Inelastic Scattering in the 2s-1d Shell. II. Odd-A Nuclei*

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The inelastic scattering of 17.5-MeV protons from F^{19}, Na^{23}, Mg^{25}, Al^{27}, and P^{31} has been studied with lithium-drifted silicon detectors with an over-all energy resolution of order 50 keV. Differential cross sections are presented for most of the low-lying levels. Using previously obtained data for the adjacent even-even nuclei, the results are compared with the predictions of strong- and weak-coupling models. The results are most consistent with strong coupling for F^{19}, Na^{23} and Mg^{25}, while Al^{27} and P^{31} seem to be more consistent with weak coupling, although extensive mixing is required in Al^{27}. The differential cross section for the most strongly excited levels are compared with a distorted-wave Born-approximation calculation using collective form factors. Reasonable fits for $l=2$ excitations in F^{11}, Al^{27}, and P^{31} and $l=3$ transitions in P^{31} are obtained, and the appropriate reduced transition strengths are extracted. Previously unreported levels are observed in Na^{23} (5.38, 5.76, and 5.94 MeV), Al^{27} (7.66, 7.79, and 7.99 MeV), and P^{31} (5.34 MeV).

I. INTRODUCTION

THIS is the second of two papers dealing with a study of the inelastic scattering of 17.5-MeV protons from nuclei in the lower reaches of the 2s-1d shell. The first paper[1] dealt with results obtained from even-A targets while this paper presents the results obtained from the odd-A targets F^{19}, Na^{23}, Mg^{25}, Al^{27}, and P^{31}. As it will be convenient to refer to the results of Ref. 1 throughout, it will be designated simply as I. The over-all purpose of these experiments is described in I; however, there are some important differences in the information obtained in the inelastic scattering from odd-A nuclei as compared to even-A nuclei. In the odd nucleus the angular momentum of the excited levels cannot be definitely determined, because even in the most favorable cases only the l value of the inelastic transition can be extracted. Thus the angular momentum of the excited state is determined only to within the vector sum of $\mathbf{l}+\mathbf{I}_0$ when I_0 is the ground-state spin. Further, the excitation of a level with spin-parity $I_f\pi_f$ from a ground state $I_0\pi_0$ can involve all multipole orders (l) of excitation consistent with $I_0+I_f+1\leq l \leq |I_0-I_f-1|$ *and* $\pi_f\pi_i=(-1)^l$. Even with these undesirable features the inelastic scattering from odd-A nuclei can be very informative and may in many cases be more useful than the scattering from even-A nuclei. This is because the extraction of any nuclear-structure information from even-A targets requires the use of an approximate theory for the reaction mechanism and the scattered waves. However, in the case of odd-A nuclei, quantitative model-independent statements about relative cross sections of particular levels are made which are relatively insensitive to details of the reaction mechanism and the distorted scattered waves. Thus gross prediction of nuclear models can be directly tested without recourse to uncertain details.

* Supported in part by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.
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[1] G. M. Crawley and G. T. Garvey, Phys. Rev. **160**, 981 (1967).

II. STRONG AND WEAK COUPLING IN ELASTIC SCATTERING

The simplest predictions of strong coupling (deformed potential) and weak coupling (time average spherical potential) are very clearly presented in a paper of Blair.[2] This paper develops the results within the framework of an adiabatic, Fraunhofer diffraction model. However, very similar results are obtained in the more realistic distorted-wave Born approximation[3] if it is assumed that the distorted waves are not strongly energy-dependent and are approximately the same for adjacent nuclei. The most important assumptions that the results to be presented below depend upon are that the inelastic transition only involve a single l value and that the interaction inducing the transition be only first order in the collective coordinates. Thus if the outgoing channels are strongly coupled, the conclusions will in general not be correct.

In the strong-coupling model the states of the target are written as

$$|IMK\rangle=\left[\frac{2I+1}{16\pi^2}\right]^{1/2}\Big\{\mathcal{D}_{M,K}{}^{I}(\theta_i)\sum_j c_j\chi_\Omega{}^j + \mathcal{D}_{M,-K}{}^{I}(\theta_i)\sum_j c_j(-1)^{I-j}\chi_{-\Omega}{}^j\Big\}, \quad (1)$$

where $\chi_\Omega{}^j$ is an intrinsic configuration with angular momentum j and projection Ω along the body-fixed symmetry axis. For the low-lying states of an odd-A deformed nucleus the coefficients c_j are in principle obtained from a Nilsson[4] model. With the assumptions mentioned in the previous paragraph this leads to an expression for the cross sections of the members of the

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[3] N. Austern, *Fast Neutron Physics*, edited by J. B. Marion and J. L. Fowler, (Interscience Publishers, Inc., New York, 1961), Vol. 2, Chap. 5; R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1962 (unpublished); R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).
[4] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 19 (1955).

ground-state band of the following form:

$$d\sigma/d\Omega(l,I_0K_0\rightarrow I_fK_0)=|\langle I_0l;K_00|I_fK_0\rangle Q_l(\theta)+\sum_j(-1)^{I_0-j}\langle I_0l;-K_02K_0|I_fK_0\rangle Q_{l,j}'(\theta)|^2, \quad (2)$$

where $\langle I_0lK_00|I_fK_0\rangle$ is a vector addition coefficient and $Q_l(\theta)$ is proportional to the scattering amplitude without change of the relative orientation of the intrinsic state relative to the body fixed axis, while $Q_l'(\theta)$ is proportional to the amplitude in which the intrinsic angular momentum is "flipped" relative to the deformed potential. In Ref. 2,

$$|Q_l(\theta)|=\left(\frac{d\sigma}{d\Omega}(0-2)\right)^{1/2}$$

(the square root of the cross section for the quadrupole transition within the ground-state band of the adjacent even-even nucleus). Clearly the second term in (2) may be neglected if $l<2K_0$ but for the case $l=2$ and $K=\frac{1}{2}$, which is often encountered, this term must, in principle, be retained. Transitions to levels in bands other than the ground-state band are governed by similar expressions within the framework of the model, but these transitions are so weak that their relative intensities are easily perturbed by small admixture in the wave functions.

In the weak-coupling model, the basis states are constructed from a particle or hole coupled to the 0^+ ground state of an adjacent even-even nucleus and this same particle or hole coupled to a collective vibration of the adjacent even-even nucleus. This latter coupling gives rise to a multiplet of states ranging in angular momentum from $l+j_p, l+j_p-1, \cdots$ to $|l-j_p|$, where l is the angular momentum of the vibration and j_p is the angular momentum of the particle or hole. In this model the relative cross sections for the levels within a particular weak-coupling multiplet are given by

$$\frac{d\sigma}{d\Omega}(l,I_0\rightarrow I_f)=\frac{2I_f+1}{(2l+1)(2j_p+1)}\frac{d\sigma}{d\Omega}(0\rightarrow l). \quad (3)$$

In this model $I_0=j_p$ and $(d\sigma/d\Omega)(0\rightarrow l)$ is the differential cross section associated with the collective l-pole vibration of the adjacent even-even nucleus.

Later in this paper results of distorted-wave calculations for the inelastic scattering will be presented. The relevant expressions necessary for understanding the analysis presented are given now. The inelastic cross sections are parametrized in terms of an equivalent deformation parameter β_l. In terms of this deformation an expression found in Ref. 3 for a $0\rightarrow l$ cross section in an even-even nucleus is of the form

$$\frac{d\sigma}{d\Omega}=\frac{\beta_l^2R_0^2V_0^2}{a^2}\frac{\sigma_{lsj}(\theta)}{5.093\times10^3}\text{ mb.} \quad (4)$$

V_0 is the depth of the Woods-Saxon well, of radius R_0 and diffusivity a, which characterizes the elastic scattering. $\sigma_{lsj}(\theta)$ is a partial differential cross section computed by a distorted-wave program.[3] In the case of odd-A nuclei the above term is appropriately modified by the factors appearing in Eqs. (2) and (3) in order to extract the relevant value of β_l. Only in the case $I_0=\frac{1}{2}$, and $Q'(\theta)=0$ do Eqs. (2) and (3) give the same statistical factor, thus in other cases the extracted value of β_l is somewhat model-dependent. The relationship between the usual parameters of strong and weak coupling is

$$\beta_l=(2l+1)h\omega_l/2C_l, \quad (5)$$

where $h\omega_l$ is the centroid energy of the vibration and C_l is the surface tension associated with the vibration.

III. EXPERIMENTAL PROCEDURE

These (p,p') experiments were carried out using an energy analyzed 17.5-MeV beam from the Princeton University F.M. cyclotron. Lithium-drifted solid-state detectors were employed and an over-all energy resolution of less than 50 keV could be obtained. The details of the experimental procedure employed are described in I and so will not be presented here apart from those features that are particular to these nuclei. The Al^{27} and Mg^{25} targets were self-supporting foils of approximately 1 mg/cm^2 thickness. The Mg^{25} foil however was heavily oxidized and quite nonuniform. The nonuniformity increased the energy resolution to 65 keV in that case and made the use of a monitor counter essential to achieve proper normalization from angle to angle. The P^{31} target was approximately 300 μg/cm^2 and was evaporated from a stainless-steel boat onto a glass slide and then floated off in water. The target used for studying F^{19} was $\frac{1}{4}$-mil Teflon sheet. The Na targets were prepared by evaporating Na metal in vacuum onto a glass slide. The slide was prepared by dipping it into a solution of hexadecylamine in hexane, and then allowing the hexane to evaporate from the plate, leaving a coating of hexadecylamine. After about 1 mg/cm^2 of Na had been evaporated onto the slide, it was moved to a new position and a thin coating of paraffin wax was evaporated on top of the Na. The evaporation chamber was then brought to atmospheric pressure with argon gas. The slide was removed and placed under mineral oil for storage. Just previous to the foil being used the slide was removed from the mineral oil and placed in a shallow vessel containing reagent-grade hexane. The hexane dissolves both the wax and the substrata of hexadecylamine, allowing the Na foil to become free. As the foil is more dense than hexane it sinks and is protected from the air. The foil is then mounted on the target holder while under hexane. In the meantime the scattering chamber is filled with argon; the argon plus the hexane wetting of the target protects it during the chamber pump down. With this technique Na foils were prepared with less than 8% (by weight) oxygen contamination.

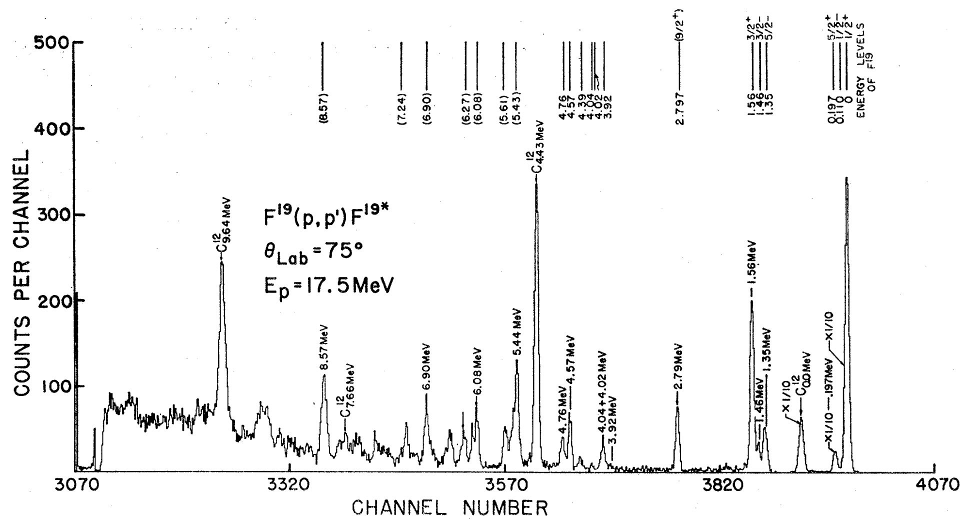


FIG. 1. The spectrum of 17.5-MeV incident protons scattered at 75° from a Teflon foil. The energies above 5 MeV appearing on the energy-level diagram are those assigned to prominent peaks observed at this and other angles.

IV. EXPERIMENTAL RESULTS

F^{19}

The spectrum of 17.5-MeV protons scattered from a Teflon $[(CF_2)_n]$ target at 75° laboratory angle is shown in Fig. 1. The accompanying level diagram is taken from the compilation of Ajzenberg and Lauritzen[5] and summaries[6] appearing in more recent work. The inelastic scattering from F^{19} has been studied with lower-energy projectiles, 13.9-MeV protons[7] and 14-MeV neutrons.[8] The energy resolution of the neutron scattering experiment is not sufficiently good as to render a direct comparison with this data useful. The 13.9-MeV proton scattering appears to yield relative intensities to all low-lying levels in good accord with those observed here. This nucleus has also been studied with 150-MeV protons[9] but the beam quality was not sufficiently good that differential cross sections for single levels could be unambiguously measured.

