mental scatter—a constant factor. This discrepancy, however, is within the corresponding experimental uncertainty[21] in the slopes of the perpendicular magnetization curves.

Conclusion (3) is most significant insofar as it indicates that the present data are entirely consistent with the "mixed-state" hypothesis; i.e., that a uniformly flux-penetrated state, terminating at H_{c2}, occurs in the present case for a bulk foil perpendicular to H_0 even though κ is less than $1/\sqrt{2}$. Data entirely similar to but less detailed than those of Table I have been observed for In–1.15 at.% Bi, where κ is still smaller and the transition to type II is at a still lower temperature.

One may question whether an intermediate-state surface-energy model can also be used to explain the present data. For example, for the slope of the magnetization curve near the critical field, Andrew and Lock[22] have derived the expression

$$4\pi(dM/dH)=1+6.1(\Delta/d)^{2/3}. \qquad (4)$$

Values of Δ (the characteristic length associated with the interphase surface energy) chosen to fit the observed slopes were found to be consistently nearly an order of magnitude larger than predicted by the Ginzburg-Landau theory.[23] Moreover, the transition field associated with Eq. (4) is supposed to be thickness-dependent. The present data, however, appear to be characteristic of a thickness-*independent* behavior and in fact indicate that the transition field is H_{c2}, not H_D. In conclusion, therefore, it appears that the mixed state, not the intermediate state, is the appropriate model to explain the present observations.

[21] This comes about because the perpendicular magnetization is at present calibrated against the *parallel* Meissner slope as is the parallel magnetization. Experiments with saturated ferromagnetic foils indicate that the calibration is correct to within a few percent, but this is the same magnitude as the discrepancy needed to be resolved here.

[22] E. R. Andrew and J. M. Lock, Proc. Roy. Soc. (London) **A63,** 13 (1949).

ACKNOWLEDGMENTS

I wish to thank D. H. Leslie for technical assistance, J. W. Savage for alloy preparation, W. M. Robertson for interferometric studies of the Cr films, and T. G. Berlincourt and R. R. Hake for their careful criticism of the manuscript.

[23] V. L. Ginzburg, Physica **24,** S42 (1958).

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Flux-Transport Noise in Type-II Superconductors

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The transport of magnetic flux in type-II superconductors under the influence of a transport current is shown to give rise to a noise voltage, superposed on the dc voltage. This flux-flow noise is analogous to shot noise limited by transit-time effects, and is caused by the motion of discrete units of flux between the edges of the superconductor. If the flux is in a circular motion, so that no transit of flux between the edges takes place, no flux-flow noise can be detected. It is shown from measurements of the frequency spectrum of the noise voltage in vanadium foils that the dc voltage is built up of voltage pulses generated during flux motion, whose duration is equal to the transit time of the flux units. Because of interaction with pinning centers, a fraction of the flux does not take part in the motion. This fraction is found to increase with decreasing temperature and current. It increases with magnetic field in cold-rolled vanadium that exhibits a resistance minimum, but decreases with field in annealed vanadium, where such a minimum is not present. The moving flux units are bundles of flux lines. The average number of flux quanta in a bundle depends on pinning conditions, as is shown for vanadium foils and an indium-thallium crystal. This number decreases with increasing temperature, current, and magnetic field. It is also found to fluctuate owing to interaction with the pinned fraction. At high fields and currents, flux transport in annealed material is in the form of single flux quanta. When the power dissipation in the specimen is increased, a noise voltage with approximately $1/f^b$ frequency dependence is found, with $2<b<3$. This flicker noise, as we call it, is shown to be generated by temperature fluctuations in the foil. These fluctuations are caused by nucleate boiling of liquid helium, and they virtually disappear below the helium λ point. Some flicker noise is left there, due to pressure variations above the liquid.

1. INTRODUCTION

If a type-II superconductor is subjected to a magnetic field greater than a critical value H_{c1} and if a transport current is passed that is greater than a critical value J_c, a dc voltage can be measured. This voltage is generally accepted as being due to the viscous motion of supercurrent vortices containing a flux quantum, under the influence of a Lorentz force.[1] The vortices are extended in the magnetic field direction and are therefore called vortex lines or flux lines.

The direct experimental evidence for this motion,

[1] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139,** A1163 (1965).

however, is restricted. Giaever[2] showed that if the flux in one specimen is coupled to the flux in a second one and if a current is applied to the first specimen, it is possible to get a voltage across the second one due to coupled motion of flux. van Ooijen and van Gurp[3] pointed out that if flux units cross a specimen, this flux motion should give rise to voltage pulses with a duration equal to the transit time of the flux. Such a process causes voltage fluctuations, superposed on the dc voltage. By measurement of this noise it was evidenced that flux transport indeed takes place in the superconductor.

A question is at what moment the voltage is generated: when the flux enters or leaves the superconductor, or during the flow in the superconductor, or both. It will be shown in the present work that the noise measurements give an unambiguous answer to that question: Voltage is generated during flow of flux in the superconductor.

Earlier noise measurements[3] have now been extended to measurements of the temperature and magnetic field dependences, which are determined by the influence of pinning of the vortices to lattice irregularities.

We will treat the noise due to viscous flux flow which we will call flux-flow noise. The average fraction of flux that is pinned as well as the average size of the moving flux entities is determined as a function of transport current, magnetic field, and temperature. This is done by measurements of the frequency spectrum of the noise voltage. The spectrum is also used to demonstrate the mechanism of voltage generation.

Thereafter, we will describe measurements on excess noise at low frequencies with frequency dependence roughly as $1/f^b$ with $2<b<3$. A model will be given for this noise in terms of temperature fluctuations giving rise to flux movements.

2. EXPERIMENTAL

The measurements were done on vanadium foils 30 μ thick, either cold-rolled or cold-rolled and subsequently annealed, and on an indium-20 at.% thallium single-crystal wire. Table I gives particulars of these materials and Fig. 1 shows the specimen arrangement. The specimens V2C and V2A were cut parallel to the rolling direction and the current was applied in the same direction to produce flux flow in a lateral direction. Current and potential contacts were spot-welded. Specimen V 29 was a superconducting so-called Corbino disc. This is an isotropic circular specimen with high-conductivity contacts at the center and around the circumference, so that the electric field is directed radially.[4] In this case there is flux flow in a tangential direction. The central current and potential contact was a spot-welded V-shaped Pt wire. The outer current and potential contacts were connected to a copper ring soldered to a platinum foil that was spot-welded to the circumference of the vanadium foil. A superconducting niobium ring could not be used as an outer contact because of interference from flux trapped in this ring.

TABLE I. Properties of specimens (dimensions and treatment).

