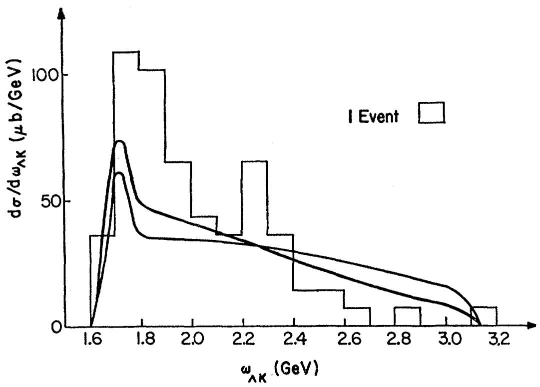

FIG. 11. The same as in Fig. 9 at 7.9 GeV/c (all events). Data taken from Ref. 7.

cutoff $\alpha_K'=19.5m_\pi^2$ mentioned earlier, which reproduces the ΛpK^+ total cross section. We do *not* present these curves as a possible fit, because there are many arguments pointing out that there must be an important contribution from pion exchange (see Sec. 5). We only want to show that, as far as the Δ^2 behavior is concerned, the results are not critical with respect to the choice of the cutoff parameters. We discuss this point in more detail in Sec. 5.

5. DISCUSSION

If we restrict ourselves to the comparison of the Δ^2 spectra (Figs. 5–8) we see that the model reproduces the data fairly well. The forward-backward peaks in the c.m.s. are mainly contributed by pion exchange; however, for the hyperon spectra [Figs. 7(b), 8(b)] the additional peaking provided by K exchange improves the agreement with the experiment. (The data showing the separate contributions of pion and kaon exchange to the forward peaks are reported in Table I for the c.m. angular spectra in the reaction $pp\to p\Lambda K^+$.) For the proton spectra, the addition of isotropic K exchange does generally fit the large momentum transfer region $|\cos\theta|<0.6$ better than anisotropic K exchange. We notice that in all the proton spectra the K-exchange contribution (especially if isotropic) plays the role of a background term, although not interpretable as phase space. The presence of such a term is needed if we want to obtain agreement with the experiment; pion exchange alone would predict too fast a decrease from low- to high-momentum transfers, especially for high-mass events [Fig. 5(b)]. However, the presence of such a background term might also receive a different explanation; for example, one could hope to reproduce the experimental data by assuming pion exchange plus Regge-pole background (e.g., through a picture like the one proposed by Chan *et al.*[30]). We have not investigated this point further; we merely state that our background term can be interpreted as a K-exchange term, without excluding other possibilities.

For the hyperon spectra [Figs. 7(b) and 8(b)], anisotropic pion exchange seems to be more satisfactory than isotropic pion exchange.[31] This is not surprising if we think the reactions $\pi p\to\Lambda K$ to occur through K^* exchange (Reggeized or not). However, the contribution of the K-exchange term to the forward-backward peak is significant. Any other acceptable explanation of the background term should reproduce this effect.

An interesting feature of our analysis is that, in all the Δ^2 spectra for the ΛpK^+ final state [with the exception of Fig. 5(a)], the best fit to the data is provided by the curves calculated under the hypothesis of diffractive K exchange alone, as explained at the end of Sec. 4. However, there are many arguments why this hypothesis cannot be accepted: namely, the clustering of events at low masses in the ΛK^+ system, which cannot be reproduced by pure K exchange, especially at the lower energies; the presence of a sizable Λ polarization[6] (K exchange predicts such a polarization to be zero); and, the most important of all, the analysis of the reactions leading to Σ production, which will be discussed later. We only like to point out that, as far as the Δ^2 distributions are concerned, for both pion- and kaon-exchange mechanisms the use of an adequate cutoff in Δ^2, plus the assumption of a strongly forward-peaked off-shell angular distribution makes the two momentum transfers to the final baryons *simultaneously small*, which is the feature experimentally observed. Therefore it is not surprising that, by taking the anisotropic off-shell angular distributions and choosing the cutoffs properly, one can "blend" the mechanisms of interaction of Fig. 1 in different proportions and still reproduce the gross features of the behavior in Δ^2. We conclude that, in the reaction $pp\to p\Lambda K^+$, the high-energy Δ^2 spectra, although quite characteristic, are not the best tools to detect the dominance of one or the other mechanism of interaction.