Important features to note in the spectrum shown are the large cross sections for the excitation of the $\frac{5}{2}^+$ level at 0.197 MeV and the $\frac{3}{2}^+$ level at 1.56 MeV. In a collective rotational model of F^{19} these should be the most strongly excited levels as they are the only members of the ground-state band $(K=\frac{1}{2})$ directly excited with an $l=2$ angular-momentum transfer. Interestingly, none of the low-lying negative parity states are observed to have appreciable cross sections. They are all less than 1 mb at all angles observed. This is a surprising result as Coulomb excitation studies on F^{19} by Litherland[10] *et al.* concluded that the reduced octupole transition strength between the ground state and the 1.35-MeV $\frac{5}{2}^-$ state is 12 ± 4 single-particle units.[11] The similarity of the matrix elements involved in electric radiative transitions and inelastic scattering[12] usually leads to a direct correspondence between enhanced electromagnetic transitions and large inelastic cross sections. However, a few exceptions[13,14] to this rule are known and understood. If the Coulomb excitation results are correct the discrepancy with this experiment is not readily explainable.

The measured differential cross sections in F^{19} are shown in Figs. 2 and 3 for the levels most pertinent to this study. In the strong-coupling model, the one believed appropriate to F^{19}, the predicted ratio of the cross sections for the $J=\frac{5}{2}$ and $\frac{3}{2}$ members of the ground band is

$$\frac{(d\sigma/d\Omega)(l=2,\frac{1}{2}\rightarrow\frac{5}{2})}{(d\sigma/d\Omega)(l=2,\frac{1}{2}\rightarrow\frac{3}{2})}=\frac{(\sqrt{3}Q+\sqrt{2}Q')^2}{(\sqrt{2}Q+\sqrt{3}Q')^2}.$$

Thus if the magnitude of the second term on the right is comparable with the first term, it will have an appreciable effect on the relative cross sections. If this second term is negligibly small then the predicted ratio is $\frac{3}{2}$, which is very near the observed ratio of 1.61 ± 0.08, based on the integrated cross sections from 50°–112°. This requires $Q'/Q=-0.08_{-0.08}^{+0.06}$. An argument as to

[5] T. Lauritzen and F. Ajzenberg-Selove, in *Nuclear Data Sheets*, compiled by K. Way *et al.* (U. S. Government Printing Office, National Academy of Science—National Research Council, Washington, D. C.), NRC60-5.6-229.

[6] J. W. Olness and D. H. Wilkinson, Phys. Rev. **141**, 966 (1966).

[7] H. F. Lutz, J. J. Wesolowski, L. F. Hansen, and S. F. Eccles, Phys. Letters **20**, 410 (1966).

[8] G. C. Bonazzola, E. Chiavassa, and T. Bressani, Nucl. Phys. **86**, 378 (1966).

[9] D. Newton, A. Clegg, and G. L. Salmon, Nucl. Phys. **55**, 353 (1964).

[10] A. E. Litherland, M. A. Clark, and C. Broude, Phys. Letters 3, 204 (1963).

[11] D. H. Wilkinson, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, pp. 852-889.

[12] W. T. Pinkston and G. R. Satchler, *Proceedings of the International Conference on Nuclear Structure, Kingston 1960*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 394.

[13] G. T. Garvey, G. M. Crawley, H. O. Funsten, N. R. Roberson, and L. Zamick, Argonne National Laboratory Report No. ANL-6848 (unpublished).

[14] G. R. Satchler, J. L. Yntema, and H. W. Broeck, Phys. Letters **12**, 55 (1964).

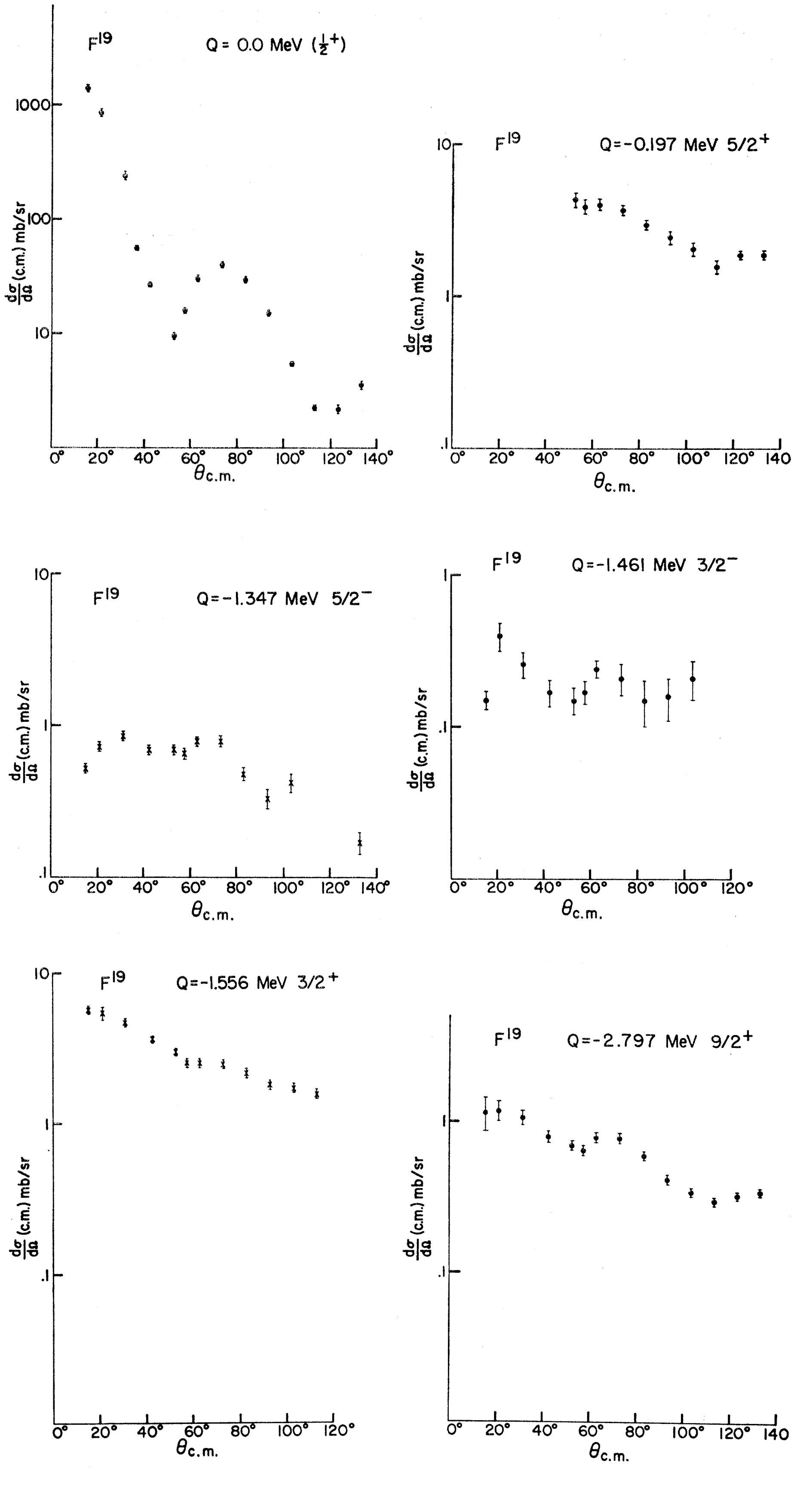


FIG. 2. Angular distribution for levels excited in F^{19} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

why $Q'(\theta)$ is small in this case has been offered.[9] The asymptotic Nilsson[15] wave function $[Nn\Lambda]$ for this intrinsic state is $[220]$ so the $K=\frac{1}{2}$ projection of the total angular momentum is due only to the spin projection and thus the $K=-\frac{1}{2}$ configuration has the nucleon spin oppositely oriented with respect to the symmetry axis. Therefore the $Q'(\theta)$ transition involves a spin flip, which is known to be weak.[16] In the model suggested by Blair[2] the Ne^{20}, $0+\rightarrow 2+$, (1.63 MeV) cross section should be substituted for $(d\sigma/d\Omega)(0\rightarrow 2)$ in Eq. (2) for this case. However, the angular distributions for the F^{19} ground-state band members and the 1.63-MeV level in Ne^{20} are not similar and the magnitude of the cross section for the 1.56 MeV at forward angles (20°–60°) is 20–30% smaller than would be predicted using the Ne^{20} cross section.[17] If the 2.797-

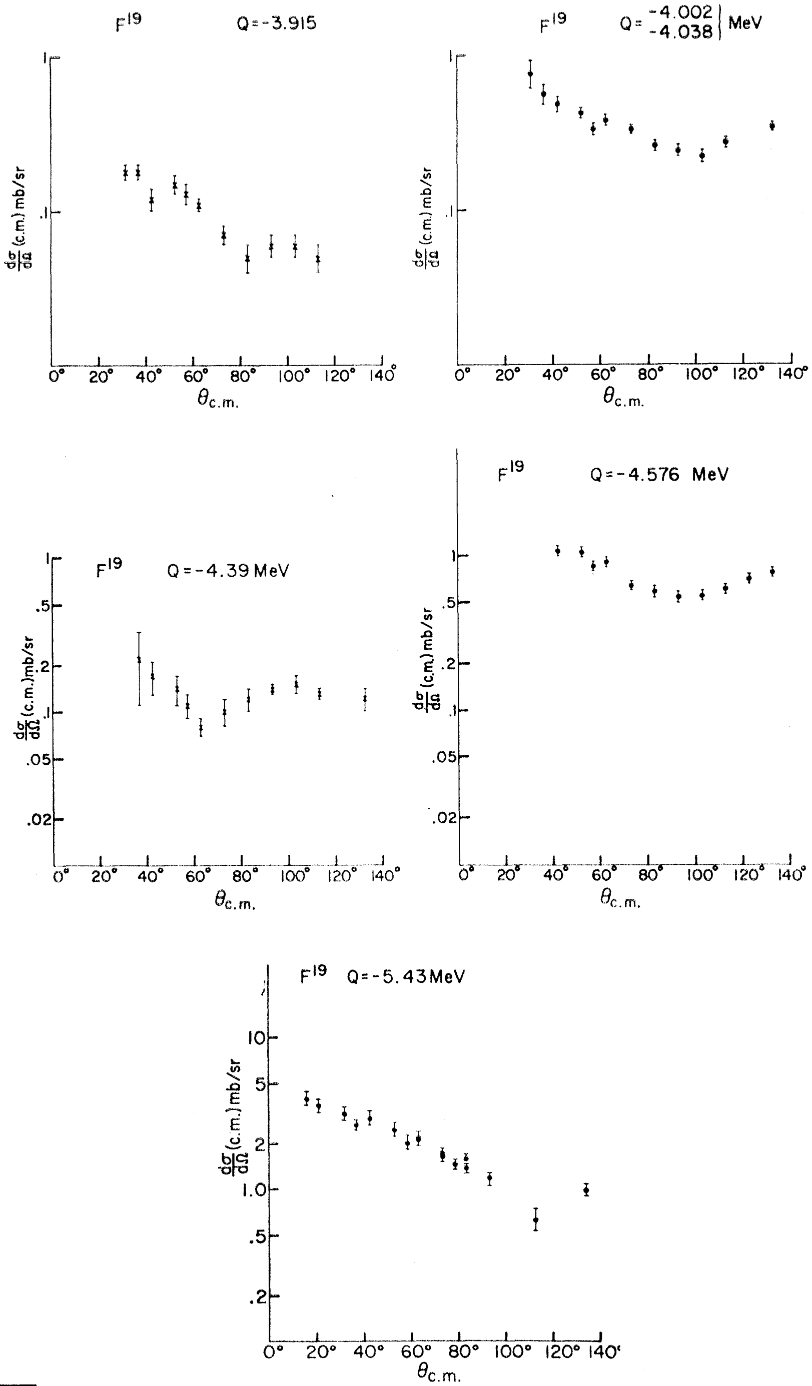


FIG. 3. Angular distribution for levels excited in F^{19} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

[15] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Skrifter, 1, No. 8 (1959).

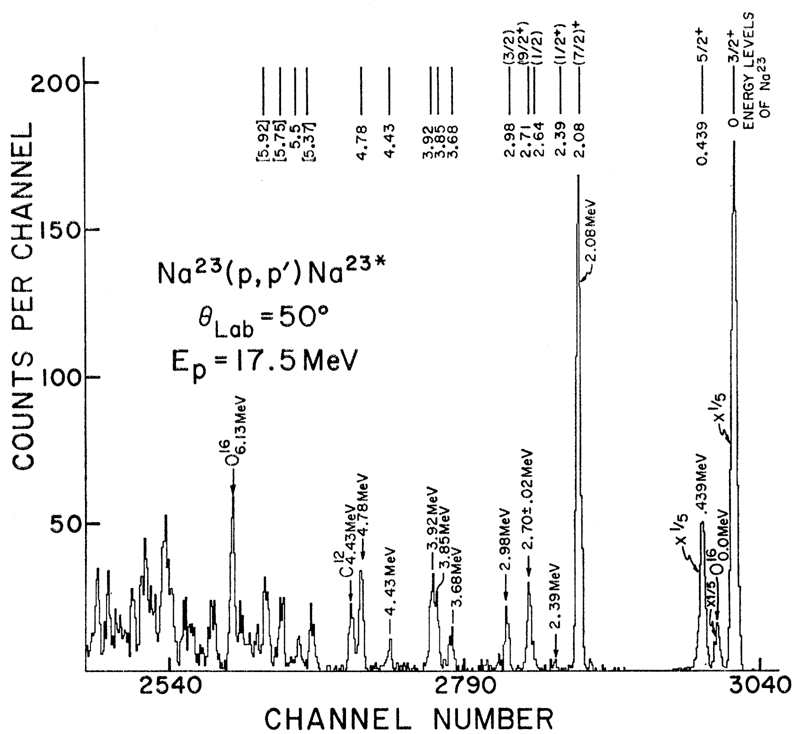


FIG. 4. The spectrum of 17.5-MeV incident proton scattered at 50° from an evaporated Na foil. The levels shown at 5.37, 5.75, and 5.92 are previously unobserved levels in this nucleus.

