Spin Assignments of the B¹² 0.95-, 1.67-, and 2.62-MeV Levels*

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 $\gamma\text{-}\gamma$ angular correlations were performed on the B12 $1.67 \rightarrow 0.95 \rightarrow 0$ and $2.62 \rightarrow 0.95 \rightarrow 0$ cascades following formation of the initial states via the $B^{11}(d,p)B^{12}$ reaction. Deuteron energies of 1.70 and 3.13 MeV, respectively, were used. The results, when combined with previous information, lead to spin-parity assignments of 2^+ , 2^- , and 1^- for the B¹² 0.95-, 1.67-, and 2.62-MeV levels. Some information on the γ -ray decay modes of these states was obtained. This information is in good accord with previously reported work. B¹¹+d yield curves were obtained for the production of 0.95- and 1.67-MeV γ rays and for the coincidence rate between these two γ rays for deuteron energies between 0.55 and 3.05 MeV. Yield curves for production of the B¹² 0.95-, 1.67-, and 2.62-MeV levels were constructed from these γ -ray yield curves.

I. INTRODUCTION

HE mass-12 system—specifically C¹²—has long enjoyed a special and important position in nuclear-structure studies of light nuclei. It has been used extensively as a testing ground for nuclear models and refinements to these models. The theoretical studies have relied on the extensive experimental studies¹ of the T=0 spectrum of C¹² and have in turn stimulated more detailed investigations of these states and of the first few MeV of the T=1 spectrum² of C¹². It has become clear that an experimental interpretation of this T=1 spectrum, which commences at an excitation energy of 15.1 MeV, is quite difficult. This is so because all the T=1 levels are unbound against α emission and all but the lowest are unbound against proton emission; furthermore, study of the T=1 spectrum is complicated by the presence of T=0 levels in the same region of excitation. However, the T=1spectrum can be studied in B12 and N12. In N12, only the ground state is bound against nucleon emission, while the first five states of B¹² are bound and decay by γ -ray emission. Thus the properties of the mass-12 T=1 states can be studied more easily and more profitably in B^{12} than in N^{12} .

The specific purpose of the work reported herein was to determine the spins of the first three excited

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states of B¹². The method used was the γ - γ correlations technique initiated by Warburton and Rose³ and developed by Litherland and Ferguson.⁴ In the present work, the γ - γ correlation measurements did not yield unambiguous results by themselves; however, when combined with previous measurements, they could be interpreted to yield definite spin assignments for the 0.95-, 1.67-, and 2.62-MeV levels of B¹².

Information available on the properties of the bound levels of B^{12} and pertinent to the present investigation is illustrated in Fig. 1. The excitation energies are from data reviewed or presented by Olness and Warburton.⁵ The lifetime values are from measurements⁶⁻⁸ reviewed by these authors. The γ -ray branching ratios are also from Olness and Warburton,⁵ but include results arising from the present measurements.

The spin parity of the B¹² ground state is fixed¹ by its β decay to C¹². The parities of the excited states were determined by analysis of $B^{11}(d,p)B^{12}$ angular distributions. In addition, these results, which have recently been reviewed by Gallmann et al.,⁷ give $J \leq 3$ for the 0.953-MeV level and J=1 or 2 for the 1.674and 2.621-MeV levels. For the 0.953-MeV level, J=3is ruled out by the lifetime⁶ of the level and J=0 is ruled out, since the $0.95 \rightarrow 0$ transition has an observed

Work performed in part while at Brookhaven National Laboratory.

[‡] Work performed in part while at Lockheed Palo Alto Research Laboratory.

¹F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959); *Nuclear Data Sheets*, compiled by K. Way et al. (U. S. Government Printing Office, National Academy of Sciences—National Research Council, Washington, D. C., 1962). ²See, e.g., R. E. Segel, S. S. Hanna, and R. G. Allas, Phys. Rev. **139**, B818 (1965).

⁸ E. K. Warburton and H. J. Rose, Phys. Rev. 109, 1199 (1958).

⁴ A. E. Litherland and A. J. Ferguson, Can. J. Phys. 39, 788 (1961); A. J. Ferguson, Angular Correlation Methods in Gamma-Ray Spectroscopy (North-Holland Publishing Co., Amsterdam, 1965).

⁸ J. W. Olness and E. K. Warburton, following paper, Phys. Rev. 166, 1004 (1968).

⁶ E. K. Warburton and L. F. Chase, Jr., Phys. Rev. 132, 2273 (1963).

⁷ A. Gallmann, F. Hibou, P. Fintz, P. E. Hodgson, and E. K. Warburton, Phys. Rev. 138, B560 (1965).

⁸ M. J. Throop, Bull. Am. Phys. Soc. 12, 484 (1967).