Considering now the invariant-mass distributions (Fig. 9–11), at the lower energies we have a fair agree-

TABLE I. Theoretical values of the "folded" c.m. angular cross section, averaged over the interval $0.9\leq\cos\theta\leq1$, predicted by π and K exchange diagrams separately (in μb).

Diagrams	5-GeV/c protons[a]	5-GeV/c protons[b]	6.6-GeV/c protons	Diagrams	6.6-GeV/c Λ	7-GeV/c Λ
π	106.4	40.4	205.5	π anis.	168.3	170.7
K anis.	30.6	21.7	97.4	π isot.	90.3	85.8
K isot.	14.5	16.4	41.4	K	104.9	120.0

[a] $\omega_{\Lambda K}<1.9$ GeV. [b] $\omega_{\Lambda K}>1.9$ GeV.

[30] H. M. Chan, K. Kajantie, and G. Ranft, Nuovo Cimento **49**, A157 (1967).

[31] This might indicate that, after all, the off-shell $\pi p\to\Lambda K$ angular distribution is not too different from the physical one, although one has no reasons of expecting it *a priori*. Additional evidence for this point may be provided by the experimental analysis of the ΛK angular distribution, made in Ref. 6.

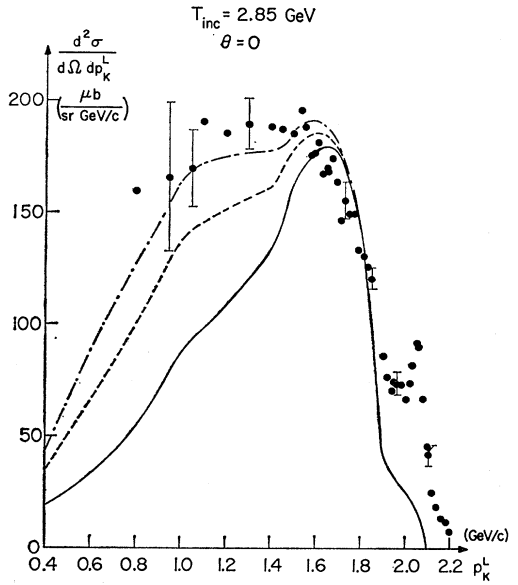

FIG. 12. Example of experimental K^+ laboratory momentum spectrum at fixed angle obtained in pp collisions at 2.85 GeV (Ref. 3), compared with the predictions of the model. The solid curve represents the theoretical contribution from three-body final states only (with isotropic off-shell angular distributions). The other curves include also a phase-space contribution from four-body final states, with a cross section of 16 μb (dashed curve) and 25 μb (dot-dashed curve), respectively. Typical statistical errors are shown for some of the experimental points.

ment between theory and experiment; at high energy, the agreement deteriorates. [For these spectra, a better agreement would be obtained by using a smoother cutoff for pion exchange (around 60–75m_π^2).] From Fig. 10(b), we see again that anisotropic pion exchange is preferred.[31] For K exchange, the influence of the assumption for the angular distribution is less marked.

Let us now discuss the behavior of the total cross sections (Figs. 3 and 4), which are the only data practically available for Σ production. The comparison of Λ and Σ production is particularly instructive. Indeed, the pion-exchange diagrams give comparable cross section for all YKN final states.[32] Instead, K exchange depresses Σ production with respect to Λ production, because the ΣpK coupling constant is likely to be much smaller than the ΛpK, and may even be consistent with zero.[33] This strong difference is not reflected in the reactions from pp collisions: the measured Σ channels have smaller production rates than the Λ channel, but not so small as should be expected from a dominant K exchange mechanism. On the other hand, pion exchange alone would predict roughly equal rates for Λ and Σ^0 production; the addition of K exchange, contributing practically only to the Λ, accounts for the observed difference in a straightforward way.[34]

Two points still remain to be cleared. The first one is the observed (and unexplained) suppression of the channel Σ^+K^0p at high energy. The other one is the predicted large cross section for the channel Σ^+K^+n; the study of this reaction may provide a significant test of the one-pion-exchange model.

