^{28}Si(d,^{3}He)^{27}Al Reaction and the Structure of ^{28}Si and ^{27}Al†

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The level structure of ^{27}Al from 0- to 6.5-MeV excitation has been studied with the ^{28}Si(d,^{3}He)^{27}Al reaction at E_d=34.4 MeV. Eight experimental angular distributions are presented. Assignments of l values and spectroscopic factors are made with a local, zero-range distorted-wave Born-approximation analysis. The deduced l values are consistent with the known spins of levels of ^{27}Al of excitation energy $\leq$3.0 MeV. Levels at 4.05- and 5.15-MeV excitation are assigned l=1, and the level at 4.40-MeV excitation is assigned l=2. The extracted spectroscopic factors are discussed in terms of some simple models for ^{27}Al and compared with results from new shell-model calculations.

INTRODUCTION

THE single-nucleon pickup reaction provides a basic experimental technique for the investigation of the wave functions of the ground and excited states of nuclei. The shapes of the angular distributions and the magnitudes of the cross sections of this reaction can be analyzed with the distorted-wave Born approximation (DWBA) to extract the values of the transferred orbital angular momenta and the spectroscopic factors for the various residual levels.[1] If data are obtained for an adequate sampling of residual levels, the sums of the spectroscopic factors yield the nucleon occupation numbers for the accessible shell-model orbits in the target nucleus ground state.[2]

The multiple experimental advantages which accrue from working with charged particles in both the entrance and exit channels are such that current studies of proton pickup generally utilize the (d,^{3}He) or, less commonly, the (t,α) reactions rather than the analogous (n,d) reaction. Until recently, however, a theoretical treatment in terms of the distorted-wave Born approximation which predicted absolute cross sections was available only for the nucleon $\rightleftarrows$ deuteron examples of the transfer reactions.[1] The element of the theory in question involved the overlap of the wave functions of the incident and exiting light particles, a term which enters the expression for the reaction cross section as an over-all normalization. The overlap calculation for the mass-1–mass-2 case is immediately tractable. The corresponding overlaps for the case of deuteron $\rightleftarrows$ ^{3}He or H^3 transitions have now been evaluated.[3] The normalizing values[3] obtained, when used in the appropriate DWBA calculations, correctly predict the measured cross sections for (d,t) and (d,^{3}He) reactions on such archetypal "closed-shell" nuclei as ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb.[4–6] Another factor which is vital to a proper treatment of mass-3 particles in the DWBA analysis is a knowledge of the general trends of the elastic scattering and, hence, of the optical potentials which enter the DW calculations. Such studies have recently become available.[7–9] Thus, the mass 2 $\rightleftarrows$ mass 3 transfer reactions now appear capable of yielding spectroscopic information of accuracy comparable to that obtained with the d $\rightleftarrows$ nucleon reactions.

This report presents results[10] of an experimental study of the ^{28}Si(d,^{3}He)^{27}Al reaction and discusses the implications of these results for the structure of the ground state of ^{28}Si and the low-lying states of ^{27}Al. The experimental investigations[4] of the ^{16}O(d,^{3}He)^{15}N and ^{40}Ca (d,^{3}He)^{39}K reactions have laid the foundations for a systematic study of proton pickup via the (d,^{3}He) reaction in 1d-2s nuclei. The central position of ^{28}Si in the s-d shell and its supposed semiclosed nature suggest the early inclusion of this nucleus in such a program. The level density of the residual nucleus ^{27}Al is such that a good number of excited states can be cleanly separated with our experimental resolution. In addition, its level structure has been relatively well studied by other techniques.[11] Several theoretical models[12–14] have been proposed for ^{28}Si and ^{27}Al. Their respective degrees of validity can be examined, in some instances, by a comparison of experimentally determined spectroscopic factors for proton transfer to the various predicted values. In addition to weak- and strong-coupling calculations and their variants, the results of recent shell-

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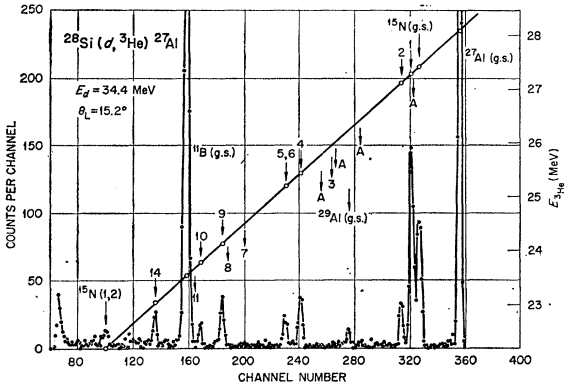

FIG. 1. Pulse-height distribution of ^{3}He particles emitted at $\theta_L = 15.2°$ from the natural silicon target bombarded with 34.4-MeV deuterons. The groups corresponding to formation of levels of ^{27}Al are labeled with the numbering scheme of Table II. Some energies corresponding to levels of ^{28}Al formed with the ^{29}Si(d,^{3}He)^{28}Al reaction are labeled by the letter A. Noted explicitly are groups from the (d,^{3}He) reactions on ^{30}Si, ^{16}O, and ^{12}C.

model calculations for these nuclei have become available.[15] Because the configuration basis used in these new calculations is restricted to the $d_{5/2}$ and $s_{1/2}$ shells, some of the observed transfer phenomena are, *a priori*, not accounted for. However, both the general level structure of ^{28}Si and ^{27}Al and the interrelationship of the observed $2s_{1/2}$ and $1d_{5/2}$ spectroscopic strength in the ^{28}Si(d,^{3}He)^{27}Al reaction are well reproduced.