FIG. 1. Energy-level diagram for B12 showing information available on the bound levels of this nucleus prior to the present work.

anisotropy in the $B^{11}(d,p\gamma)B^{12}$ reaction.^{6,9} Previously, it was argued⁶ that proton- γ angular-correlation measurements¹⁰ ruled out J=1 for the 0.953-MeV level. However, a recent repetition⁹ of this experiment is in strong disagreement with the earlier results and does not discriminate against J=1. Thus, we retain J=1and 2 as possibilities for the B^{12} 0.953-MeV level.

A preliminary report¹¹ was previously given of part of this work, which was commenced in 1962. At that time the γ -ray yield from the B¹¹ $(d, p\gamma)$ B¹² reaction was measured for deuteron energies between 0.55 and 3.05 MeV. These data have been valuable for planning subsequent investigations^{5,6,11} of B¹² via the $B^{11}(d, \dot{\rho}\gamma)B^{12}$ reaction. These measurements are described in Sec. II.

The γ - γ angular-correlation measurements are described in Sec. III. Angular correlations were measured between the two γ rays in the B¹² $1.67 \rightarrow 0.95 \rightarrow 0$ or $2.62 \rightarrow 0.95 \rightarrow 0$ cascade following population of the B^{12} 1.67- or 2.62-MeV state by the $B^{11}(d,p)B^{12}$ reaction. Since this reaction results in alignment of the initial B¹² state, the correlation is a triple one. We use the procedure and method of analysis designated as method I by Litherland and Ferguson.⁴ The great advantage of this method is that it is independent of the reaction mechanism of the $B^{11}(d,p)B^{12}$ reaction.

II. YIELD OF THE $B^{11}(d, p\gamma)B^{12}$ REACTION

When B¹¹ is bombarded by deuterons with energies greater than 1 MeV, γ rays with energies of 0.95 and 1.67 MeV are easily observed, as illustrated in Fig. 2. These arise from decay of the 0.95-, 1.67-, and 2.62MeV levels. Because of the accidental energy matching, 0.95+1.67=2.62, the contribution to these γ -ray intensities from the decay of the three levels in question is not easily determined by γ -ray singles measurements. Our method was to measure the relative yields of the 0.95- and 1.67-MeV γ rays and also the coincidences between them as a function of deuteron energy.

The relative yields of the 0.95- and 1.67-MeV γ rays and the 1.67-0.95 coincidences were measured in 100keV steps for deuteron energies between 0.55 and 3.05 MeV, using the Brookhaven National Laboratory electrostatic accelerator. Two 3×3 -in. NaI crystals, both at 90° to the beam direction, and 2 in. from the target, were used. At each deuteron energy the pulse-height spectra from one of the crystals was displayed on a 400-channel analyzer and the coincidences were recorded between a gate centered on the 1.67-MeV full-energy-loss peak in the displayed spectrum and a gate centered on the 0.95-MeV full-energy-loss peak in the spectrum from the other crystal. The relative yields of the 0.95- and 1.67-MeV γ rays were determined from the areas under the full-energy-loss peaks, and the calculated efficiency of the NaI crystal. The results for the relative yield of the two γ rays are given by the closed circles in Fig. 3. In this figure, the solid lines drawn through the data points are for purposes of connecting the points. The relative yields of both γ



FIG. 2. γ -ray spectra from 2.8-MeV deuterons incident on a 0.3-mg/cm² boron target enriched to 99.9% in B¹¹. The spectra are from a 4×4 -in. NaI(Tl) crystal at 90° to the beam direction and 13 cm from the target. γ -ray full-energy-loss peaks are labeled by their energies in MeV. Both a singles spectrum and a spectrum in coincidence with 0.95-MeV pulses from a second 90° NaI(Tl) detector are shown In the spectrum in concidence with 0.95-MeV pulses from a second 90° NaI (TI) detector are shown. In the coincidence spectrum, the 0.72-MeV peak arises from the B¹² 1.67 \rightarrow 0.95 \rightarrow 0 cascade, while the 1.67-MeV peak arises from the 2.62 \rightarrow 0.95 \rightarrow 0 and 2.62 \rightarrow 1.67 \rightarrow 0 cascades. The 0.95-MeV peak arises from the 0.05 1.67 \rightarrow 0 cascades. 0.95-1.67 coincidences, since the pulse-height gate for the second detector included Compton pulses from 1.67-MeV γ rays. The 1.67- and 0.95-MeV peaks are not present in coincidence spectra recorded for $E_d < 2.1$ MeV.

⁹ A. P. Borden and R. C. Ritters, Phys. Rev. 159, 875 (1967).
¹⁰ S. Gorodetzky, A. Gallmann, P. Fintz, and G. Bassompierre, J. Phys. Radium 22, 573 (1961).
¹¹ L. F. Chase, Jr., W. W. True, and E. K. Warburton, Bull. Am. Phys. Soc. 9, 56 (1964).



