the observed elemental ratios of Li, Be, and B relative to C, N, and O in cosmic rays of energies higher than 1.5 GeV/nucleon[30,31] strengthens the hypothesis of a spallation origin of Li, Be, and B in the cosmic radiation and points out to the stability of ^{7}Be in cosmic rays.

Finally, the relatively low value of the production cross section of ^{10}Be in ^{16}O (and the preliminary value in ^{12}C) will make it more difficult than initially expected to use this isotope as a "clock" for the determination of cosmic-ray "ages," at least in connection with the expected difference in the ratio of Be/B between laboratory experiments and cosmic-ray measurements at similar energies.[32–34]

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Intrinsic Matrix Elements in the *s-d* Shell

CARLA ABULAFFIO
Department of Physics, Technion, Israel Institute of Technology, Haifa, Israel
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A new table of the intrinsic matrix elements of a two-body scalar Hamiltonian independent of spin is presented. The space spanned is that subspace of the *s-d* shell which contains the more deformed intrinsic states. The table is shown to be especially useful for the study of the relative position of bands belonging to the same $SU(3)$ representation.

I. INTRODUCTION

THE aim of this work is to present a new table of the intrinsic matrix elements of a two-body scalar Hamiltonian in the *s-d* shell. The table does not contain more information than that in Tables VII and IX of Ref. 1, from which it was derived, but the information is more readily available.

The table will be found useful for the study of the dependence of the intrinsic matrix elements on the two-body interaction. It has already been utilized in our work[2,3] on the order of levels in ^{22}Ne and lately in the work being done on ^{24}Mg.

The usefulness of the table was first suggested by Levinson in 1961, and the table was then computed by Kugler and myself. Unfortunately, we performed this calculation with decimals, and because of the accumulation of round-off errors, the final results were too inaccurate to be used in quantitative work. We have now precisely recomputed the parameters appearing in the table (as quotients of integers) by making use of the Elliott 503 Computer.

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II. DEFINITIONS AND NOTATIONS

With the exception of Eq. (1), all the quantities and symbols used in this and the following sections are defined in Ref. 1. Nevertheless, for the sake of readability, we repeat here some of the definitions.

The Hamiltonian of a N-nucleon system in the space of the *s-d* shell can be written as

$$H=\sum_{\rho=1}^{10} M_\rho H_\rho\,, \tag{1}$$

if the N nucleons interact through a two-body scalar interaction independent of spin. The matrix elements M_ρ depend only on the two-body interaction and not on the nuclear configuration. They are defined as follows:

$$\begin{aligned} &M_1\equiv\langle s^2,0|H|s^2,0\rangle\,, && M_6\equiv\langle d^2,2|H|d^2,2\rangle\,,\\ &M_2\equiv\tfrac{1}{5}^{1/2}\langle s^2,0|H|d^2,0\rangle_s\,, && M_7\equiv\langle d^2,4|H|d^2,4\rangle\,,\\ &M_3\equiv\langle d^2,0|H|d^2,0\rangle\,, && M_8\equiv\langle d^2,1|H|d^2,1\rangle\,,\\ &M_4\equiv\langle sd,2|H|sd,2\rangle_s\,, && M_9\equiv\langle sd,2|H|sd,2\rangle_a\,,\\ &M_5\equiv(1/14)^{1/2}\langle sd,2|H|d^2,2\rangle_s\,, && M_{10}\equiv\langle d^2,3|H|d^2,3\rangle\,. \end{aligned} \tag{2}$$

TABLE I. Intrinsic matrix elements of the 10 operators H_ρ and the connected operators $F_{\alpha\beta\gamma}H_\rho$. Upon changing the sign of the numerical coefficient of K, one obtains the matrix elements $\langle|F_{-455}H_\rho|\rangle/\langle|F_{-44}|\rangle$ and $\langle|F_{-41555}H_\rho|\rangle/\langle|F_{-44}|\rangle$ from the matrix elements $\langle|F_{411}H_\rho|\rangle/\langle|F_{4-4}|\rangle$ and $\langle|F_{45111}H_\rho|\rangle/\langle|F_{4-4}|\rangle$, respectively.