MeV state is assigned $J=\frac{9}{2}$, $K=\frac{1}{2}$ and is used in Eq. (2) to predict the $l=4$ differential cross section for the $J=\frac{7}{2}$, $K=\frac{1}{2}$ level either the 4.00- or 4.57-MeV levels are acceptable candidates.

A large cross section is observed at an excitation energy of 5.430±0.02 MeV, which does not coincide with any of the levels listed for F^{19}.[5] Actually the excitation energy seems to agree better with a level at 5.420±0.010 MeV at forward angles and 5.440±0.010 MeV at backward angles. Perhaps there is a doublet at this energy in F^{19} with a separation the order of 20 keV. However, the angular distribution for this "level" at 5.43±0.02 MeV looks very similar to the $l=2$ distribution observed for $\frac{5}{2}+$ state at 0.197 MeV and the $\frac{3}{2}+$ state at 1.56 MeV. The F^{19} cross sections are normalized relative to the C^{12} cross sections.[18]

Na^{23}

The spectrum of 17.5-MeV protons scattered from Na^{23} at 50° in the laboratory is shown in Fig. 4. The associated level diagram is taken from the compilation of Endt and Van der Leun.[19] The spins and parities of the low-lying levels are taken from a recent article by Polletti and Start.[20] The success of the target-making technique discussed above is illustrated by the relatively low yields from C^{12} and O^{16}. The yields at 5.38, 5.76, and 5.94 MeV are from previously unreported levels in Na^{23}. The angular distributions for all levels up to 5.5 MeV are shown in Figs. 5 and 6. The levels at 2.71 and 2.64 MeV are not clearly resolved; however, the mean energy for this doublet is 2.70±0.02, indicating that the level at 2.64 MeV must be very weakly excited. A level at 2.768 MeV in Mg^{23} has been observed in the pickup of an $l=1$ neutron in $Mg^{24}(p,d)Mg^{23}$ [21] and $Mg^{24}(He^3,\alpha)$ Mg^{23} reactions.[22] It is not clear which Na^{23} level this $\frac{1}{2}-$ or $\frac{3}{2}-$ state corresponds to. Reference 22 assigns this level as the 2.64-MeV level, the fact that this level is a hole state is quite consistent with it having greater excitation energy in Mg^{23} than it does in Na^{23}. The shape of the 2.391 angular distribution is uniquely characteristic of $l=0$; however, it appears from the d,n experiment[23] mentioned above that this level has $J\pi=\frac{1}{2}+$. If this level is associated with the next Nilsson orbital (No. 9) then in the asymptotic limit of the Nilsson model both levels have the same $[Nn_Z\Lambda]$. They are therefore related by spin flip. The small cross section observed in this case is then consistent with the explanation given for the small size of $Q'(\theta)$ in F^{19}.

Though the observed energies of the lowest lying $\frac{5}{2}+$ and $\frac{7}{2}+$ are quite different from what would be expected in a pure rotational model it is still believed that these states are relatively pure $K=\frac{3}{2}$ levels with smaller admixtures from $K=\frac{1}{2}$ and $\frac{5}{2}$ bands. The mixing alters

[16] J. M. Soper, *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 565.
[17] G. Schrank, E. K. Warburton, and W. W. Daehnick, Phys. Rev. **127**, 2159 (1962).
[18] W. W. Daehnick and R. Sherr, Phys. Rev. **133**, B934 (1964).
[19] P. N. Endt and C. Van der Leun, Nucl. Phys. **34**, 1 (1962).
[20] A. R. Poletti and D. F. H. Start, Phys. Rev. **147**, 800 (1966).
[21] E. Kashy and R. L. Kojub (private communication).
[22] J. Dubois and L. G. Earwaker, Phys. Rev. **160**, 925 (1967).
[23] E. B. Paul and J. H. Montague, Nucl. Phys. **54**, 497 (1964).

the energy spacings but the wave functions are still similar to those of the axially symmetric rotational model. In this model the ratio of the cross section for the $\frac{7}{2}+$ state to that of the $\frac{5}{2}+$ state is

$$\frac{\langle \frac{3}{2}\, 2\, \frac{3}{2}\, 0 \,|\, \frac{7}{2}\, \frac{3}{2}\rangle^2}{\langle \frac{3}{2}\, 2\, \frac{3}{2}\, 0 \,|\, \frac{5}{2}\, \frac{3}{2}\rangle^2}=\frac{9}{5}\,.$$

Taking the integral of the cross sections between 15.8° and 98° the experimental ratio is found to be 1.72 ± 0.08 in excellent agreement with the model prediction. The differential cross section has a shape quite different from that observed[1] for the $0^+ \rightarrow 2^+$, 1.37-MeV level in Mg^{24} so that direct comparison of magnitudes is difficult. If integrated cross sections between 15° and 90° are compared it is found that the Na^{23}, 0.439 and 2.08-MeV states are some 20% smaller than would be obtained

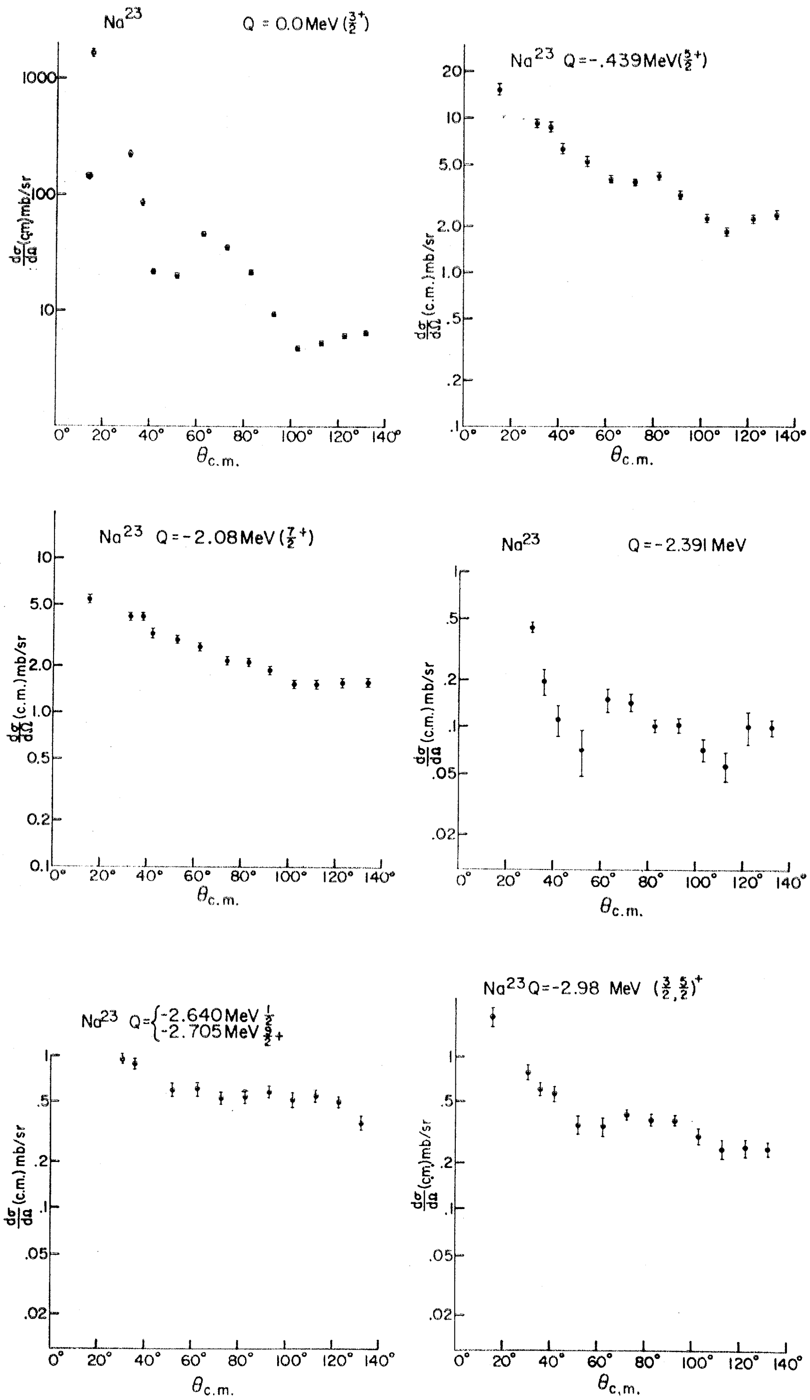


FIG. 5. Angular distribution for levels excited in Na^{23} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

substituting the Mg^{24} cross section into Eq. (2). The Na^{23} cross sections were normalized using a thin sliver of Corning glass No. 0.0088, which has the highest Na content of readily available glasses. The elemental composition of the glass is known to 0.2%. The Na elastic peak is readily separated from all constituents of the glass except Mg^{24}, whose cross section is well known. Thus the Na^{23} cross section is obtained by comparing the Na plus Mg^{24} cross section to the well-known O^{16} cross section.[24]

Mg^{25}

A spectrum of protons scattered from Mg^{25} is shown in Fig. 7. The energy-level diagram is taken from the 1962 compilation[19] and the more recent work of Cujec.[25] The energy levels above 5.01 MeV which show up

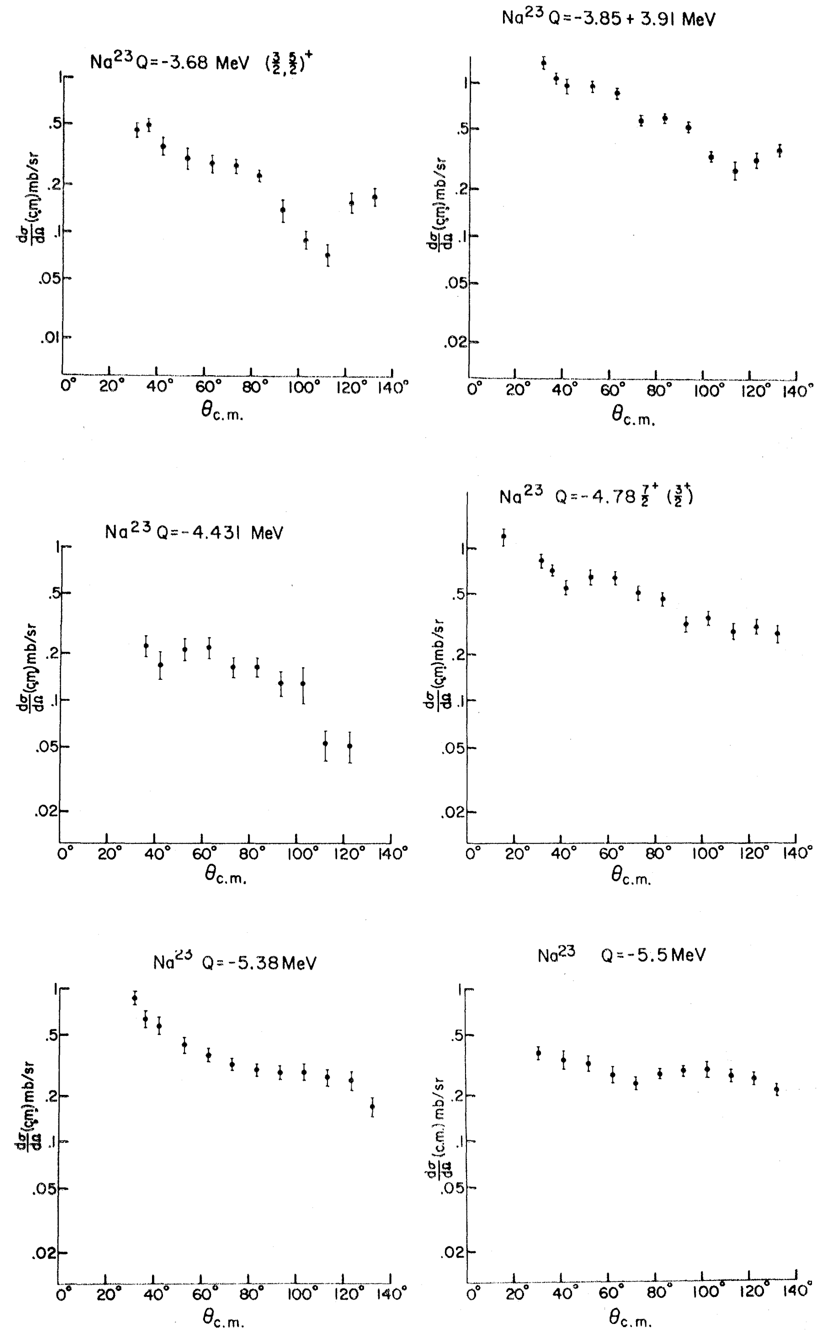


FIG. 6. Angular distribution for levels excited in Na^{23} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

[24] W. W. Daehnick, Phys. Rev. **135**, B1168 (1964).
[25] B. Cujec, Phys. Rev. **136**, B1305 (1965).