FIG. 3. Yield curves for 0.95-MeV γ rays (closed circle), 1.67-MeV γ rays (closed circles), and 1.67-0.95 coincidences (solid curve). The open squares are data of Sellschop and Mingay (Ref. 12) for the yield of the B¹¹(d,p)B¹² (0.95-MeV level) reaction normalized to the 0.95-MeV γ -ray yield for deuteron energies below 2.1 MeV. The absolute cross-section scale, which is uncertain to 15%, is obtained from measurements by Pullen *et al.* (Ref. 13) of the B¹¹(d,p)B¹² (0.95-MeV level) and B¹¹(d,p)B¹² (1.67-MeV level) cross sections at a deuteron energy of 3 MeV. The dashed curves are the constructed (see text) yield curves for production of the B¹² 0.95- and 1.67-MeV levels, while the 1.67-0.95 coincidence curve, if increased by 6%, gives the cross section for formation of the B¹² 2.62-MeV level.

rays were also measured between deuteron energies of 1.7 and 2.2 MeV in 25-keV steps. The results showed no structure and are in good agreement with the curves shown in Fig. 3.

The yield curve shown in Fig. 3 for the 1.67-0.95 coincidences was obtained by drawing a smooth curve through the experimental points. The open squares are obtained from integrated angular distributions measured by Sellschop and Mingay¹² for the B¹¹(d,p)B¹² (0.95-MeV level) reaction. From Fig. 3 it is seen that for deuteron energies below 2.1 MeV the B¹² 2.62-MeV level is not formed. Thus the yield of the 0.95-MeV γ ray for $E_d < 2.1$ MeV is almost entirely due to formation of the 0.95-MeV level, with only a small contribution from the 3% $1.67 \rightarrow 0.95 \rightarrow 0$ cascade (see Fig. 1). Using this fact, the data of Sellschop and Mingay were normalized to our yield curve for the 0.95-MeV γ ray, taking cognizance of the 1.67 \rightarrow 0.95 \rightarrow 0 cascade. The result is shown in Fig. 3. The absolute cross-section scale of Fig. 3 was then obtained by integrating the absolute differential cross section for the $B^{11}(d,p)B^{12}$ (0.95-MeV level) measured by Pullen et al.¹³ at 3.0 MeV. Pullen quoted an accuracy of $\pm 10\%$ for the absolute cross section. The absolute crosssection scale of Fig. 3 has an estimated accuracy of ±15%.

The dashed curve in Fig. 3 connecting the open squares and connecting with the yield curve of the 0.95-MeV γ ray below a deuteron energy of 2.1 MeV represents the yield curve for formation of the B¹² 0.95-MeV level. For deuteron energies greater than 2.1 MeV, the difference between this dashed curve and the 0.95-MeV γ -ray yield represents the sum of the yields of the $1.67 \rightarrow 0.95 \rightarrow 0$, $2.62 \rightarrow 0.95 \rightarrow 0$, and $2.62 \rightarrow 1.67 \rightarrow 0$ cascades (see Fig. 1). From this difference, the yield curve for direct formation of the 1.67-MeV level was constructed. It is indicated by the dashed curve connecting with the yield curve for the 1.67-MeV γ ray. The cross section for the B¹¹(d,p)B¹² (1.67-MeV level) reaction at a deuteron energy of 3.0 MeV was also measured by Pullen et al., who found 160 mb. This result is in good agreement with that given by the yield curve of Fig. 3, which is 166 mb. The absolute cross-section scale for the 1.67-0.95 coincidences (Fig. 3) is obtained from the difference between the yield of the 1.67-MeV γ ray and that for formation of the 1.67-MeV level. The coincidence yield represents the yield of the $2.62 \rightarrow 0.95 \rightarrow 0$ and $2.62 \rightarrow 0.95 \rightarrow 0$ $1.67 \rightarrow 0$ cascades, so that the yield for direct formation of the 2.62-MeV level can be obtained by dividing the coincidence cross section by the sum of the branching ratios, $94\pm1\%$ of these two cascades.

III. γ - γ CORRELATION MEASUREMENTS

A. The $1.67 \rightarrow 0.95 \rightarrow 0$ Correlation

Experimental Procedure and Results

Preliminary experiments on the decay of the B¹² 1.67-MeV level, using the B¹¹($d,p\gamma$)B¹² reaction, yielded a branching ratio of $(3.0\pm0.6)\%$ for the $1.67 \rightarrow 0.95$ cascade. This is in good agreement with the value of $(3.2\pm0.5)\%$ recently obtained by Olness and Warburton.⁵ We adopt a value of $(3.2\pm0.4)\%$. The smallness of this branching ratio is the chief obstacle to an accurate measurement of the triple correlation

$$\mathbf{B}^{11}(d,p)\mathbf{B}^{12} \ (1.67 \xrightarrow{\gamma_2} 0.95 \xrightarrow{\gamma_2} 0)$$

and was the determining factor in the design of the experiment.