	$\langle\|H_\rho\|\rangle$
H_1	$(1/72)N^2+(1/36)N\xi+(1/72)\xi^2-(1/36)N-(1/36)\xi$
H_2	$-(5/36)[(\lambda+\mu)(\lambda+\mu+3)-\lambda\mu]+(265/576)N^2$ $+(143/288)N\xi+(185/576)\xi^2-(353/288)N+(1037/1440)\xi$ $+(17/48)\mu(\mu+2)+(37/240)\epsilon-\Sigma P_{ij}{}^x$
H_3	$(17/720)N^2-(1/360)N\xi+(17/720)\xi^2-(17/360)N$ $+(77/1800)\xi+(1/20)\mu(\mu+2)-\frac{1}{10}K^2-(1/100)\epsilon$
H_4	$(1/288)N^2+(7/144)N\xi-(23/288)\xi^2+(71/144)N$ $-(263/720)\xi-\frac{1}{8}\mu(\mu+2)-(1/40)\epsilon+\frac{1}{2}\Sigma P_{ij}{}^x$
H_5	$(7/9)[(\lambda+\mu)(\lambda+\mu+3)-\lambda\mu]-(119/144)N^2-(73/72)N\xi$ $-(55/144)\xi^2-(161/72)N-(71/72)\xi$ $-(7/12)\mu(\mu+2)-(7/12)\epsilon$
H_6	$(17/288)N^2-(31/1008)N\xi+(71/2016)\xi^2-(17/144)N$ $+(779/5040)\xi+(1/56)\mu(\mu+2)+(1/14)K^2-(1/40)\epsilon$
H_7	$(3/20)N^2-(3/70)N\xi+(1/140)\xi^2-(11/20)N+(137/700)\xi$ $+(2/35)\mu(\mu+2)+(1/35)K^2+(3/50)\epsilon$
H_8	$(9/160)N^2-(1/80)N\xi-(7/160)\xi^2+(3/16)N-(7/80)\xi$ $-(1/40)\mu(\mu+2)-(1/40)\epsilon$
H_9	$(13/96)N^2+\frac{1}{16}N\xi+(5/96)\xi^2-(29/48)N+(61/240)\xi$ $+\frac{1}{8}\mu(\mu+2)+(1/40)\epsilon-\frac{1}{2}\Sigma P_{ij}{}^x$
H_{10}	$(7/120)N^2-(1/20)N\xi-(1/120)\xi^2+\frac{1}{6}N-\frac{1}{6}\xi-\frac{1}{10}\mu(\mu+2)$

	$\langle\|F_{15}H_\rho\|\rangle$
H_1	$-\frac{1}{9}N\xi-\frac{1}{9}\xi^2+(2/9)\xi$
H_2	$-(217/384)N^2-(493/576)N\xi-(4039/5760)\xi^2+(15/32)N$ $+(1493/1440)\xi-\frac{1}{12}\mu(\mu+2)+(5/12)K^2$ $+(37/480)\epsilon^2-(37/240)\epsilon$
H_3	$-(7/480)N^2+(29/720)N\xi-(889/7200)\xi^2+(13/120)N$ $+(53/1800)\xi-(2/15)\mu(\mu+2)+(19/60)K^2$ $-(1/200)\epsilon^2+(1/100)\epsilon$
H_4	$(7/192)N^2-(77/288)N\xi+(601/2880)\xi^2-(5/48)N$ $+(163/720)\xi+(1/24)\mu(\mu+2)+\frac{1}{8}K^2-(1/80)\epsilon^2+(1/40)\epsilon$
H_5	$(119/96)N^2+(227/144)N\xi+(277/288)\xi^2-(11/24)N$ $-(71/72)\xi+\mu(\mu+2)-(8/3)K^2-(7/24)\epsilon^2+(7/12)\epsilon$
H_6	$\frac{1}{64}N^2+(185/2016)N\xi+(587/20160)\xi^2+(1/48)N$ $-(289/5040)\xi+(17/84)\mu(\mu+2)-(11/21)K^2$ $-(1/80)\epsilon^2+(1/40)\epsilon$
H_7	$-(3/80)N^2+(69/280)N\xi-(9/2800)\xi^2-(1/40)N$ $-(589/1400)\xi-(31/280)\mu(\mu+2)+(23/280)K^2$ $+(3/100)\epsilon^2-(3/50)\epsilon$
H_8	$-(7/320)N^2+\frac{3}{32}N\xi+(41/320)\xi^2-(13/80)N+\frac{1}{16}\xi$ $-\frac{1}{10}\mu(\mu+2)+\frac{1}{10}K^2-(1/80)\epsilon^2+(1/40)\epsilon$
H_9	$-(7/192)N^2-(17/96)N\xi+(13/960)\xi^2+(5/48)N-(1/240)\xi$ $-(1/24)\mu(\mu+2)-\frac{1}{8}K^2+(1/80)\epsilon^2-(1/40)\epsilon$
H_{10}	$(7/120)N^2+\frac{1}{12}N\xi-(17/120)\xi^2+(7/120)N-(7/120)\xi$ $+(17/120)\mu(\mu+2)+(1/40)K^2$

