Here, 1 stands for $j_1\tau_1$, -1 stands for $j_1-\tau_1$, and the quantities M and L are defined by

$$M(12,34;J)=\langle j_1\tau_1 j_2\tau_2;J|v|j_3\tau_3 j_4\tau_4;J\rangle \\ -(-)^{j_3+j_4-J}\langle j_1\tau_1 j_2\tau_2;J|v|j_4\tau_4 j_3\tau_3;J\rangle, \quad (A5)$$

and

$$L_{J_0J_0'}{}^J(12;34,56)=\delta_{j_1,j_2}\delta_{\tau_1,\tau_2} \times\sum_k [k]\begin{Bmatrix} j_1 & j_3 & J_0' \\ j_5 & k & j_6 \\ J_0 & j_4 & J \end{Bmatrix} M(34,56;k). \quad (A6)$$

The symbols { } in (A6) are ordinary 9-j symbols.

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Proton Hole States in ^{29}Al, ^{31}P, and ^{33}P†

R. C. BEARSE, D. H. YOUNGBLOOD,* AND J. L. YNTEMA
Argonne National Laboratory, Argonne, Illinois
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The low-lying states in ^{29}Al, ^{31}P, and ^{33}P have been studied via the reactions ^{30}Si$(d,{}^3$He$)^{29}$Al, ^{26}Mg$(\alpha,p)^{29}$Al, ^{32}S$(d,{}^3$He$)^{31}$P, ^{34}S$(d,{}^3$He$)^{33}$P, and ^{30}Si$(\alpha,p)^{33}$P. Spectroscopic factors have been extracted for the $(d,{}^3$He) reactions leading to two final states in ^{29}Al, three in ^{31}P, and two in ^{33}P. In the ^{30}Si$(d,{}^3$He$)^{29}$Al reaction, we observed $l=2$ and $l=0$ angular distributions from reactions leading to the ground state and first excited state, respectively. For the ^{34}S$(d,{}^3$He$)^{33}$P reaction, we obtained an $l=0$ distribution for the reaction to the ground state and an $l=2$ distribution for the reaction to the second excited state.

1. INTRODUCTION

ALTHOUGH the *sd*-shell nuclei are among the most thoroughly studied, there are several nuclei in this region about which little is known. Two such cases are ^{29}Al and ^{33}P. Currie[1] has identified the first three excited states in ^{33}P but has assigned no spins or parities. The level order of ^{29}Al was investigated by Jaffe *et al.*,[2] who used the ^{27}Al$(t,p)^{29}$Al reaction and assigned $\frac{5}{2}^+$ for the ground state. Interest in the mass-29 isobars has been aroused by the recent experiments of Youngblood *et al.*[3] at Argonne, and Teitelman *et al.*[4] at Rutgers, in which they studied the $T_>$ states in ^{29}P that are the analogs of the low-lying states in ^{29}Al.

The present work consisted of spectrograph studies to determine level positions in ^{33}P and in ^{29}Al. In addition, the $(d,{}^3$He) reaction on ^{30}Si, ^{32}S, and ^{34}S was investigated to find the proton hole states in ^{29}Al, ^{31}P, and ^{33}P. The orbital angular momentum of the transferred proton and the spectroscopic factor of the transition were obtained by comparing the experimental angular distributions with the theoretical curves obtained from distorted-wave theory.[5]

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* Present address: The Cyclotron Institute, Texas A & M University, College Station, Tex. 77843.
[1] W. M. Currie and J. E. Evans, Phys. Letters **24B**, 399 (1967).
[2] A. A. Jaffe, F. DeS. Barros, P. D. Forsyth, J. Muto, I. J. Taylor, and S. Ramavataram, Proc. Phys. Soc. (London) **76**, 914 (1960).
[3] D. H. Youngblood, G. C. Morrison, and R. E. Segel, Phys. Letters **22**, 625 (1966).
[4] B. Teitelman, J. P. F. Sellschop, G. M. Temmer, and G. T. Garvey, Bull. Am. Phys. Soc. **12**, 570 (1967).
[5] We are indebted to Dr. R. M. Drisko for the use of the code JULIE.