The measurements were carried out with the Brookhaven National Laboratory 3.5-MV electrostatic accelerator, using a deuteron energy of 1.70 MeV. At this energy the B¹² 2.62-MeV level is not formed (see Fig. 3). The target consisted of 0.3 mg/cm² of 99.9% enriched B¹¹ evaporated onto a gold backing. The beam current was about 0.4 nA. Three 3×3 -in. NaI(Tl) γ -ray detectors were used. These were placed with their front faces 10 cm from the B¹¹ target. Two of the detectors were at 90° to the beam and opposite each other. The third detector (movable) was placed either at 0° to the beam or at 90° to the beam and to the other two detectors. Two separate coincidence circuits

¹² D. W. Mingay, thesis, University of the Witwaterstrand, Johannesburg, South Africa, 1964 (unpublished); J. P. F. Sellschop (private communication).
¹³ D. J. Pullen, D. H. Wilkinson, and A. B. Whitehead, in

¹³ D. J. Pullen, D. H. Wilkinson, and A. B. Whitehead, in *Proceedings of the Rutherford Jubilee International Conference, Manchester*, edited by J. B. Birks (Academic Press Inc., New York, 1961), p. 565; D. J. Pullen, thesis, Oxford University, 1963 (unpublished).

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FIG. 4. χ^2 -versus-arctan x_2 curves for the B¹² 1.67 $\rightarrow 0.95 \rightarrow 0$ γ - γ correlation. For each value of x_2 , the E2/M1 mixing ratio of the $0.95 \rightarrow 0$ transition, the minimum of the envelope of χ^2 for $|x_1| < 0.032$ is shown, where x_1 is the M2/E1 mixing ratio of the $1.67 \rightarrow 0.95$ transition. χ^2 curves are shown for the four possible combinations of the spins of the three levels involved. The vertical dashed lines indicate the constraint $|x_2| < 0.124$ imposed for the lifetime of the 0.95-MeV level (see Ref. 5 and text). It is seen that the two χ^2 curves for a J = 1 assignment to the 1.67-MeV level give unacceptable solutions, i.e., χ^2 is greater than the 0.1% limit for $|x_2| < 0.124$. Thus the 1.67-MeV level has J=2. The curve designated 2-1-1 (A) is the normal χ^2 curve for a 2-1-1 spin sequence. It gives acceptable solutions. However, when the angular distribution of the crossover $1.67 \rightarrow 0$ distribution is included in the least-squares fit, the curve labeled 2-1-1 (B) results. This does not have an acceptable solution. It is seen that the 2-2-1 spin sequence does give an acceptable fit and thus the 0.95-MeV level has J=2.

with resolving times of ~ 70 nsec allowed coincidences to be formed between one of the 90° detectors (fixed detector) and the other two detectors. An electronic gate selected a pulse-height region in the fixed detector. The γ -ray spectra from the other two detectors were displayed simultaneously in two separate 200channel pulse-height analyzers. An example of such a coincidence spectrum, recorded using 100 channels, is shown in Fig. 2.

A complete set of measurements proceeded as follows. The fixed detector was gated on the full-energy-loss peak of the 0.95-MeV γ ray (γ_2) and coincidence spectra from the other two detectors, at 90° and 0° to the beam, were recorded for a known integrated beam charge. Then the fixed detector was gated on the fullenergy-loss peak of the 0.72-MeV γ ray (γ_1), and again spectra from the other two detectors were recorded for a known integrated beam charge. This procedure was then repeated with the movable detector in its 90° position. Two such sets of data were taken, so that a total of 16 coincidence spectra were recorded.

From each, the intensity of the appropriate full-energyloss peak (either γ_1 or γ_2) was extracted by conventional spectrum "stripping" techniques. These intensities were then normalized to the integrated charge and corrected for analyzer dead-time losses and for the difference between the efficiencies of the two coincidence systems. (The relative efficiencies of the two coincidence systems was determined by measurements using Na²² and Co⁶⁰ radioactive sources.)

The results of these measurements were the anisotropies for 5 of the 7 geometries specified by Litherland and Ferguson.⁴ Or, in other words, the relative counting rates at the corners of the two "octants" specified by Broude and Gove.14

The triple angular-correlation function for each of the 7 geometries of Litherland and Ferguson can be expressed as a Legendre polynomial expansion in one angle. As has been discussed, both γ_1 and γ_2 are predominantly dipole and can have very little contribution from quadrupole radiation.⁵ This means that the Legendre polynomial expansions will have the form $A_0 + A_2 P_2(\theta)$, with insignificant contributions from $P_k(\theta)$ terms with k > 2. Thus, the anisotropies measured for the five geometries are sufficient to specify the five angular-correlation functions, i.e., the ratio A_2/A_0 for each of the five geometries. These ratios are listed in Table I, together with an explanation of the geometries used. The result of a measurement of the angular distribution of the $1.67 \rightarrow 0$ transition is also given in Table I.