EXPERIMENTAL PROCEDURE

The present experiment was performed with a 34.4-MeV deuteron beam from the Oak Ridge isochronous cyclotron. The target was a layer of natural silicon evaporated onto a very thin carbon foil. The ^{3}He particles were detected and distinguished from the other reaction products with a $\Delta E-E$ solid-state detector telescope and summing electronics. The coincident ΔE and $E+\Delta E$ signals were ultimately recorded in a two-dimensional multichannel display. Complete separation of the ^{3}He and ^{4}He bands was thus achieved and constantly monitored.

The two-dimensional spectra were reduced off-line to conventional particle energy versus intensity plots with computer programs. Such a spectrum, taken at $\theta_L = 15.2°$, is shown in Fig. 1. The energy calibration of these spectra was obtained by reference to the known energies of the low-lying levels of ^{27}Al, to the Q values of the ground state (d,^{3}He) reactions on ^{12}C, ^{16}O, and ^{28}Si, and to the calculated kinematic dependence of the energies of the ^{3}He particles with respect to the angle of observation. The differential calibration thus obtained was 23 keV per channel, while the full width at half-maximum (FWHM) values for the particle groups from the Si reaction were approximately 3–½ channels. The energies of the higher ^{27}Al levels excited with the present reaction could be assigned on this basis to within better than ±50 keV and subsequently related to the level structure of ^{27}Al tabulated in Ref. 11. These more precise energy values are used in the remainder of the paper. As can be seen from Fig. 1, there are seven particle groups attributed to excited levels of ^{27}Al, which, at forward angles, have intensities ≥5% of the ground-state intensity. The differential cross sections of these groups were measured as a function of angle and the results are presented subsequently. Upper limits are also placed on the cross sections for several other known levels of ^{27}Al. The span of excitation energy in ^{27}Al which is covered by the present data extends to 6.5 MeV.

The absolute cross sections given for the various ^{28}Si(d,^{3}He)^{27}Al transitions are based on an average of cross-section values obtained from a comparison with elastic deuteron scattering and from a direct measurement with a SiO_2 foil. For the elastic scattering comparison, the Si target and the apertures defining the solid angle of the detector system were left undisturbed after the (d,^{3}He) measurements, and the $\Delta E-E$ telescope was replaced by a detector capable of stopping the 34.4-MeV deuterons. Elastic scattering was measured under these conditions at angles between 30° and 38° in the lab system, where the Si scattering is easily resolved from that of oxygen and the cross section is relatively constant with respect to angular variations. The connection between detected reaction events per unit incident charge and millibarns per steradian was made by assuming the c.m. cross section for the ^{28}Si(d,d)^{28}Si reaction at 34.4 MeV, $\theta_{cm}=32°$, to be 56 mb/sr. This is the value given by an optical-model calculation which employs potential well parameters obtained from formulas based on the optical-model analysis of extensive 34.4-MeV deuteron elastic scattering measurements.[16] In the other measurement, the yield of the ground-state ^{3}He group from a blown glass foil whose areal density was determined by subsequent weighing, was measured directly. The resulting two values for the cross-section normalization differed by 10% and the average value used is assigned a total 15% uncertainty.

DWBA CALCULATIONS

The predictions of the local zero-range DWBA for the pickup of protons of given sets of the quantum numbers n, l, and j were calculated with the computer code JULIE[1] for the transitions to the various levels of ^{27}Al. It has been shown[4] that at the energies of the present experiment the additions of the refinements of nonlocality and finite range to the basic DWBA calculations do not significantly affect the shapes of the predicted differential cross sections. As the effects that do occur from these refinements have been thoroughly

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TABLE I. DWBA parameters used in the calculations from which the curves in Figs. 2, 3, and 4 and the spectroscopic factors in Table II were obtained.

	V (MeV)	r_0 (F)	r_{0c} (F)	a (F)	W (MeV)	r_0' (F)	a' (F)	W_D (MeV)	V_{so} (MeV)
deuteron	94.3	1.046	1.3	0.807	0	1.357	0.733	11.0	7.0
^{3}He	143.4	1.14	1.4	0.723	23.8	1.60	0.81	0	0
bound state		1.20	1.20	0.65		$\lambda=25$			

described,[4] we did not include finite-range and nonlocal calculations in the present study.