2. METHODS

Spectrograph Studies

For a study of the ^{26}Mg$(\alpha,p)^{29}$Al reaction, rolled targets (80 μg/cm^2 thick and enriched to 99.4% in ^{26}Mg) were exposed to a 10.980-MeV beam of ^{4}He^{++} ions from the Argonne tandem Van de Graaff. The proton groups from the reaction were measured at three angles by use of the Argonne magnetic spectrograph.[6] The protons were detected by Kodak NTB-50 emulsions covered with acetate foil to stop all other particles. The beam energy was determined by measuring the position of the α-particle group elastically scattered through 150° by a thin Au foil. The experiment was repeated on separate days, and the beam energy was determined for each run. Figure 1 shows a typical spectrum taken at 60°. The only appreciable contamination is from the ^{12}C$(\alpha,p_0)^{15}$N reaction. In Table I, the energies determined in the present work are compared with those of Jaffe *et al.*[2] It should be noted that agreement is good for all of the states we have measured except the third excited state. A comparison between their Fig. 5 and the table accompanying it indicates that their value is a misprint. The ground-state Q value we obtained was -2.875 ± 0.010 MeV, in fair agreement with the value -2.860 ± 0.009 MeV calculated from the mass tables of Mattauch *et al.*[7]

A 35-μg/cm^2-thick SiO target, enriched to 95.55% in ^{30}Si and evaporated onto a 10-μg/cm^2 carbon foil, was used to observe the levels in ^{33}P. Other conditions were

[6] J. R. Erskine, Phys. Rev. **135**, B110 (1964).
[7] J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, Nucl. Phys. **67**, 1 (1965).

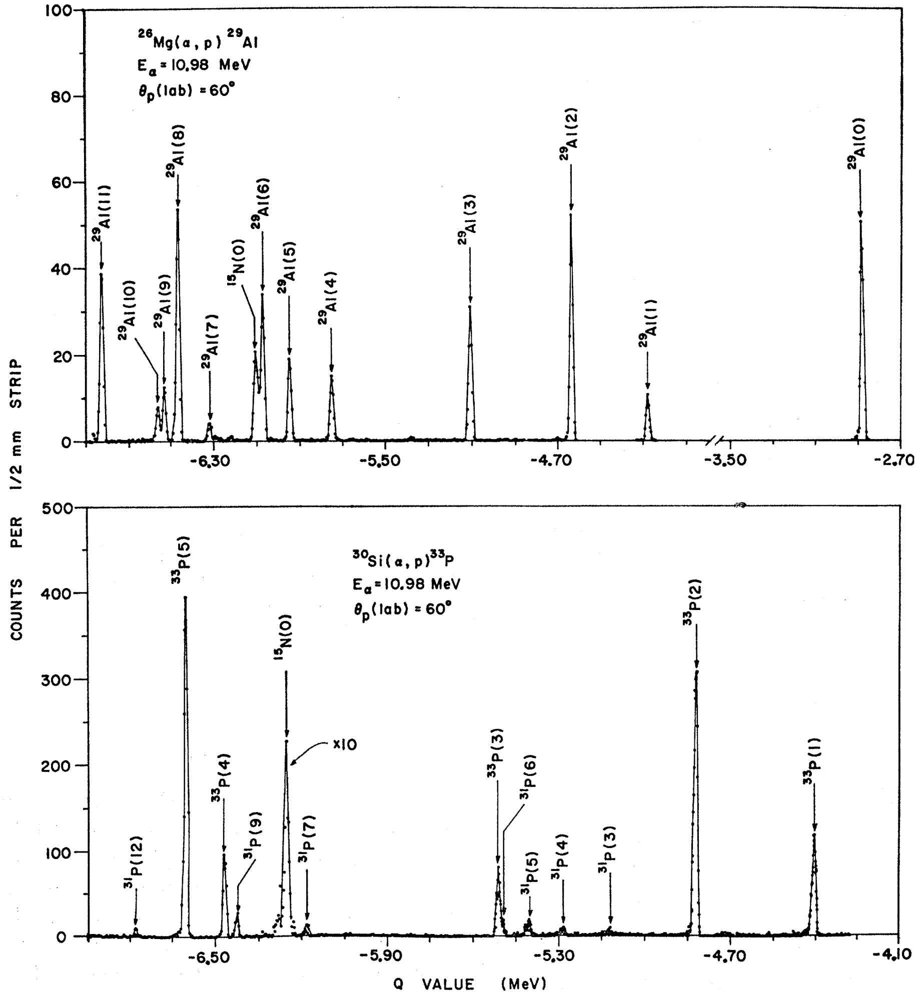

FIG. 1. Spectra from the ^{26}Mg(^{4}He,p)^{29}Al and ^{30}Si(^{4}He,p)^{33}P reactions. The energies measured with the Brown-Buechner magnetic spectrograph have been converted to Q values. The regions of no data were prescanned only and contained not more than one count per 0.5-mm strip. The ^{33}P ground-state group was not included in the exposure taken at 60°.