Analysis

The analysis of the triple correlation $B^{11}(d,p)B^{12}$ $(1.67 \rightarrow 0.95 \rightarrow 0)$ was carried out with the aid of a computer program written by one of us (W. W. T.). The five correlations were fitted simultaneously by the method of linear least-squares¹⁴ for discrete values of the mixing ratios of the two transitions with the population parameters of the 1.67-MeV level as variables. This was done for all the allowed values of the spins of the B12 states involved. Since the B12 ground

TABLE I. Angular-correlation coefficients determined from the triple γ - γ correlation measurements of the $1.67 \rightarrow 0.95 \rightarrow 0$ cascade γ rays in B¹². The coefficients have not been corrected for the finite solid angle subtended by the detectors. In subsequent analysis these were, however, taken as $Q_2 = 0.844$ and 0.850 for the 0.72- and 0.95-MeV γ rays.

Geometry ^a	θ_1	θ_2	ϕ	A_{2}/A_{0}
Ι	VAR	90°	180°	$+0.215\pm0.034$
II	90°	VAR	180°	$+0.049\pm0.037$
V	90°	90°	VAR	-0.092 ± 0.030
VI	90°	VAR	90°	-0.045 ± 0.034
VII	VAR	90°	90°	$+0.115\pm0.031$
Angular d	$+0.001\pm0.006$			

Defined in Ref. 4. **b** θ_1 and θ_2 are the polar angles of γ_1 and γ_2 and ϕ is the azimuthal angle tween γ_1 and γ_2 . The variable angle is designated by VAR.

¹⁴ C. Broude and H. E. Gove, Ann. Phys. (N. Y.) 23, 71 (1963).

state has $J^{\pi}=1^+$, and the 0.95- and 1.67-MeV levels both have J=1 or 2 (see Fig. 1), four such spin combinations are possible. The results of this procedure are illustrated in Fig. 4, which shows, for all four spin combinations, the variation of the mixing ratio of the $0.95 \rightarrow 0$ transition as a function of χ^2 —which represents the goodness of fit and has an expectation value of unity since it is normalized to the degree of freedom.¹⁵

The curves of Fig. 4 correspond to the envelope of the minimum value of X^2 resulting from the constraint $-0.032 \le x_1 \le +0.032$, where x_1 is the mixing ratio (ratio of the amplitudes of quadrupole and dipole radiation) of the $1.67 \rightarrow 0.95$ transition. This constraint follows⁵ from consideration of the smallest possible radiative width and the largest possible M2strength for this E1, M2 transition. Likewise, we have indicated by the vertical dashed lines in Fig. 4 the constraint $-0.124 \le x_2 \le +0.124$ for the M1, $E2 \ 0.95 \rightarrow$ 0 transition, this constraint following from consideration⁵ of the smallest possible radiative width and the largest possible E2 strength for this transition.

We reject a given spin combination if it gives a χ^2 which is everywhere above the 0.1% probability limit for allowed values of x_2 . It is seen from Fig. 4 that the spin sequences $1 \rightarrow 2 \rightarrow 1$ and $1 \rightarrow 1 \rightarrow 1$ are eliminated by this criterion. Thus the 1.67-MeV level has $J^{\pi}=2^{-}$, regardless of whether the 0.95-MeV level has $J^{\pi} = 1^+$ or 2^+ . The spin sequence $2 \rightarrow 1 \rightarrow 1$, and thus $J^{\pi} = 1^{+}$ for the 0.95-MeV level, can be rejected by consideration of the angular distribution of the $1.67 \rightarrow 0$ transition. This was done as follows. For the E1, M2 $1.67 \rightarrow 0$ transition we have the constraint $|x_3| \leq 0.045.^{5}$ If now we include the measured angular distribution of this transition in the least-squares fit and vary x_3 within the above constraint so as to minimize χ^2 for each $x_1 x_2$ set, we obtain the χ^2 curve for the spin sequence $2 \rightarrow 1 \rightarrow 1$, labeled B in Fig. 4. (The $2 \rightarrow$ $1 \rightarrow 1$ curve labeled A is without the inclusion of this angular distribution.) This curve lies above the 0.1%probability limit and so we reject the spin sequence $2 \rightarrow 1 \rightarrow 1$ and therefore $J^{\pi} = 1^+$ for the 0.95-MeV level. The reason why inclusion of the angular distribution of the $1.67 \rightarrow 0$ transition in the least-squares fit has such a strong effect is that for the $2 \rightarrow 1 \rightarrow 1$ spin sequence the least-squares fit corresponds to rather strong alignment of the 1.67-MeV level and thus a rather anisotropic $1.67 \rightarrow 0$ transition $(A_2/A_0 \simeq 0.12)$ which is at variance with the observed isotropic distribution (Table I). On the other hand, the solution for the $2 \rightarrow 2 \rightarrow 1$ spin sequence is consistent with no alignment of the 1.67-MeV level and predicts an angular distribution for the $1.67 \rightarrow 0$ transition in excellent agreement with experiment for $-0.124 \le x_2 \le +0.05$. This latter range for x_2 is obtained by combining the constraint from lifetime measurements,⁵ $|x_2| \le 0.124$, with the 0.1% probability limit of Fig. 4 for the $2 \rightarrow 2 \rightarrow 1$ spin sequence. We consider this range as a firm bound on x_2 . The 10% probability limit on x_2 from Fig. 4 corresponds to $x_2 \le 0.0$ and thus x_2 most probably lies in the range $-0.124 \le x_2 \le 0.0$.

