Polarization Effects in Slow-Neutron Scattering. III. Nuclear Polarization*

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The theory of polarized-neutron scattering is extended to include the effects of nuclear polarization. General formulas for the cross section and the polarization of the scattered beam are given for arbitrary spin orderings. Several experiments which allow the determination of the spin dependence of the nuclear scattering amplitude are discussed.

INTRODUCTION

In two previous papers,[1,2] one of us has discussed various aspects of the scattering of slow neutrons from magnetically ordered solids. By making use of the density-matrix description for the polarized beam, expressions were derived for the cross section and for the polarization of the scattered beam resulting from a combination of nuclear, magnetic, and spin-orbit scattering from crystals with arbitrary magnetic ordering. Elastic scattering from particular arrays was considered in more detail. Some of these results have also been given by other authors.[3–5]

None of these authors has explicitly considered the effects of nuclear polarization on the scattering. Since many of the interesting features of the scattering are characteristically observed at low temperatures, such effects are possible, especially as the experiments are performed at ever decreasing temperatures and increasing magnetic field. For instance, Shull and Ferrier[6] have been able to detect a scattering contribution due to the small "brute force" polarization of V^{51} at temperatures below 15°K. In the particular case of the rare earths, the hyperfine splittings are large enough so that nuclear-polarization-dependent terms should be clearly observable in the 1–4°K range.

The occurrence of nuclear-polarization-dependent terms raises two, somewhat separate problems. First, in many experiments they may represent only a nuisance, for which a correction must be made. The expressions to be derived in this paper should facilitate this correction. Second, several of the terms are of interest in connection with the problem of the spin dependence of the nuclear-scattering amplitude. Experiments with unpolarized nuclei always lead[7] to two possible pairs (a_+, a_-) for the scattering amplitude due to compound nuclear states of spin $(I+\frac{1}{2}, I-\frac{1}{2})$, respectively. This ambiguity can only be resolved, and the values a_+ and a_- determined separately, by taking advantage of the nuclear-polarization-dependent terms. This has previously been investigated only for the simplest cases of purely nuclear scattering[8] and scattering from a colinear ferromagnet.[9]

The principal results of this paper are contained in Eq. (20), which gives the cross section for scattering of a polarized beam, and Eq. (21), which gives the polarization of the scattered beam.

GENERAL EXPRESSIONS FOR SCATTERING CROSS SECTION AND FINAL POLARIZATION OF THE SCATTERED BEAM

We will make use of the previously employed formalism and notation.[1,2] The cross section in Born approximation for scattering of a polarized beam is given by Eq. (I.1)

$$\frac{d^2\sigma}{d\Omega' d\epsilon'}=\frac{k'}{k}\left(\frac{m_0}{2\pi\hbar^2}\right)^2\sum_{qq'}p_q\,\mathrm{tr}[\mathcal{V}_{qq'}{}^{\dagger}(\mathbf{K})\mathcal{V}_{q'q}(\mathbf{K})\rho]\times\delta\left(\frac{\hbar^2}{2m_0}(k'^2-k^2)+E_{q'}-E_q\right)\quad(1)$$

and the polarization of the final beam $\mathbf{P}_f$ is given by Eq. (I.16)

$$\tfrac{1}{2}\mathbf{P}_f\frac{d^2\sigma}{d\Omega' d\epsilon'}=\frac{k'}{k}\left(\frac{m_0}{2\pi\hbar^2}\right)^2\sum_{qq'}p_q\,\mathrm{tr}[\mathcal{V}_{qq'}{}^{\dagger}(\mathbf{K})\mathbf{S}\mathcal{V}_{q'q}(\mathbf{K})\rho]\times\delta\left(\frac{\hbar^2}{2m_0}(k'^2-k^2)+E_{q'}-E_q\right).\quad(2)$$

Here $\mathbf{k}$ and $\mathbf{k}'$ are, respectively, the initial and final wave vectors of the neutron, and $\mathbf{K}=\mathbf{k}-\mathbf{k}'$. The scattering system, which has probability p_q of being in initial state q with energy E_q, goes to final state q' with energy $E_{q'}$. The quantum numbers q and q' refer to such properties as the state of magnetization, distribution of phonons, and arrangement of nuclear spins. $\mathcal{V}(\mathbf{K})$ is the Fourier transform of the interaction be-

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[1] M. Blume, Phys. Rev. **130**, 1670 (1963); to be referred to as I.

[2] M. Blume, Phys. Rev. **133**, A1366 (1964); to be referred to as II.

[3] O. Halpern and M. H. Johnson, Phys. Rev. **55**, 898 (1939).

[4] Yu. A. Izyumov, Zh. Eksperim. i Teor. Fiz. **42**, 1673 (1962) [English transl.: Soviet Phys.—JETP **15**, 1162 (1962)]; Usp. Fiz. Nauk. **80**, 41 (1963) [English transl.: Soviet Phys.—Uspekhi **16**, 3 (1963)].

[5] A. W. Overhauser, Bull. Am. Phys. Soc. **7**, 241 (1962).

[6] C. G. Shull and R. P. Ferrier, Phys. Rev. Letters **10**, 295 (1963).

[7] G. E. Bacon, *Neutron Diffraction* (Oxford University Press, Oxford, England, 1955), p. 33, *et seq.*

[8] M. E. Rose, Atomic Energy Commission Report No. AECD-2183 (unpublished).

[9] R. I. Schermer, Phys. Rev. **130**, 1907 (1963). Two typographical errors appear in the paper. In the second term of Eq. (1), replace $(I+1+f_N f_n)$ by $[1+If_N f_n/(I+1)]$; in Eq. (8), add a factor of $\frac{1}{2}$ to the last term.

tween the neutron and the scatterer. The density matrix ρ is given by

$$\rho=\tfrac{1}{2}\mathbf{1}+\mathbf{P}\cdot\mathbf{S}, \tag{3}$$

where $\mathbf{1}$ is the 2×2 unit matrix, $\mathbf{S}$ the neutron spin operator, and $\mathbf{P}$ the neutron-beam polarization. The trace in (1) and (2) is over neutron spin coordinates.