We present now two additional pieces of comparison between theory and experiment, where the application of the model is more doubtful. One is the K^+ counter experiment at an incident kinetic energy of the protons of 2.85 GeV[3]. For strange particle production, this is quite a low energy, and the predictions of the one-boson-exchange model do not differ too much from phase space, especially for K exchange. Furthermore, the experimental spectra contain also the contributions from four-particle final states. However, we have attempted a fit to the observed energy spector at fixed angle, by summing the contributions predicted by the model for ΛK^+p, Σ^0K^+p, and Σ^+K^+n,[35] and a phase space distribution for the K^+'s from four-body processes (equally shared between Λ and Σ production). Figure 12 shows one example of the spectra obtained; the others can be found in the experimental paper.[3] The theoretical curve fits the high-energy tail of the spectrum fairly well,[36] where four-body processes are not kinematically allowed. In the low-momentum region, the statistical contribution from four-body reactions can account for the difference between the data and the one-boson-exchange curve. (The cross section for K^+ production in four-body final states, determined in Ref. 1, is 16±9 μb.) However, we should

[32] Except for Σ^+K^+n, for which the model predicts a larger production rate. However, this reaction has not yet been analyzed experimentally.

[33] There are two contradictory evaluations of the coupling constants ($G_{\Lambda pK^+}{}^2/4\pi=G_\Lambda{}^2$ and $G_{\Sigma^0pK^+}{}^2/4\pi=G_\Sigma{}^2$), both from KN dispersion relations. The first determination, by M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters **21**, 229 (1966), yields $G_\Lambda{}^2=4.8\pm1.0$, $G_\Sigma{}^2\leq3.2$. A more recent analysis by J. K. Kim [G. Goldhaber, in *Proceedings of the Rochester Theoretical Conference on Particles and Fields, 1967* (Interscience Publishers, Inc., New York, 1967), p. 57] uses a different theoretical treatment of the unphysical region for $\bar{K}N$ scattering, and obtains $G_\Lambda{}^2=16\pm2$, $G_\Sigma{}^2=0.3\pm0.5$. In our opinion, the strong dependence of the values obtained on the theoretical manipulations allows practically all values between 5 and 16 for $G_\Lambda{}^2$ and between 0 and 3 for $G_\Sigma{}^2$. However, there is a strong indication that $G_\Sigma{}^2$ must be considerably smaller than $G_\Lambda{}^2$, perhaps even by an order of magnitude. We have used the older values in our calculations, because they were the only ones available at that time: $G_\Lambda{}^2=4.8\pm1.0$, $G_\Sigma{}^2=1.6\pm1.6$. Therefore, all our K-exchange curves are affected by large errors, especially those for Σ production; however, in this case, the K-exchange contributions are small, so that the uncertainties are not too significant. Of course, if a larger value of $G_\Lambda{}^2$ were to be taken, one should correspondingly decrease the cutoff parameter α_K in order to maintain the agreement with the experiment.

[34] It should be pointed out that other possible background mechanisms are not likely to discriminate so strongly as K exchange between Λ and Σ production. Rejecting the K-exchange hypothesis would probably make it necessary to assume markedly different cutoffs for π exchange in Λ and Σ production.

[35] For K exchange, we have retained the interference term between the two diagrams with the initial protons interchanged, because at such a low energy this term is small but not completely negligible. We have assumed the off-shell K^+p interaction to occur in an s wave, as is the case for low-energy physical scattering.

[36] Except for the small peak due to the Λp interaction (Ref. 2), which obviously cannot be reproduced by our model.

keep in mind that both the theoretical curves and the experimental data are affected by large uncertainties.

Finally, we shall discuss the distributions in the Treiman-Yang (TY) angle for the reaction $pp \to p\Lambda K^+$. (We can define two independent TY angles; we consider the azimuthal angle of the Λ measured in the ΛK^+ c.m., for which the pion-exchange mechanism predicts an isotropic distribution.) The study of this type of angular correlation makes sense also in the present formulation of the model, because the isotropy requirement is a consequence not of the dynamics of the off-shell $\pi p \to \Lambda K$ reactions, but of the fact that the exchanged particle is spinless. It is true that the absorption corrections (one should call them more properly rescattering corrections) may alter the isotropy; however in most of the quasi-two-body reactions explained by pion exchange with absorption corrections, the induced anisotropy has proved to be small.[37] Therefore, also in our case one would expect substantial isotropy if pion exchange dominates. Deviations from isotropy can be contributed by K-exchange diagrams, which are isotropic in a different TY angle.[38] Some of the available experimental distributions are presented in Fig. 13, together with the predictions of our model.[39] We see that, in spite of the K-exchange contribution, the theoretical curves are practically still isotropic. The experimental data presented here do not seem to be consistent with isotropy. However, the data at different energies are also not consistent among themselves; in our opinion, further experimental analysis is needed in order to clarify the situation. If we accept the observed anisotropies as statistically significant, our simplified version of the model cannot account for the experimental behavior. The most natural explanation is that the rescattering corrections affect the TY angle more strongly here than in the quasi-two-body final states. A quantitative answer will be possible only when the absorption model will be extended to effective three-particle final states.