Specimen	Thickness (μ)	Width (mm)	
V2C	30	1.1	Cold-rolled vanadium
V2A	30	1.3	Cold-rolled and annealed vanadium
V29	30	diameter 4.9	
In 20% Tl	...	2	[111] single-crystal wire

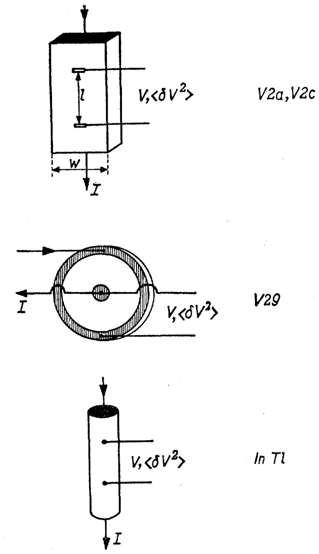

FIG. 1. Specimen arrangements.

The InTl specimen had the current applied in the axial direction and contacts soldered to it.

The noise voltage was measured by a conventional technique.[3] After amplification the signal was fed into a wave analyzer, which is essentially a sensitive tuned amplifier for frequencies up to 16 kHz with bandwidth $df=4.4$ Hz. The output signal of the wave analyzer was detected by a thermocouple valve to give a thermal emf proportional to the noise power. This thermal emf was measured with an electronic μV meter. The setup was calibrated with an ac signal of known frequency and amplitude. The noise voltage was measured as the difference between the readings with transport current switched on and off. At the lowest temperatures (1.5°K)

[2] I. Giaever, Phys. Rev. Letters **15**, 825 (1965).

[3] D. J. van Ooijen and G. J. van Gurp, Phys. Letters **17**, 230 (1965); in *Proceedings of the Tenth International Conference on Low-Temperature Physics, Moscow, 1966* (Proizuodstrenno-Izdatel'skii Kombinat, VINITI, Moscow, USSR, 1967); Philips Res. Rept. **21**, 343 (1966).

[4] See A. C. Beer, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1963), Suppl. 4, p. 71.

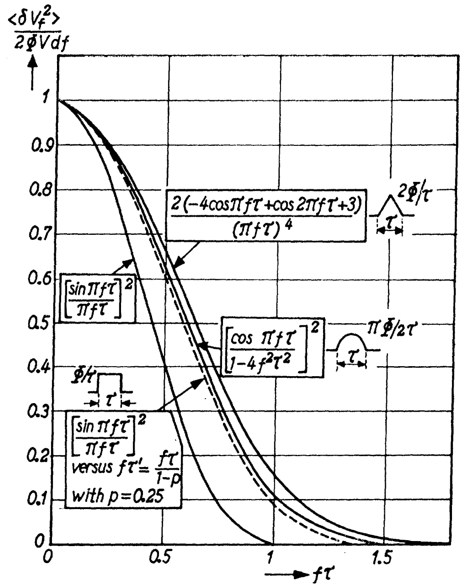

FIG. 2. Theoretical flux-flow noise spectra for different voltage pulses.

the magnetic field was switched on and off, because switching the current influenced the temperature of the helium bath.

The sensitivity of the measurement is of the order of 10^{-18} V^2 across the specimen resistance of the order of 5×10^{-3} Ω if proper care is taken.

3. FLUX-FLOW NOISE—THEORY

The motion of vortices in a superconductor causes an electric field given by[5]

$$\mathbf{E}=-\mathbf{v}\times\mathbf{B}, \quad (1)$$

where $\mathbf{B}$ is the average magnetic induction and $\mathbf{v}$ is the flux-line velocity. One can write for the balance of forces per unit length of a vortex line the approximate expression

$$(J-J_c)\phi_0=-\eta v, \quad (2)$$

where J is the current density, J_c is the critical current density for flux movement, ϕ_0 is the flux quantum, and η is the viscosity coefficient per flux line. The effective driving force on the vortices which causes them to cross the superconductor is balanced by a viscous drag.

This equation has been shown[5] to be justified in the case of dirty superconductors, where the Hall effect is negligible so that the line velocity is perpendicular to the supercurrent flow. In the present work the Hall field, which was found to be about 10^{-3} times the longitudinal field, will be neglected.

In the description of the noise measurements we use the following model.

In the presence of an external perpendicular field the superconductor carries a Meissner shielding current along the edge that is proportional to the magnetic field strength. When a large enough transport current is applied, the vortices that are present in the bulk start moving in one direction. This would result in field gradients on either side of the superconductor. Because of interaction of the surface current with the external magnetic field, new vortices are created by introversion of the Meissner current and subsequent splitting off. This creation is presumably due to fluctuations[6] and therefore a random process. On the opposite side of the superconductor the reverse process takes place. Interaction with the Meissner current which, on this side, flows in the other direction, causes the vortex to open on one side so that the circulating current dies out and with it the flux inside it. In this picture no flux enters or leaves the superconductor. It is contained within supercurrent vortices but only as long as these circulating currents flow. Dying out of a circulating current annihilates the flux inside it. The dc voltage is therefore not caused by voltage pulses when vortices are created or annihilated, as the total flux in the circuit is constant, which is different from the superconducting dynamo.[7]

The creation and destruction of vortices will probably not cause any voltage pulses and consequently also no noise, since this process is not dissipative, as was pointed out by Yntema.[8]

The voltage is generated by moving vortices and the contribution to this voltage by one vortex line (or bundle of vortex lines) is present as long as the vortex is moving. This can also be demonstrated by the voltage measurement on the Corbino disc (V29). If the magnetic field is perpendicular to the foil and the material is isotropic, the flux lines move in concentric circles. This circular movement does not involve the creation or destruction of vortices at the edges and the dc voltage can only be produced by vortices during the movement in the superconductor. Due to dissipation caused by friction of the normal core during the motion, energy is taken from the source and a voltage is measured.

Equation (1) assumed that no pinning forces are present. If there is pinning of vortices, the number of vortices per unit area that take part in the motion is reduced. If a fraction p is pinned, this number is $n(1-p)$, and since $B=n\phi_0$, we suppose that the electric field can be given by

$$E=vB(1-p). \quad (3)$$

Earlier measurements[3] showed that the flux moves in bundles of flux lines, with total flux Φ. The transport

[5] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965); A. G. van Vijfeijken and A. K. Niessen, Philips Res. Rept. **20**, 505 (1965); P. Nozières and W. F. Vinen, Phil. Mag. **14**, 667 (1966); B. D. Josephson, Phys. Letters **16**, 242 (1965).

[6] V. P. Galaiko, Zh. Eksperim. i Teor. Fiz. **50**, 1322 (1966) [English transl.: Soviet Phys.—JETP **23**, 878 (1966).