We consider a potential $\mathcal{V}(\mathbf{K})$ which is the sum of nuclear, magnetic, and spin-orbit contributions,

$$\mathcal{V}(\mathbf{K})=\mathcal{V}_n(\mathbf{K})+\mathcal{V}_m(\mathbf{K})+\mathcal{V}_{so}(\mathbf{K}), \tag{4a}$$

where

$$\mathcal{V}_n(\mathbf{K})=(2\pi\hbar^2/m_0)(T_0+\mathbf{T}_1\cdot\mathbf{S}), \tag{4b}$$

$$\mathcal{V}_m(\mathbf{K})=(2\pi\hbar^2/m_0)(2\gamma e^2/mc^2)\mathbf{Q}\cdot\mathbf{S}, \tag{4c}$$

$$\mathcal{V}_{so}(\mathbf{K})=i(2\pi\hbar^2/m_0)(2\gamma e^2/m_0c^2)\mathbf{R}\cdot\mathbf{S}, \tag{4d}$$

and the detailed form of the operators T_0, $\mathbf{T}_1$, and $\mathbf{Q}$ are given in (I.6). We have introduced $\mathbf{R}$ for the operator appearing in (II.7). The electron charge and mass are e and m, respectively. while the neutron mass and gyromagnetic ratio are m_0 and $\gamma=-1.91$, respectively.

Upon inserting (4a) into (1) and (2) we are led to consider six types of terms:

(i) purely nuclear scattering,
(ii) purely magnetic scattering,
(iii) nuclear-magnetic interference,
(iv) pure spin-orbit scattering,
(v) spin-orbit-magnetic interference,
(vi) spin-orbit-nuclear interference.

Of these, (i)–(iii) have been treated in (I), except for terms involving nuclear polarization, i.e., terms involving $\mathbf{T}_1$. Terms (iv)–(vi) are treated in II, and may be taken over with no change except for the components of (vi) involving $\mathbf{T}_1$. We will not consider spin-orbit effects any further since terms involving $\mathcal{V}_{so}$ are smaller than their magnetic counterparts by a factor of m/m_0. However, the operator structure exhibited in (4b)–(4d) gives a very simple recipe for adding spin-orbit terms to the general expressions (5) and (6) derived below. Terms (iv) follow from terms (ii) with the replacement $\mathbf{Q}\rightarrow i(m/m_0)\mathbf{R}$. Terms (vi) follow from (iii) in the same way. Terms (v) are found by considering those parts of (iii) involving $\mathbf{T}_1$ and replacing $\mathbf{T}_1\rightarrow i(2\gamma e^2/m_0c^2)\mathbf{R}$. The factor i appearing in these replacements changes the experimental conditions under which spin-orbit terms are observed relative to their magnetic counterparts. For instance, we may write (iii) as twice the real part of the product of nuclear and magnetic amplitudes. In passing to (vi), then, this would become minus twice the imaginary part of the equivalent products. The conditions under which such a term may be observed are discussed in II.

We proceed now by inserting (4a)–(4c) into (1) and (2), using (I.7) to evaluate the indicated traces. At this stage we do not explicitly repeat the terms previously derived, which are given in (I.8) for the cross section and (I.17) for the polarization. We find

$$\begin{aligned}d^2\sigma/d\Omega'd\epsilon'=(k'/k)\sum_{qq'}p_q\{&\mathrm{Re}(\langle q|T_0^\dagger|q'\rangle\langle q'|\mathbf{T}_1\cdot\mathbf{P}|q\rangle)+\tfrac{1}{4}i\mathbf{P}\cdot\langle q|\mathbf{T}_1^\dagger|q'\rangle\times\langle q'|\mathbf{T}_1|q\rangle\\&+(\gamma e^2/mc^2)\,\mathrm{Re}(\langle q|\mathbf{T}_1^\dagger|q'\rangle\cdot\langle q'|\mathbf{Q}|q\rangle+i\mathbf{P}\cdot\langle q|\mathbf{T}_1^\dagger|q'\rangle\times\langle q'|\mathbf{Q}|q\rangle)\}\\&\times\delta[(\hbar^2/2m_0)(k'^2-k^2)+E_{q'}-E_q]+(\mathrm{I}.8),\end{aligned} \tag{5}$$

$$\begin{aligned}\tfrac{1}{2}\mathbf{P}_f(d^2\sigma/d\Omega'd\epsilon')=(k'/k)\sum_{qq'}p_q\{&\tfrac{1}{2}\,\mathrm{Re}(\langle q|T_0^\dagger|q'\rangle\langle q'|\mathbf{T}_1|q\rangle+i\langle q|T_0^\dagger|q'\rangle\langle q'|\mathbf{T}_1|q\rangle\times\mathbf{P})\\&-\tfrac{1}{8}i\langle q|\mathbf{T}_1^\dagger|q'\rangle\times\langle q'|\mathbf{T}_1|q\rangle-\tfrac{1}{2}(\gamma e^2/mc^2)\,\mathrm{Re}(i\langle q|\mathbf{T}_1^\dagger|q'\rangle\times\langle q'|\mathbf{Q}|q\rangle)\\&+\tfrac{1}{2}(\gamma e^2/mc^2)\,\mathrm{Re}(\langle q|\mathbf{T}_1^\dagger|q'\rangle\langle q'|\mathbf{P}\cdot\mathbf{Q}|q\rangle+\langle q|\mathbf{P}\cdot\mathbf{T}_1^\dagger|q'\rangle\langle q'|\mathbf{Q}|q\rangle\\&-\mathbf{P}\langle q|\mathbf{T}_1^\dagger|q'\rangle\cdot\langle q'|\mathbf{Q}|q\rangle)\}\delta[(\hbar^2/2m_0)(k'^2-k^2)+E_{q'}-E_q]+(\mathrm{I}.17).\end{aligned} \tag{6}$$

We should point out that both (I.8) and (I.17) contain terms which are second order in $\mathbf{T}_1$. These lead to both nuclear-polarization-dependent and -independent terms. Only the latter have been carried through the remainder of the analysis in I.

ELASTIC SCATTERING

An inelastic scattering event is one in which $E_{q'}\neq E_q$. This may occur by means of an interaction between the incident neutron and any combination of the phonon, electron spin, and nuclear spin systems. We make rather different approximations for these three systems in order to proceed with the calculation of elastic scattering. The approximations all have the character that they give the coherent scattering and all of the nuclear polarization dependence properly. This is presumably everything of experimental interest as far as the present work is concerned. The purely magnetic, inelastic scattering will not be included. This is treated in other work[10] and will be unchanged by the presence of nuclear polarization. In detail, we proceed as follows. We write the lattice states $|q\rangle$ as a product of phonon, electron spin, and nuclear spin parts, so that the sum over q and q' is really a multiple sum over the quantum numbers of these systems individually.

(a) For the nuclear spin system we make a quasi-elastic approximation, $E_{q'}=E_q$ for all q'. Since this is a complete set of states we may immediately sum over final nuclear states, and the answer will appear as a thermal average over initial nuclear states, denoted by diagonal braces, e. g., $\langle \mathbf{I}_\nu\rangle$.