In conclusion, we find that the 0.95- and 1.67-MeV levels of B¹² have $J^{\pi}=2^+$ and 2⁻, respectively, and that the E2/M1 mixing ratio for the $0.95 \rightarrow 0$ transition is in the range $-0.124 \le x \le +0.05$ and most probably in the range $-0.124 \le x \le 0.0$. These conclusions were obtained by combining the present angular-correlation and angular-distribution measurements with the previous information discussed in Ref. 5 and outlined in Fig. 1 and Sec. I.

B. The $2.62 \rightarrow 0.95 \rightarrow 0$ Correlation

Experimental Procedure and Results

Preliminary experiments, using both proton- γ and γ - γ - γ coincidence techniques, showed that the main decay of the B¹² 2.62-MeV level is to the 0.95-MeV level.^{11,16} Limits of <10 and <16% were obtained for the 2.62 \rightarrow 0 and 2.62 \rightarrow 1.67 transitions. These results are in accord with those of Olness and Warburton,⁵ which are shown in Fig. 1. Both the 2.62 \rightarrow 0.95 \rightarrow 0 and 2.62 \rightarrow 1.67 \rightarrow 0 cascades give rise to 0.95- and 1.67-MeV γ rays. This complicates the analysis of the angular correlation between the 0.95-and 1.67-MeV γ rays. In the initial discussion, we shall neglect the presence of the 14% 2.62 \rightarrow 1.67 \rightarrow 0 cascade; but in the final analysis we shall include the possible effects of this cascade.

The measurements were carried out at a deuteron energy of 3.13 MeV, using the Lockheed Palo Alto Research Van de Graaff accelerator. The beam current was about 0.4 nA. Two 4×4-in. NaI(Tl) detectors were used with their front faces 13 cm from the B¹¹ target. One detector was fixed at 90° to the beam. The other detector, mounted on a goniometer, was movable about the edges of an octant.¹⁴ The fixed detector was gated on the full-energy-loss peak of either the 1.67- or the 0.95-MeV γ ray. Spectra from the movable detector in coincidence with this gate were recorded. The coincidence spectra obtained were similar to those shown in Fig. 2 and 5.

Spectra were taken with the movable detector placed every 15° about the edges of the octant for both the 0.95- and 1.67-MeV γ rays detected in the fixed detector. This sequence was repeated once. In all, 38 coincidence spectra were recorded. From each spectrum the intensity of the appropriate full-energy-loss

¹⁵ Actually, χ^2 is expected to be considerably less than unity in this case, since the angular-correlation coefficients of Table I are not all independent. If the angular correlations have no A_k terms with k>2 and experimental points are measured at the corners of the octant only as in the present case, three of the five geometries listed in Table I are independent, the other two are redundant.

¹⁶ Reference 11 contains a transcription error. The main γ -ray branch of the 2.62-MeV level was erroneously reported to be to the 1.67-MeV level rather than to the 0.95-MeV level,



FIG. 5. $B^{11}(d, p\gamma\gamma)B^{12} \gamma$ -ray coincidence spectrum showing a portion of a 4×4-in. NaI(Tl) spectrum in coincidence with an electronic gate set on the full-energy-loss peak of the 1.67-MeV γ ray viewed by a second NaI(Tl) detector. The spectrum was obtained with both detectors at 90° to the deuteron beam and opposite each other with their front faces 13 cm from the B¹¹ target. This is one of the 38 spectra taken in the measurement of the B¹² 2.62 \rightarrow 0.95 \rightarrow 0 angular correlation. The 1.67-MeV peak arises from random coincidences. The dashed and full curves show a least-squares fit to the 0.95-MeV peak assuming a Gaussian peak superimposed on an exponential background.

peak was extracted by two methods: either by standard spectrum "stripping" or by least-squares fitting of a Gaussian peak plus exponential background. The latter procedure is illustrated in Fig. 5. The results of these two techniques were in very good agreement. The peak intensities were normalized to the counts recorded by a gate set on the 1.67-MeV peak as viewed by the fixed NaI(Tl) detector. Corrections were made for analyzer dead time and for geometrical misalignments. The latter were determined by singles measurements using radioactive sources.