The parameters used for the deuteron optical-model potential in the DWBA calculations were those of Newman and co-workers,[16] which were mentioned in the preceding section. Preliminary elastic scattering investigations on *s-d* shell nuclei indicate that this potential is adequate in the present application. The parameters for the ^{3}He potential were obtained from a search with the optical-model code HUNTER[17] over the data of Baugh *et al.*[7] for the scattering of 29-MeV ^{3}He from ^{27}Al. The search was constrained so as to hold the geometry of the well to different values from those used by Baugh *et al.*, our choice being consistent with a recent analysis of ^{3}He scattering by Bassel and collaborators.[8] This technique resulted in real potentials in the 145- and 175-MeV regions for nuclei from ^{24}Mg to ^{40}Ca, in contrast to the 100-MeV potentials obtained by Baugh *et al.* It transpires that the ^{3}He potentials characterized by the 100-MeV depth yield quite different shapes when used in the DWBA calculations than do the equivalent 145- and 175-MeV potentials. The DWBA predictions based on the latter potentials agree much better with the present experimental shapes for the measured l transfers of 0, 1, and 2. This preference for the deeper potentials is consistent with $(d,^3He)$ results on other nuclei.[4] A real potential of depth of 175 MeV, which would be consistent[8] with the results of ^{3}He scattering from nuclei heavier than 40Ca, produces DWBA curves which fit the data as well over all as do the curves calculated from the 145-MeV potential obtained from a best fit of the ^{27}Al data. In particular, the second maxima and minima of the $l=2$ distributions are best reproduced by the deepest potential. The maximum cross sections resulting from calculations with the 100-, 145-, and 175-MeV potentials agree to within about 10%.

The values of the various optical-model and bound-state wave-function parameters that were employed in the JULIE calculations for the present study are listed in Table I. The presence of spin-orbit terms in the deuteron channel and particularly in the bound-state potentials results in a dependence of the predictions upon the spin of the residual state. The effects of this j dependence in regard to the shapes of the angular distributions are insignificant in the angular range studied, but over all magnitudes are appreciably affected. Because of limitations in computer storage space, the spin-orbit potentials were not included for the distorted wave $l=3$ and 4 calculations but were kept in the bound-state potentials. It might be noted here that the over-all magnitude (but not the angular variations) of the differential cross section as computed in the DWBA is very sensitive to the bound-state wave function. A change of 4% in the radius parameter (1.20 to 1.25 F) results in ≈20% increases in magnitudes. The dimensions chosen for the bound-state well are those employed in the study of the $(d,^3He)$ reaction on ^{16}O and ^{40}Ca. Together with the value[3] of 2.95 for the $(d,^3He)$ normalizing factor, they produce, as will be seen, spectroscopic strengths which satisfy the sum-rule limit to well within the uncertainties involved. The bound-state wave functions were calculated in all instances such that the extracted particles were bound by their separation energies as given by the excitations of the residual levels.

These are various problems inherent in the process of relating experimental nucleon transfer cross sections to amplitudes in nuclear wave functions with a DWBA analysis. Several discussions appear in the literature[18–21] and the general conclusion is that the limit of accuracy of the current DWBA analysis itself is ±25%. The deformations which are probable for the nuclei studied in the present instance doubtless magnify these uncertainties.[22,23] In consideration of a point of particular interest for a pickup experiment, it might be noted that that attribution of admixtures of higher configurations to a a nuclear ground state on the basis of observed pickup strength is particularly questionable if multistep processes are capable of generating the requisite cross sections. Contributions of this sort are difficult to rule out.[24,25] There is experimental evidence,[26] however, that the angular distributions resulting from multistep or target excitation processes differ significantly from the usual shapes, and theoretical work on the problem suggests that the strengths of such processes are perhaps

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TABLE II. Levels of ^{27}Al below 5.2 MeV.

No.	Level energy[a] (MeV)	j^{π} [a]	l[b]	C^2S_{exp}[b]	C^2S_{sm}[c]		$\sum C^2S(j)$ exp	$\sum C^2S(j)$ Nilsson $\eta=-4$	$\eta=-2$	$\eta=+2$	$\eta=+4$
0	0.000	$\frac{5}{2}^+$	2	3.76	3.92	$\sum C^2S(\frac{1}{2}^+)$	0.49	1.11	0.74	0.31	1.06
1	0.842	$\frac{1}{2}^+$	0	0.49	0.83	$\sum C^2S(\frac{3}{2}^+)$	0.56	0.42	0.09	0.07	1.12
2	1.013	$\frac{3}{2}^+$	2	0.56	...	$\sum C^2S(\frac{5}{2}^+)$	4.77	4.47	5.17	5.62	3.82
3	2.212	$\frac{7}{2}^+$	...	($\leq$0.4)	...						
4	2.731	$\frac{5}{2}^+$	2	0.61	0.54						
5	2.976	$\frac{3}{2}^+$	(2)	$\leq$0.4[d]	...						
6	3.000	$\frac{9}{2}^+$	(4)	($\leq$0.6)	...						
7	3.674	$\frac{1}{2}^+$	...	$\leq$0.02	0.06						
8	3.951	$\frac{3}{2}$	...								
				$n=1$ $n=2$							
9	4.052	$\frac{1}{2}^-$, $(\frac{3}{2})^-$	1	$j=\frac{1}{2}$: 1.8 0.43							
				$j=\frac{3}{2}$: 1.5 0.36							
10	4.403	$(\frac{5}{2})^+$	2	0.35							
11	4.504		...								
12	4.58		...								
13	4.81		...								
				$n=1$ $n=2$							
14	5.149	$(\frac{1}{2},\frac{3}{2})^-$	1	$j=\frac{1}{2}$: 1.2 0.29							
				$j=\frac{3}{2}$: 1.0 0.24							