as described for ^{29}Al. Spectra were taken at five angles between 45° and 150°. Figure 1 shows a typical spectrum taken at 60°, the resolution width (full width at half-maximum) being about 15 keV. The strongest contaminant is from the ^{12}C(α,p_0)^{15}N reaction. The lines from the ^{28}Si(α,p)^{31}P reaction are also in evidence but could easily be separated from ^{33}P lines by tracking their energies as a function of angle.

TABLE I. Energy levels in ^{29}Al from the ^{26}Mg(^{4}He,p) reaction, as measured with the Argonne magnetic spectrograph. The second column lists the data of Jaffe *et al.*,[a] who used the ^{27}Al(t,p) reaction. Note that their value for the third excited state (in parentheses) is apparently a misprint.

Level No.	Excitation energy (MeV) Present work	Jaffe *et al.*[a]
1	1.405±0.010	1.406
2	1.759±0.006	1.762
3	2.228±0.006	(2.334)
4	2.873±0.006	2.875
5	3.069±0.006	3.071
6	3.193±0.006	3.191
7	3.439±0.010	3.434
8	3.584±0.008	3.584
9	3.647±0.010	3.646
10	3.679±0.010	3.676
11	3.947±0.010	3.941

[a] Reference 2.

Table II lists the five excited states of ^{33}P observed in the present work as well as the three previously known from the work of Currie.[1] Our energies are in good agreement for the first excited state and in fair agreement for the second; but for the third excited

TABLE II. Energies of the excited states of ^{33}P from the ^{30}Si(α,p) reaction, as measured with the Argonne magnetic spectrograph. The second column lists the data of Currie and Evans,[a] who studied the same reaction with a solid-state detector.

Level No.	Excitation energies (MeV) Present work	Currie and Evans[a]
1	1.436±0.008	1.43±0.01
2	1.852±0.008	1.81±0.01
3	2.540±0.008	2.42±0.03
4	3.500±0.008	Not observed
5	3.638±0.010	Not observed

[a] Reference 1.