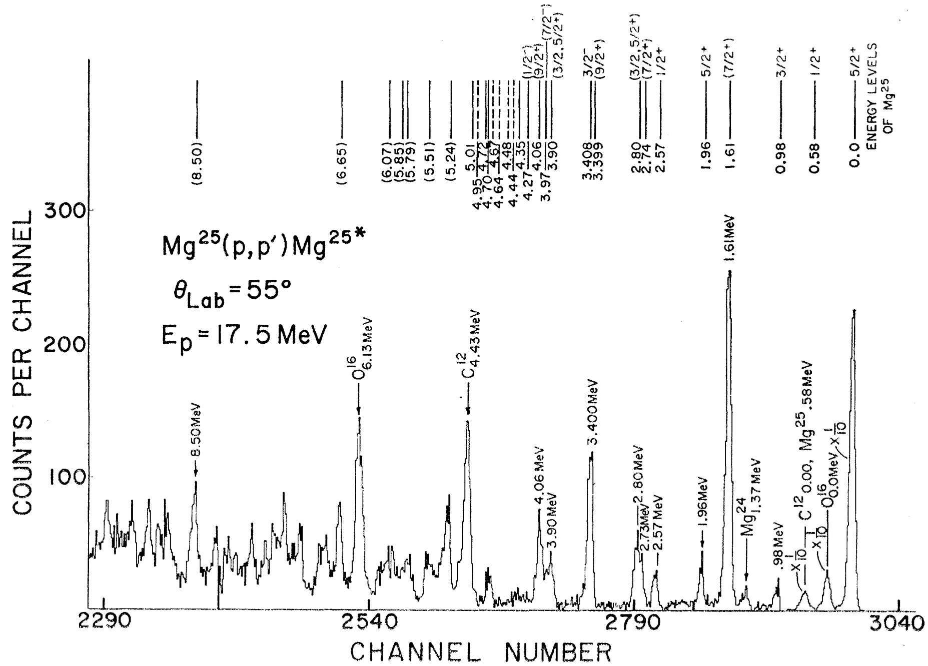


FIG. 7. The spectrum of 17.5-MeV incident protons scattered at 55° from a Mg^{25} foil.

strongly in this reaction are noted in brackets. Inelastic scattering from Mg^{25} has been studied with protons,[17] deuterons,[26] and α particles[27] and a preliminary report of the results presented here has been published.[28] The relative intensities observed in all cases are very similar. Figures 8 and 9 show the differential cross sections measured for this nucleus. As was observed for Na^{23} the largest yields are to members of the ground-state band ($K=\frac{5}{2}$) which can be reached with an $l=2$ transition ($J=\frac{7}{2},\frac{9}{2}$). The $\frac{7}{2}+$ level is at 1.611 while the $\frac{9}{2}+$ level is part of a doublet at 3.400 MeV, the other member of the doublet seemingly weakly excited. The angular distributions for these two members of the ground-state band are quite similar to one another but quite different from the collective $0^+\rightarrow 2^+$ differential cross section observed in $Mg^{24}(p,p')Mg^{24*}$. In fact these distributions are quite suggestive of the $0^+\rightarrow 2^+$, $Q=-1.805$-MeV angular distribution observed in Mg^{26}.[1] As was pointed out in I there does not seem to be a simple characteristic $l=2$ shape so that similarity of the shape of differential cross section implies additionally a similarity of form factor.

In a strict rotational model of the differential cross sections of the $\frac{7}{2}+$ and $\frac{9}{2}+$ members of the ground-state band is given by Eq. (2) as

$$\frac{\langle \frac{5}{2}\,2\,\frac{5}{2}\,0\,|\,\frac{7}{2}\,\frac{5}{2}\rangle^2}{\langle \frac{5}{2}\,2\,\frac{5}{2}\,0\,|\,\frac{9}{2}\,\frac{5}{2}\rangle^2}=\frac{20}{7}\,.$$

The observed ratio for the total cross section over the range of angles investigated is 2.4 ± 0.15, which probably reflects some small contribution to the yield attributed to the 3.399-MeV $\frac{9}{2}+$ level from the nearby 3.408-MeV level. Using 40-MeV α particles and comparing the maximum in the differential cross sections for these same two levels a ratio of 2.86 was obtained[27] in excellent agreement with the prediction of strong coupling.

Al^{27}

Figure 10 shows the spectrum of 17.5-MeV protons scattered from an Al target at an angle of 90° to the incident beam. The energy-level diagram is taken from Ref. 19 with additions from the more recent literature.[29–31] The spin-parity assignment of $\frac{9}{2}+$ to the 3.00-MeV level was the most commonly accepted one. Some evidence[32] was offered that it might be $\frac{7}{2}+$, (however, some recent experiments[31] confirm that the spin of this level is in fact $\frac{9}{2}+$). Table I lists the energy of the levels observed and their cross section at 30° in the laboratory. In the region 6.9 to 8.2 MeV, where no energy levels are given in Ref. 19, five levels, shown with asterisks, have been observed in this experiment; the three higher ones do not correspond to previously observed levels. Angular distributions for the lower-lying levels are presented in Figs. 11 and 12. The in-

[26] A. G. Blair and E. W. Hamburger, Phys. Rev. **122**, 566 (1961).
[27] J. S. Blair, Argonne National Laboratory Report No. ANL-6878 (unpublished).
[28] G. M. Crawley and G. T. Garvey, Phys. Letters **19**, 228 (1965).

[29] T. Ophel and B. T. Lawergren, Nucl. Phys. **52**, 417 (1964).
[30] C. Van der Leun, D. M. Sheppard, and P. M. Endt, Nucl. Phys. **A100**, 316 (1967).
[31] D. M. Sheppard and C. Van der Leun, Nucl. Phys. **A100**, 333 (1967).
[32] T. Wakatsuki and B. D. Kun, *Nuclear Spin-Parity Assignments*, edited by N. B. Gove (Academic Press Inc., New York, 1966), p. 186.

elastic scattering of deuterons[33] and α particles[27,34] has been performed with results similar to those observed here for the low-lying levels. In Ref. 28 a preliminary version of this study was presented; the only modification to those results is the increase of the Si and Al cross sections by approximately 15%. This does not change any conclusions drawn in Ref. 28.

The 45-keV energy resolution is not sufficient to allow a separation of the yields of the 3.000- and 2.976-MeV levels. However the average energy found for the posi-

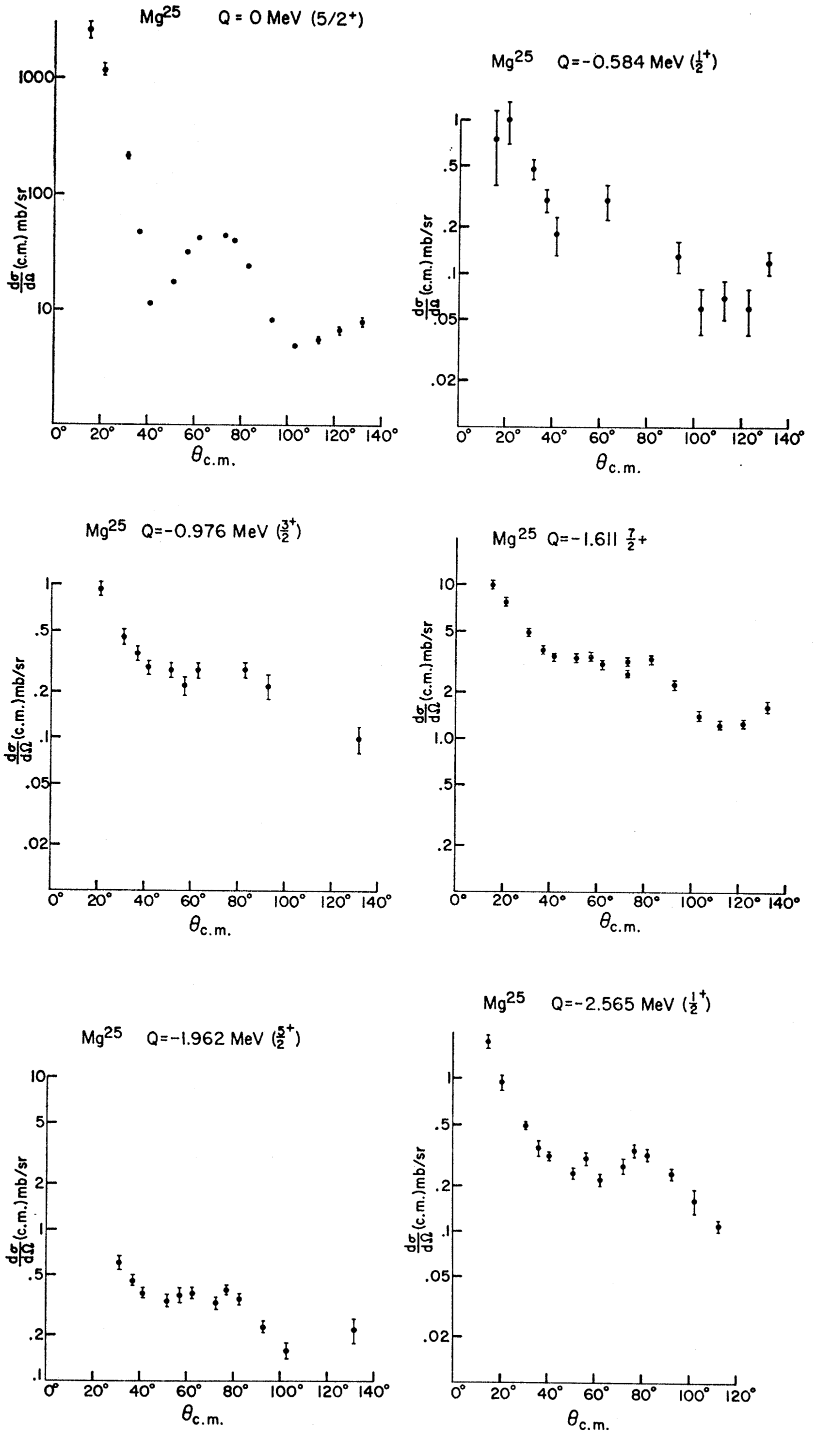


FIG. 8. Angular distribution for levels excited in Mg^{25} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

[33] H. Niewodniczanski, J. Nurzynski, A. Strzalkowski, J. Wilczynski, J. R. Rook, and P. E. Hodgson, Nucl. Phys. **55**, 386 (1964).
[34] J. Kokame, K. Fukunaga, and H. Nakamura, Phys. Letters **14**, 234 (1965).

tion of this peak is 3.000±0.002 MeV, showing that the principle contribution to this peak is from the 3.000-MeV level.

As the odd-particle count is the same for Mg^{25} and Al^{27} there is some similarity in the angular-momentum sequence of the low-lying levels which might lead one to believe that in Al^{27} there is a $K=\frac{5}{2}$ ground-state band and an excited $K=\frac{1}{2}$ band, as is the case in Mg^{25}. However, the energies do not work out as favorably in Al^{27} as in Mg^{25} for such a model. A comparison of Figs. 8 and 11 shows immediately a large difference in the cross sections observed in Al^{27} and Mg^{25} to the corresponding angular-momentum states. First, the $\frac{9}{2}+$ 3.00-MeV level has a cross section 20% larger than the $\frac{7}{2}+$ level rather than being a factor of 2.4 times smaller as it is in Mg^{25} and further the cross section for the $\frac{1}{2}$ and $\frac{3}{2}$ states are both larger in Al^{27} than in Mg^{25}. The cross section for the lowest-lying $\frac{7}{2}^+$ state in Al^{27} is a factor of 2 smaller than in Mg^{25}. These results are completely inconsistent with a simple strong-coupling model but can be understood[27,28,33] in terms of a weak-coupling model with large mixing between the ground and 2.731-MeV states. This model will be pursued further, later in this paper. The angular distribution to the 4.81-MeV level is that associated with $l=0$ in agreement with the recent[30] spin-parity assignment for this level. The cross sections were normalized by comparison to the elastic scattering data on Al^{27} by Dayton and Schrank[35] at 17.5 MeV. The work of Ref. 35 has been independently checked and found correct to within 2%.[36]

P^{31}

A spectrum of 17.5-MeV incident protons scattered at 50° laboratory angle from an evaporated phosphorus target is shown in Fig. 13. The level diagram is taken from Ref. 19 and the recent work of Harris and Breitenbecker.[37] The energies shown in this level diagram are those determined in this experiment. The level

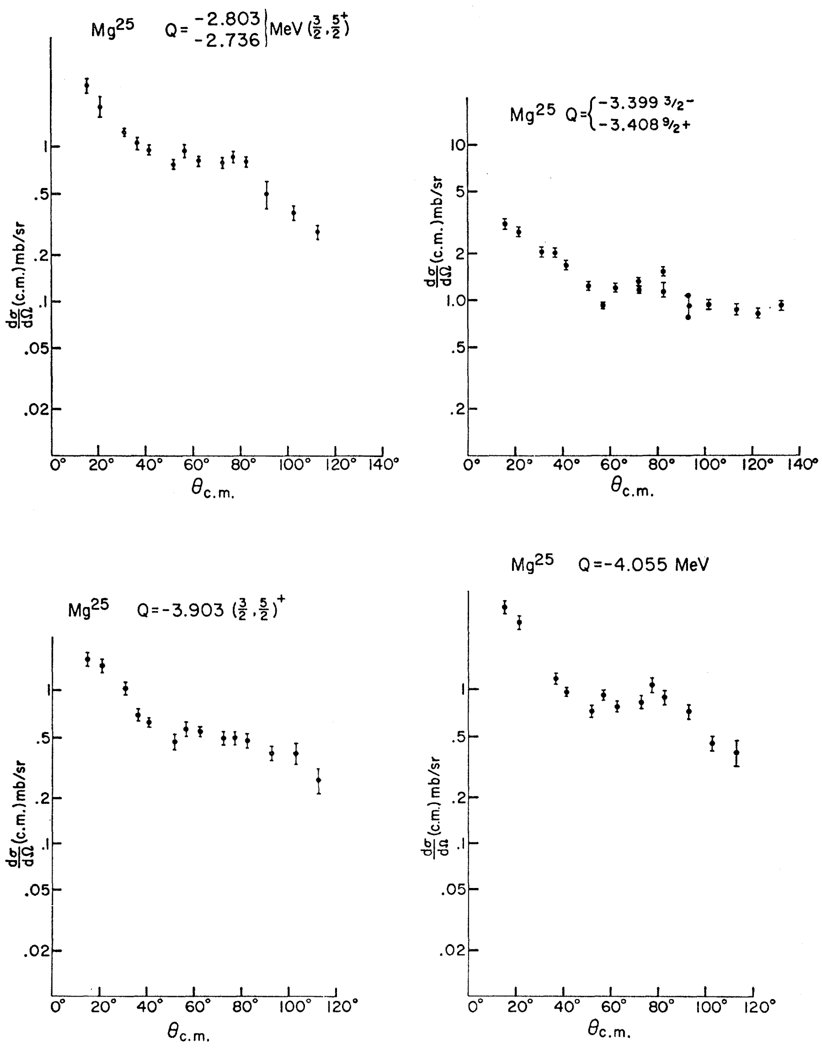


FIG. 9. Angular distribution for levels excited in Mg^{25} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

[35] I. E. Dayton and G. Shrank, Phys. Rev. **101**, 1358 (1956).
[36] R. E. Pollock and S. Shutz (private communication).
[37] G. I. Harris and D. V. Breitenbecker, Phys. Rev. **145**, 866 (1966).