	$\dfrac{\langle\|F_{411}H_\rho\|\rangle}{\langle\|F_{4-4}\|\rangle}$
H_1	$-(2/9)N-(2/9)\xi+4/9$
H_2	$-(143/144)N-(1621/720)\xi+(37/120)\epsilon+4/9$
H_3	$(7/180)N-(241/900)\xi+\frac{1}{5}K-(1/50)\epsilon-8/45$
H_4	$-(31/72)N+(349/360)\xi-(1/20)\epsilon+10/9$
H_5	$(73/36)N+(127/36)\xi-(7/6)\epsilon+16/9$
H_6	$(115/504)N-(247/2520)\xi-(1/7)K-(1/20)\epsilon-4/63$
H_7	$(27/70)N-(267/700)\xi-(2/35)K+(3/25)\epsilon-46/35$
H_8	$\frac{1}{8}N+(3/40)\xi-(1/20)\epsilon$
H_9	$-(11/24)N-(21/40)\xi+(1/20)\epsilon-\frac{2}{3}$
H_{10}	$\frac{1}{3}N+(9/20)\xi+\frac{2}{3}$

	$\langle\|F_{1155}H_\rho\|\rangle$
H_1	$(8/9)\xi^2-(4/9)N-(4/9)\xi+(4/9)\mu(\mu+2)-(4/9)K^2$
H_2	$(32/9)\xi^2-(16/9)N-(16/9)\xi+(22/9)\mu(\mu+2)-(34/9)K^2$
H_3	$(32/45)\xi^2-(16/45)N-(16/45)\xi$ $+(28/45)\mu(\mu+2)-(52/45)K^2$
H_4	$-(16/9)\xi^2+(8/9)N+(8/9)\xi-(8/9)\mu(\mu+2)+(8/9)K^2$
H_5	$-(64/9)\xi^2+(32/9)N+(32/9)\xi-(56/9)\mu(\mu+2)+(104/9)K^2$
H_6	$-(32/63)\xi^2+(16/63)N+(16/63)\xi$ $-(40/63)\mu(\mu+2)+(88/63)K^2$
H_7	$(24/35)\xi^2-(12/35)N-(12/35)\xi$ $+(16/35)\mu(\mu+2)-(24/35)K^2$
H_8	0
H_9	0
H_{10}	0

	$\dfrac{\langle\|F_{45111}H_\rho\|\rangle}{\langle\|F_{4-4}\|\rangle}$	$\dfrac{\langle\|F_{441111}H_\rho\|\rangle}{\langle\|F_{44-4-4}\|\rangle}=\dfrac{\langle\|F_{-4-45555}H_\rho\|\rangle}{\langle\|F_{-4-444}\|\rangle}$
H_1	$(8/3)\xi$	16/3
H_2	$(32/3)\xi-4K+4$	40/3
H_3	$(32/15)\xi-(8/5)K+8/5$	16/15
H_4	$-(16/3)\xi$	−32/3
H_5	$-(64/3)\xi+16K-16$	−32/3
H_6	$-(32/21)\xi+(16/7)K-16/7$	32/21
H_7	$(72/35)\xi-(24/35)K+24/35$	96/35
H_8	0	0
H_9	0	0
H_{10}	0	0

The definition of the operators H_ρ, which are independent of the residual interaction, results from the coupling of Eqs. (1) and (2). H_ρ is the Hamiltonian defined by

$$M_i(H_\rho)=\delta_{i\rho}. \tag{3}$$

The importance of Eq. (1) lies in two factors:

(i) The M_ρ are the quantities which are most immediately computed from the two-body interaction.

(ii) For a definite nuclear configuration, the matrix elements of the H_ρ can easily be calculated once and for all.

Table I gives the dependence of the intrinsic matrix elements of the H_ρ and the connected operators

$$\begin{array}{l} F_{15}H_\rho \quad F_{411}H_\rho \quad F_{-455}H_\rho \\ F_{1155}H_\rho \quad F_{45111}H_\rho \quad F_{-41555}H_\rho \quad F_{441111}H_\rho \quad F_{-4-45555}H_\rho \end{array} \tag{4}$$

on the nuclear configuration. [The operators $F_{\alpha\beta\gamma} = F_\alpha F_\beta F_\gamma$ are defined in Ref. 1 as productions of $SU(3)$ generators].