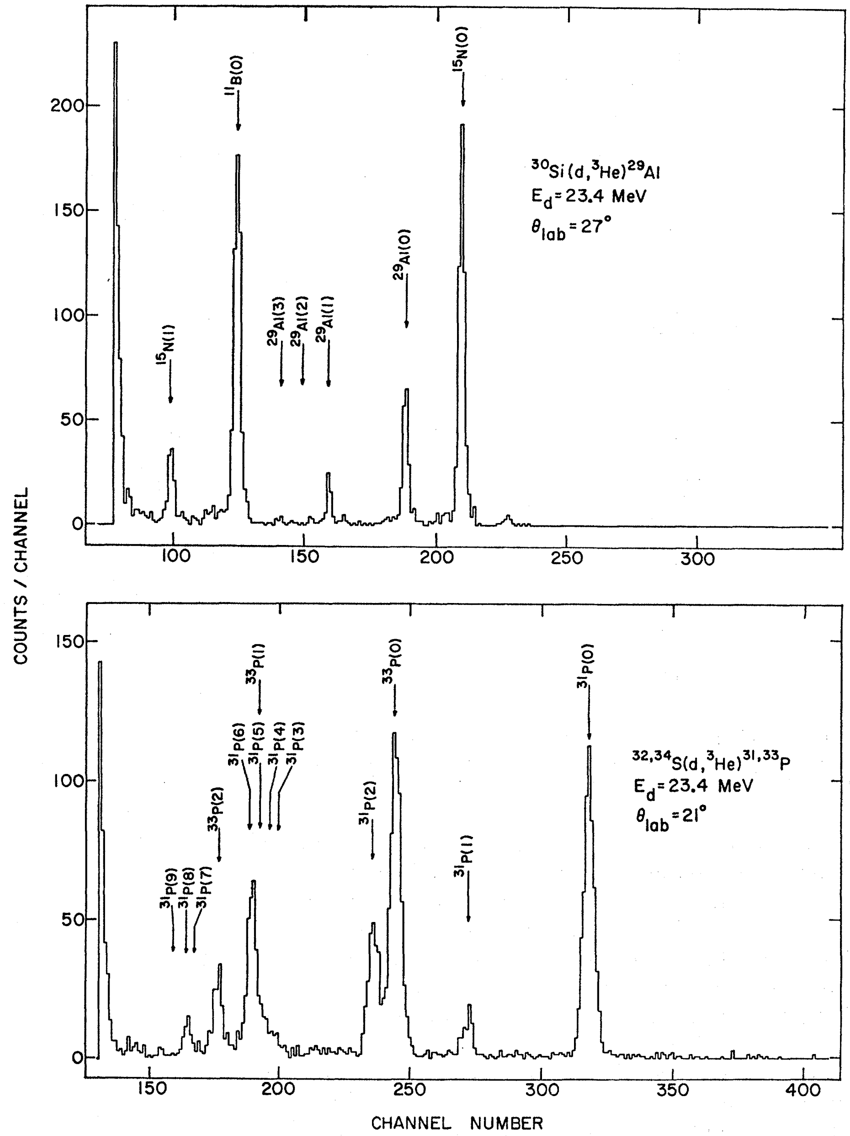

FIG. 2. Spectra obtained for the ^{30}Si(d,^{3}He)^{29}Al and S(d,^{3}He)P reactions by use of the 60-in. scattering chamber. The sulfur targets were enriched to 50% ^{34}S.

state the agreement is quite poor. Lines from the 3.41- and 3.505-MeV states of ^{31}P are quite close to the ^{33}P line, however; and while they were resolved in our spectrograph studies, they undoubtedly were not resolved in the work of Currie (who used a solid-state detector) and may very well be the cause of the discrepancy. The group from the fourth excited state at 3.500±0.005 MeV was observed at all five angles. The group which we attribute to the fifth excited state at 3.638±0.010 MeV was observed at only three angles. These states have not been previously reported. We obtained a ground-state Q value of −2.965±0.010 MeV for the ^{30}Si(α,p)^{33}P reaction, in excellent agreement with the value −2.969±0.005 MeV calculated from the mass tables of Mattauch *et al.*[7]

Angular Distributions

Angular distributions for the ^{30}Si(d,^{3}He)^{29}Al and ^{32,34}S(d,^{3}He)^{31,33}P reactions were taken in the 60-in. scattering chamber[8] with the 23.4-MeV deuteron beam of the Argonne cyclotron. The particles were identified with a surface-barrier E–dE/dx telescope. The dE/dx detector had a thickness of 65 μ. Targets of SiO evaporated onto carbon to an estimated thickness of 60 μg/cm^2 of ^{30}Si were used for the ^{29}Al studies. PbS with the natural isotopic concentration of ^{32}S was evaporated onto carbon for the ^{31}P measurements. These targets were estimated to have a thickness of 35 μg/cm^2 in ^{32}S. Similar targets enriched to 50% in ^{34}S were used for the

[8] J. L. Yntema and H. W. Ostrander, Nucl. Instr. Methods **16**, 69 (1962).

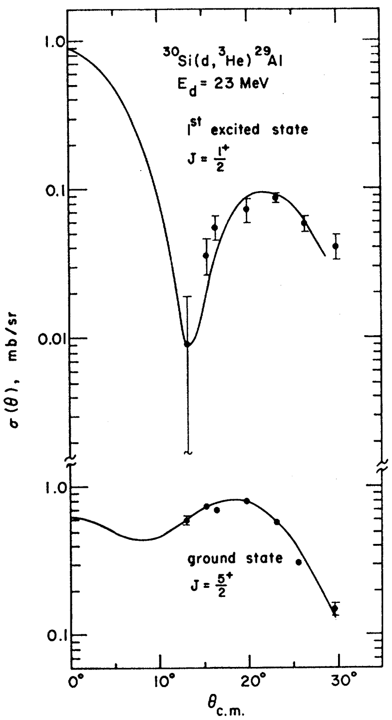

FIG. 3. The angular distributions obtained for the $^{30}Si(d,^{3}He)^{29}Al$ reaction. The solid lines are the curves obtained when the parameters listed in Table III were used in the code JULIE.