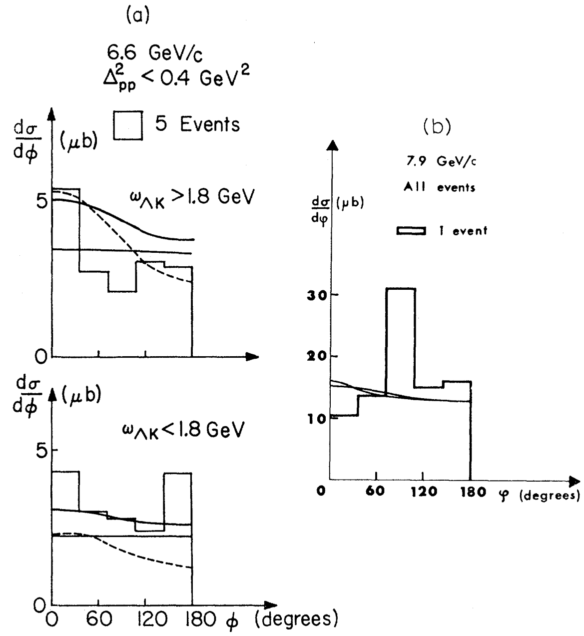

FIG. 13. Experimental distributions in the Treiman-Yang angle (defined as in Ref. 39), compared with the predictions of the model: (a) at 6.6 GeV/c (data taken from Ref. 6), (b) at 7.9 GeV/c (data taken from Ref. 7). In Part (a), the curves have the same meaning as in Fig. 5; the anisotropic case corresponds to the higher solid curves. In Part (b), the anisotropic and isotropic cases are practically indistinguishable.

6. CONCLUDING REMARKS

We have studied the reactions $pp \to YKN$ as a typical example of true three-partical final states in which peripheral interactions are expected to be important. Our ignorance of the rescattering effects and of the off-shell interaction for $\pi p \to YK$ and $KN \to KN$ has led us to adopt an empirical cutoff in order to describe the damping effects at large momentum transfers. A somewhat universal choice of cutoffs (different for pion and kaon exchange) has given a satisfactory description of the Δ^2 distributions and the mass distributions, for incident proton momenta between 3 and 8 GeV/c. In our opinion, the best interpretation of the data is obtained by assuming an admixture of elementary pion and kaon exchange; however, alternative interpretations, e.g., using Regge poles, cannot be ruled out. The admixture hypothesis seems to account fairly well for the relative magnitudes of Λ and Σ cross sections: a large cross section is predicted for the reaction $pp \to \Sigma^+K^+n$, not analyzed so far.

Some discrepancies are still present in the comparison of theory and experiment, and may easily be attributed to the approximate character of the model. This is particularly the case of the Treiman-Yang angular distributions. However, on the whole the simple picture presented by us can help the experimentalists in analyzing their data, and can be easily extended to other three-particle-final state reactions, for which a more rigorous theoretical treatment is not yet available.

Note added in proof. After the paper was accepted for publication, we became aware of another very interesting experimental investigation [W. Chinowsky, R. R. Kinsey, S. L. Klein, M. Mandelkern, J. Schultz, F. Martin, M. L. Perl, and T. H. Tan, Rept. UCRL 17619 (August, 1967)], where our reactions are studied in detail at 6 GeV/c. {An experimental investigation at

[37] See, e.g., the reactions $\pi p \to \rho p$, $\pi p \to \rho N_{33}^*$ (Jackson *et al.*, Ref. 19), where the parameters $\rho_{1,-1}$, Reρ_{10} (for the ρ meson), Reρ_{31}, Re$\rho_{3,-1}$ (for the N^*) are very close to zero, both theoretically and experimentally.

[38] For a detailed discussion of this effect, see E. Ferrari, Phys. Letters **2**, 66 (1962).