(b) Except for very slow neutrons, the same approximation is valid for the electron spin system in a paramagnet situated in any presently attainable magnetic field. However, in an ordered magnetic system we consider only coherent elastic scattering, for which $|q'\rangle=|q\rangle$. These approximations only affect the purely magnetic scattering terms. The nuclear operators do not affect the electron spin system in the uncoupled, approximate representation we are using. Therefore, in both the purely nuclear scattering and in the nuclear-magnetic interference terms we in fact need only consider those cases in which the initial and final states of the magnetic system are the same.

(c) Since nuclear-polarization experiments are performed at low temperatures, the phonon inelastic cross section is expected to be small. We will ignore the phonons entirely. For the coherent scattering, this amounts simply to dropping a Debye-Waller temperature factor common to all terms. The "isotropic" background scattering will, however, not be given correctly below. Our results will give this background as due to only isotopic and nuclear spin incoherence, with a certain dependence upon nuclear polarization. Phonon inelastic scattering will add to this background a part of what will appear below as coherent scattering, with a different dependence upon nuclear polarization.

We now give explicity the only nonvanishing matrix elements which, in the light of the above approximations, will appear in (5) and (6). For the magnetic system we follow (I.9)–(I.11),

$$\langle q'|\mathbf{Q}|q\rangle=\delta_{q'q}\sum_\nu \exp(i\mathbf{K}\cdot\mathbf{R}_\nu)f_\nu(\mathbf{K})S_\nu\mathbf{q}_\nu, \quad (7a)$$

where

$$\mathbf{q}_\nu=\hat{K}\times(\boldsymbol{\eta}_\nu\times K). \quad (7b)$$

Here, S_ν is the magnitude of the electronic spin for the ion at site ν, with position vector $\mathbf{R}_\nu$ from the origin, $\boldsymbol{\eta}_\nu$ is a unit vector in the direction of this spin, and $f_\nu(\mathbf{K})$ is the magnetic form factor for this ion. K is a unit vector in the direction of $\mathbf{K}$.

For the nuclear operator T_0 we have

$$\langle q'|T_0|q\rangle=\delta_{q'q}\sum_\nu \exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\{a_\nu\}, \quad (8a)$$

where

$$\{a_\nu\}=[a_\nu^+(I_\nu+1)+a_\nu^-I_\nu]/(2I_\nu+1) \quad (8b)$$

is the familiar coherent scattering length for the nucleus at position $\mathbf{R}_\nu$ with spin I_ν. The matrix elements of $\mathbf{T}_1$ are given by

$$\langle q'|\mathbf{T}_1|q\rangle=\langle q'|\sum_\nu \exp(i\mathbf{K}\cdot\mathbf{R}_\nu)C_\nu\mathbf{I}_\nu|q\rangle, \quad (9a)$$

where

$$C_\nu=2[(a_\nu^+-a_\nu^-)/(2I_\nu+1)]. \quad (9b)$$

The physical significance of C_ν may be seen as follows. The strength of the nuclear coherent scattering is given by $|\{a_\nu\}|^2$. On the other hand, the scattering cross section for an isolated atom is given by

$$\{|a_\nu|^2\}=[(I_\nu+1)|a_\nu^+|^2+I_\nu|a_\nu^-|^2]/(2I_\nu+1). \quad (10)$$

The spin incoherent scattering appears as the difference between these two quantities:

$$\{|a_\nu|^2\}-|\{a_\nu\}|^2=\tfrac{1}{4}I_\nu(I_\nu+1)|C_\nu|^2, \quad (11)$$

so that C_ν might be called the "incoherent scattering length." An experimental determination of C_ν will resolve the ambiguity in the spin-dependent scattering lengths, (a_ν^+, a_ν^-). Equation (11) shows that C_ν^2 is known from measurements with unpolarized nuclei, so that it is sufficient to determine simply the algebraic sign of C_ν.

If we insert (7)–(9) into (5) we obtain for the elastic scattering cross section

$$d\sigma/d\Omega'=\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})][\{a_{\nu'}^*\}\{a_\nu\}+\tfrac{1}{4}c_{\nu'}^*c_\nu(\langle\mathbf{I}_\nu\cdot\mathbf{I}_{\nu'}\rangle+i\mathbf{P}\cdot\langle\mathbf{I}_{\nu'}\times\mathbf{I}_\nu\rangle)]$$
$$+\mathrm{Re}\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]\{a_{\nu'}^*\}c_\nu\mathbf{P}\cdot\langle\mathbf{I}_\nu\rangle\}+2\,\mathrm{Re}\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_\nu(\mathbf{K})[\{a_{\nu'}^*\}\mathbf{q}_\nu\cdot\mathbf{P}$$
$$+\tfrac{1}{2}c_{\nu'}^*\langle\mathbf{I}_\nu'\rangle\cdot\mathbf{q}_\nu+\tfrac{1}{2}ic_{\nu'}^*\mathbf{P}\cdot(\langle\mathbf{I}_{\nu'}\rangle\times\mathbf{q}_\nu)]\}+\text{magnetic terms}, \quad (12)$$

where we have defined the magnetic scattering amplitude $p_\nu(\mathbf{K})$ by

$$p_\nu(\mathbf{K})=(\gamma e^2/mc^2)S_\nu f_\nu(\mathbf{K}). \quad (13)$$

[10] A. W. Saenz, Phys. Rev. **119**, 1542 (1960).

The purely magnetic terms in (12) are given by the last line of (I.12). The final beam polarization is given by inserting (7)–(9) into (6):

