PHYSICAL REVIEW VOLUME 167, NUMBER 4 20 MARCH 1968

Self-Consistent Binding Energies and Densities of Protons and Neutrons in ^{10}Be and ^{10}C

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(Received 28 August 1967)

Binding energies, root-mean-square radii, and deformations of the protons and neutrons are calculated for the prolate and the oblate energy minima in ^{10}Be and ^{10}C, utilizing the Hartree-Fock approach. The nucleons interact via the Volkov and the Coulomb force. The energies are corrected for the c.m. motion. The mass and the charge distribution are surprisingly different: The theoretical charge deformation in ^{10}Be is $\beta_C=0.62$ [$\beta_C(\text{expt})=\pm0.7\pm0.1$], while the calculated mass deformation has the value $\beta_M=0.41$. A division of the total binding energy into a proton and a neutron part yields for ^{10}Be twice as large a contribution for the protons as for the neutrons. This puzzle can be understood by means of Wigner's SU_4 model.

IN the last few years several Hartree-Fock (HF) calculations[1–7] have been performed on light nuclei. In only a few of these publications[7] has the isospin-violating Coulomb force been taken into account. Furthermore, most authors utilized the proton-neutron symmetry of the force to reduce the HF problem by a factor of 2, so they were only able to calculate for self-conjugate nuclei. The few HF treatments of other nuclei are done in such a restricted space that the protons and neutrons can not show a different behavior. The calculation by Volkov[1] is limited to variation of the oscillator parameter in the symmetry axis and in the perpendicular plane. The results, which are reported by Ripka[1], have been obtained by an effective nonsaturating force and under the restriction to the *sd* shell.

We want to show here with the example of the mirror nuclei ^{10}Be and ^{10}C that a HF calculation allowing a sufficiently large space as a basis can yield different values for the deformations and for the mean square radii R_{rms} of the proton and the neutron distributions in the same nucleus. The Hamiltonian has the form

$$H=\sum_i \frac{P(i)^2}{2m}-\frac{1}{2mA}[\sum_i P(i)]^2 + \sum_{i<k}\left[V(i,k)+\frac{e^2[1+\tau_z(i)][1+\tau_z(k)]}{4r_{ik}}\right]. \quad (1)$$

We subtract the kinetic energy of the whole nucleus to extract the c.m. motion. We chose the Volkov force[1]

$$V(i,k)=(0.4+0.6P_x)V(r_{ik}), \quad (2)$$

with

$$V(r)=S_1\exp(-r^2/\mu_1^2)+S_2\exp(-r^2/\mu_2^2),$$
$$S_1=-83.34\text{ MeV},\quad \mu_1=1.6\text{ F},$$
$$S_2=144.86\text{ MeV},\quad \mu_2=0.82\text{ F},$$

for the interaction between the nucleons. This force was adapted to give approximately the singlet and triplet scattering length and to fit the binding energies and the rms radii R_{rms} of ^{4}He and ^{16}O. We utilize as the basis the oscillator functions up to the $N=2n+l=3$ shell or the Nilsson functions calculated on this basis, including $N=\pm2$ mixing. The $N=3$ shell has to be included to describe correctly the deformation in the p shell because the quadrupole operator connects the p shell with the $N=3$ wave functions. The force is independent of the spins of the interacting particles, and it is naturally also invariant under time reversal. Assuming that the states connected by the spin flip operator θ_Σ and the time-reversal operator $T=\theta_\Lambda\theta_\Sigma$ are equally occupied, one is able to lower the order of the matrices involved by a factor of 2 or 4.

$$\theta_\Sigma|\tau_z,\pi,\Lambda,\Sigma,k\rangle=(-)^{\frac{1}{2}-\Sigma}|\tau_z,\pi,\Lambda,-\Sigma,k\rangle,$$
$$\theta_\Lambda|\tau_z,\pi,\Lambda,\Sigma,k\rangle=(-)^{\Lambda}|\tau_z,\pi,-\Lambda,\Sigma,k\rangle. \quad (3)$$

The good quantum numbers are the isospin projection τ_z, the parity π, the angular momentum projection Λ onto the symmetry axis, and the spin projection Σ. The quantum number k labels the different Nilsson states for the same good quantum numbers.