[7] J. van Suchtelen, J. Volger, and D. van Houwelingen, Cryogenics **5**, 256 (1965); R. Weber, Z. Angew. Physik **22**, 449 (1967).

[8] G. B. Yntema, Phys. Rev. Letters **18**, 642 (1967).

of this amount of flux across the superconductor takes place in a transit time τ and gives rise to a voltage pulse. The actual shape of this pulse is not known, but we will use for convenience a rectangular voltage pulse as shown in Fig. 2:

$$V(t)=\Phi/\tau, \quad \text{for} \quad 0\leq t\leq \tau$$
$$V(t)=0, \quad \text{for} \quad t>\tau. \tag{4}$$

If the flux bundles are random and moving independently at a rate of N per unit time and if all the pulses are identical, it can be shown[9] that this process gives rise to a noise voltage with a mean square value in the frequency band between f and $f+df$:

$$\langle \delta V_f^2 \rangle = 2\Phi V[(\sin\pi f\tau)/\pi f\tau]^2 df, \tag{5}$$

where the dc voltage

$$V=N\int_0^\infty V(t)dt=N\Phi. \tag{6}$$

For comparison we also calculated a sinusoidal voltage pulse

$$V(t)=(\pi\Phi/2\tau)\sin\pi t/\tau, \quad \text{for} \quad 0\leq t\leq\tau \tag{7}$$
$$V(t)=0, \quad \text{for} \quad t>\tau,$$

for which the noise spectrum is

$$\langle \delta V_f^2 \rangle = 2\Phi V[\cos\pi f\tau/(1+4f^2\tau^2)]df. \tag{8}$$

These spectra are shown in Fig. 2 as a function of $f\tau$.

The value of the transit time τ for a constant velocity v (rectangular pulse) is given by

$$\tau=lwB(1-p)/V=\tau'(1-p), \tag{9}$$

where l and w are the length and the width of the sample, respectively. By comparing the experimental noise spectra with the theoretical expressions, it is possible to determine the value of $(1-p)$ from the dc results.

Figure 2 also shows spectrum (5) versus $f\tau'$ for $p=0.25$ and the spectrum for a triangular pulse.[3] These spectra and spectrum (8) cannot be distinguished from each other experimentally. This shows that the value of p as determined from a comparison between experimental and theoretical spectra depends critically on the assumed shape of the elementary pulse. Experiment can therefore give no conclusive evidence as to the precise shape of the voltage-time function and p can only be determined qualitatively.[10]

[9] D. K. C. MacDonald, *Noise and Fluctuations* (John Wiley & Sons, Inc., New York, 1962).

[10] After submission of this paper Fournet and Baixeras [Phys. Letters **25A**, 552 (1967)] introduced a velocity distribution for the flux lines, to account for the discrepancy between the simple model with $p=0$ and experiment. In this way it would not be necessary to introduce a pinned fraction. There is however a pinned fraction when local values of the critical current exceed the value of the transport current, i.e., for those currents where the I-V characteristic is not linear.

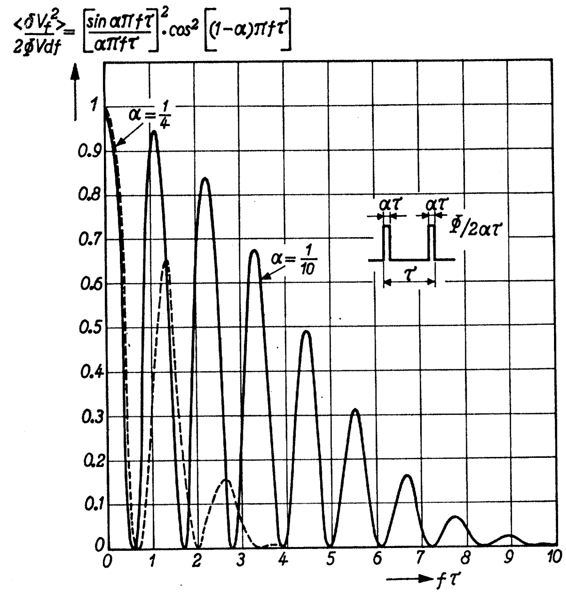

FIG. 3. Theoretical noise spectra for a combination of two narrow voltage pulses.

We have also calculated the noise spectrum of a voltage time function, consisting of two narrow pulses of width $\alpha\tau$, separated by the transit time τ.

This would be a possible voltage if the flux would cause voltage pulses when entering and leaving the superconductor. For convenience we took the narrow pulses as rectangular.

The spectrum can be written as

$$\langle \delta V_f^2 \rangle = 2\Phi V[(\sin\alpha\pi f\tau)/\alpha\pi f\tau]^2 \cos^2(1-\alpha)\pi f\tau \, df \tag{10}$$

and is drawn in Fig. 3 for two values of α. It is an oscillating function with cutoff frequency as governed by $\alpha\tau$ and with a period governed by $(1-\alpha)\tau$.

This spectrum is completely different from those in Fig. 2 and it will be possible to conclude from the experiments which voltage time-function is right.

The noise amplitude yields the size of the moving flux bundles Φ. This value can be found by extrapolating the measured noise spectrum to $f=0$. The value of Φ as determined from

$$\langle \delta V_0^2 \rangle = 2\Phi V df \tag{11}$$

is expressed as the number of single flux quanta ϕ_0 in a bundle.

In the case of the Corbino disc (V29) the flux is flowing in circles. If one potential contact is in the center and the other one is around the whole edge of the specimen, no transit time can be defined, so that no voltage pulses are generated and consequently no flux-flow noise voltage should be found.

4. FLUX-FLOW NOISE—RESULTS

Noise spectra were measured for various combinations of current, magnetic field, and temperature.

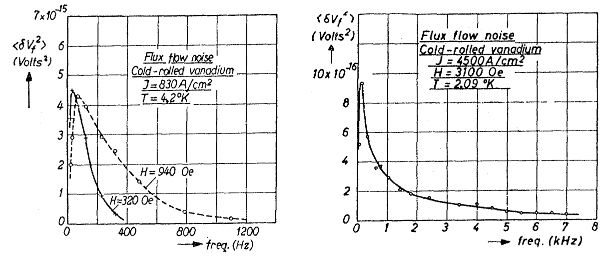

FIG. 4. Experimental flux-flow noise spectra. Cold-rolled vanadium. (a) $T=4.2°K$, (b) $T=2.09°K$.

Figure 4(a) shows two examples of flux-flow noise spectra for cold-rolled vanadium foil. The corresponding oscillograms of the noise are shown in Fig. 5.

The spectra agree with the model as given above for a single voltage pulse of duration τ. They are in disagreement with the spectrum (10) of a double pulse, so that experimental evidence shows that a model leading to this spectrum is incorrect.