$$\tfrac{1}{2}\mathbf{P}_f(d\sigma/d\Omega') = \sum_{\nu\nu'} \exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})][\tfrac{1}{2}\mathbf{P}\{a_{\nu'}{}^*\}\{a_\nu\} - \tfrac{1}{8}\mathbf{P}c_{\nu'}{}^*c_\nu\langle\mathbf{I}_{\nu'}\cdot\mathbf{I}_\nu\rangle - \tfrac{1}{8}ic_{\nu'}{}^*c_\nu\langle\mathbf{I}_{\nu'}\times\mathbf{I}_\nu\rangle]$$
$$+\operatorname{Re}\{\sum_{\nu\nu'} \exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})][\tfrac{1}{2}\{a_{\nu'}{}^*\}c_\nu\langle\mathbf{I}_\nu\rangle + \tfrac{1}{2}i\{a_{\nu'}{}^*\}c_\nu\langle\mathbf{I}_\nu\rangle\times\mathbf{P} + \tfrac{1}{4}c_{\nu'}{}^*c_\nu\langle(\mathbf{I}_{\nu'}\cdot\mathbf{P})\mathbf{I}_\nu\rangle]\}$$
$$+\operatorname{Re}\{\sum_{\nu\nu'} \exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]\{a_{\nu'}{}^*\}p_\nu(\mathbf{K})[\mathbf{q}_\nu + i(\mathbf{q}_\nu\times\mathbf{P})]\}$$
$$+\tfrac{1}{2}\operatorname{Re}\{\sum_{\nu\nu'} \exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]c_{\nu'}{}^*p_\nu(\mathbf{K})[\langle\mathbf{I}_{\nu'}\rangle(\mathbf{q}_\nu\cdot\mathbf{P}) + (\langle\mathbf{I}_{\nu'}\rangle\cdot\mathbf{P})\mathbf{q}_\nu$$
$$-\mathbf{P}(\langle\mathbf{I}_{\nu'}\rangle\cdot\mathbf{q}_\nu) - i\langle\mathbf{I}_{\nu'}\rangle\times\mathbf{q}_\nu]\} + \text{magnetic terms.} \quad (14)$$

The purely magnetic terms are given by the last two lines of (I.18).

To perform the thermal averages over the nuclear magnetic substates, we assume that the nuclei are uncorrelated. This is an excellent approximation for most magnetic materials; the internuclear coupling is the Suhl-Nakamura interaction, which is quite weak compared to the usual hyperfine interaction at each individual nuclear site. We will not discuss such cases as liquid He^3 or solid hydrogen, although they are very interesting. In these the effective internuclear coupling is very strong as a result of the correlation between electronic and nuclear states, which is brought about by Fermi statistics, and there is also the possibility of nuclear "spin waves."

The thermal averages in (12) and (14) may then be written as[11]

$$\langle\mathbf{I}_{\nu'}\cdot\mathbf{I}_\nu\rangle = \langle\mathbf{I}_{\nu'}\rangle\cdot\langle\mathbf{I}_\nu\rangle(1-\delta_{\nu'\nu}) + I(I+1)\delta_{\nu'\nu}, \quad (15a)$$

$$\langle\mathbf{I}_{\nu'}\times\mathbf{I}_\nu\rangle = \langle\mathbf{I}_{\nu'}\rangle\times\langle\mathbf{I}_\nu\rangle + i\langle\mathbf{I}_\nu\rangle\delta_{\nu'\nu}, \quad (15b)$$

$$\langle(\mathbf{I}_{\nu'}\cdot\mathbf{P})\mathbf{I}_\nu\rangle = (\langle\mathbf{I}_{\nu'}\rangle\cdot\mathbf{P})\langle\mathbf{I}_\nu\rangle(1-\delta_{\nu'\nu}) + \langle\mathbf{I}_\nu(\mathbf{I}_\nu\cdot\mathbf{P})\rangle\delta_{\nu'\nu}. \quad (15c)$$

The last term of (15c) can only be calculated if the nuclear hyperfine Hamiltonian is known. For unoriented nuclei it equals $(1/3)I(I+1)\mathbf{P}$. Since it appears only in the expression for the polarization of the incoherent scattering, it is presumably of minor experimental importance. It is, however, interesting to note that this term depends upon the second moment of the nuclear spin distribution. It will be present even if the nuclei are aligned but not polarized, as would occur, for instance, as a result of the interaction of the nuclear electric-quadrupole moment with a crystalline electric field gradient. Such a term can only occur for S-wave neutrons if one observes a vector quantity, i.e., the final polarization.

We define the vector-nuclear polarization $\mathbf{P}_\nu{}^N$ by the relation

$$\mathbf{P}_\nu{}^N = \langle\mathbf{I}_\nu\rangle/I_\nu. \quad (16)$$

The magnitude of $\mathbf{P}_\nu{}^N$ varies between unity for complete polarization and zero for no polarization. It will be convenient to define a new symbol

$$\mathbf{d}_\nu = C_\nu I_\nu \mathbf{P}_\nu{}^N. \quad (17)$$

Before (12) and (14) can be applied, they must be averaged over isotopic distributions. To be consistent with (I) we shall use angular brackets $\langle\ \rangle$ to denote isotopic averages. There should be no confusion with the use of such brackets to also represent thermal averages in (12) and (14). The thermal averages will no longer explicitly appear, except in the term $\langle\mathbf{I}_\nu(\mathbf{P}\cdot\mathbf{I}_\nu)\rangle$, for which we will use a double set of brackets to indicate the combined isotopic and thermal average.

The averaging is performed exactly as in I. We define, for instance,

$$\langle\{a\}\rangle = \sum_\alpha r_\alpha\{a_\alpha\}, \quad (18)$$

where r_α is the concentration of the αth isotope with coherent scattering length $\{a_\alpha\}$. Similar definitions apply to the isotopic averages of all other nuclear amplitude combinations appearing in (12) and (14).

Further, we have a set of relations similar to

$$\langle\{a_{\nu'}{}^*\}\{a_\nu\}\rangle = \langle\{a_{\nu'}{}^*\}\rangle\langle\{a_\nu\}\rangle, \qquad \nu\neq\nu'$$
$$= \langle|\{a_\nu\}|^2\rangle, \qquad \nu=\nu'. \quad (19)$$

We use (11) to express $|C_\nu|^2$ in terms of more familiar quantities. We also include at this point the purely magnetic scattering found from (I.15) for the cross section and (I.19) for the polarization. The

[11] In (15b), a factor $(1-\delta_{\nu'\nu})$ in the first term of the right-hand side would be extraneous. After thermal averaging, $\langle\mathbf{I}_\nu\rangle$ is no longer an operator, so that $\langle\mathbf{I}_\nu\rangle\times\langle\mathbf{I}_\nu\rangle\equiv 0$.