[a] References 11, 28, 29, 30, and present work.
[b] Recent direct reaction studies which succeed in resolving the 3-MeV doublet [H. E. Grove *et al.*, Proceedings of the International Conference on Nuclear Structure, Tokyo, 1967 (unpublished)] indicate that none of the observed pickup cross section can be attributed to a single-step $l=2$ transition.
[c] Reference 15.
[d] Present work.

a factor of 10 less than the corresponding single-step mechanism.[27]

EXPERIMENTAL RESULTS

The levels of ^{27}Al having excitation energies less than 5.2 MeV[11] are presented in Table II. The numbering of the levels in the first column is the same as that employed to label the particle groups in Fig. 1. The spins and parities of the first seven levels are taken from Ref. 11 and the current literature.[28–30] The l values are those assigned from the present work. The spectroscopic factors C^2S_{exp} listed in Table II were extracted from the data with the DWBA analysis described in the preceding section. There are "experimental" errors in the values of the spectroscopic factors which result from the uncertainty (15%) in the over-all cross-section normalization and from the latitude available in normalizing the theoretical predictions to the data. These latter may be readily evaluated by reference to the figures of the angular distributions. As mentioned earlier, there are inherent uncertainties in the DWBA analysis itself of the order of 25%.

The experimental angular distributions of the transitions assigned $l_p=2$ are presented in Fig. 2, along with the $l=2$ DWBA predictions. The 4.403-MeV level constitutes the only new assignment in this group. The statistical errors in the yield from this level are large at all of the angles and at $\theta_L=10.4°$ the yield to this level is obliterated by the group from the carbon contaminant. However, comparison with $l=3$ and $l=0$ predictions, the most likely alternatives, clearly indicates the preference for an $l=2$ assignment. The agreement between the calculations and the data for the ground-state distribution is quite good except for the 45° point where experiment is lower than theory by about a factor of 2. This same feature occurs in the 1.013- and 2.731-MeV distributions, but since 45° represents a minimum in the distribution, this was not considered a serious deficiency. It might be noted that the Q value of the ground-state transition is such that the exiting ^{3}He particles have about 29 MeV of kinetic energy, the energy at which the ^{3}He elastic scattering data was taken.

The angular distributions of the transitions to the $\frac{1}{2}^+$, 0.842-MeV level and to the unresolved doublet at 3.0 MeV are shown in Fig. 3, together with DWBA predictions for l transfers of 0, 2, and 4. The distribution of the transition to the doublet was fitted with an incoherent sum of $l=2$ and $l=4$ predictions. The amounts of the respective contributions were determined by minimizing the RMS deviation of the sum curve from the experimental points. The smooth curve through the data points is the result. The upper limits on the spectroscopic factors listed for the 2.976- and 3.000-MeV levels were extracted on this basis.

The angular distributions of the remaining two transitions that possessed strength sufficient to permit

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data analysis are presented in Fig. 4, along with DWBA predictions for $l=0$, 1, and 2. As can be seen in the figure, the data are in good agreement with the predictions for $l=1$ and are completely distinctive from the $l=0$ and 2 curves. Thus, we conclude that the transitions to the 4.052- and 5.149-MeV levels proceed by pickup of $l=1$ protons.

There are three known levels below 4-MeV excitation in ^{27}Al which are not populated with the present reaction with sufficient intensity to make them consistently distinguishable from the background of 1–2 counts per channel. These are the levels at 2.212, 3.674, and 3.951 MeV in Table II. Upper limits on the spectroscopic factors for the first two of these levels are listed in Table II. It is to be emphasized that the numbers given are upper limits and not "best estimates."

DISCUSSION

A. Previous Transfer Results

Neutron pickup from ^{28}Si leading to some of the lower levels of ^{27}Si has been studied with the (^{3}He,^{4}He) reaction at bombarding energies of 10 and 15 MeV.[31–33] The more recent experimental results were analyzed with the DWBA theory and spectroscopic factors for the levels thus extracted. These results and the corresponding spectroscopic factors obtained in the present study are grouped for comparison in Table III. The respective numbers are in qualitative agreement. The present results are felt to be more reliable than the 10-MeV (^{3}He,^{4}He) values because, as is discussed in Ref. 31, ^{4}He scattering from ^{27}Al at $\sim$10 MeV is not consistent with an optical-model interpretation and, in addition, the (^{3}He,^{4}He) cross sections exhibit significant statistical-like fluctuations with changes in the incident energy. It is not known to what extent these phenomena are present at 15 MeV, but the agreement of these results with the present work is better than for the 10-MeV numbers. The spectroscopic factors given in Table III for the present work are in good agreement with preliminary results of recent (d,^{3}He) and (d,t) experiments at lower deuteron energies.[34,35]