The results of least-squares Legendre polynomial fits to the angular correlations measured in the five geometries are listed in Table II. As in the case of the $1.67 \rightarrow 0.95 \rightarrow 0$ correlation, the two transitions, $2.62 \rightarrow 0.95$ and $0.95 \rightarrow 0$, must be predominantly dipole, with upper limits on any quadrupole contributions such that terms $P_k(\cos\theta)$ with k>2 must be negligible.⁵ Thus only A_2/A_0 is listed. The measured correlations are also shown in Fig. 6. The curve through the experimental points in Fig. 6 are the results of a least-squares fit which we shall now describe.

Analysis

The experimental results for the $2.62 \rightarrow 0.95 \rightarrow 0$ were fitted simultaneously by the method of linear

TABLE II. Angular-correlation coefficients determined from the triple γ - γ measurements of the $2.62 \rightarrow 0.95 \rightarrow 0$ cascade γ rays in B¹². The coefficients have not been corrected for the finite solid angle subtended by the detectors. In subsequent analysis these were, however, taken as $Q_2=0.919$ and 0.921 for the 0.95- and 1.67-MeV γ rays, respectively.

Angular variables ^b							
Geometry ^a	θ_1	θ_2	$oldsymbol{\phi}$	A_2/A_0			
I	VAR	90°	180°	-0.075 ± 0.016			
II	90°	VAR	180°	-0.125 ± 0.016			
V	90°	90°	VAR	$+0.111\pm0.011$			
VI	90°	VAR	90°	-0.029 ± 0.017			
VII	VAR	90°	90°	-0.011 ± 0.016			

* Defined in Ref. 4.

 θ_1 and θ_2 are the polar angles of γ_1 and γ_2 and ϕ is the azimuthal angle between γ_1 and γ_2 . The variable angle is designated by VAR.

least-squares as described in Sec. III A. In a preliminary fit, the $2.62 \rightarrow 1.67 \rightarrow 0$ branch was neglected; the spin sequence $1 \rightarrow 2 \rightarrow 1$ was assumed for the $2.62 \rightarrow$ $0.95 \rightarrow 0$ cascade, and the mixing ratio of the $2.62 \rightarrow$ 0.95 transition was constrained to be 0. An acceptable fit to the angular correlations resulted for $-0.12 \le x_2 \le$ -0.02, where x_2 , the mixing ratio of the $0.95 \rightarrow 0$ transition, was varied. This is within the range allowed for x_2 as discussed in Sec. III A. The solid curve through the experimental points in Fig. 6 is the test fit for $x_2 = -0.08$.

The $(14\pm3)\%$ $2.62 \rightarrow 1.67 \rightarrow 0$ branch was taken into account as follows. Since γ_1 and γ_2 from this cascade were unresolved from γ_2 and γ_1 , respectively, in the $(80\pm3)\%$ $2.62 \rightarrow 0.95 \rightarrow 0$ cascade, the theoretical angular-distribution coefficients for these two cascades were combined by summing those for geometries *I*, *II*, *V*, *VI*, and *VII* from one cascade to those



FIG. 6. Results of the γ - γ triple-correlation measurement for the 2.62 \rightarrow 0.95 \rightarrow 0 cascade γ rays in B¹². The experimental points are shown for the five indicated geometries. The geometries are explained in the text and in Ref. 4. The distributions are plotted against cos³ because only terms in $P_2(\cos\theta)$ are expected. The solid curves show the fit to these data for the spin values and mixing ratios indicated in the insert. In later analysis the presence of the 2.62 \rightarrow 1.67 \rightarrow 0 cascade was taken into account (see text), the difficulty being that the first and second γ rays from this latter cascade are degenerate in energy with the second and first (respectively) γ rays from the 2.62 \rightarrow 0.95 \rightarrow 0 cascade.



FIG. 7. χ^2 versus arctan x_2 for the B¹² 2.62 \rightarrow 0.95 \rightarrow 0 plus 2.62 \rightarrow 1.67 \rightarrow 0 correlations. For each value of x_2 , the minimum of the envelope of χ^2 is shown for the indicated constraints on x_1, x_3, x_4 , and the branching ratios of the 2.62 \rightarrow 0.95 and 2.62 \rightarrow 1.67 transitions. The construction of these χ^2 plots is explained further in the text. The constraint $|x_2| < 0.124$, resulting from the lifetime of the 0.95-MeV level (see Ref. 5 and text), is indicated. It is seen that J=1 for the 2.62-MeV level gives an acceptable solution while J=2 does not. The spin sequence 2-1-1 (not shown) is also excluded. Thus the B¹² 2.62-MeV level has J=1.