In the vanadium foils the transit time was found to lie in between 0.5 and 10 msec. The general behavior of $\langle \delta V_f^2 \rangle$ follows the frequency dependence as was calculated for flux-flow noise spectra irrespective of specimen width.

The flux-line velocity v can be found directly as the product of the cutoff frequency $f_c=1/\tau$ and the foil width: $v=wf_c$. In the measurements on the vanadium foils this velocity had a value between 10 and 150 cm/sec.

Flux-flow noise will certainly also be present on the Hall voltage. Since it is proportional to the dc voltage, the effect is very small and we did not try to detect it.

No noise was found in the fully superconducting or in the normal state.

Pinned Fraction

The determination of the flux-line fraction p that is pinned was done by comparing the measured with the theoretical spectrum for a rectangular pulse. The spectrum (5) was drawn as a function of $f\tau'$ for different values of p. The experimental results were also plotted

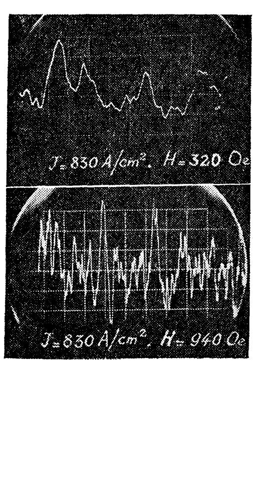

FIG. 5. Oscillograms of total noise voltage, corresponding to the spectra of Fig. 4(a). Vertical deflection 0.5 μV/div, horizontal deflection 5 msec/div.

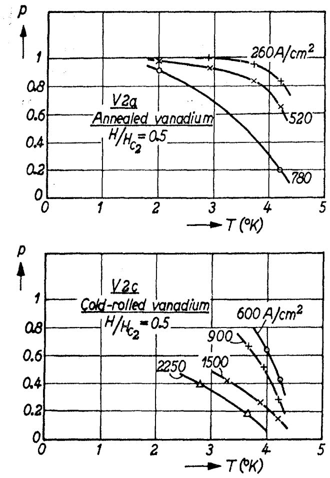

FIG. 6. Pinned flux fraction versus temperature for different values of the transport current, cold-rolled and annealed vanadium.

as a function of $f\tau'$. By matching the theoretical to the experimental curves, the value of p was then determined. As was discussed above, the value of p depends on the shape of the voltage pulse. For a given noise spectrum p is much smaller for a triangular or sinusoidal pulse than for a rectangular one, as Fig. 2 illustrates. The absolute value of p is therefore not known with certainty but the dependence on temperature, magnetic field, and current can be measured in a qualitative way.

Figures 6 and 7 show p as a function of temperature and magnetic field at different values of transport-current density for cold-rolled and annealed vanadium (V2C and V2A, respectively). The temperature dependence is such that at a given value of H/H_{c2}, p decreases

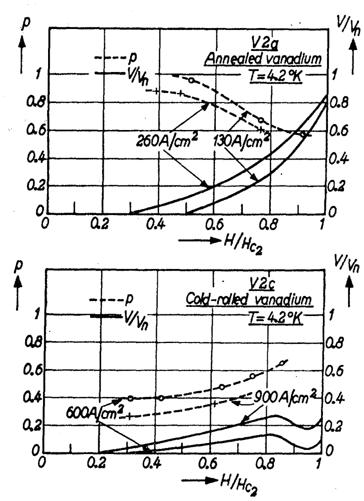

FIG. 7. Pinned flux fraction and dc voltage versus magnetic field for two values of the transport current, cold-rolled and annealed vanadium.

on warming up. Increasing the current density always results in a lower value of p.

This temperature and current dependence can be understood in terms of the effective driving force on the flux lines $(J-J_c)\phi_0$. As J_c goes down with increasing temperature an increase of both current density and temperature causes a higher value of the driving force and consequently a lower value of the pinned flux-line fraction.

A current-dependent value of p substituted in Eqs. (2) and (3) would give rise to a nonlinear I-V characteristic. Since the absolute value of p is not known, a quantitative calculation of this characteristic could not be carried out. It was found that the I-V curve was not linear for those currents at which p differed significantly from zero.

The field dependence of p is different for the two materials. With rising magnetic field p is found to increase in the cold-rolled foil, but to decrease in the annealed foil.

This behavior agrees with the different field dependence of critical current density.[11] In the cold-rolled material $J_c(H)$ goes through a broad minimum and increases towards H_{c2}, so that apparently pinning increases with field. This is also shown in the dc field transition, where the dc voltage goes through a maximum and then decreases with increasing field. Figure 7 shows that this process is due to a continuous increase in pinning starting at low fields.

In the annealed vanadium the field dependence of p is in agreement with the continuously decreasing value of J_c with increasing magnetic field. Apparently the pinning decreases with field.

So far it has been assumed that the moving flux bundles are not halted for a certain time on their way across the superconductor. If this would happen, the result would be a number of shorter voltage pulses instead of one pulse, and thus a higher cutoff frequency f_c and therefore also an apparently higher value of p. This process may happen when the pinning centers are distributed inhomogeneously over the cross section of the foil. This is the case in the annealed vanadium, where a relatively small number of grain boundaries all lie parallel to the field direction. Flux bundles may therefore sometimes be halted at these grain boundaries. This may cause the rather high values of p at low fields and currents in the annealed foil.

Noise Reduction

The foregoing treatment shows that the experiments agree with the simple model. There is, however, a deviation which is the decrease towards low frequencies that is often found in the flux-flow noise spectrum, as Fig. 4 shows. A mechanism that would cause such a decrease is one in which a positive voltage pulse would be followed by a negative pulse, as is found in shot-noise reduction in vacuum tubes due to space charge where the noise at low frequencies is reduced, so that the noise spectrum exhibits a maximum.[9,12] The noise reduction is there caused by a positive fluctuation of the average number of crossing electrons giving rise to increased space charge and therefore causing a decrease of the over-all current. Similarly, in the case of flux flow, a positive fluctuation in the number of crossing flux lines could have an inhibiting effect on the over-all flux flow. This inhibiting effect on the flux-flow process could be caused by an increase of the viscosity coefficient η. A positive fluctuation in the vortex density would bring the vortices closer to each other. Due to their interaction with the pinned vortices, this increases the viscosity. According to Eq. (2) this would make the over-all velocity go down and consequently the number of crossing vortices. The positive fluctuation in the number of crossing flux lines is thus followed by a negative fluctuation and the low-frequency noise is reduced.