^{33}P measurements. The high intensity of the elastically scattered deuterons from the lead required the use of small solid angles for the detector system and low beam currents to avoid excessive pulse pileup with resulting loss of resolution. Therefore, each datum required an exposure of several hours. Representative spectra from both the $^{30}Si(d,^{3}He)^{29}Al$ and the $S(d,^{3}He)P$ reactions are shown in Fig. 2.

Relative cross sections were determined by calculating the area under each peak after suitable subtraction of background. To obtain absolute cross sections, it is necessary to know the target thickness. The thickness can be reasonably approximated by comparing the measured angular distribution of deuterons scattered elastically from the Si and S in the targets with the angular distributions obtained from the deuteron optical-model-potential parameters used in the distorted-wave calculations. The experimental angular distributions for the $(d,^{3}He)$ reaction on ^{30}Si, ^{32}S, and ^{34}S are shown in Figs. 3–5, respectively. The solid lines are the angular distributions predicted from distorted-wave calculations. The optical-model-potential parameters used were the same for all three nuclei and are listed in Table III. In single-nucleon-pickup reactions at this deuteron energy, all useful spectroscopic information (except J dependence) is contained in the angular range from 10° to 30°. The angular distributions were taken from 12° to 30° in 3° steps. Occasionally, a somewhat closer spacing at forward angles was desirable. The values of c^2S for the observed transitions are listed in Table IV.

3. RESULTS

$^{30}Si(d,^{3}He)^{29}Al$

For the ground state of ^{29}Al we obtain an $l=2$ angular distribution, which agrees with the $\frac{5}{2}^+$ spin assignment that Jaffe *et al.*[2] obtained for this state. For the first excited state we see an $l=0$ distribution requiring a $\frac{1}{2}^+$ assignment. Teitelman[4] has examined the excitation function for elastic scattering of protons from ^{28}Si in a search for the analogs of the ground state and first excited state of ^{29}Al in the compound system ^{29}P. His identification of the ground-state analog is unambiguous, but in the region where he expects the analog of the first excited state he observes two anomalies with a separation of about 16 keV. This observation possibly suggests that the first excited state of ^{29}Al may be a doublet. Our angular distributions for the reaction $^{30}Si(d,^{3}He)^{29}Al$ to the first excited state do not support the existence of this second state; but this evidence is inconclusive since the $^{30}Si(d,^{3}He)^{29}Al$ reaction excites only those states that are mainly single-proton holes in the *sd* shell. In a search for a doublet at 1.4 MeV excitation in ^{29}Al, we used thin ^{26}Mg targets in the magnetic spectrograph as described above. The spectrum showed no indication of a second state with a separation greater than about 15 keV and a strength greater than 10% of that of the first state.

$^{32}S(d,^{3}He)^{31}P$

We are able to extract angular distributions and spectroscopic factors for the first three states in ^{31}P. Higher states were not resolved. The data are well fitted by angular distributions calculated for $l=0$, $l=2$, and $l=2$ for the ground state, first excited state, and second excited state, respectively—as would be expected from the spin assignments[9] of $\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$.

TABLE III. Parameters used in the code JULIE.

	V (MeV)	r (F)	a (F)	W (MeV)	W' (MeV)	r' (F)	a' (F)	r_c (F)
Target nucleus+d	61.2	1.416	0.571	0	69.6	1.088	0.847	1.40
Final nucleus+He^3	172.6	1.40	0.603	33.5	0	1.70	0.80	1.40

[9] P. M. Endt and C. Van der Leun, Nucl. Phys. 34, 1 (1962).

There is no indication of the state that Colli *et al.*[10] have suggested in the region of 450 keV.

^{34}S$(d,{}^3$He$)^{33}$P

The ground-state distribution (Fig. 5) of ^{33}P is well fitted by $l=0$. This implies that it is a $\frac{1}{2}^+$ state, as expected. The first excited state was obscured by several levels in ^{31}P. The distribution for the second excited state of ^{33}P is well fitted by $l=2$. No other groups corresponding to states in ^{33}P were observed. We have extracted spectroscopic factors for the second excited state under the alternative assumptions of $d_{5/2}$ and $d_{3/2}$ assignments for this state. The $d_{3/2}$ assignment requires that c^2S be 50% greater than the limit for the $d_{3/2}$ strength. Hence we would tend to assign $J^\pi=\frac{5}{2}^+$ to this state—as would be expected if the first few levels in ^{33}P follow those in ^{31}P. The first excited state (for which we have no angular distribution) should then be the $\frac{3}{2}^+$ state.