B. $\frac{7}{2}^+$ and $\frac{9}{2}^+$ Levels

The ^{27}Al level at 2.212-MeV excitation is securely assigned[11,29] as $\frac{7}{2}^+$. One member of the 3.0-MeV doublet is assigned $\frac{3}{2}^+$ while the other has been assigned[36,28] both as $\frac{7}{2}^+$ and $\frac{9}{2}^+$. It would be extremely valuable to have this latter uncertainty settled with finality, since it bears directly on the success of the various nuclear models applied to ^{27}Al. We will, in accord with the recent work[29] of Sheppard and Van der Leun, refer to the high-spin member of the 3.0-MeV doublet as a $\frac{9}{2}^+$ level.

The present reaction populates the $\frac{7}{2}^+$ level at 2.212 MeV very weakly if at all. The nearness of groups from the isotopic contamination in the natural Si target (labels "A" in Fig. 1) prevent the setting of the limit on its possible spectroscopic value at a lower value than is given in Table II. In contrast, the angular distribution of the 3.0-MeV doublet group differs significantly from the $l=2$ distribution to be expected for the $\frac{3}{2}^+$ state alone, indicating a significant cross section

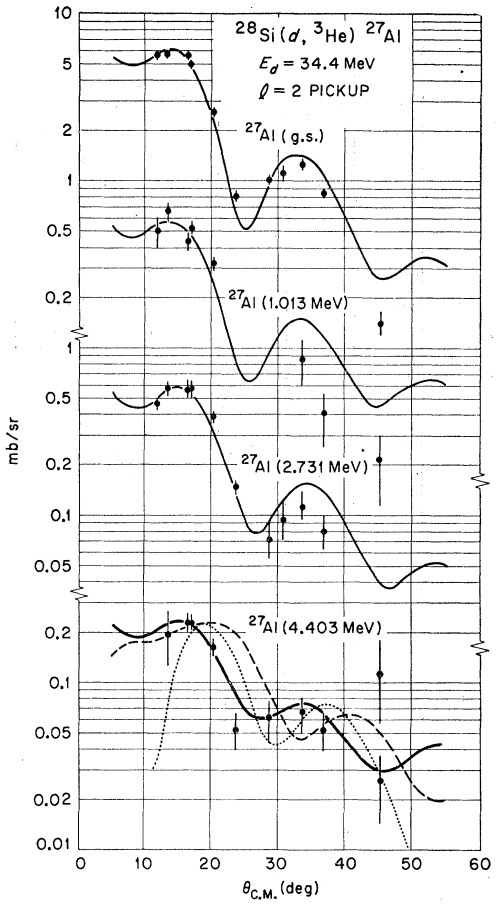

FIG. 2. Angular distributions of ^{28}Si(d,^{3}He)^{27}Al transitions that are assigned $l=2$. The curves are DWBA predictions based on the parameters of Table I.

TABLE III. Comparison of ^{28}Si(d,^{3}He)^{27}Al and ^{28}Si(^{3}He,α)^{27}Si spectroscopic factors.

Level energy (MeV)	l	(^{3}He,α)[a] $E_{^3He}=10$ MeV	(^{3}He,α)[b] $E_{^3He}=15$ MeV	(d,^{3}He) $E_d=34.4$ MeV
0.000	2	2.0	2.99	3.76
0.84–0.78	0	0.7	0.42	0.49
1.01–0.96	2	0.5	0.38	0.56
2.73–2.65	2	1.3	0.78	0.61

[a] Reference 32. [b] Reference 33.

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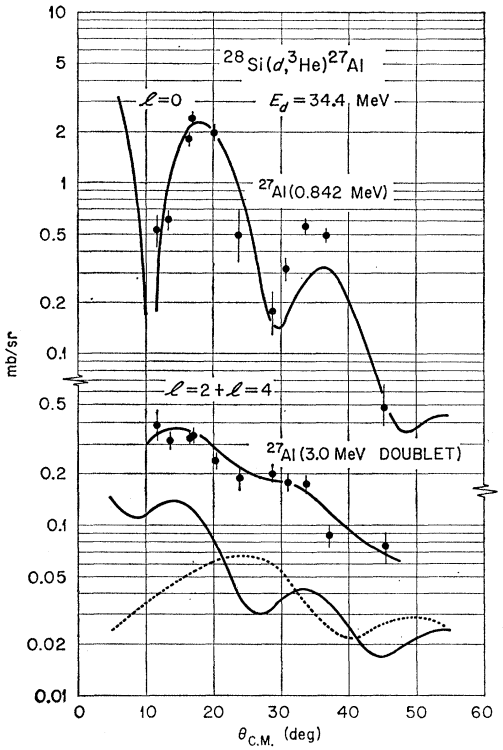

FIG. 3. Angular distributions of the $l=0$ level at 0.842 MeV and of the $\frac{3}{2}^+$-$\frac{5}{2}^+$ doublet at 3.0 MeV.

for the high-spin member. As was mentioned and as illustrated in Fig. 3, the experimental distribution for this doublet can be well-fitted with a sum of $l=2$ and 4 DWBA curves, and the resultant spectroscopic factor implies for the $\frac{5}{2}^+$ state is rather large. We do not attach great significance to this number, however, as the $l=4$ DWBA curve is so indistinctive that similar results might be obtained with various "semi-isotropic" shapes combined with an $l=2$ shape.