for geometries II, I, V, VII, and VI from the other. These coefficients were added in the ratio $(14\pm 6)/$ (80 ± 6) ; that is, two fitting procedures were followed with this ratio, two standard deviations each side of its mean and the lowest value of χ^2 of the two was taken. For each of these two fitting procedures, the mixing ratios of the $2.62 \rightarrow 1.67$, $1.67 \rightarrow 0$, and $2.62 \rightarrow 0$ 0.95 transitions were fixed and the $0.95 \rightarrow 0$ mixing ratio was varied from $-\infty$ to $+\infty$. This procedure was repeated for different mixing ratios of the $2.62 \rightarrow$ 1.67, $1.67 \rightarrow 0$, and $2.62 \rightarrow 0.95$ transitions until the best fit (minimum value of χ^2) was found for each value of the $0.95 \rightarrow 0$ mixing ratio and for all values of the other mixing ratios allowed by the constraints imposed by the limits on the partial lifetimes of these transitions.5

The χ^2 -versus-arctan x_2 curves for the spin sequence $1 \rightarrow 2 \rightarrow 1$ and $2 \rightarrow 2 \rightarrow 1$ shown in Fig. 7 were generated by this procedure. Thus, for each value of x_2 in Fig. 7, the χ^2 shown is the lowest value generated for x_1 , x_3 , and x_4 within the specified ranges and the branching ratios within their specified ranges. It is seen that the $2 \rightarrow 2 \rightarrow 1$ spin sequence for the $2.62 \rightarrow 0.95 \rightarrow 0$ cascade does not give an acceptable solution for x_2 within its allowed range and we exclude this sequence. Likewise, the spin sequence $2 \rightarrow 1 \rightarrow 1$ (not shown) was excluded. On the other hand, both spin sequences $1 \rightarrow 1 \rightarrow 1$ (not shown) and $1 \rightarrow 2 \rightarrow 1$ gave acceptable solutions. Thus the B¹² 2.62-MeV level,

which was $J^{\pi}=1^{-}$ or 2^{-} , is found to have $J^{\pi}=1^{-}$ regardless of whether the 0.95-MeV level has J=1 or 2. For the $1 \rightarrow 1 \rightarrow 1$ spin sequence we find that x_2 , the mixing ratio of the $0.95 \rightarrow 0$ transition, is in the range $-0.124 \le x_2 \le +0.02$, where the lower limit is from the lifetime of the 0.95-MeV level and the upper limit is the 0.1% limit of Fig. 7. The 10% upper limit is -0.03, so that x_2 is most probably in the range $-0.124 \le x_2 \le -0.03$. For x_2 in this range the magnetic substate populations of the 2.62-MeV level are nearly equal; that is, this level is formed with very little alignment as was the case for the 1.67-MeV level.

In conclusion, the 2.62-MeV level is found to have $J^{\pi}=1^{-}$ by combining this work with that summarized in Sec. I and Fig. 1 and in Refs. 5 and 7. In addition the E2/M1 mixing ratio of the $0.95 \rightarrow 0$ transition is found to be in the range $-0.124 < x_2 < +0.02$ (0.1% limit) or $-0.124 \le x_2 \le -0.03$ (10% limit). These limits are close to those found in the analysis of the $1.67 \rightarrow$ $0.95 \rightarrow 0$ results (Sec. IIA) and since we have two independent limits, we combine them and adopt $-0.124 \le x_2 \le +0.00$ as a firm limit on the possible range of the $0.95 \rightarrow 0$ mixing ratio.

IV. CONCLUSIONS

By combining the measurements reported in this work with those of previous work, summarized in Fig. 1 and in Refs. 5 and 7, we have obtained firm spinparity assignments of 2^+ , 2^- , and 1^- for the B¹² 0.95-, 1.67-, and 2.62-MeV levels. The present measurements are consistent with the constraints imposed by lifetime limits⁵ on the M2/E1 mixing ratios of the $1.67 \rightarrow 0.95$ and $2.62 \rightarrow 0.95$ transitions. We obtained a firm (0.1%limit) constraint of -0.124 < x < +0.00 for the E2/M1 $0.95 \rightarrow 0$ transition by combining the constraint from the lifetime of the level⁵ with the present work. This mixing ratio is probably in the range -0.124 < x <-0.03. Our phase convention for mixing ratios is that of Rose and Brink,¹⁷ which is the same as that of Litherland and Ferguson⁴ for E2/M1 mixtures.

The mixing ratio of the $0.95 \rightarrow 0$ transition was previously measured by Beck.¹⁸ He obtained + (0.085±0.063) by combining angular-distribution and linear polarization measurements on the $0.95 \rightarrow 0$ transition produced by the B¹¹(d,p)B¹² reaction at $E_d = 700$ keV. This result has the opposite sign to the range allowed by our measurements. However, decreasing Beck's result by two standard deviations gives x = -0.04, so that the disagreement is not too serious.

Results of the present measurement, as well as previous results, have been discussed and compared to theory by Olness and Warburton.⁵

¹⁷ H. J. Rose and D. M. Brink, Rev. Mod. Phys. **39**, 306 (1967).

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