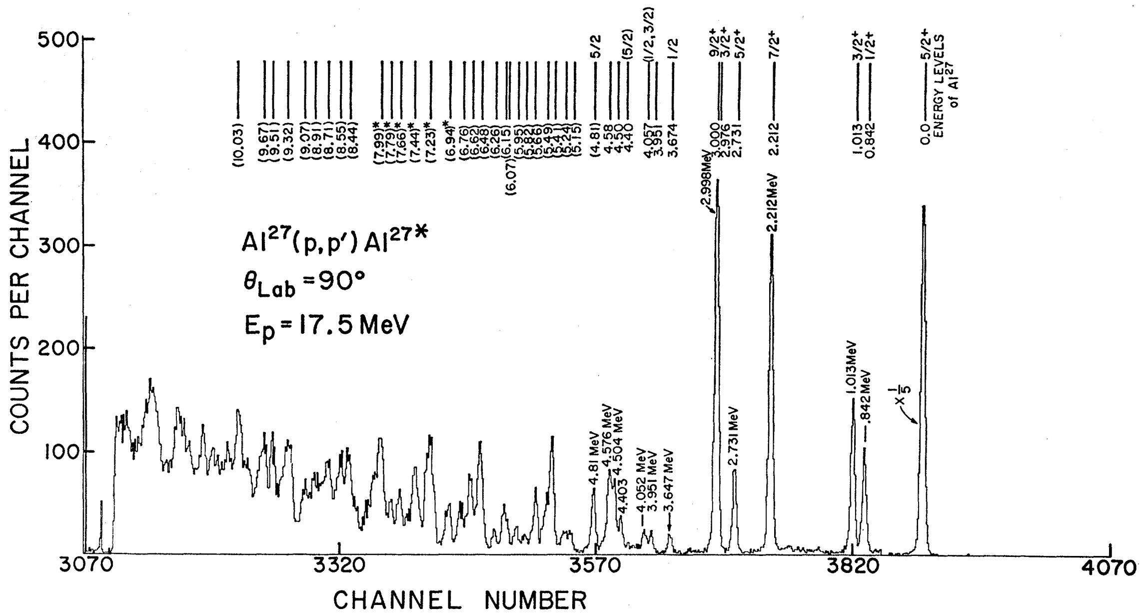


FIG. 10. The spectrum of 17.5-MeV incident protons scattered at 90° in the laboratory. The levels with asterisk (*) between 6.9 and 8.2 MeV are previously unreported levels in this nucleus.

at 5.34 MeV is a previously unobserved level in P^{31} if it does not correspond to the level at 5.25 MeV observed by Cujec *et al.*,[38] who studied $Si^{30}(d,n)P^{31}$. This possibility has been suggested by Harris *et al.*[37] but it does not seem to be very likely as the agreement between Ref. 37 and this work is within 20 keV for energy levels both above and below this excitation energy. Evidence will be presented for making a $\frac{9}{2}+$ assignment to this level; thus with such a high spin it would not be populated in the d,n reaction with 5.5-MeV incident deuteron energy.

Table II lists the excitation energies of the levels observed in this experiment. The energies of levels below 6 MeV are accurate to ±15 keV while those above are good to ±25 keV.

Figures 14, 15, and 16 show the angular distributions for all the levels observed below 5.67 MeV. Of these 15 lowest-lying levels 9 are excited with appreciable cross section and with angular distributions that are related to those observed for 4 strongly excited levels S^{32}.[1] In a true weak-coupling picture only 8 strongly excited levels would be observed in the P^{31} inelastic scattering. P^{31} has been studied with high-energy inelastic electron scattering[39] and there is a good deal of similarity between those results and what is observed here. However, in some cases the energy resolution of the electron-scattering experiment was not sufficient to allow separation of the yields of close-lying doublets so that some of their peaks are due to two or more levels. One of the difficult problems encountered in interpreting this (p,p') data and properly relating these cross sections to those observed in S^{32} arises from the apparent similarity between $l=2$ and $l=4$ angular distributions for 17.5-MeV protons in this region of the periodic table.[1] Discussion of a weak-coupling scheme for P^{31} based on S^{32} will be presented later in this paper..

V. DISTORTED-WAVE ANALYSIS

The lack of a unique l dependence in the differential cross section which was observed and discussed in I also applies to those odd-A nuclei. Although the $l=2$ differential cross sections within the same band or same weak-coupling multiplet look very similar there seems to be no universal $l=2$ shape in this region $(16\leq A\leq 32)$. However, those transitions in the odd-A nuclei that are assigned as weak-coupling states show angular distributions that are similar even in details to those of the core transitions in the adjacent even-even nuclei. This similarity of angular distribution, it must be pointed out, is not the trivial matter that it is for 40-MeV inelastic α scattering where the distributions are characteristic of the l value only. In the case of 17.5-MeV proton scattering, similar form factors must also be involved.

In no case were the angular distributions within the ground-state bands of an odd-A rotational nucleus (F^{19}, Na^{23}, Mg^{25}) similar to the $l=2$ distribution observed for the adjacent deformed even-even nucleus.

Because the values extracted for β_l the "deformation parameter" from the even nuclei[1] were in such good agreement with electromagnetic values, where they were known, it was felt worthwhile to try to extract these values for the strongest transitions observed in the odd A nuclei. The fits to the differential cross section

[38] B. Cujec, W. G. Davies, W. K. Dawson, T. B. Grandy, G. C. Neilson, and K. Ramavataram, Phys. Letters **15**, 266 (1965).

[39] P. Kossanyi-Demay, R. M. Lombard, and G. R. Bishop, Nucl. Phys. **62**, 615 (1965).

obtained using the Perey optical-model parameters[40] are shown in Fig. 17. The agreement between calculation and the observed differential cross sections for the $l=2$ transitions in F^{19} and Al^{27} is reasonable while these calculations clearly fail to reproduce the forward peaking observed for Na^{23}, Mg^{25}, and P^{31}. Small variation of the optical-model parameters were made to try to obtain better fits to the inelastic scattering while still preserving relatively good agreement with the elastic data. These fits are shown in Fig. 18 while the optical-model parameters used to obtain them are shown in Table III. Table IV gives the β_l values in cases where the fit to the differential cross section allowed reasonable extraction of this parameter; the deformation

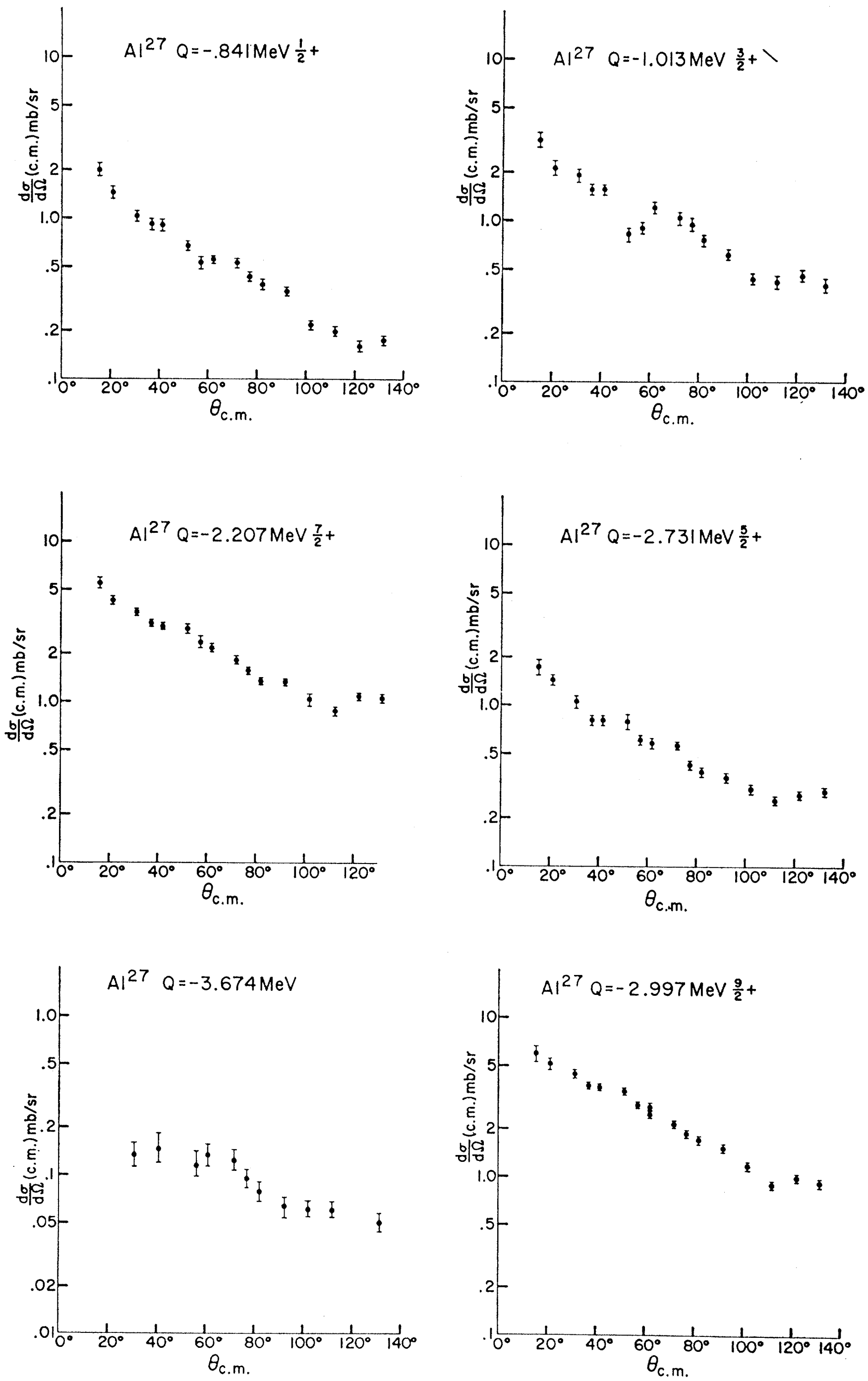


FIG. 11. Angular distribution for levels excited in Al^{27} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

[40] F. G. Perey, Phys. Rev. **131**, 745 (1963).

distances $\beta_l R_0$ are also given. The extraction of a reduced transition strength in an odd-A nucleus with $J_0 > \frac{1}{2}$ is model-dependent as follows from the discussion in Sec. I. Thus in the case of Al^{27} the extraction of β_l was made within the framework of the weak-coupling model using the relationship $\beta_2{}^2 = 5\hbar\omega/2C_2$. Figure 19 shows the fits obtained for 3 $l=3$ transitions. The poor fit for the F^{19} 1.35-MeV transition is shown to further emphasize how large the disagreement is between these results and Coulomb excitation. The direct component of the $l=3$ distribution must be smaller than is shown here. The Coulomb excitation[10] results assign this $E3$ transition 12 ± 4 single-particle units. In the neighboring nuclei O^{16} and Ne^{20} enhanced $E3$ transitions have been

TABLE I. Energies of the levels observed in the inelastic scattering of 17.5-MeV protons from Al^{27}. The previous results are due to the reference cited in the text. The results of this experiment are given in column 2. The levels below 5.00 MeV are accurate to 10 keV while those above 5.00 MeV are reliable to 25 keV.