[39] Our definition of the TY angle (for the Λ) is the same as currently done in the experimental papers; accordingly, we have $\varphi = 0$ or π when all five particles are coplanar in the ΛK center-of-mass system. (We define $\varphi = 0$ when the momenta of the Λ and of both the incident protons lie in the same half-plane.)

the same energy has been performed also by I. J. Bloodworth [Oxford thesis, 1967 (unpublished)]}. Of particular interest are the values of the total cross sections for the Σ channels: $\sigma(\Sigma^+K^0p)=26\pm4$, $\sigma(\Sigma^0K^+p)=17_{-2}^{+4}$, $\sigma(\Sigma^+K^+n)=57\pm7$ (in μb). The latter value disagrees with our prediction by a factor of 2. However, the other two values tend to remove the difference in magnitude which seemed to exist between the cross sections for the corresponding channels. Thus the model is still working satisfactorily, provided a stronger cutoff ($\alpha_\pi\sim20$–$22m_\pi^2$) is chosen for pion exchange in the case of Σ production. However, this value of the cutoff cannot be used also for the Λ, as is apparent from the analysis performed in this paper. Therefore the model must contain three cutoff parameters instead of two. Note that the fits to the spectra presented in this work (which refer all to the Λ) are not affected by the above considerations. A more detailed discussion of Σ production will be given in a forthcoming paper.

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One of us (E. F.) thanks Professor Y. Nambu, for the kind hospitality in the theoretical group of The Enrico Fermi Institute for Nuclear Studies in Chicago, and for the use of the computer facilities. We acknowledge interesting discussions with Professor P. G. O. Freund and Professor J. D. Jackson, and with Dr. A. C. Melissinos and Dr. M. Firebaugh who communicated to us the results of their experiments prior to publication. We thank also Professor R. E. Marshak and Professor S. Okubo for their interest in this work.

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Muon Tridents

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Under the assumption that muons are heavy electrons, the total cross section for muon tridents on lead is calculated for 12.0-BeV incident muons including the effect of exchange for identical particles in the final state; various differential cross sections are presented. The positron spectrum for 31.5-MeV electron tridents on copper is also calculated and found to agree with Criegee's experimental results. It is found that the entire effect of statistics is confined to a region of phase space where the two leptons of like charge in the final state have an invariant mass of less than 3.5 times their rest mass.

THE muon is a very well measured[1] but very poorly understood particle. In every conceivable property observed to date, muons have behaved like heavy electrons.[2]

Yet, somehow, this is unreasonable. How can two particles, identical in all other respects, still manage to have unequal masses? There must exist a property, some *other* property, no matter how seemingly insignificant or obscure, that is different for muons and electrons, and as such gives rise to their difference in mass. Thirty-five years of searching for this elusive property have given not even a hint of success. This discourages some people, but spurs others on to look for still another unmeasured property, the one perhaps that might at long last be different.

One outstanding property that is very well known for electrons but not at all known for muons is their statistics.[3] States in which two identical electrons exist must be totally antisymmetric to the exchange of these particles. Although one fervently believes that this is also the case for muons,[4] there is a complete lack of experimental information on the subject because no one has yet been able to obtain a final state in which two identical muons were present.[5]

One reaction in which two identical muons can occur in the final state is the direct production of a muon pair by a muon in the field of a nucleus, or muon tridents. The trident process is also interesting from the point of view of checking quantum electrodynamics at small distances, so it has attracted considerable attention over the years.

The first calculation of direct pair production by electrons was performed almost immediately after the

* Alfred P. Sloan Research Fellow.
† Work partially supported by U. S. Atomic Energy Commission Contract No. AT(30-1) 2752.

[1] G. Feinberg and L. M. Lederman, Ann. Rev. Nucl. Sci. **13**, 431 (1963); see, also, F. J. M. Farley, Progr. Nucl. Phys. **9**, 259 (1964).

[2] New York Times, Jan. 22, 1961 (p. 33).

[3] W. Pauli, Z. Physik **31**, 765 (1925).

[4] G. F. Dell'Antonio, Ann. Phys. (N. Y.) **16**, 153 (1961); see, however, S. Kamefuchi and Y. Takahashi, Nucl. Phys. **36**, 177 (1962).

[5] See, however, R. O. Stenerson, Bull. Am. Phys. Soc. **12**, 31 (1967); and M. L. Morris and R. O. Stenerson, Nuovo Cimento (to be published).