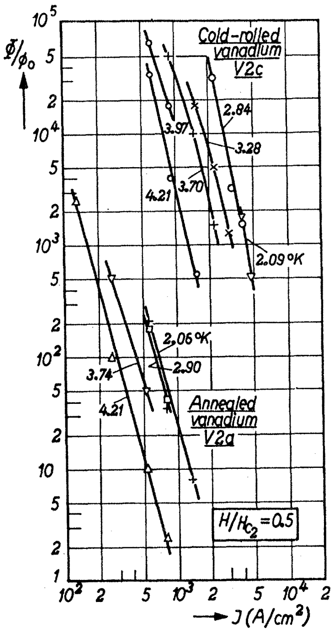

FIG. 8. Flux bundle size versus transport current density for different temperatures, cold-rolled and annealed vanadium.

Bundle Size

The size of the moving flux units Φ is determined using Eq. (11). For the extrapolation of $\langle\delta V_f^2\rangle$ to $f=0$ the low-frequency flux-flow noise reduction is disregarded. The average bundle size is expressed as the number of single flux quanta ϕ_0. This number is shown in Fig. 8 as a function of transport-current density for different temperatures at a given value of H/H_{c2}.

[11] G. J. van Gurp, Philips Res. Rept. **22**, 10 (1967).

[12] A. van der Ziel, *Noise* (Prentice-Hall Inc., Englewood Cliffs, N.J., 1956).

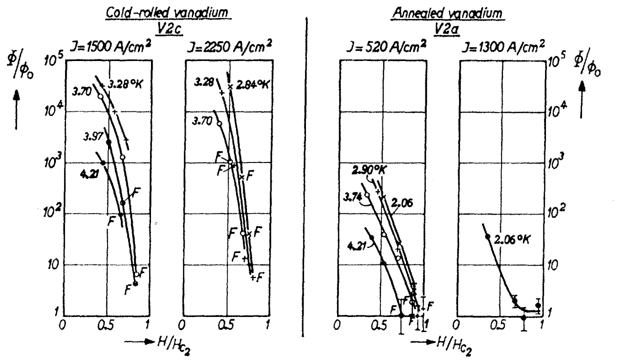

FIG. 9. Flux bundle size versus reduced field for different temperatures and transport currents, cold-rolled and annealed vanadium. For small bundles the uncertainty is indicated by vertical bars. F means presence of flicker noise.

The bundling of flux lines is due to pinning effects. If pinning barriers are too high for one flux line to overcome, the flux travels in bundles, so that the pinned flux lines will be pushed over the barrier by the other ones in the bundle. The bundle size expresses the amount of coherence that is left in the vortex lattice. Apparently this is broken up into smaller domains when movement takes place. In other words, the two-dimensional single crystal becomes polycrystalline with the different crystals moving independently. These bundles have also been observed[13] for a stationary situation in the electron microscope.

The influence of increasing current and temperature is such as to increase the driving force $(J-J_c)\phi_0$ on the flux lines. The size of the bundles, necessary for movement, is then decreased. In the case of weak pinning, as in annealed vanadium, the value of Φ and thus the noise level is several orders of magitude smaller.

The effect of magnetic field on the bundle size is shown in Fig. 9. Here Φ/ϕ_0 is plotted versus H/H_{c2} for different values of J and T. The general behavior is that at low fields Φ decreases slowly with increasing field, but at higher fields this decrease is rapid, and near H_{c2} the bundle size is of the order of $10\phi_0$ in cold-worked samples and ϕ_0 in annealed vanadium.

An explanation of the field dependence may be given in terms of overlapping vortices. When the distance between the vortices becomes comparable to their diameter, i.e., about 2λ, the vortices experience their mutually repulsive forces.

The interaction energy $\sum_i U_i$ between one vortex and its z nearest neighbors at distance d can be written as[14]

$$\sum_i U_i = z(\phi_0/4\pi\lambda)^2(\pi\lambda/2d)^{1/2}\exp(-d/\lambda) \quad (12)$$

for $d \gg \lambda \gg \xi$. This interaction energy is now compared to the flux-line self-energy ϵ, which determines the pinning energy. This results in

$$\frac{\sum_i U_i}{\epsilon} = \frac{z(\pi\lambda/2d)^{1/2}\exp(-d/\lambda)}{\ln(\lambda/\xi)}. \quad (13)$$

For the triangular lattice $z=6$ and $B=(2/\sqrt{3})(\phi_0/d^2)$. This expression can be calculated for different values of H/H_{c2} as follows.

The value of λ is calculated from the expression[14] for H_{c1}:

$$H_{c1} = (\phi_0/4\pi\lambda^2)\ln(\lambda/\xi). \quad (14)$$

In cold-rolled vanadium[11] at 4.2°K, $\kappa=\lambda/\xi=2.9$ and $H_{c1}=0.4$, $H_c=150$ Oe and we find $\lambda=1100$ Å and $\xi=375$ Å. From the temperature dependence $(1-t^4)^{-1/2}$ we find at 2°K a value $\lambda=830$ Å. In annealed vanadium[11] at 4.2°K, $\kappa=2.2$ and $H_{c1}=0.5H_c$, so that $\lambda=910$ Å and $\xi=410$ Å.

The result of this calculation is shown in Fig. 10, where $(\sum_i U_i)/\epsilon$ is shown as a function of H/H_{c2}. The interaction energy may be considerable compared with the line energy. At a given value of H/H_{c2} it is lower at a higher temperature. It is also lower for annealed material.

As was remarked earlier, in the case of pinning the flux lines are being pushed over the barriers by a cooperative pressure of the other flux lines in a bundle. Depending on pinning conditions, current, and temperature, a certain pressure P is needed for flux movement. When the magnetic field is increased the flux lines get closer together so that they experience a greater repulsive force on each other. The total force on a flux line is, therefore, for a given bundle size, greater when the field is increased. In order to build up the pressure P a smaller bundle is now sufficient. As a consequence the average bundle size necessary for flux movement is decreased.

The value of Φ/ϕ_0 tends to unity near H_{c2} and at large current densities. Under these circumstances a different type of noise with frequency dependence roughly at $1/f^b$ appears where b is between 2 and 3, as indicated in Fig. 9 by the symbol F. Since the value of Φ is found from extrapolation of $\langle \delta V_f^2 \rangle$ to $f=0$, the determination of Φ is inaccurate when this noise is

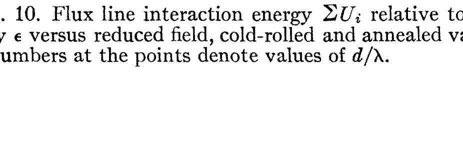

FIG. 10. Flux line interaction energy ΣU_i relative to the line energy ϵ versus reduced field, cold-rolled and annealed vanadium. The numbers at the points denote values of d/λ.

[13] H. Träuble and U. Essmann, Phys. Status Solidi **20**, 95 (1967).