Previous results E_x (MeV)	J^π	Present experiment E_x (MeV)	(p,p') cross section at 30° lab. (mb/sr)
0.00	$\frac{5}{2}+$	0.00	178
0.842	$\frac{1}{2}+$	0.841	1.0
1.013	$\frac{3}{2}+$	1.013	1.8
2.212	$\frac{7}{2}+$	2.207	3.5
2.731	$\frac{5}{2}+$	2.731	1.0
2.976	$\frac{3}{2}+$		
3.000	$(\frac{9}{2}+)$	2.997	4.3
3.674	$\frac{1}{2}$	3.677	0.1
3.951		3.952	0.1
4.052	$\frac{1}{2}$	4.051	0.2
4.40		4.410	0.5
4.504		4.502	0.7
4.576		4.569	0.6
4.805		4.802	0.5
5.149		5.149	0.2
5.24		5.239	0.2
5.410			
5.424		5.42	0.5[a]
5.49		5.50	0.5[b]
5.54		5.53	0.3[a]
5.66		5.66	0.2
5.75		not seen	
5.82		5.81	
5.95		5.95	0.1
6.07		6.07	
6.15		6.14	
6.26		6.26	
6.52		6.47	
6.60		6.64	
6.76		6.76	
6.81		not seen	
6.99		6.94	
↑		7.23	
		7.66	
		7.79	
		7.99	
		8.44	
28		8.55	
levels		8.71	
in Al^{27}		8.91	
		9.07	
		9.32	
		9.51	
		9.67	
↓		10.03	

[a] Cross section at 35° lab.
[b] Cross section at 60° lab.

TABLE II. Energies of the levels observed in the inelastic scattering of 17.5-MeV protons from P^{31}. The previous results are due to references cited in the text under P^{31}. The levels below 5 MeV are accurate to 10 keV while those above 5.00-MeV excitation are reliable to 25 keV.

Previous results E_x (MeV)	J^π	Present experiment E_x (MeV)	(p,p') cross section at 30° lab. (mb/sr)
0.00	$\frac{1}{2}+$	0.00	254
1.265	$\frac{3}{2}+$	1.264	4.6
2.232	$\frac{5}{2}+$	2.232	7.8
3.13	$\frac{1}{2}+$	3.15	0.5
3.29	$\frac{5}{2}+$	3.30	0.7
3.41	$\frac{7}{2}+$	3.41	1.3
3.51	$\frac{3}{2}+$	3.51	2.2
4.19	$\frac{5}{2}+$	4.19	0.5[a]
4.26	$\frac{3}{2}+$	4.23	0.4[a]
4.43	$\frac{7}{2}-$	4.43	1.9
4.59	$(\frac{5}{2})$	4.62	0.9
4.63	$\frac{3}{2}$		
4.78		4.78	2.0
5.01	$(\frac{1}{2},\frac{3}{2}+)$	5.01	1.5
5.12			
5.25	$\frac{1}{2}$		
		5.34	2.5
5.53			
5.66		5.67	2.0
5.77			
5.89			
(6.05)		6.05	
6.18			
(6.25)		6.25	
6.38			
6.43		6.43	0.4
6.55		6.56	
7.15			
7.77		7.78	1.4
7.89		7.89	
18 levels in P^{31}			
8.72		8.72	

[a] Cross section at peak 55° lab.

TABLE III. Optical-model parameters used in the analysis of inelastic-scattering data. These parameters gave the best fit to the $l=2$ transitions shown in Fig. 18.

Nucleus	V_R (MeV)	W_R (MeV)	r_0 (F)	r_c (F)	a (F)	V_{s0} (MeV)	r_I (F)	a_I (F)	V_I (MeV)
F^{19}	48.5	0	1.25	1.25	0.65	7.5	1.25	0.47	39.3
Na^{23}	46.1	0	1.25	1.25	0.65	7.5	1.25	0.47	35.5
Mg^{25}	46.1	0	1.25	1.25	0.65	7.5	1.25	0.47	37.2
Al^{27}	42.7	0	1.30	1.30	0.62	7.5	1.30	0.44	42.2
P^{31}	41.5	0	1.30	1.30	0.62	7.5	1.30	0.44	31.9

observed with $B(E3)$ values 14 ± 1.5 [41] and 7_{-2}^{+3} [42] times the single-particle unit, respectively. Taking into account the relevant statistical factors $(2J_f+1/2J_i+1)$ the cross section expected for this 1.35-MeV level in F^{19} should be greater than 3 mb/sr at forward angles if the magnitude of these inelastic cross sections are related to their $B(E3)$ values. The cross section to the 1.35-MeV level in F^{19} is less than 1 mb at all angles observed. Thus there seems to be a serious disagreement in the relative enhancements of the octupole matrix elements between the electromagnetic measurements and the inelastic-scattering results presented here. The β_3 extracted for the 1.35-MeV to ground-state transition corresponds to a value 3.8 ± 0.6 times the single-particle value. In light of the angular distribution to this level, this should be taken as an upper limit.

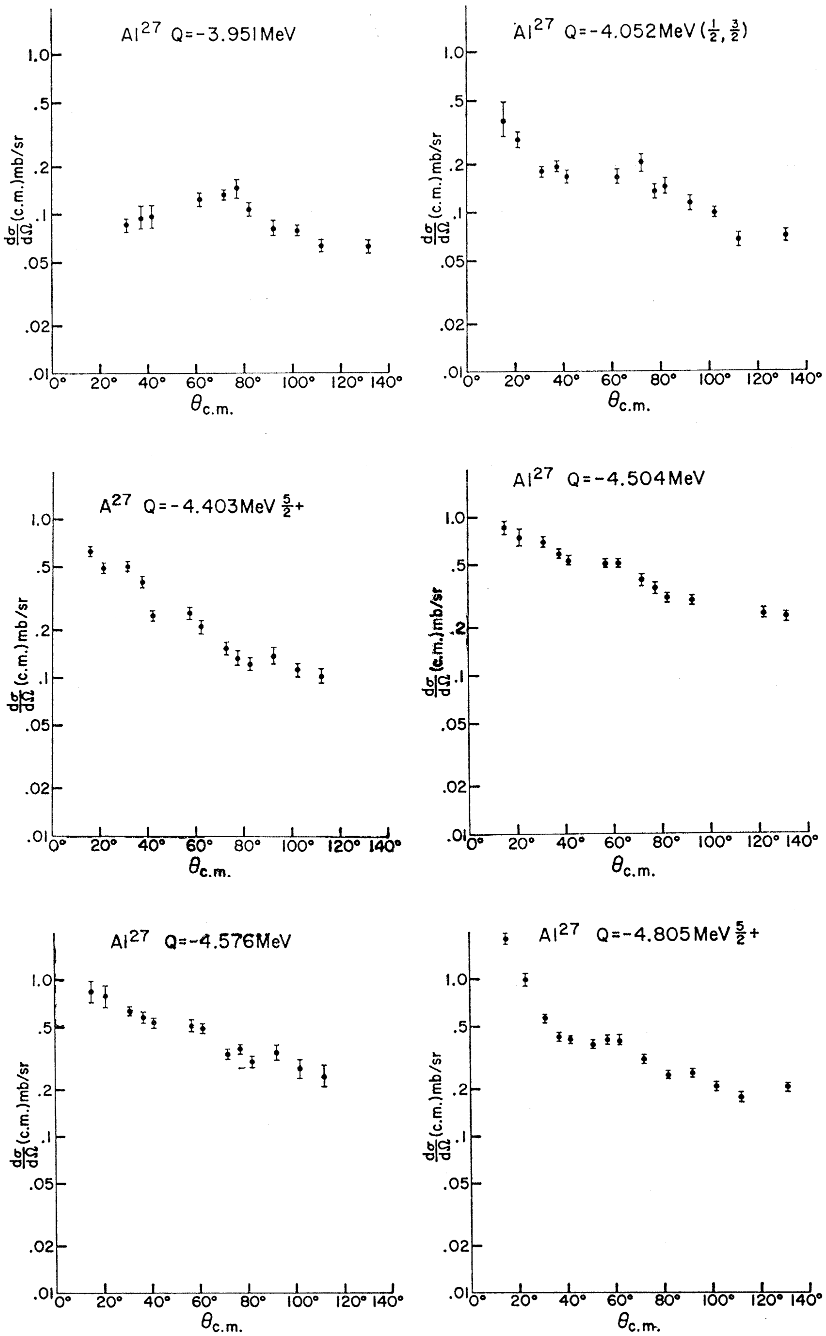


FIG. 12. Angular distribution for levels excited in Al^{27} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

[41] T. Alexander and K. W. Allen, Can. J. Phys. 43, 1564 (1965).
[42] C. Broude, A. E. Litherland, and J. D. Pearson, Phys. Letters 11, 321 (1964).

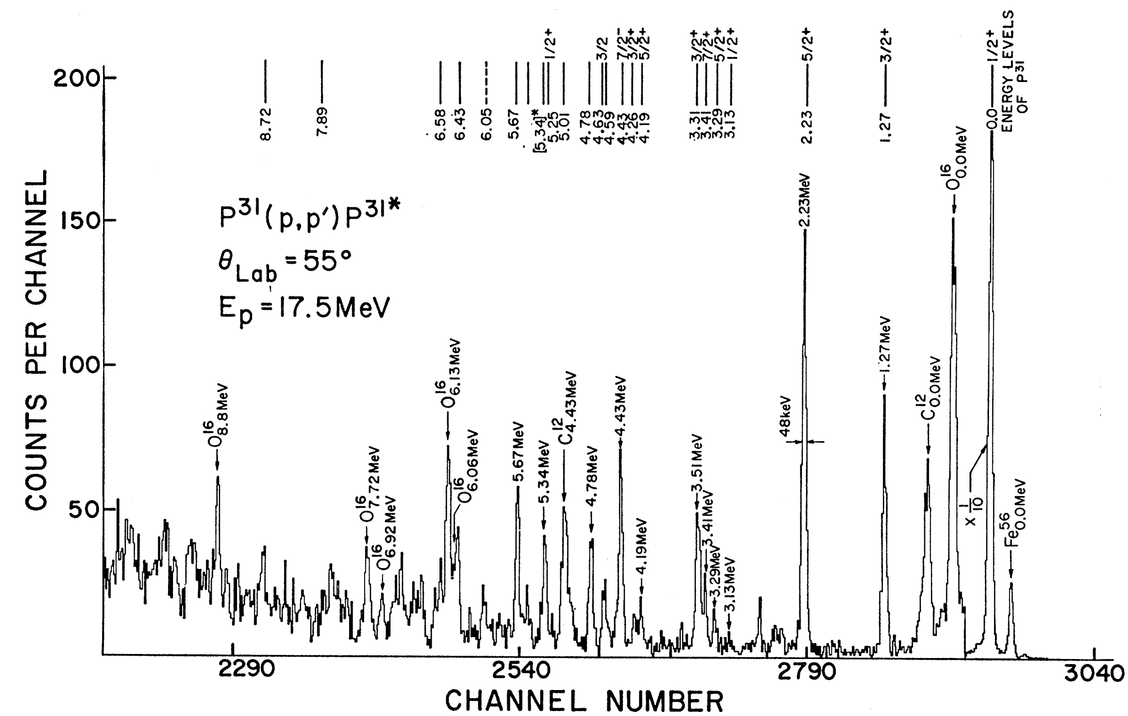


FIG. 13. The spectrum of 17.5-MeV incident protons scattered at 55° in the laboratory system. The level at 5.34 MeV is a previously unreported level in this nucleus.

VI. DISCUSSION: RELATION TO NUCLEAR MODELS

F^{19}

The results in F^{19} are consistent with a rotational model but as is mentioned in Sec. I, the relative cross-section test is not very strong for $K=\frac{1}{2}$. The present results would indicate that either the 4.00- or 4.57-MeV levels are the most likely candidates for being the $\frac{7}{2}+$ member of the ground-state band. The cross section observed for the 1.35 MeV, $\frac{5}{2}-$ state is smaller than would be predicted from the observed[10] $B(E3)$.

TABLE IV. The effective transition probabilities expressed in terms of a deformation parameter β_l. These values are obtained from distorted-wave Born-approximation fits to the experimental data as shown in Figs. 18 and 19. Column 3 lists the "deformation distance," $\beta_l R_0$.

Nucleus	Level (MeV)	J^π	β_l $l=2$	$\beta_l R_0$ (F) $l=2$
F^{19}	0.197	$\frac{5}{2}+$	0.49±0.02	1.63±0.07
	1.56	$\frac{3}{2}+$	0.51±0.02	1.70±0.07
Al^{27}	0.842	$\frac{1}{2}+$	0.57±0.02	2.22±0.08
	1.013	$\frac{3}{2}+$	0.53±0.02	2.07±0.08
	2.212	$\frac{7}{2}+$	0.52±0.02	2.03±0.08
	2.731	$\frac{5}{2}+$	0.33±0.02	1.29±0.08
	3.000	$(\frac{9}{2}+)$	0.48±0.02	1.87±0.08
P^{31}	1.27	$\frac{3}{2}+$	0.32±0.02	1.31±0.08
	2.23	$\frac{5}{2}+$	0.36±0.02	1.47±0.08
	3.51	$\frac{3}{2}+$	0.26±0.02	1.06±0.08
	4.78	$(\frac{5}{2}+)$	0.20±0.01	0.82±0.04
			$l=3$	
F^{19}	1.35	$\frac{5}{2}-$	0.35±0.04	
P^{31}	4.43	$\frac{7}{2}-$	0.37±0.01	
	5.67	$(\frac{5}{2}-)$	0.40±0.01	

Na^{23}

The results obtained here are consistent with a strong-coupling model in so far as the cross-section ratio within the ground-state band is well satisfied. However, the 0.439- and 2.080-MeV levels are not properly spaced according to the rotation model so that some mixing certainly must be involved. This being the case, our results must be understood as having the amount of extraband mixing in each of the two levels equal to within 10% and not exceeding 20% in the squared amplitude.