A further illustration of the complexities involved in evaluating evidence for possible $l=4$ pickup in this region can be obtained from a recent study[37] of the (d,t) and (^{3}He,^{4}He) reactions on ^{26}Mg. In this instance the low-lying $\frac{7}{2}^+$ level in ^{25}Mg was excited about 10 times more strongly than the neighboring $\frac{5}{2}^+$ level, in direct contrast with the present results for ^{27}Al. The inelastic scattering of protons from ^{25}Mg and ^{27}Al also seem to show this reversal of strengths between the $\frac{7}{2}^+$ and $\frac{5}{2}^+$ levels.[13] Thus, while various $\frac{7}{2}^+$ and $\frac{5}{2}^+$ levels in this region are appreciably excited with pickup reactions, there is in this work no conclusive evidence that these cases proceed via single-step direct transitions and, hence, none for $1g_{9/2}$ or $1g_{7/2}$ configuration admixtures in the ground states of these middle-sd shell nuclei.

C. $l=1$ Levels

The $l=1$ transitions to the levels at 4.052 and 5.149 MeV present an interesting problem in interpretation. Alternate values for the spectroscopic factors of these levels are given in Table II, corresponding to the assumptions that the extracted proton is from the $1p$ or $2p$ shell. For the assumption of $1p$ origin, which would imply a spin of $\frac{1}{2}^-$ for at least the lower level, the resultant spectroscopic factors account, at a minimum, for essentially all of the total strength available from the $1p_{1/2}$ shell. The 4.052-MeV level would, under this assumption, be analogous to, say, the low-lying[38] $\frac{1}{2}^-$ level in ^{19}F and, perhaps, the 2.77–2.64 MeV level in ^{23}Mg-^{23}Na, which has been assigned as $J=\frac{1}{2}$ by particle-γ correlation techniques[39,40] and as $l=1$ by pickup experiments.[40,41] Unless the remainder of the strength lies very high, the implication of this assumption of $1p$ origin of the $l=1$ transition would be to center the energy of the $1p$ single-hole configuration in ^{27}Al at close to 5-MeV excitation. This is in contradiction to the relevant proton knockout experiments[42] and to current Hartree-Fock calculations,[43] both of which yield a splitting of the $1p$ and $1d$-$2s$ shells of around 15 MeV for the Al-Si region.

The alternative to the assumption of a purely $1p$ origin for the $l=1$ levels is that the transitions proceed to some extent via pickup from $(sd)^{12-2n}(2p)^{2n}$ components in the ^{28}Si ground-state wave function. The deformation usually associated with the ^{28}Si nucleus

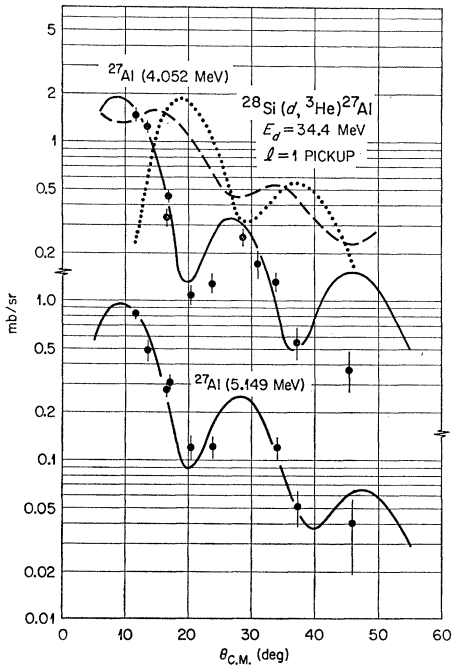

FIG. 4. Angular distributions of the two ^{28}Si(d,^{3}He)^{27}Al transitions assigned as $l=1$. The solid, dashed, and dotted curves show respectively, the $l=1$, 2, and 0 DWBA predictions.

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would imply extensive mixing of higher-lying configurations into the dominant s-d basis. The amount of such admixtures that would be allowed by assuming a pure $2p$ origin for the $l=1$ pickup strength (see Table II) seems very high, however. Of course, in this particular instance, the ubiquitous problem in DWBA calculations of the bound-state wave function, specifically the question of how to bind $2p$ particles in the ^{28}Si potential, could have very significant effects on the spectroscopic factors. The use of the separation energies in the bound-state calculations produces what should be in effect minimum cross sections and maximum spectroscopic factors for $l=1$, $n=2$ pickup.

Realistically, one might expect that for $l=1$ levels at this energy of excitation in ^{27}Al the $(1p)^{-1}$ and $(2p)^1$ configurations are considerably admixed. It thus might be possible to find an acceptable middle ground by distributing the observed spectroscopic strength between pickup of $1p$ and $2p$ particles. Unfortunately, it is essentially impossible to distinguish between the two cases from an analysis of the observed angular distribution other than on the basis of the over-all intensities as we have attempted.