Figures 1 and 2 show the results for the minimum of the binding energy at a positive deformation in the nuclei ^{10}Be and ^{10}C. The data for the relative minimum

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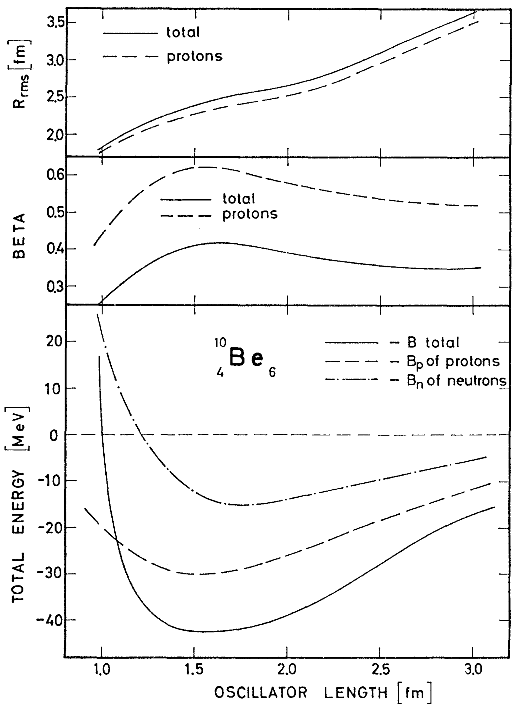

FIG. 1. The figure shows in the lower part the total binding energy of ^{10}Be for the prolate minimum dependent on the oscillator length. The distribution of the total energy to the protons and neutrons is also displayed. The middle part gives the deformation in the usual definition (Ref. 12) for the charge and the mass distribution. The rms radii are displayed in the upper part. The curves are mean values for the ground-state rotational band.

TABLE I. Data of the prolate and oblate energy minima in ^{10}Be and ^{10}C. Theoretical and experimental values are listed for the prolate ($\beta>0$) and oblate ($\beta<0$) energy minima in ^{10}Be and ^{10}C. B is the theoretical and B(expt) is the experimental total binding energy. B_p and B_n show how the energy is distributed between the protons and the neutrons. All the energies are corrected to the band head by utilizing the experimental moment of the inertia of ^{10}Be and the mean value of the total angular momentum squared $\langle J^2\rangle$. R_C is the theoretical rms radius for the charge, R_C (expt) is the experimental value extracted from electron scattering data (see Refs. 8 and 9) on neighbouring nuclei, and R_M is the rms radius of the mass density. The remaining rows list the quadrupole moments Q_C, Q_M and the deformations β_C, β_M which yield the same quadrupole moments for a homogeneous distribution. The mean value of the angular momentum in the rotational band is calculated by the equation $\bar{J}(\bar{J}+1)=\langle J^2\rangle$.

	^{10}Be		^{10}C	
	$\beta>0$	$\beta<0$	$\beta>0$	$\beta<0$
$-B$ (MeV)	-46.4	-43.9	-41.8	-39.5
$-B$ (MeV)	-65.0	$\cdots$	-60.4	$\cdots$
$-B_p$ (MeV)	-32.1	-27.7	-6.8	-8.8
$-B_n$ (MeV)	-14.3	-16.2	-34.8	-30.7
R_C (F)	2.28	2.28	2.47	2.43
R_C (expt)	2.2 ± 0.2	$\cdots$	$\cdots$	$\cdots$
R_M (F)	2.39	2.36	2.40	2.37
Q_C (F^2)	16.2	-9.0	13.6	-16.1
Q_M (F^2)	29.7	-24.8	29.9	-25.1
β_C	0.62	-0.34	0.30	-0.36
β_C (expt)	$\pm0.7\pm0.1$	$\cdots$	$\cdots$	$\cdots$
β_M	0.41	-0.35	0.41	-0.35
$\langle J^2\rangle$	7.51	6.86	7.54	6.86
$\bar{J}$	2.29	2.17	2.29	2.17

at a negative deformation are compared with them in Table I.[8,9] The value shown in Figs. 1 and 2 are mean values for the whole rotational band. The average angular momentum in both nuclei is $\bar{J}_{>}=2.29$ for the positive deformation and $\bar{J}_{<}=2.17$ for the negative deformation. If one takes the momentum of inertia of the ground-state rotational band in ^{10}Be, then this means a lowering of the total energy for the band heads by $E_J-E_0=4.2$ MeV for the prolate deformation and $E_J-E_0=3.8$ MeV for the oblate nuclei. These corrected values are listed in Table I for the minima. Contrary to Volkov's results, this calculation yields the prolate deformation as the ground state. The radiative capture ^{9}Be$(n,\gamma)^{10}$Be feeds with 65% the ground state[10] in ^{10}Be and not at all the 0^+ state at 6.18 MeV, which corresponds probably to the oblate state. A similar behavior is shown by the reaction ^{9}Be$(d,p)^{10}$Be, which produces the 0^+ and the 2^+ states in the ground-state band[11] but fails to yield the 0^+ state at 6.18 MeV. This indicates that ^{10}Be has the same deformation as ^{9}Be, which is probably prolate because it is in the neighborhood of ^{8}Be. The comparison of the experimental binding