[14] P. G. de Gennes, *Superconductivity of Metals and Alloys* (W. A. Benjamin, Inc., New York, 1966).

present. Moreover, since this excess noise is caused by temperature fluctuations, as will be discussed in the next section, the value of H_{c2} will also fluctuate, so that in a constant field the specimen undergoes fluctuations into the normal state, where Φ is no longer defined. Separation of the two different noise spectra is possible by the difference in frequency dependence, but is difficult when the flux-flow noise is small. It is, however, possible to remove the $1/f^b$ noise by cooling the helium bath through the λ transition. By doing this, it is found that when Φ has reached the value ϕ_0, it does not decrease any further with increasing field. Figure 9 shows for annealed vanadium at 2.06°K that at high enough field, $\Phi=\phi_0$, within the accuracy of measurement.

This means that the flux lines are moving more or less independently of each other. In other words, the flux-line lattice, which is polycrystalline when moving at low fields, is amorphous under these conditions.

The number of flux bundles crossing the specimen per unit time can be found from the expression $N=V/\Phi$. Practical values in the vanadium foils lie in between 5×10^4 sec^{-1} at low currents and fields, where Φ is large, and 3×10^{11} sec^{-1} at high currents and fields, where Φ is small.

We have assumed so far that the bundle size Φ and velocity v are constant. In reality, however, both quantities are fluctuating because of interaction of the moving flux lines with the pinning centers and the pinned flux lines. The moving bundles are in a dynamic equilibrium with the pinned flux lines, so that a bundle loses flux lines to pinning centers and picks up others as it moves past the centers. This is shown by a high-frequency tail in the flux-flow noise spectrum, as shown in Fig. 4(b). This is found mainly at low temperatures, especially at low current densities and high fields. The noise in the tail is of the order of 10% of the noise at low frequencies, which means that the bundle size or the velocity undergo fluctuations of about 1%. From the cutoff frequency a relaxation time of the order of 10^{-4} to 10^{-5} sec is estimated, so that this interaction of the moving bundles with the pinning centers is a fast process.

Corbino Disc

No flux-flow noise was found on the circular specimen at 4.2°K, within the measuring accuracy (2×10^{-18} V^2). The only noise present was excess noise at the higher current densities. In order to get rid of this noise, measurements were also taken at $T=2.15$°K. Some noise was found here, but it was an order of magnitude less than for specimen V2A at a comparable field. This noise may be caused by the fact that the central current contact is not exactly in the middle, so that on one side of the specimen flux may cross from one side to another. Since the current density is very low at the edge, this flux moves in the form of large bundles and may then give rise to a measurable noise voltage. Furthermore velocity fluctuations of the moving flux bundles also cause voltage fluctuations.

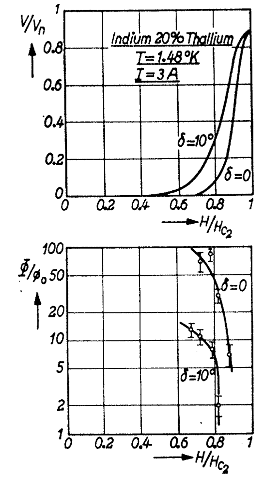

Fig. 11. Flux bundle size and dc voltage versus reduced field for two orientations of In-20% Tl [111] crystal with respect to field direction.

This result agrees with the model that the flux moves in circles and should therefore not give flux-flow noise as there is no transit time.

Indium-Thallium

Some noise measurements were also taken on the indium-20% thallium [111] crystal. This material undergoes a martensitic transformation[15] at 80°C, resulting in the formation of twin boundaries lying in [110] directions on (110) planes. The critical current of such a crystal is a sharp maximum as a function of the angle of rotation δ when the flux lines are parallel with the twin boundaries.[16] Noise measurements were taken with $I=3$ A at $T=1.5$°K as a function of field for two orientations. The results are shown in Fig. 11, together with the dc voltage transitions.

The field dependence is very similar to that for vanadium foils. For the flux lines parallel with the twin boundaries $(\delta=0)\Phi$ is almost an order of magnitude larger than for $\delta=10°$. This agrees with the dependence of Φ on pinning conditions in vanadium: Much pinning requires large bundles for flux movement.

5. FLICKER NOISE

The excess noise at low frequencies with roughly a a $1/f^b$ frequency dependence will be called flicker noise, because of the similar frequency dependence in vacuum tubes[12] and because the effect is due to temperature fluctuations. This was indicated by the virtual disappearance of this noise on cooling through the helium λ transition. The flux-flow noise was not affected by this transition. At the λ point helium undergoes a transition to the superfluid state and the heat conductivity jumps to a very high value. The heat transfer from a heated surface to the bath is therefore increased con-

[15] Z. S. Basinski and J. W. Christian, Acta Met. **2**, 148 (1954).
[16] G. J. van Gurp, Phys. Status Solidi **17** K135 (1966).

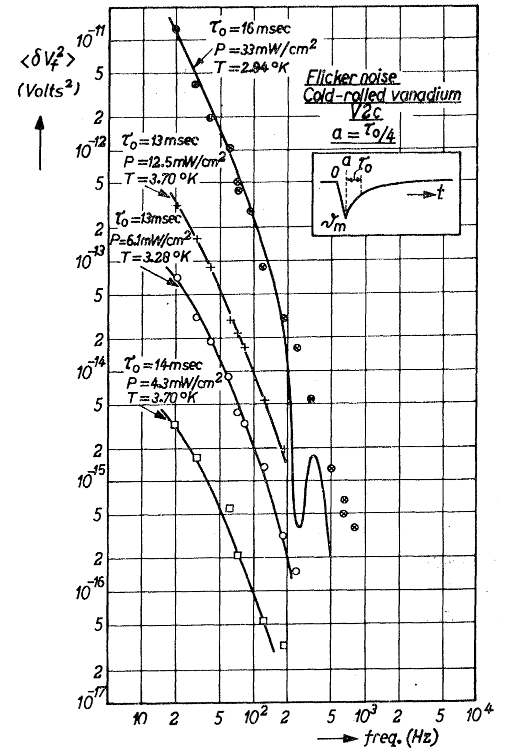

FIG. 12. Some flicker-noise spectra for various values of power dissipation P and temperature T, cold-rolled vanadium. Full lines are calculated with (16a) for $a=\tau_0/4$.

siderably. We propose a model in which the flicker noise is caused by temperature fluctuations in the specimen, due to fluctuations in the Joule heat transfer from the specimen to the helium bath. At low heat flux q, this heat transfer is by convection and is proportional to the average temperature difference ϑ with the bath. At higher heat flux the predominant process is nucleate boiling.[17] Heat transfer is improved due to the formation of vapor bubbles on the surface of the heater, caused by evaporation of a thin layer of liquid on the surface. The heat flux is proportional to $(\vartheta)^\gamma$ with $1\leq\gamma\leq 2$ depending on surface roughness.[18] The value of ϑ is about 0.1°K. During nucleate boiling the heat transfer at a particular site increases during formation and growth of a bubble and decreases after departure of the bubble. This causes a temperature fluctuation as was measured in water boiling, where these fluctuations could be identified with bubble formation.[19] This formation takes place at certain nucleation sites, the number of which increases with dissipated power. At low-heat flux the bubbles at a particular site are formed at irregular intervals, but at high-heat flux it occurs more or less periodically. The process has been studied in detail in water. Less evidence is available for boiling helium, but the mechanism appears to be the same. We observed the boiling process in helium with a microscope and measured the power dissipation in an InPb foil simultaneously at $T=4.2$°K. Formation of bubbles on the specimen was observed with certainty to occur at heat transfer values down to about 0.3 mW/cm². The specimen was glued on one side to a polystyrene holder and the heat transfer through this side was neglected in this calculation.