Some recent results from other experiments have some bearing on the character of these levels. The 4.052-MeV level has been assigned a spin of $\frac{1}{2}$ on the basis of γ-ray correlation experiments.[29,44] In another experiment, the ^{26}Mg(d,n)^{27}Al reaction at 3 MeV has shown a strong, characteristic $l=0$ distribution to the 3.764-MeV level but only a weak and anomalous distribution for the 4.052-MeV level,[30] thus mitigating against a positive parity, $J=\frac{1}{2}$ assignment for the 4.052-MeV level. These two pieces of information, together with our data, thus indicate a $\frac{1}{2}^-$ assignment for the 4.052-MeV level. Secondly, a recent report[45] on the ^{30}Si(d,t)^{29}Si reaction shows that the $\frac{7}{2}^-$ and $\frac{3}{2}^-$ "single-particle" states in ^{29}Si at 3.62 and 4.93 MeV are very weakly excited, thus indicating very small sd-fp configuration mixing in the 16 neutrons in ^{30}Si. The additional two particles could possibly anneal some of the deformation of ^{28}Si and thus the admixtures in ^{28}Si might be expected to be higher than in ^{30}Si, but on the whole, the ^{30}Si results make large $2p$ admixtures in ^{28}Si less likely. A more extensive body of stripping and pickup data to the higher excited states of several nuclei and their interpretation with a DWBA analysis which incorporates such refinements as considerations of the deformations of the states involved will perhaps permit a more definitive delineation of the effective roles of the $1p$ and $2p$ shells in this region.

D. $l=2$ and $l=0$ Levels

Five of the angular distributions measured in the present experiment show the characteristics of the $l=2$ and 0 distributions that are expected for the transfer of $1d$- and $2s$-shell nucleons. The sum of the spectroscopic factors extracted from these distributions is 5.8 (see Table II). This number agrees very well with the sum-rule limit of 6 which results from the assumption that all of the protons in ^{28}Si exterior to the ^{16}O core occupy s-d shell orbits. To the extent that excitations from the s-d shell to the $1f$, $2p$, and $1g$ shells exist in the ^{28}Si ground state (as discussed above), this limit of 6 for the s-d shell must be reduced. The experimental sum is also subject to change by the addition of strength from unobserved higher lying $l=2$ and 0 levels. To within the limits of current experimental and theoretical techniques, however, the absolute magnitudes of the spectroscopic strengths of Table II are consistent to well within the experimental uncertainties with Bassel's normalization for the (d,^{3}He) reaction and also with the studies of the ^{40}Ca(d,^{3}He)^{39}K and ^{16}O(d,^{3}He)^{15}N reactions at 34.4 MeV, the analyses of which closely corresponded to that employed in the present work.

Recent γ-ray correlation measurements[29] suggest a spin of $\frac{5}{2}$ for the 4.40-MeV level assigned here as $l=2$. If we assume the correctness of this probable $\frac{5}{2}$ assignment and exclude from consideration questions concerning excitations to higher major shells, the spectroscopic factors of Table II can be grouped according to the spins of the residual levels and the percentage occupancies of the protons in the ^{28}Si ground state obtained. These numbers are: $\langle 1d_{5/2}\rangle=82\%$, $\langle 2s_{1/2}\rangle=8\%$, and $\langle 1d_{3/2}\rangle=10\%$.

The detailed distribution of the observed spectroscopic strength is difficult to reconcile with the predictions of simple weak coupling (or vibrational) and strong coupling (or Nilsson) collective models for the ^{27}Al and ^{28}Si systems. A weak-coupling model[12,13] in which the low excited states of ^{27}Al are explained as couplings of a $d_{5/2}$ hole to the 2^+ first excited state of ^{28}Si cannot account for the significant $l=0$ strength to the $\frac{1}{2}^+$ 0.814-MeV level. Thus it would seem that such weak-coupling calculations should be expanded to include the remaining s-d orbitals, a change which, however, removes much of the attractive simplicity of this approach. It might be inserted here that the weak-coupling model would be incompatible with a $\frac{7}{2}^+$ assignment to the high-spin member of the 3.0-MeV doublet, that is to say, incompatible with two low-lying $\frac{7}{2}^+$ levels.