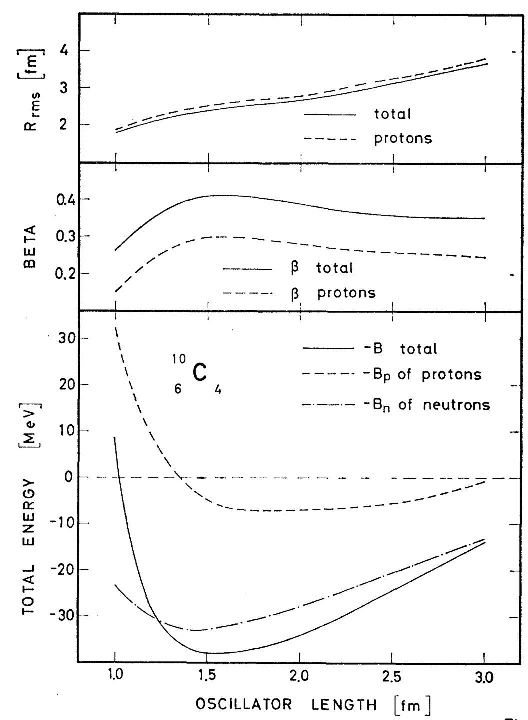

Fig. 2

FIG. 2. The binding energy, the deformation, and the rms radius are displayed for the prolate energy minimum in ^{10}C depending on the oscillator length. The details are the same as in Fig. 1.

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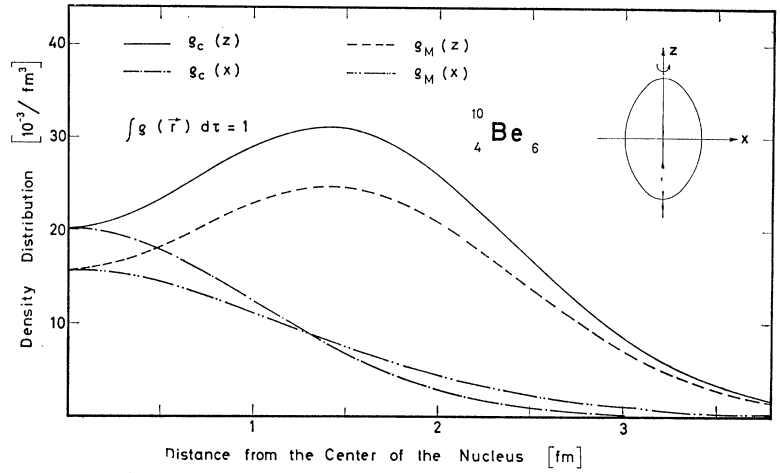

FIG. 3. The charge and the mass distribution are exhibited for the prolate energy minimum in ^{10}Be. The four curves display the charge distribution along the symmetry axis $\rho_C(z)$ and in the x-y plane $\rho_C(x)$ and the mass density along the same axes $\rho_M(z)$ and $\rho_M(x)$. Both densities are normalized to unity.

energies with the theoretical values of Table I shows that the Volkov force underbinds the nuclei. Indeed, Volkov has already found that his force gives the correct energy only for $4n$ nuclei and that the $(4n+2)$ nuclei are too weakly bound. Nevertheless, the mean square radius and the deformation in ^{10}Be are described in good agreement with the experimental results. The deformation was calculated from the lifetime measurement of Warburton *et al.*[12] by the formula[13]

$$B(E2;2^+\rightarrow 0^+)=\tfrac{1}{5}[(3Z/4\pi)R_0^2]^2\beta^2(1+0.4\beta)^2. \quad (4)$$

The distribution of the total binding energy between the protons and the neutrons can be easily understood if one considers the SU_4 state of maximum spatial symmetry [442]. One finds 13 spatially symmetric and eight antisymmetric pairs. In ^{10}Be, they are divided up into seven even and six odd pairs for the neutrons and six even and two odd for the protons. If one takes into account that the even pairs are strongly bound but that the force between the odd pairs is repulsive, then one can understand why the contribution of the neutrons to the total binding energy is so small although ^{10}Be has only four protons and six neutrons.

Two smaller effects are superimposed on this main effect: First is the Coulomb repulsion of the protons. This can be seen by considering the proton-neutron symmetry between ^{10}Be and ^{10}C in Table I. Second, one can observe that the deformation affects the binding energies. One has only to realize that four particles of the same charge prefer the prolate shape as in ^{8}Be and that six particles of the same charge prefer the oblate shape as in ^{12}C. This effect changes the binding energy from 2 to 5 MeV.

Figure 3 shows the charge distributions $\rho_C(z)$, $\rho_C(x)$, and the mass distributions $\rho_M(z)$, $\rho_M(x)$ for the prolate ^{10}Be along the symmetry axis and along an axis in the x-y plane. Both densities are normalized to unity. The larger binding energy for the protons is indicated by the higher density in the center of the nucleus. This explains also the smaller mean square radius of the charge for ^{10}Be $(\beta>0)$ in Table I, although the protons have a larger deformation than the mass distribution.

ACKNOWLEDGMENT

The authors wish to thank Professor H. Marschall for interesting discussions.

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