final results are

$$
\begin{aligned}
d\sigma/d\Omega' = {} & \Big|\sum_{\nu}\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\{a_\nu\}\rangle\Big|^2+\tfrac{1}{4}\Big|\sum_{\nu}\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\mathbf{d}_\nu\rangle\Big|^2+\operatorname{Re}\Big\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]\langle\{a_{\nu'}^*\}\rangle\langle\mathbf{d}_\nu\cdot\mathbf{P}\rangle\Big\} \\
& +\tfrac{1}{4}i\mathbf{P}\cdot\Big\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]\langle\mathbf{d}_{\nu'}^*\rangle\times\langle\mathbf{d}_\nu\rangle\Big\} \\
& +\sum_{\nu}\Big\{\langle\{|a_\nu|^2\}\rangle-|\langle\{a_\nu\}\rangle|^2-\tfrac{1}{4}|\langle\mathbf{d}_\nu\rangle|^2-\langle(I_\nu+1)^{-1}\{|a_\nu|^2\}\mathbf{P}_\nu^N\cdot\mathbf{P}\rangle \\
& +\langle(I_\nu+1)^{-1}|\{a_\nu\}|^2\mathbf{P}_\nu^N\cdot\mathbf{P}\rangle+\operatorname{Re}[\langle\{a_\nu^*\}\mathbf{d}_\nu\cdot\mathbf{P}\rangle-\langle\{a_\nu^*\}\rangle\langle\mathbf{d}_\nu\cdot\mathbf{P}\rangle]\Big\} \\
& +2\operatorname{Re}\Big\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_\nu(\mathbf{K})[\langle\{a_{\nu'}^*\}\rangle\mathbf{q}_\nu\cdot\mathbf{P}+\tfrac{1}{2}\langle\mathbf{d}_{\nu'}^*\rangle\cdot\mathbf{q}_\nu+\tfrac{1}{2}\mathbf{P}\cdot(\langle\mathbf{d}_{\nu'}^*\rangle\times\mathbf{q}_\nu)]\Big\} \\
& +\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_{\nu'}^*(\mathbf{K})p_\nu(\mathbf{K})[\mathbf{q}_{\nu'}\cdot\mathbf{q}_\nu+i\mathbf{P}\cdot(\mathbf{q}_{\nu'}\times\mathbf{q}_\nu)]. \qquad (20)
\end{aligned}
$$

The first four terms give the nuclear coherent scattering. This is followed by the nuclear incoherent scattering. The next to last line gives the coherent nuclear-magnetic interference terms. The last line gives the coherent, pure, magnetic scattering.

$$
\begin{aligned}
\tfrac{1}{2}\mathbf{P}_f(d\sigma/d\Omega') = {} & \tfrac{1}{2}\mathbf{P}\Big|\sum_{\nu}\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\{a_\nu\}\rangle\Big|^2-\tfrac{1}{8}\mathbf{P}\Big|\sum_{\nu}\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\mathbf{d}_\nu\rangle\Big|^2-\tfrac{1}{8}i\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]\langle\mathbf{d}_{\nu'}^*\rangle\times\langle\mathbf{d}_\nu\rangle \\
& +\tfrac{1}{2}\operatorname{Re}\Big\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})][\langle\{a_{\nu'}^*\}\rangle\langle\mathbf{d}_\nu\rangle+i\langle\{a_{\nu'}^*\}\rangle\langle\mathbf{d}_\nu\times\mathbf{P}\rangle+\tfrac{1}{2}\langle\mathbf{d}_{\nu'}^*\cdot\mathbf{P}\rangle\langle\mathbf{d}_\nu\rangle]\Big\} \\
& +\sum_{\nu}\Big\{-\tfrac{1}{2}\mathbf{P}[\langle\{|a_\nu|^2\}\rangle-2\langle|\{a_\nu\}|^2\rangle+|\langle\{a_\nu\}\rangle|^2-\tfrac{1}{4}|\langle\mathbf{d}_\nu\rangle|^2] \\
& +\tfrac{1}{2}\langle(I_\nu+1)^{-1}\{|a_\nu|^2\}\mathbf{P}_\nu^N\rangle-\tfrac{1}{2}\langle(I_\nu+1)^{-1}|\{a_\nu\}|^2\mathbf{P}_\nu^N\rangle \\
& +\tfrac{1}{2}\operatorname{Re}[\langle\{a_\nu^*\}\mathbf{d}_\nu\rangle-\langle\{a_\nu^*\}\rangle\langle\mathbf{d}_\nu\rangle+i\langle\{a_\nu^*\}\mathbf{d}_\nu\times\mathbf{P}\rangle-i\langle\{a_\nu^*\}\rangle\langle\mathbf{d}_\nu\times\mathbf{P}\rangle+\tfrac{1}{2}\langle\langle|c_\nu|^2\mathbf{I}_\nu(\mathbf{I}_\nu\cdot\mathbf{P})\rangle\rangle-\tfrac{1}{2}\langle\mathbf{d}_\nu^*\rangle\langle\mathbf{d}_\nu\cdot\mathbf{P}\rangle]\Big\} \\
& +\operatorname{Re}\Big\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})][\langle\{a_{\nu'}^*\}\rangle p_\nu(\mathbf{K})(\mathbf{q}_\nu+i(\mathbf{q}_\nu\times\mathbf{P})) \\
& +\tfrac{1}{2}p_\nu(\mathbf{K})(\langle\mathbf{d}_{\nu'}^*\rangle(\mathbf{q}_\nu\cdot\mathbf{P})+\langle\mathbf{d}_{\nu'}^*\cdot\mathbf{P}\rangle\mathbf{q}_\nu-\mathbf{P}(\langle\mathbf{d}_{\nu'}^*\rangle\cdot\mathbf{q}_\nu)-i\langle\mathbf{d}_{\nu'}^*\rangle\times\mathbf{q}_\nu)]\Big\} \\
& -\tfrac{1}{2}\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_{\nu'}^*(\mathbf{K})p_\nu(\mathbf{K})[\mathbf{P}(\mathbf{q}_{\nu'}\cdot\mathbf{q}_\nu)+i(\mathbf{q}_{\nu'}\times\mathbf{q}_\nu)] \\
& +\operatorname{Re}\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_{\nu'}^*(\mathbf{K})p_\nu(\mathbf{K})(\mathbf{q}_{\nu'}\cdot\mathbf{P})\mathbf{q}_\nu. \qquad (21)
\end{aligned}
$$

The order of the terms is the same as in (20): nuclear coherent, nuclear incoherent, nuclear-magnetic interference, and pure magnetic scattering.

DISCUSSION AND EXAMPLES

Let us consider experiments designed to determine the sign of C_ν. The simplest experiments would be those in which the coherent scattering is observed. It should be remembered that besides the isotropic elastic background due to nuclear incoherence, calculated above, there is an inelastic background which we have ignored. Part of this, the phonon inelastic scattering, will have the nuclear-polarization dependence of the coherent nuclear scattering. This will further complicate the over-all polarization dependence of the background, which is already quite complicated enough.

As far as coherent scattering is concerned, nuclear polarization enters only through the quantity $\langle\mathbf{d}_\nu\rangle$. The terms linear in $\langle\mathbf{d}_\nu\rangle$ are those of interest. It can be seen that the vector $\frac{1}{2}\langle\mathbf{d}_\nu\rangle$ plays the same role in the scattering from polarized nuclei as the vector $p_\nu(\mathbf{K})\mathbf{q}_\nu$ plays in the scattering from a magnetized electronic system. The experimental conditions under which the magnetic terms containing a factor i can be observed were discussed in I. The corresponding nuclear-polarization terms will be observed under identical conditions.