Mg^{25}

This nucleus is known to be well described as a strong coupling rotor and our results do little more than confirm this picture.

Al^{27}

There have been several recent attempts[27,28,33,34,43,44] to describe the low-lying states of Al^{27} within the framework of a weak-coupling model. The states are formed by coupling a $d_{5/2}$ hole to the ground and first excited

[43] V. K. Thankappen, Phys. Rev. **141**, 957 (1966).
[44] D. Evers, J. Hertel, T. W. Ritz-Schmidt, and S. J. Skorka, Nucl. Phys. **A91**, 492 (1967).

state of Si^{28}. The work of Thankappen[43] takes an approach quite different from that taken in papers analyzing inelastic scattering results[27,28,33,34] but the conclusions are happily the same. The key point at issue is the amount of mixing between the 0 phonon and 1 phonon $\frac{5}{2}+$ states. Thankappen[43] gives 0.174 as the square of the amplitude of the one-phonon state mixed into the ground state while the results of the inelastic proton scattering yields 0.19. A procedure for obtaining this mixing from inelastic scattering is set down in Ref. 28 but is presented here for completeness and to facilitate the discussion of the results. As discussed above the ground state is taken to be a mixture of zero- and one-phonon state. Thus

$$\psi_{5/2}(\mathrm{G.S})=(1-A^2)^{1/2}|0\,\tfrac{5}{2},\tfrac{5}{2}\rangle+A\,|2\,\tfrac{5}{2},\tfrac{5}{2}\rangle, \qquad \text{(6a)}$$

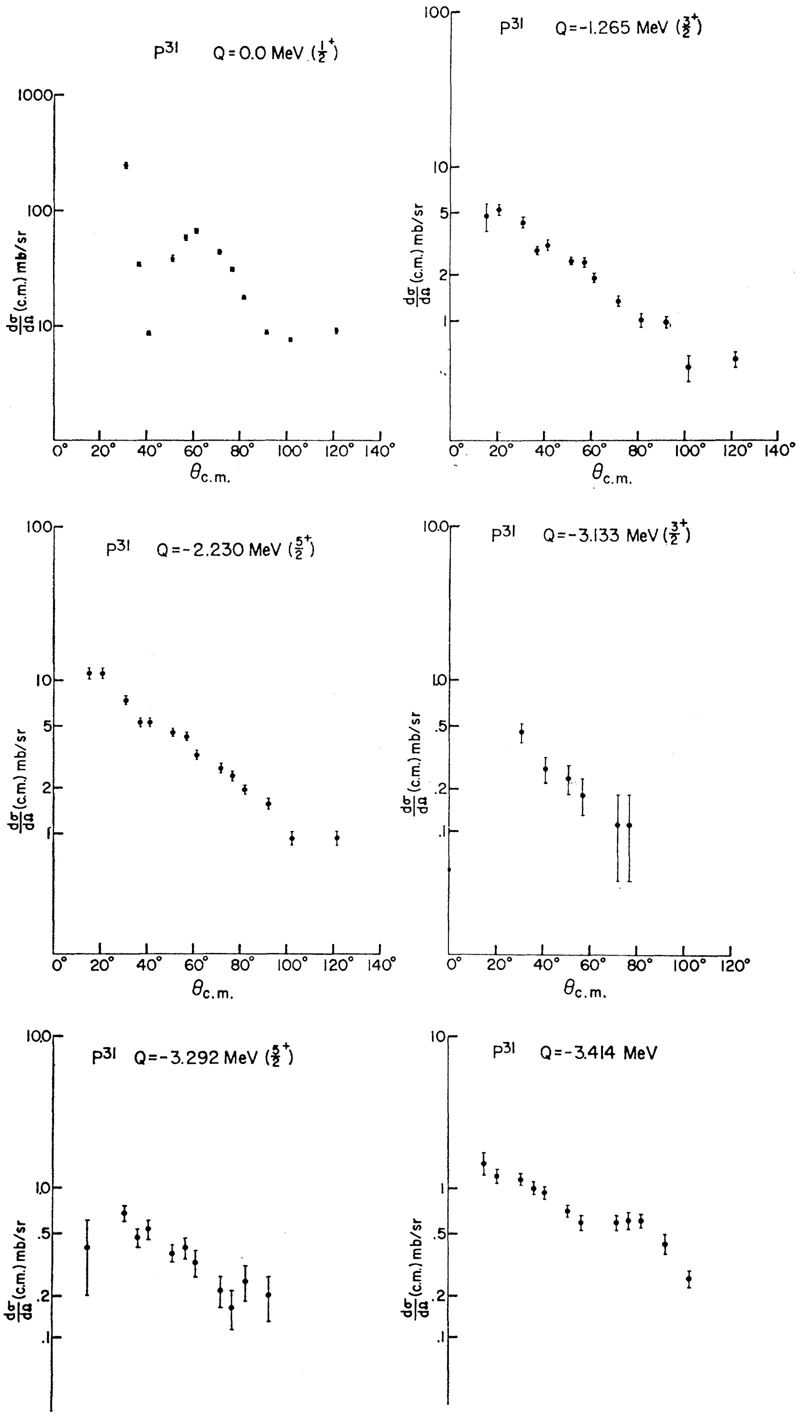


FIG. 14. Angular distribution for levels excited in P^{31} by inelastic protons cattering. The errors shown do not include the uncertainty in absolute normalization (10%).

$$\psi_{5/2+}(2.73\text{ MeV})=-A\,|0\,\tfrac{5}{2},\tfrac{5}{2}\rangle+(1-A^2)^{1/2}|2\,\tfrac{5}{2},\tfrac{5}{2}\rangle, \quad (6b)$$

while the other states of the one-phonon multiplet are given by

$$\psi_J(2\,\tfrac{5}{2},J)=|2\,\tfrac{5}{2},J\rangle,$$

where $J=\frac{1}{2}$, $\frac{3}{2}$, $\frac{7}{2}$, and $\frac{9}{2}$. In the simple weak-coupling model presented in Sec. I the expected differential cross sections for the various members of the multiplet are given by

$$\frac{d\sigma}{d\Omega}(\tfrac{5}{2}+\rightarrow J)=\frac{2J+1}{30}\frac{d\sigma}{d\Omega}(0\rightarrow 2+,\ 1.772\text{ MeV Si}^{28}),$$

where J takes the values $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$ for the members of the weak-coupling multiplet. Though the angular distributions for the 1.772-MeV level in Si^{28} and states of the weak-coupling multiplet are similar their total

TABLE V. Cross sections for the low-lying states of Al^{27} comprising the members of the weak coupling multiplet. The cross sections shown are integrated over the angular range 15°–130°. Column 2 gives the expected cross sections in the weak coupling model if there is no mixing between the ground and 2.731-MeV level. Column 3 gives the result for a mixing amplitude of 0.435. Column 4 gives the observed results.

Nucleus	Excited level MeV	J_π	Predicted cross section No mixing	Predicted cross section Mixing ($A^2=0.19$)	Observed cross section (mb)
Al^{27}	0.841	$\frac{1}{2}^+$	0.87±0.07	0.71±0.06	0.80±0.03
	1.013	$\frac{3}{2}^+$	1.75±0.14	1.42±0.1	1.45±0.04
	2.207	$\frac{7}{2}^+$	3.50±0.24	2.85±0.2	3.00±0.09
	2.731	$\frac{5}{2}^+$	2.63±0.18	1.00±0.09	0.85±0.03
	3.000	$(\frac{9}{2}^+)$	4.38±0.32	3.55±0.3	3.55±0.11
			13.1 ±1.0	9.53±0.8	9.65±0.4
Si^{28}	1.772^+				13.1 ±1.0

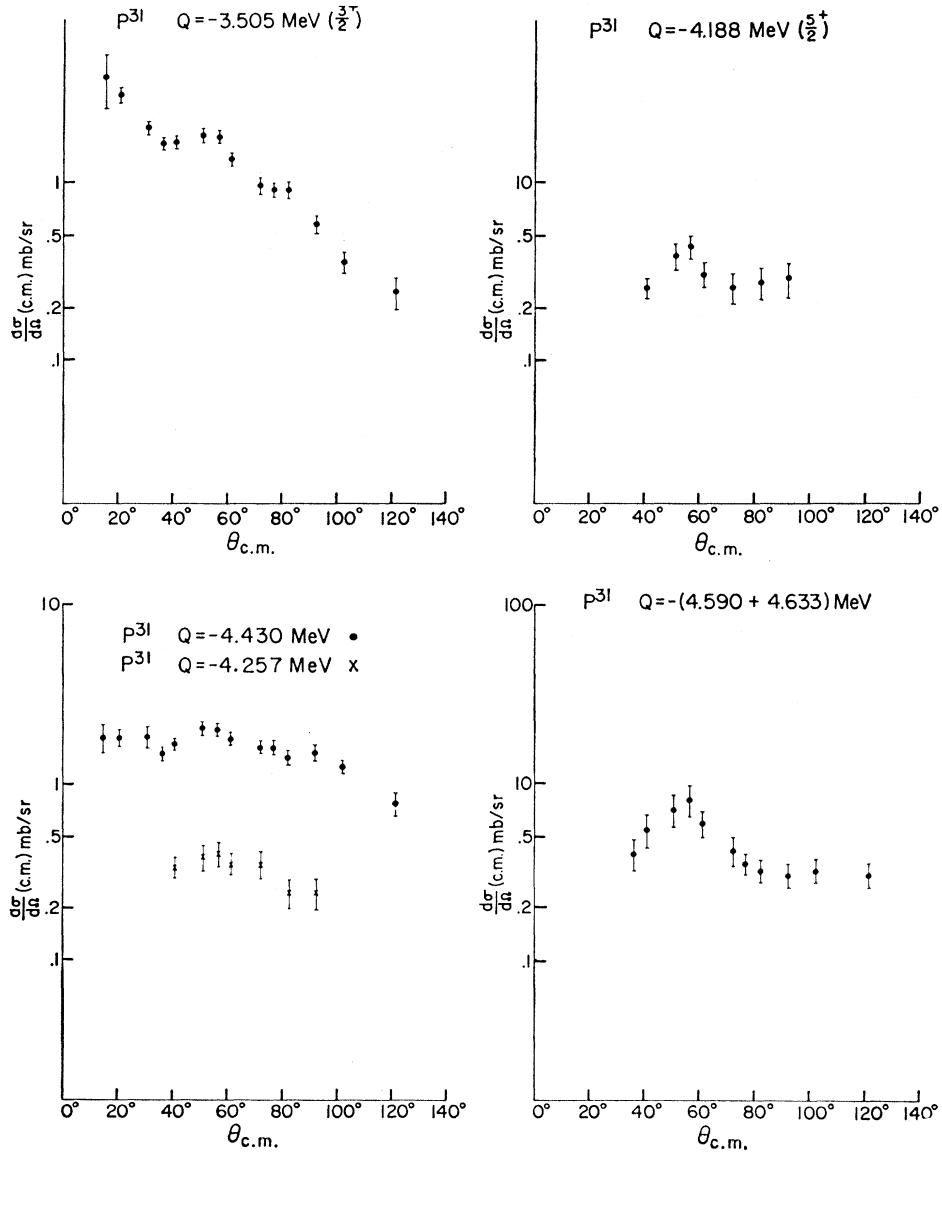


FIG. 15. Angular distribution for levels excited in P^{31} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

cross sections over the range of angles observed 15°–130° is used in Table V as being most representative of their relative intensities. Column 4 of Table V shows all the states of the multiplet approximately obeying the $(2J+1)$ intensity ratio predicted in weak coupling except the excited $\frac{5}{2}+$ state, which is much smaller. This is due to mixing between the $\frac{5}{2}+$ ground state and this $\frac{5}{2}+$ excited state, which causes destructive interference in the transition amplitude. If the reduced matrix element $\langle J_c|O_2|J_c'\rangle$ is set equal to 0 when $J_c=J_c'=2$, where J_c is the angular momentum associated with the collective core and O_2 represents some generalized quadrupole operator responsible for the inelastic transition, it can be readily shown that the cross sections are given by

$$\frac{d\sigma}{d\Omega}(\tfrac{5}{2}+\to\tfrac{5}{2}+)=(1-2A^2)^2\frac{1}{5}\frac{d\sigma}{d\Omega}(0-2+\mathrm{Si}^{28})$$

and

$$\frac{d\sigma}{d\Omega}(\tfrac{5}{2}+\to J+)=(1-A^2)\frac{2J+1}{30}\frac{d\sigma}{d\Omega}(0\to 2+\mathrm{Si}^{28}).$$

This assumption, that $\langle 2|Q_2|2\rangle=0$, is strictly correct within the confines of a vibrational model and is empirically justified in this case by the work of Thankappan[43] and Evers *et al.*[44] The results for $A^2=0.19$ are shown in Table V and the agreement with experiment is seen to be quite good. Further, this value for A^2 is in good agreement with other values cited[27,33,43,44] in the literature. It would predict a relative spectroscopic factor for proton pickup from Si^{28} of $0.19/0.81=0.235$ between the excited $\frac{5}{2}+$ state and the ground state. The observed ratio is 0.24.[45] As no assumptions are made about the microscopic nature of the core state, this model makes no predictions about relative spectroscopic