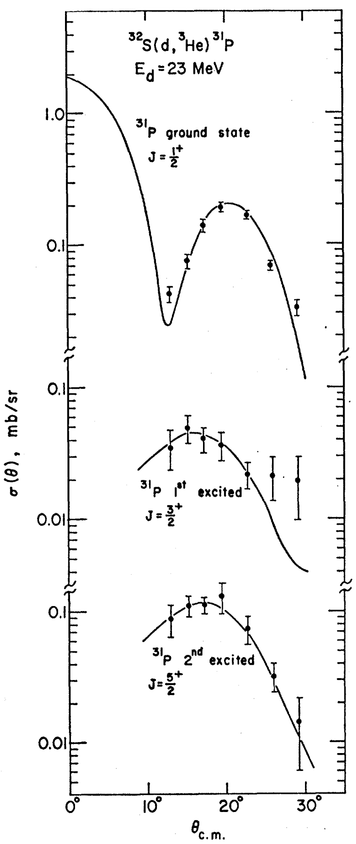

FIG. 4. The angular distributions obtained for the ^{32}S$(d,{}^3$He$)^{31}$P reaction. The fits were obtained with the code JULIE and the parameters listed in Table III.

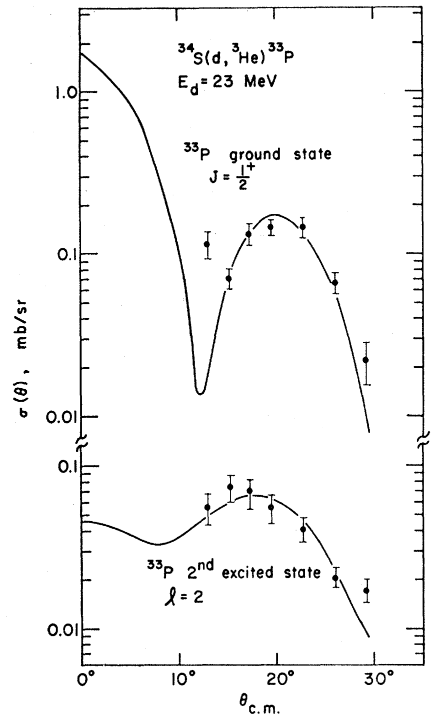

FIG. 5. The angular distributions obtained for the ^{34}S$(d,{}^3$He$)^{33}$P reactions. The solid lines are the distributions obtained with the code JULIE and the parameters listed in Table III.

4. DISCUSSION

The sum of the spectroscopic factors for the $\frac{5}{2}^+$ ground state ($S=5.6$) and the $\frac{1}{2}^+$ first excited state ($S=0.8$) of ^{29}Al is 6.4. Although this analysis is uncertain to ±25%, this suggests that most of the protons in ^{30}Si outisde of the $1p$ shell are in the $1d_{5/2}$ and $2s_{1/2}$ shells and there can be, at most, only one $d_{3/2}$ proton. The $d_{5/2}$ shell itself seems to be 70–90% full. Davies[11]

TABLE IV. Spectroscopic factors, l, and J^π obtained by comparing the results from the code JULIE with the data.

Nucleus	State	E_x (MeV)	l	J^π	Values of c^2S Present	Values of c^2S Glaudemans[a]
^{29}Al	Ground	0	2	$\frac{5}{2}^+$ [b]	5.6	...
	First	1.40	0	$\frac{1}{2}^+$	0.8	...
^{31}P	Ground	0	0	$\frac{1}{2}^+$ [b]	0.78	1.409
	First	1.27	2	$\frac{3}{2}^+$ [b]	0.83	0.536
	Second	2.23	2	$\frac{5}{2}^+$ [b]	2.0	
^{33}P	Ground	0	0	$\frac{1}{2}^+$	1.8	1.55
	First	1.4	Obscured by ^{31}P groups			...
	Second	1.8	2	$\frac{5}{2}^+$	3.4	...
			2	$\frac{3}{2}^+$	5.0	...

[a] Reference 14.
[b] Previously known, e.g., from Ref. 9.