To illustrate the effect of the various terms in (20) and (21), we shall give a few specific examples. Many of the results have already been given elsewhere.

(a) We consider first the case of "brute-force nuclear polarization," which has been discussed previously by Rose.[8] This is polarization due to the interaction of the nuclear magnetic moment with an externally applied field in a nonmagnetic material. For this situation, (20) gives for the coherent scattering

$$
\begin{aligned}
(d\sigma/d\Omega') = {} & \Big|\sum_{\nu}\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\{a_\nu\}\rangle\Big|^2 \\
& +\tfrac{1}{4}\Big|\sum_{\nu}\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\mathbf{d}_\nu\rangle\Big|^2 \\
& +\operatorname{Re}\Big\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]\langle\{a_{\nu'}^*\}\rangle\langle\mathbf{d}_\nu\cdot\mathbf{P}\rangle\Big\}. \qquad (22)
\end{aligned}
$$

Let $\hat{H}$ be a unit vector in the direction of the external field. It is convenient to consider separately the components of the neutron beam which are parallel (subscript +) and antiparallel (subscript −) to $\hat{H}$. The nuclear polarization may also be parallel or antiparallel to $\hat{H}$, depending upon the sign of the nuclear gyromagnetic ratio g_N.

The coherent nuclear scattering cross section may then be written,

$$(d\sigma/d\Omega')_{\pm} = |\sum \exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\{a_\nu\}\pm\tfrac{1}{2}\mathbf{d}_\nu\cdot\hat{H}\rangle|^2.$$

Substituting from (8b) and (17) we find, for each isotope, that the neutrons which are parallel to the direction of nuclear polarization see a coherent scattering amplitude given by the upper pair of signs in

$$\left(\frac{I+1}{2I+1}\right)a^+\left(1\pm\frac{IP^N}{I+1}\right)+\left(\frac{I}{2I+1}\right)a^-(1\mp P^N). \quad (23)$$

As $P^N\to1$, this coherent amplitude $\to a^+$, as it must, since the compound state in this process is just that of total spin $I+\frac{1}{2}$ and z component $I+\frac{1}{2}$. We will expect to find that the spin incoherent scattering vanishes in this instance. The neutrons which are antiparallel to the nuclear polarization see a coherent scattering amplitude given by the lower set of signs in (23). As $P^N\to1$, this amplitude $\to(a^++2Ia^-)/(2I+1)$. The amplitude is still a mixture because the initial state of nucleus "up" and neutron "down" does not lead to an eigenstate of the compound system.

We note that C_ν always occurs in the combination $C_\nu I_\nu \mathbf{P}_\nu{}^N=\mathbf{d}_\nu$. Therefore the sign of $P_\nu{}^N$ must be known before the sign of C_ν can be determined. Further, the presence of even-even isotopes, for which $\mathbf{d}_\nu=0$, may have a large effect on the experimental result. The physical basis for this is as follows: As the nuclear polarization is changed, the coherent amplitude of each polarized isotope is effectively changed. This in turn changes the relative amount of isotopically coherent and incoherent scattering. It is possible for instance to arrange an experimental situation in which the isotopic incoherence would vanish at a particular nuclear polarization for one of the isotopes. The complicated expressions for the nuclear incoherent scattering are a result of this interplay of spin and isotopic incoherence.

Let us continue the example by determining the incoherent scattering for a monoisotopic scatterer. We find

$$(d\sigma/d\Omega')_{\text{inc}}=[\{|a|^2\}-|\{a\}|^2]$$
$$\times\{1-[I/(I+1)](P^N)^2-[1/(I+1)]\mathbf{P}^N\cdot\mathbf{P}\}.$$

As argued above, we find that if $P_N=1$ and $\mathbf{P}^N\cdot\mathbf{P}=+1$, the incoherent scattering vanishes. We note that the incoherent scattering is reduced if $P^N\neq0$, even for unpolarized neutrons, while the coherent scattering is increased. The coefficient $\{|a|^2\}-|\{a\}|^2$ is known from experiments with unpolarized neutrons so that the experimental determination of $(d\sigma/d\Omega')_{\text{inc}}$ adds no new information on the scattering amplitudes. The term in $\mathbf{P}^N\cdot\mathbf{P}$, however, permits a determination of the sign of $\mathbf{P}^N$ relative to the applied field, which is often an unknown quantity.

Finally, we consider the polarization of the scattered beam:

$$\tfrac{1}{2}\mathbf{P}_f(d\sigma/d\Omega')=\tfrac{1}{2}\mathbf{P}|\sum_\nu\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\{a_\nu\}\rangle|^2-\tfrac{1}{8}\mathbf{P}|\sum_\nu\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\mathbf{d}_\nu\rangle|^2$$
$$+\tfrac{1}{2}\,\mathrm{Re}\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})][\langle\{a_{\nu'}^*\}\rangle\langle\mathbf{d}_\nu\rangle+\tfrac{1}{2}\langle\mathbf{d}_{\nu'}^*\cdot\mathbf{P}\rangle\langle\mathbf{d}_\nu\rangle+i\langle\{a_{\nu'}^*\}\rangle\langle\mathbf{d}_\nu\times\mathbf{P}\rangle]\}, \quad (24)$$

which must be divided by (22). The simplest method of determining $\langle\mathbf{d}_\nu\rangle$ is clearly to use an unpolarized incident beam and analyze the final polarization, since in this case the desired term is the only one which contributes. The last term in (24), which contains a factor i, is the analog of a magnetic term discussed in I. It can be observed if the nuclear amplitude contains a large imaginary part (strong absorption) or if the nuclear structure factor is complex.

(b) Nuclear polarization may often be enhanced by placing the nucleus in a magnetic material. A large number of possibilities are then opened up which we cannot investigate here. We shall simply write down the result for a simple colinear magnetic system with only one type of magnetic ion; that is, either a ferromagnet, antiferromagnet, or saturated paramagnet in which all the electron spins are either parallel or antiparallel to a unit vector $\boldsymbol{\eta}$. Then $\mathbf{q}_\nu=\pm\mathbf{q}$, where $\mathbf{q}=\hat{K}\times(\boldsymbol{\eta}\times\hat{K})$ is independent of ν. Further we assume that $\mathbf{I}_\nu$ is colinear with $\boldsymbol{\eta}$. The nuclear coherent scattering is again given by (22), and the coherent cross section is

$$d\sigma/d\Omega'=(22)+|\sum_\nu\exp(i\mathbf{K}\cdot\mathbf{R}_\nu)p_\nu(\mathbf{K})\mathbf{q}_\nu|^2+2\,\mathrm{Re}\{\sum_{\nu\nu'}\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_\nu(\mathbf{K})\mathbf{q}_\nu\cdot\langle\{a_{\nu'}^*\}\mathbf{P}+\tfrac{1}{2}\mathbf{d}_{\nu'}^*\rangle\} \quad (25)$$

which is the result given in Ref. 9. In analogy with the previous discussion, this result may be interpreted as a coherent mixture of nuclear and magnetic scattering with a modified nuclear coherent scattering length.