The nuclear configuration is *not* the most general one in the *s-d* shell. We restrict ourselves to nuclear states, in which only the first three of the six available intrinsic single-particle states $\phi_0, \phi_1, \phi_{-1}, \phi_2, \phi_{0'}, \phi_{-2}$ are occupied. The same limitation held, of course, in Ref. 1. If the number of nucleons in state ϕ_i is indicated by n_i, our restriction can be stated as

$$n_i = 0 \quad \text{for} \quad i = 2, 0', -2. \tag{5}$$

The nuclear configuration is described by the six parameters

$$(\lambda\mu), K, \epsilon, \sum_{i<j} P_{ij}^x, N, \text{and } \xi. \tag{6}$$

$(\lambda\mu)$ is the $SU(3)$ representation, $K = \mu, \mu-2, \cdots, 1$ or 0 is the band number, $\epsilon = 2\lambda + \mu = 4n_0 + n_1 + n_{-1}$ is a measure of the intrinsic quadrupole distortion, and P_{ij}^x is the Majorana operator for particles i and j. Therefore, $\sum_{i<j} P_{ij}^x$ depends only on the partition $[f] \equiv [f_1, f_2, \cdots, f_s]$, which describes the symmetry of the wave function under space permutations. It is well known[4] that

$$\langle [f_1, f_2, \cdots, f_s] | \sum_{i<j} P_{ij}^x | [f_1, f_2, \cdots, f_s] \rangle = \tfrac{1}{2} \Big[\sum_{i=1}^{s} f_i^2 - \sum_{i=1}^{s} (2i-1) f_i \Big]. \tag{7}$$

$N = n_0 + n_1 + n_{-1}$ is the total number of nucleons in the *s-d* shell and $\xi = n_0 - n_1 - n_{-1}$.

III. DERIVATION OF TABLE

Table I is derived immediately (in principle) from Tables VII and IX, Eqs. (IV A3) and (IV A32) of Ref. 1, if one remembers the meaning of the H_ρ. Equation (IV A3) of Ref. 1, namely,

$$g_\sigma = \sum_{\rho=1}^{10} S_\rho^\sigma M_\rho,$$

becomes in fact $g_\sigma = S_\rho^\sigma$ for $H = H_\rho$. If we now set $H = H_\rho$ in Eq. IV A32 of Ref. 1, we obtain

$$\begin{aligned} H_\rho = {} & 9S_\rho^1 C + (S_\rho^2 + 23 S_\rho^7) \mathbf{L}\cdot\mathbf{L} \\ & + [(S_\rho^2 + 6S_\rho^9)(N-2) + S_\rho^3(N-1)] \sum_i l_i^2 \\ & + S_\rho^9 (\sum_i l_i^2)^2 + S_\rho^7 [\sum_{i<j} X_{ij}^7 - 23 \mathbf{L}\cdot\mathbf{L}] \\ & + \tfrac{1}{9} S_\rho^8 \sum_{i<j} [(Q\times Q)_i^{(4)} \cdot (Q\times Q)_j^{(4)}] \\ & + S_\rho^{10} \sum_{i<j} [(Q\times L)_i^{(3)} \cdot (Q\times L)_j^{(3)}] \\ & + S_\rho^{11} \sum_{i<j} [(Q\times Q)_i^{(2)} \cdot (Q\times Q)_j^{(2)}] + S_\rho^5 \sum_{i<j} P_{ij}^x \\ & + [10 S_\rho^1 N(N-2) + \tfrac{1}{2} S_\rho^6 N(N-1)], \end{aligned} \tag{8}$$

which is the basis for the construction of Table I. In the table, the intrinsic diagonal matrix elements $\langle [f](\lambda\mu)K\epsilon | O | [f](\lambda\mu)K\epsilon \rangle$ of any operator O is indicated by the symbol $\langle | O | \rangle$.

IV. CONCLUSION

We have found the table particularly useful in studying the relative positions of levels belonging to different bands of the same $SU(3)$ representation. This is so because only a few of the matrix elements of the H_ρ are K-dependent and hence contribute to the separation between such bands.

The usefulness of the table is of course limited by the restriction $n_2 = n_{-2} = n_{0'} = 0$. This restriction means that we can deal only with the most deformed intrinsic states for every nucleus. In the cases of 2, 3, and 4 nucleons in the *s-d* shell, the intrinsic states included in our basis derive from two $SU(3)$ representations—(40) and (21), (60) and (41), (80) and (61), respectively. In the other cases, the intrinsic states included derive from a single $SU(3)$ representation. There is little hope of extending in a simple way the validity of Table I to the case in which n_2, n_0, and n_{-2} are different from zero, because in this case the n_i are no longer good quantum numbers. The intrinsic state is then a mixture of wave functions having the same ϵ and K, but different n_i.

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