[10] L. Colli, P. Forti, and E. Gadioli, Nucl. Phys. 54, 253 (1964).
[11] W. G. Davies, thesis, University of Alberta, 1966 (unpublished).
[12] J. B. French, in *Proceedings of the International Conference on Nuclear Spectroscopy with Direct Interactions*, edited by F. E. Throw (Argonne National Laboratory, Argonne, Illinois, 1964), Report No. ANL 6878, p. 181.

has measured spectroscopic factors for the $^{30}Si(d,n)^{31}P$ reaction. To compare those results with the present work, it is necessary to make some assumptions about the distribution of neutron holes in ^{30}Si. The spectroscopic factors for stripping are related to the hole distribution of the target nucleus by the expression[12]

$$\sum_{T<} [(2J_f+1)/(2J_i+1)]c^2S = \langle\text{proton holes}\rangle_j - [1/(2T_j+1)]\langle\text{neutron holes}\rangle_j .$$

The $^{30}Si(d,{}^3He)^{29}Al$ reaction indicates that the probable populations of proton holes for the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ shells are about 1.0, 1.5, and 3.5, respectively. If the neutron-hole populations are assumed to be 1.0, 0.5, and 2.5 for the same orbits, then the relative spectroscopic factors would be expected to be in the ratio 1.0:2.0:0.5 for the $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ states. This appears to be in excellent agreement with the ratio 1.0:1.98:0.76 which Davies[11] obtained for the states in question; however, it is probable that the $d_{5/2}$ strength is spread over several states and hence the total $d_{5/2}$ spectroscopic factor for all the $d_{5/2}$ states seen in the $^{30}Si(d,n)^{31}P$ reaction may be somewhat larger than the value suggested by the present work.

Our $^{32}S(d,{}^3He)^{31}P$ spectroscopic factors for the $\frac{1}{2}^+$ ground state ($S=0.78$) and $\frac{3}{2}^+$ first excited state ($S=0.83$) indicate that the $d_{3/2}$ and $2s_{1/2}$ shells in ^{32}S are about equally populated. Their sum of 1.6 for these two shells accounts for most of the proton strength beyond ^{28}Si. The distribution of the neutron strength in ^{32}S, observed by Fou and Zurmühle,[13] is similar to the distribution of the proton strength. In addition, they observe that the $d_{5/2}$ strength in ^{31}S, the mirror of ^{31}P, is spread over at least four states. Our results indicate that this may also be true for ^{31}P, but we are unable to resolve the higher $d_{5/2}$ states. With the addition of two neutrons to ^{32}S, the $2s_{1/2}$ proton shell becomes almost full. Also, the $d_{5/2}$ strength in the first $\frac{5}{2}^+$ state of ^{33}P seems to be 50% greater than that for the first $\frac{5}{2}^+$ state in ^{31}P. Since the total $d_{5/2}$ strength could be as large as 6.0, there may be other $\frac{5}{2}^+$ hole states that we are not observing. It is unfortunate that we cannot resolve the first excited state, since this is probably the $d_{3/2}$ state; it would be interesting to see how the strength in this state has changed, i.e., whether it has given up its strength to the $2s_{1/2}$ state or whether this extra strength in the $s_{1/2}$ state has come at the expense of the $d_{5/2}$ strength.

Glaudemans *et al.*[14] have treated this region of the *sd* shell in some detail by considering those states formed by the coupling of the nucleons outside the $d_{5/2}$ shell, which they assume to be closed. Their semiempirical calculation of the energies and spin parities of the various states in ^{31}P and ^{33}P is in good agreement with observed level schemes for the low-lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states. One would expect agreement for ^{31}P, since they used the properties of this nucleus in determining their parameters by a least-squares fit. We have used the wave functions and prescriptions of their treatment in our calculation of the spectroscopic factors for those transfers that we observe. These values are given in Table IV along with the experimental values. For the $^{32}S(d,{}^3He)^{31}P$ reaction, their model predicts a larger value for the ground-state S and a smaller value for the first-excited-state S than is experimentally observed. For the $^{34}S(d,{}^3He)^{33}P$ reaction, theory and experiment are in much better agreement for the ground-state S, but no experimental results are available for comparison with the theoretically predicted value of S for the first excited state. The $d_{5/2}$ strengths in ^{31}P and ^{33}P are not, of course, correctly predicted by the model; they arise from excitation of the $d_{5/2}$ core, which is assumed to be closed in the calculations of Glaudemans *et al.*

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[13] C. M. Fou and R. W. Zurmühle, Phys. Rev. **151**, 927 (1966).
[14] P. W. M. Glaudemans, G. Wiechers, and P. J. Brussaard, Nucl. Phys. **56**, 548 (1964).