A particularly interesting case of (25) occurs for the superlattice reflections in an antiferromagnet. Since $\langle\mathbf{d}_\nu\rangle$ as well as $\mathbf{q}_\nu$ reverses sign between antiferromagnetically coupled neighboring atoms, the nuclear polarization terms enter into these reflections. For an unpolarized beam, and for only the superlattice reflections

$$d\sigma/d\Omega'=|\sum_n\exp(i\mathbf{K}\cdot\mathbf{n})|^2|\sum_j\exp(i\mathbf{K}\cdot\mathbf{r}_j)[p_j(\mathbf{K})\mathbf{q}_j+\tfrac{1}{2}\langle\mathbf{d}_j\rangle]|^2,$$

where we have written $\mathbf{R}_\nu=\mathbf{n}+\mathbf{r}_j$, $\mathbf{n}$ being the vector from the origin to the origin of a unit cell, and $\mathbf{r}_j$ the position vector within the unit cell.

Once again, a term linear in $\langle \mathbf{d}_\nu \rangle$ may be isolated by analyzing the final polarization for an initially unpolarized beam. We have, again for the superlattice reflections only,

$$\tfrac{1}{2}\mathbf{P}_f(d\sigma/d\Omega') = \mathrm{Re}\{\sum_{\nu\nu'} \exp[i\mathbf{K}\cdot(\mathbf{R}_\nu - \mathbf{R}_{\nu'})][\tfrac{1}{2}\langle\{a_{\nu'}^*\}\rangle\langle \mathbf{d}_\nu\rangle + \langle\{a_{\nu'}^*\}\rangle p_\nu(\mathbf{K})\mathbf{q}_\nu]\}.$$

The experimental geometry may then be chosen so that $\mathbf{q}=0$, i.e., $\mathbf{K}$ parallel to $\boldsymbol{\eta}$.

(c) The nuclear polarization tends to be colinear with the electronic magnetization; this led, above, to nuclear polarization effects in the superlattice reflections of an antiferromagnet, which are normally thought of as being of pure magnetic character. The same phenomenon occurs in more complicated magnetic systems. Let us for instance consider a magnetic spiral. As shown in I, this is a case in which the vector product terms in (20) and (21) contribute.

We consider a spiral which propagates with a ferromagnetic component in the direction $\hat{u}_3$, while spiraling about in a plane defined by the vectors $\hat{u}_1$ and $\hat{u}_2$. The unit vectors $\boldsymbol{\eta}_\nu$ giving the direction of the electron spin are then expressible as

$$\begin{aligned}\boldsymbol{\eta}_\nu &= \eta_{||}\hat{u}_3 + \eta_\perp\{\hat{u}_1 \cos\boldsymbol{\varepsilon}\cdot\mathbf{R}_\nu + \hat{u}_2 \sin\boldsymbol{\varepsilon}\cdot\mathbf{R}_\nu\} \\ &= \eta_{||}\hat{u}_3 + \tfrac{1}{2}\eta_\perp[\mathbf{u}_- \exp(i\boldsymbol{\varepsilon}\cdot\mathbf{R}_\nu) + \mathbf{u}_+ \exp(-i\boldsymbol{\varepsilon}\cdot\mathbf{R}_\nu)], \quad (26)\\ &\qquad \eta_{||}^2 + \eta_\perp^2 = 1,\end{aligned}$$

where $\boldsymbol{\varepsilon} = (2\pi/\lambda_s)\hat{u}_3$, λ_s being the wavelength of the spiral and $\mathbf{u}_\pm = \hat{u}_1 \pm i\hat{u}_2$.

Similarly, for the nuclear polarization we write

$$\begin{aligned}\mathbf{P}_\nu^N = P_{\nu||}^N\hat{u}_3 + \tfrac{1}{2}P_{\nu\perp}^N[\mathbf{u}_- \exp(i\boldsymbol{\varepsilon}\cdot\mathbf{R}_\nu) \\ + \mathbf{u}_+ \exp(-i\boldsymbol{\varepsilon}\cdot\mathbf{R}_\nu)],\\ (P_{\nu||}^N)^2 + (P_{\nu\perp}^N)^2 = (P_\nu^N)^2.\end{aligned}$$

We define

$$\langle d_{\nu||}\rangle = \langle C_\nu I_\nu P_{\nu||}^N\rangle;$$

$$\langle d_{\nu\perp}\rangle = \langle C_\nu I_\nu P_{\nu\perp}^N\rangle.$$

The necessary vector relations are

$$\mathbf{u}_\pm\cdot\mathbf{u}_\pm = 0; \quad \mathbf{u}_+\cdot\mathbf{u}_- = 2;$$

$$\hat{u}_3\cdot\mathbf{u}_\pm = 0; \qquad \hat{u}_3\cdot\hat{u}_3 = 1;$$

$$\mathbf{u}_\pm\times\mathbf{u}_\pm = \hat{u}_3\times\hat{u}_3 = 0;$$

$$\mathbf{u}_+\times\mathbf{u}_- = -2i\hat{u}_3; \qquad \hat{u}_3\times\mathbf{u}_\pm = \mp i\mathbf{u}_\pm;$$

and the vector identity

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C}).$$

In the course of the calculation, terms appear of the form

$$\sum_\nu \exp(i\mathbf{K}\cdot\mathbf{R}_\nu) \sum_{\nu'} \exp[i(\mathbf{K}+\boldsymbol{\varepsilon})\cdot\mathbf{R}_{\nu'}]$$

which vanish unless $\boldsymbol{\varepsilon}$ is a reciprocal lattice vector. Ignoring this pathological case we find, for the coherent scattering, a central peak involving the ferromagnetic components of $\boldsymbol{\eta}$ and $\mathbf{P}^N$:

$$\begin{aligned}(d\sigma/d\Omega')_{\text{central peak}} &= |\sum_\nu \exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle\{a_\nu\}\rangle|^2 + \tfrac{1}{4}|\sum_\nu \exp(i\mathbf{K}\cdot\mathbf{R}_\nu)\langle d_{\nu||}\rangle|^2 \\ &+ \mathrm{Re}\{\sum_{\nu\nu'} \exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]\langle\{a_{\nu'}^*\}\rangle\langle d_{\nu||}\rangle(\mathbf{P}\cdot\hat{u}_3)\} + 2\,\mathrm{Re}\{\sum_{\nu\nu'} \exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_\nu(\mathbf{K})\eta_{||} \\ &\times[\langle\{a_{\nu'}^*\}\rangle\mathbf{P}\cdot(\hat{u}_3 - \hat{K}(\hat{K}\cdot\hat{u}_3)) + \tfrac{1}{2}\langle d_{\nu'||}^*\rangle(1-(\hat{K}\cdot\hat{u}_3)^2) - \tfrac{1}{2}i\langle d_{\nu'||}^*\rangle\mathbf{P}\cdot(\hat{u}_3\times\hat{K})(\hat{u}_3\cdot\hat{K})]\} \\ &+ |\sum_\nu \exp(i\mathbf{K}\cdot\mathbf{R}_\nu)p_\nu(\mathbf{K})\eta_{||}|^2[1-(\hat{K}\cdot\hat{u}_3)^2]\end{aligned}$$

to which are added satellites involving the spiral, or $\perp$, components of $\boldsymbol{\eta}$ and $\mathbf{P}^N$,

$$\begin{aligned}(d\sigma/d\Omega')_{\text{satellites}} &= \tfrac{1}{8}|\sum_\nu \exp[i(\mathbf{K}\pm\boldsymbol{\varepsilon})\cdot\mathbf{R}_\nu]\langle d_{\nu\perp}\rangle|^2(1\pm\mathbf{P}\cdot\hat{u}_3) \\ &+ \tfrac{1}{4}\mathrm{Re}\{\sum_{\nu\nu'} \exp[i(\mathbf{K}\pm\boldsymbol{\varepsilon})\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_\nu(\mathbf{K})\eta_\perp\langle d_{\nu'\perp}^*\rangle[1+(\hat{K}\cdot\hat{u}_3)^2 \pm \mathbf{P}\cdot(\hat{u}_3+\hat{K}(\hat{K}\cdot\hat{u}_3)) + i\mathbf{P}\cdot(\hat{u}_3\times\hat{K})(\hat{u}_3\cdot\hat{K})]\} \\ &+ \tfrac{1}{4}|\sum_\nu \exp[i(\mathbf{K}\pm\boldsymbol{\varepsilon})\cdot(\mathbf{R}_\nu-\mathbf{R}_{\nu'})]p_\nu(\mathbf{K})\eta_\perp|^2[1+(\hat{K}\cdot\hat{u}_3)^2 \pm 2(\mathbf{P}\cdot\hat{K})(\hat{K}\cdot\hat{u}_3)].\end{aligned}$$

This reduces to Eq. (27) of I if we set $P_v^N=0$, $\eta_{\|}=0$, $\eta_{\perp}=1$. Although we will not calculate it, the nuclear polarization also affects the final neutron polarization in all three peaks.

It may be in order to point out several problems involved in performing the experiments we have indicated. In a typical sample, diffraction occurs from many "mosaic blocks," rather than from a single large crystallite. A portion of the beam diffracted by the upper layers of the crystal may be rediffracted into the incident direction by deeper layers; this is, in fact, part of the familiar phenomenon of secondary extinction. Since the polarization of the diffracted beam may be changed in magnitude and direction, the analysis of the rediffraction process may be extremely complicated, and it is thus important to ensure that the experimental crystal is extinction free.

While the above diffraction process occurs only at occasional crystallites, absorption will occur throughout the body of the crystal. Thus, even though the absorption is small enough so that the nuclear amplitudes in (20) and (21) may be taken as real, absorption may still play a significant role in the experiment. The absorption is, in general, spin-dependent—that is, it is different for the $I+\frac{1}{2}$ and $I-\frac{1}{2}$ nuclear states. Further, the spin dependence of the absorption cross section may be opposite to that of the scattering, leading to possibly erroneous results. Again, as in secondary extinction, the analysis becomes very complicated if the crystal is too thick. However, if the crystal is extinction-free, then each spin component of the incident beam may be treated individually. The usual thin crystal-diffraction solutions then apply to each component separately, each with its own appropriate absorption coefficient. In principle, this coefficient should include both incoherent scattering and true absorption, but in practice only true absorption fiill be important. The spin dependence of this quantity is given by[8]

$$\sigma_a=\sigma_{0a}+\sigma_{pa}\mathbf{P}^N\cdot\mathbf{P}, \tag{27}$$

where

$$\sigma_{0a}=[(I+1)/(2I+1)]\sigma_a^+ +[I/(2I+1)]\sigma_a^-,$$

$$\sigma_{pa}=[I/(2I+1)](\sigma_a^+-\sigma_a^-),$$

and $\sigma_a^{\pm}$ are the absorption cross sections for the two possible compound states. Although σ_{pa} is unknown for most nuclei, it may be determined by studying the transmission of the incident beam with the crystal rocked out of the reflecting position (see, for instance, Ref. 9).

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Elastic Moduli and Ultrasonic Attenuation of Polycrystalline Europium from 4.2 to 300°K

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The longitudinal and transverse acoustic velocities and the ultrasonic attenuation of high-purity polycrystalline europium metal have been measured by a pulse technique between 4.2 and 300°K. The variations with temperature of the Young modulus E, shear modulus G, adiabatic compressibility K_s, and Debye temperature Θ_D, have been determined. The Néel point at 91°K is marked by drastic changes in the elastic moduli and ultrasonic attenuations. Anomalies in the attenuation and in the compressibility are observed at about 150°K. The limiting value of Θ_D is 117°K.

INTRODUCTION

Several physical properties of europium differ from those of the other rare-earth metals: for instance, its bcc structure, in contrast to the mostly hexagonal metals of the series, the large atomic volume,[1] and the high compressibility at room temperature.[2]

The magnetic susceptibility[3] exhibits a small antiferromagnetic-type maximum at 90°K. No remanence was observed, i.e., no ferromagnetic state occurs. Nevertheless, europium does not display a typical antiferromagnetic behavior. The high-temperature susceptibility is consistent with the fact that the Eu ions are in a divalent state. At lower temperatures, however, the susceptibility fits a trivalent model.

Neutron diffraction measurements[4] show that Eu undergoes an antiferromagnetic transition, with a Néel point at 91°K, followed by a continuing ordering

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