The low-lying positive parity states of ^{27}Al are interpreted in the single-particle Nilsson model[14,46] as couplings of a proton in either Nilsson orbit No. 5 ($K=\frac{5}{2}$) or No. 9 ($K=\frac{1}{2}$) with the rotational states of the (positively) deformed ^{26}Mg core. (In this model the existence of an experimental $\frac{9}{2}^+$ state at an energy as low as 3 MeV represents a discrepancy with, or at least an inconsistency in, the theory.[28,36]) This description of ^{27}Al implies that the protons in ^{28}Si (g.s.) which are active in the pickup transitions to these same states of ^{27}Al are the last two, distributed over orbits Nos. 5 and 9. This simple version of the situation is quite

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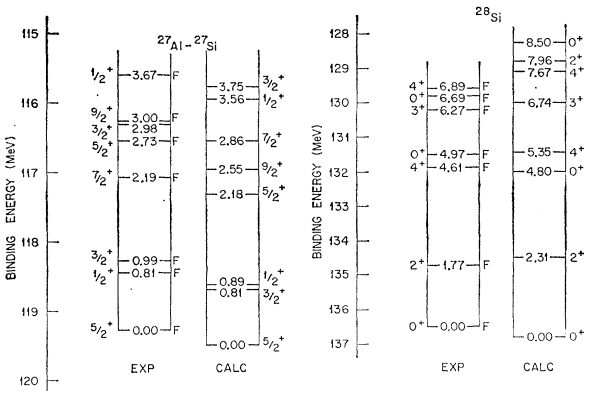

FIG. 5. Experimental energy-level sequences for ^{28}Si and ^{27}Al-^{27}Si and the sequences calculated with the $d_{5/2}$-$s_{1/2}$ shell model discussed in the text. Those levels marked with F were among the 78 levels of nuclei with masses between $A=20$ and 28 that were used in a least-squares-fit procedure to obtain the shell-model residual interaction.

inadequate to account for the observed spectroscopic factor intensities and their distribution. This may be seen most readily by noting that the sum-rule limit for this model is 2, while the ground-state spectroscopic factor is almost 4. It is evident from the results presented in Table II that essentially all of the protons in ^{28}Si which are exterior to the ^{16}O core participate in the pickup transitions to the lowest-energy states of ^{27}Al and that hence the proper description of these states must involve five protons distributed over at least four Nilsson orbits. While this sort of complexity rules out the easy calculations of spectroscopic factors for individual levels[2,47] that can be performed with the single-particle version of the Nilsson model, sum-rule arguments are still simple to apply.[48]

To do this we consider various deformations of ^{28}Si and assume that the *sd*-shell particles of the ground state occupy the lowest three available Nilsson orbits. Then, using the tables of Chi,[47] the total spectroscopic strength available for pickup to $\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$ levels can be calculated. The results are listed in Table II. The values calculated for the various deformations are not radically different, and, because of the uncertainties in the experimental C^2S values, an unambiguous choice of the deformation of ^{28}Si (g.s.) cannot be made.

Shell-model calculations have recently been made[15] for nuclei between $A=20$ and 28 with the Oak Ridge-Rochester Multishell Code.[49] The shell-model effective interaction was determined by adjusting the pertinent 2-body matrix elements and single-particle energies to best-fit 77 binding energies of nuclear energy levels in the $20\leq A\leq 28$ region. These calculations were carried out in a $1d_{5/2}$-$2s_{1/2}$ configuration basis and hence the $1d_{3/2}$ effects that are measured for these nuclei, such as the pickup to the $\frac{3}{2}^+$ levels in ^{27}Al, are beyond the scope of this model. However, full play is allowed to the complexities that can arise from $d_{5/2}$ and $s_{1/2}$ couplings. The calculated spectra for ^{27}Al and ^{28}Si are compared with the average experimental level energies in Fig. 5. It should be emphasized that the same shell-model effective interaction used to calculate these two spectra gave comparably good agreement with experiment for almost all nuclei between $A=20$ and 28, the worst exception being the odd-odd nucleus ^{26}Al. That is to say, these calculations were not particularly concerned with ^{27}Al and ^{28}Si but rather were directed at a comprehensive and simultaneous treatment of all 20-odd nuclei in the region.

These calculations strongly suggest that there are only two low-lying levels of high spin, of $j^\pi=\frac{7}{2}^+$ and $\frac{9}{2}^+$, and that hence the 3.0-MeV state is $\frac{9}{2}^+$. The spectroscopic factors $C^2S_{\rm sm}$ calculated in this model for single-nucleon transfer between ^{28}Si (g.s.) and the various levels of ^{27}Al are listed in Table II. It is seen that the agreement between the theoretical and experimental numbers is quite acceptable. These calculations are presently being extended in an attempt to being in $d_{3/2}$ configurations. The qualitative success of the $d_{5/2}$-$s_{1/2}$ model indicates that such an expansion might yield a reasonably complete description of the low-lying positive-parity states of ^{27}Al.

SUMMARY

The angular distributions of the ^{28}Si$(d,^3$He$)^{27}$Al reactions corresponding to l transfers of 0, 1, and 2 are reasonably well fitted with local, zero-range DWBA calculations that employ optical-model parameters inferred from elastic scattering analyses. The discrepancies between theory and experiment that do exist are not serious and appear to be such that minor modifications in the optical-model parameters could correct them. Minor modifications in the sense used means those changes which would not appreciably affect the agreement between the optical-model predictions and the elastic scattering data. The spectroscopic factors extracted from the experimental cross sections indicate that the ^{28}Si ground-state wave function has significant components from each of the three $2s$-$1d$ subshells. These data further indicate that a minimum requirement for a "complete" description of the levels of ^{27}Al and ^{28}Si is consideration of essentially all extra-^{16}O nucleons and the full $2s$-$1d$ configuration space.

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