The time dependence of the temperature fluctuation is assumed to be similar to that for water. In order to treat the problem analytically we assume the time function of the local temperature ϑ_l to be

$$\vartheta_l=\vartheta-\vartheta_m t/a, \qquad \text{for} \quad 0\leq t\leq a$$

$$\vartheta_l=\vartheta-\vartheta_m \exp[(a-t)/\tau_0], \quad \text{for} \quad t\geq a, \qquad (15)$$

where a is the bubble departure time and τ_0 is the surface heating time constant, as shown in Fig. 12. The form of this pulse is close to what was measured for water. If these pulses are random and independent, the noise spectrum can be calculated to be

$$\langle\delta\vartheta_f^2\rangle=2N\vartheta^2{}_m\tau_0^2 \times\frac{\omega^2a^2-2\omega a\sin\omega a+2(1-\cos\omega a)(1+\omega^2\tau_0^2+\omega^2a\tau_0)}{\omega^4a^2\tau_0^2(1+\omega^2\tau_0^2)}df, \qquad (16a)$$

with N = number of pulses per unit time and $\omega=2\pi f$. The value for $f\rightarrow 0$ is

$$\langle\delta\vartheta_0^2\rangle=\tfrac{1}{2}N\vartheta_m^2[a^2+4(\tau_0^2+a\tau_0)]df, \qquad (16b)$$

so that, if we write $a=\alpha\tau_0$,

$$\langle\delta\vartheta_0^2\rangle=\tfrac{1}{2}N\vartheta_m^2\tau_0^2(\alpha+2)^2df. \qquad (16c)$$

For $a=0$ the spectrum reduces to

$$\langle\delta\vartheta_f^2\rangle=(2N\vartheta_m^2\tau_0^2/1+\omega^2\tau_0^2)\,df. \qquad (16d)$$

For frequencies higher than $1/\tau_0$, this noise has a $1/f^2$ frequency dependence.

Figure 12 shows experimental results for specimen V2C.

The drawn curves are calculated with (16a), assuming $\alpha=\frac{1}{4}$ and matched to the experimental points. Reasonable agreement exists between experiment and this model at not too high frequencies. The value of τ_0 is of the order of 13 to 16 msec in vanadium and of the order of 30 to 50 msec in the InTl specimen. Below the λ point no nucleate boiling takes place, so that no temperature fluctuations of this kind are found there.

Flicker noise was found for power dissipations between 0.5 and 40 mW/cm², which is well within the nucleate boiling regime. The noise was found to be a

[17] D. N. Lyon, Advan. Cryog. Eng. **10**, 371 (1965).

[18] R. D. Cummings and J. L. Smith, Jr., Bull. Inst. Intern. Froid, Annexe **5**, 85 (1966).

[19] F. B. Moore and R. B. Mesler, Am. Inst. Chem. Engrs. J., **7**, 620 (1961); T. F. Rogers and R. B. Mesler, *ibid.* **10**, 656 (1964); M. G. Cooper and A. J. P. Lloyd, in *Proceedings of the Third International Heat Transfer Conference, Chicago, Illinois* (American Institute of Chemical Engineers, New York, 1966) III, p. 193.

strong function of current I and power P. Figure 13 shows $\log\langle\delta V_f^2\rangle$ at 20 Hz as a function of $\log(IP)$ for vanadium foils. The mean-square (ms) noise voltage varies roughly as $(IP)^2$, irrespective of temperature. If we want to calculate the voltage fluctuations from the temperature fluctuations, we can only give a rough estimate, based on knowledge about the nucleate boiling process in water.

If ν is the mean frequency of bubble formation and A is the number of nucleation sites, it can be shown that if heat is transferred by nucleate boiling $N=\nu A$ is proportional to q if the bubble size is constant. The characteristic time τ_0 over which the temperature rises after the bubble departure is mainly determined by the thermal diffusivity of the, liquid which we take to be approximately independent of temperature. If we take $q\propto(\vartheta_m)^\gamma$ and write $P=q$, we get from Eq. (16c)

$$\langle\delta\vartheta_0^2\rangle\propto P^{(1+2/\gamma)}. \tag{17}$$

If we take[18] $\gamma=2$, as is found for a smooth surface, we get

$$\langle\delta\vartheta_0^2\rangle\propto P^2. \tag{18}$$

The dependence of electrical resistance R on temperature at a given field and current is not known exactly, but in the region of interest the average resistance change ΔR was found for the vanadium foil to be roughly proportional to the temperature change ΔT. The expression for the ms voltage fluctuations then is

$$\langle\delta V_0^2\rangle\propto I^2\langle\delta\vartheta_0^2\rangle, \tag{19}$$

so that

$$\langle\delta V_0^2\rangle\propto(IP)^2. \tag{20}$$

This is in agreement with what is found experimentally for $\langle\delta V_f^2\rangle$ at $f=20$ Hz, as shown in Fig. 13.

As a function of field the flicker noise passes through a maximum. The place and the height of the maximum is mainly determined by the value of the resistance variation $\Delta R/\Delta T$. In the fully superconducting and in the normal state $\Delta R/\Delta T=0$, so that no noise is found.

Flicker noise was also found on the InTl crystal, where the noise was reduced by a factor of 10^3 on cooling through the λ point. Some noise was left at low temperature (1.5°K) at fields where $\Delta R/\Delta T$ was large. The frequency dependence of this noise was rather different from the ordinary flicker noise and resembles a resonance curve with a maximum at 8 Hz. It was found that the resistance of an aquadag film close to the specimen exhibited an identical noise spectrum, so that this noise is likely to be caused by temperature fluctuations in the liquid. Such fluctuations are probably due to pressure oscillations above the liquid with frequency governed by the rotary pump (510 rpm).