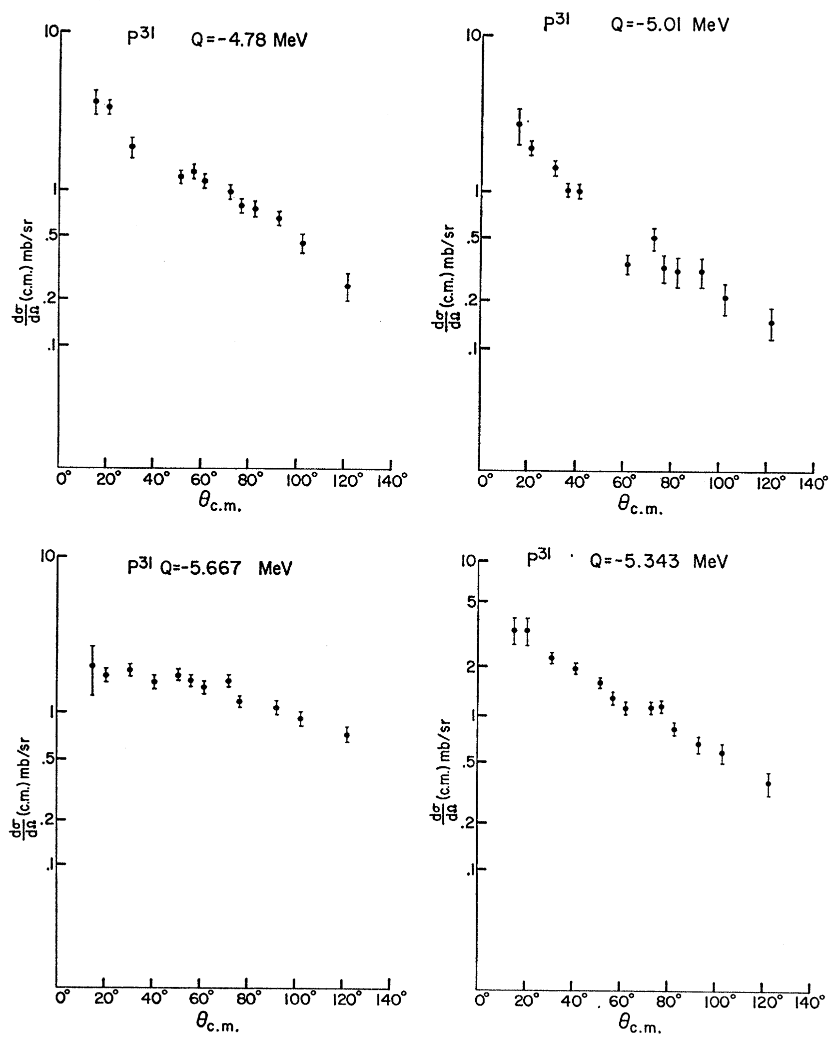
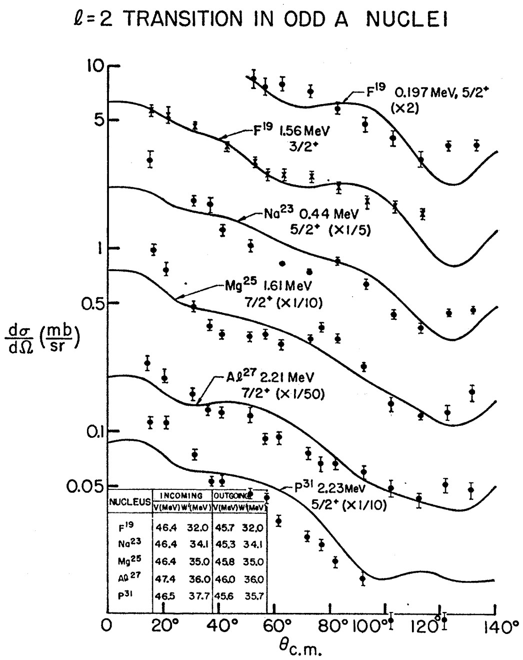


FIG. 16. Angular distribution for levels excited in P^{31} by inelastic proton scattering. The errors shown do not include the uncertainty in absolute normalization (10%).

[45] H. E. Gove, K. H. Purser, J. J. Schwartz, W. P. Alford, and D. Cline, Contribution to the International Conference on Nuclear Structure, Tokyo, 1967 (unpublished).



NUCLEUS	INCOMING		OUTGOING	
	V(MeV)	W′(MeV)	V(MeV)	W′(MeV)
F^{19}	46.4	32.0	45.7	32.0
Na^{23}	46.4	34.1	45.3	34.1
Mg^{25}	46.4	35.0	45.8	35.0
$A\ell^{27}$	47.4	36.0	46.0	36.0
P^{31}	46.5	37.7	45.6	35.7

FIG. 17. Distorted-wave Born-approximation fits to the strongest $l=2$ differential cross sections observed in these experiments. The optical-model parameters used were those of Perey (Ref. 40) with the real and imaginary potentials taking on the values indicated in the table in the lower-left corner.

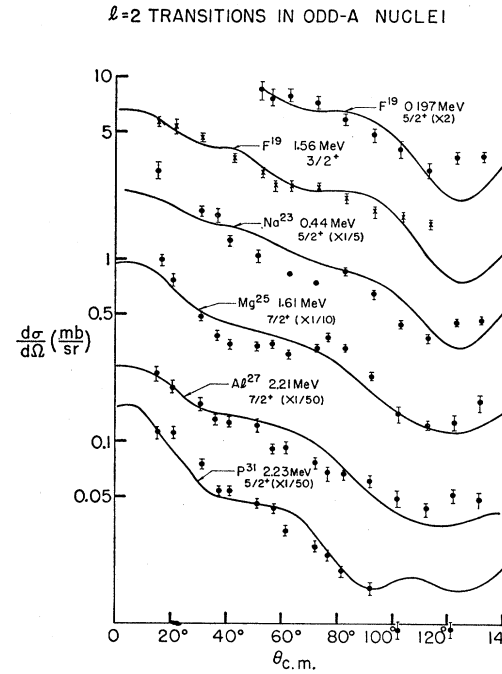


FIG. 18. The "best" distorted-wave Born-approximation fits obtained for the strongest $l=2$ differential cross section observed in these experiments. The optical-model parameters used are those shown in Table III.

factors other than the one indicated, contrary to some reports in the literature.[46]

This very simple picture works very well and a somewhat more complicated model is discussed in some detail in Refs. 43 and 44. As the model is made more realistic it seems to work less well. The transitions between the single-hole states have been completely neglected as all of the transition strength has been attributed to the core. From the order of magnitude of the $Si^{28}\,0\rightarrow 2^+$, 1.772-MeV yield this is not strictly justifiable as the *amplitude* of that transition is at most 5 times that for a single particle so that the effects of the single-hole state can not be neglected particularly since the hole transition enters with different phases relative to the core transition in some of the cases. This effect modifies the transition probabilities and would make agreement with experiment less good. These effects have been noted in the electromagnetic transition probabilities where the near equality of the $B(E2)$ values for the $J=\frac{1}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ and $\frac{9}{2}$ levels is not reproduced with a more realistic calculation.[44]

P^{31}

As there seems to be general agreement that the levels of P^{31} are not[47] consistent with results obtained from a Nilsson model with Coriolas mixing this nucleus will be discussed in terms of a weak-coupling picture based on the strongly excited levels of S^{32}. Admittedly the evidence presented here does not differentiate between a rotational or vibrational scheme for the low-lying levels because $J=\frac{1}{2}$ in the ground state; however, the higher-lying state give evidence of behavior that is suggestive of weak coupling. As the angular momentum of the core states we shall treat is $J_c=2$, 3, and 4 there can be no mixing between the ground state and either

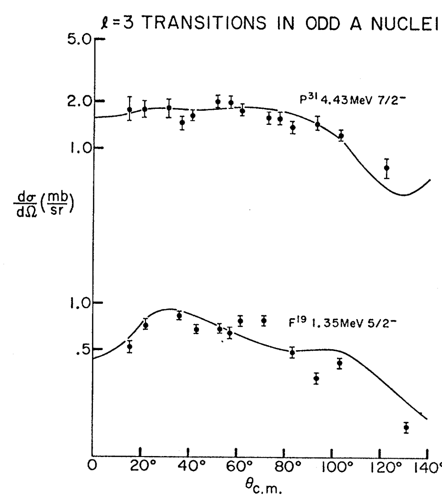


FIG. 19. Distorted-wave Born-approximation fits to the strongest $l=3$ differential cross sections. The F^{19} parameters used are taken from Table III while the P^{31} fit is obtained using the Perey (Ref. 40) parameters indicated in Fig. 17.

[46] B. Lawergren, Nucl. Phys. **A90**, 311 (1967).

[47] G. R. Bishop, A. Bottino, and A. M. Lombard, Phys. Letters **15**, 323 (1965).

of the pair of states resulting from coupling $j_p=\frac{1}{2}$ to J_c. Thus the states should be purer than was the case observed in Al^{27}.

The first difficulty with such a picture is in the observed value of the magnetic moment of the ground state which is less than $\frac{1}{2}$ the value for the proton so that a model making the P^{31} ground state simply a $S^{1/2}$ hole in the S^{32} ground state cannot be correct. However, the angular distribution to the first 2+ state in S^{32} is very similar to that for the transitions to the P^{31} levels at 1.265 ($\frac{3}{2}+$) and 2.23 ($\frac{5}{2}+$) MeV. The ratio of the $\frac{5}{2}+$ to $\frac{3}{2}+$ cross sections is 1.8±0.2 whereas both strong and weak coupling predict 1.5. The summed $\frac{3}{2}$ and $\frac{5}{2}+$ cross sections is only 16% smaller than that for S^{32} and as the absolute normalization is uncertain to 10% in both cases this must be considered good agreement. The strong $l=3$ transition observed in S^{32} at 5.01 MeV seems to be the parent of two $l=3$ transitions in P^{31}, one at 4.430 to a known $\frac{7}{2}-$ level while the other occurs at an energy of 5.667 MeV. Both of these $l=3$ transitions are in accord with observations in inelastic electron scattering. Assigning the upper level spin-$\frac{5}{2}$ the spin-weighed energy average of these levels is 4.960 MeV, which is close to the 5.01 MeV observed in S^{32}. The ratio of the cross sections is

$$\frac{d\sigma}{d\Omega}(4.430)\bigg/\frac{d\sigma}{d\Omega}(5.667)=1.1\pm0.15\,,$$

while the weak-coupling model gives 1.33, the summed cross sections for the two P^{31} levels is some 70% of that observed in S^{32}.

There are five other strong transitions observed in P^{31}. From Figs. 14, 15, and 16 they are seen to occur at 3.414($\frac{7}{2}+$), 3.505($\frac{3}{2}+$), 4.78, 5.01, and 5.434 MeV. Only the spins of the first two are known and similarity of the $l=4$ and 2 distributions makes unique identification of these levels relative to two strong $l=2$ and 4 transitions observed[1] in S^{32}, at 4.29 and 4.46 MeV, respectively, difficult. Two possible weak-coupling schemes are shown in Table VI.

TABLE VI. Possible weak-coupling relationships between levels of P^{31} and those of S^{32}. The S^{32} parent core states are listed in column 1 with their energy and spin parity. The next column contains the spin parity (either known or assigned) and energy of multiplet members in P^{31}. The next column gives the spin-weighted energy average of the multiplet and the last two columns compare the expected and measured ratios for the cross sections assuming the levels to have the spins assigned in column 2.

S^{32} state Energy (MeV)	J^π	P^{31} state J^π	Energy (MeV)	Mean energy of P^{31} states	Ratio of P^{31} states model	exp't
2.24	2^+	$\frac{3}{2}^+$	1.27	1.85	1.50	1.80±0.2
		$\frac{5}{2}^+$	2.23			
4.29	2^+	$\frac{3}{2}^+$	3.51	4.27	1.50	0.9 ±0.2
		($\frac{5}{2}^+$)	4.78			
4.47	(4^+)	$\frac{7}{2}^+$	3.41	4.49	1.25	2.0 ±0.2
		($\frac{9}{2}^+$)	5.34			
5.01	3^-	$\frac{7}{2}^-$	4.43	4.96	1.33	1.1 ±0.2
		($\frac{5}{2}^-$)	5.67			
			Alternatively			
4.29	2^+	$\frac{3}{2}^+$	3.51	4.61	1.50	1.0 ±0.2
		($\frac{5}{2}^+$)	5.34			
4.47	(4^+)[a]	$\frac{7}{2}^+$	3.41	4.19	1.25	1.5 ±0.2
		($\frac{9}{2}^+$)	4.78			

[a] Parentheses refer to uncertain spins and parities.

It would be most interesting to try to extend this model, for if the $\frac{1}{2}+$ state at 3.13 can be related to the 0+ state in S^{32} at 3.78 MeV a rather complete picture of the low-lying states of P^{31} would be obtained. Thus either proton- or neutron-pickup experiments on S^{32} would be helpful as would definite assignments of the l values for the P^{31} inelastic scattering and additionally a firm spin assignment for the 4.46-MeV level in S^{32} is needed.