The flicker noise has been discussed for convenience in terms of a resistance change ΔR. In fact this resistance is caused by flux motion and it would be more appropriate to treat it as a flux-flow voltage. Using

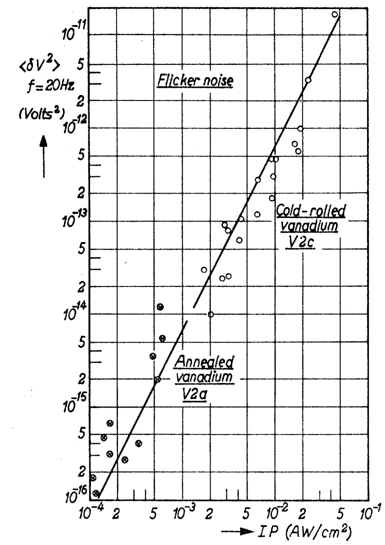

FIG. 13. Mean-square flicker-noise voltage at $f=20$ Hz versus product of transport current I and power dissipation P. The full line has slope 2.

Eqs. (2) and (3) variations in voltage V with temperature at constant current and field can be expressed as variations of J_c, η and p. All three give rise to an increase of V for an increase of temperature. The flicker-noise voltage can thus be described as being due to fluctuations in pinning conditions and in viscosity, caused by temperature fluctuations.

It should be possible to reduce the flicker noise by improving the heat transfer to the helium bath, e.g., by roughening the surface or by connecting the specimen to a heat sink.

6. JOHNSON NOISE

The spontaneous charge fluctuations of a conductor with resistance R at a temperature T, so-called Johnson noise, can be expressed as the mean-square voltage fluctuations

$$\langle\delta V_f^2\rangle=4kTRdf. \tag{21}$$

In a superconductor in the mixed state a similar noise voltage should be present due to spontaneous flux fluctuations. By application of Langevin's equation

$$Mdv/dt+\eta v=F(t), \tag{22}$$

where $F(t)$ is the fluctuating force on a flux line with effective mass M, this expression becomes

$$\langle\delta V_f^2\rangle=4kTR'df, \tag{23}$$

where R' is defined as dV/dI.

In the derivation of Eqs. (21) and (23) thermal equilibrium is assumed. This means that they are valid only for $I=0$. Because of pinning, the value of dV/dI

is then very small or zero. For $I \neq 0$, this type of noise is still less than 10^{-24} V^2 per unit bandwidth, which is about 10^5 times smaller than the limit of detection in the present experiment.

7. DISCUSSION AND CONCLUSIONS

The treatment of the flux-flow noise is based on a model in which the noise voltage is generated by identical (rectangular) and independent voltage pulses, generated at random times. The validity of these assumptions and possible deviations need some discussion.

It was already shown that nonrectangular voltage pulses (i.e., a variable flux-line velocity) with the same duration time as the rectangular pulse shift the noise spectrum to higher frequencies, so that the value of the pinned fraction p is uncertain, but yield the same value for the bundle size Φ.

If the pulses are not identical, this may be caused either by a distribution of pulse heights or by a distribution of pulse duration or both. Different heights may be due to different values of Φ. The noise voltage gives then the average value of Φ, but no information about the possible deviation from the average value. Since ϕ_0 is the lower limit to the bundle size, the average value of Φ will always be higher than ϕ_0, when there is a distribution of Φ values. A distribution of pulse duration may be due to interruptions of the movement by pinning or to a velocity distribution. In both cases the spectrum is somewhat shifted to higher frequencies. Since the area under the elementary voltage-time function must be equal to Φ, a velocity distribution will be accompanied by a distribution of pulse heights.

The movement of the flux bundles is probably not completely independent. Interaction of a moving bundle with other bundles or with pinning centers leads to fluctuations of the bundle size or the velocity. These fluctuations are an extra source of noise with high cutoff frequency as was discussed in the section on bundle size [Fig. 4(b)]. The resulting value of Φ is therefore a time average for each bundle.

The generation of the voltage pulses might not be entirely random by the mechanism that was described in the section on noise reduction. This does not affect the value of Φ, as this was determined by extrapolation to zero frequency, disregarding the low-frequency reduction of the noise.

A different mechanism is that whereby there is correlation between successive bundles, created at the same place. It can be shown that by this correlation the noise level is reduced and the spectrum gets a high cutoff frequency that is governed by the average value of the time interval between successive pulses. This mechanism may take place in the case of large flux bundles, where the departure of a bundle would leave a flux-free area behind that may only partly be filled by a relaxation of the surrounding flux lattice. In such a case, a just created bundle must be followed by another one. The departure of a small bundle only leaves a small gap behind, that can be filled in by the lattice, and this is particularly so for single flux lines. The experiments show that under the conditions where small bundles are found, the noise-cutoff frequency agrees reasonably well with the theoretical one, so that the correlation effect does not take place there. Moreover, as is shown in Fig. 9, the value of $\Phi(H)$ does not drop below ϕ_0, but remains constant slightly above that level. This suggests that here indeed single flux lines are moving.

The present experiments on flux-flow noise have shown that the simple model of moving flux lines causing a dc electric field $E=vB$ during their movement is applicable irrespective of temperature, current, field, pinning conditions, or geometry. This is supported by the experiments on a Corbino disc, where circular flux flow produces a dc voltage but no noise. Surprisingly the measurements justify the assumption that the creation of flux bundles and the subsequent flow is generally a random process and that the flux bundles move more or less independently, even when their size is reduced to one flux quantum.

The experiments yield values for the pinned flux-line fraction and the bundle size. Both quantities decrease with increasing magnetic field, current, and temperature, with the exception of cold-rolled vanadium, exhibiting a voltage minimum, where the pinned fraction was found to increase with field.

As is the case with flicker noise in vacuum diodes and with $1/f$ noise in semiconductors, flicker noise in superconductors is due to a surface effect. The flicker effect, which usually disappears below the helium λ point, shows that the temperature of the material is fluctuating. A simple model for these fluctuations in terms of nucleate boiling accounts reasonably well for the spectrum and power dependence of the flicker noise. The noise below the λ point is also caused by temperature fluctuations, probably due to vapor pressure fluctuations.

ACKNOWLEDGMENT

It is a pleasure to thank D. J. van Ooijen for many stimulating discussions.

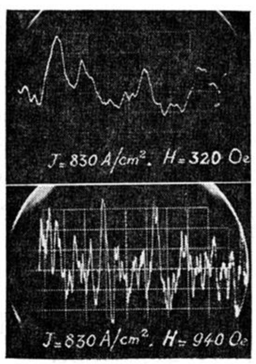

FIG. 5. Oscillograms of total noise voltage, corresponding to the spectra of Fig. 4(a). Vertical deflection 0.5 μV/div, horizontal deflection 5 msec/div.